EOT GRADE 10 g

1. Define the sample space, *S*, of a fair coin being tossed once.

   SOLUTION:

   The sample space *S* includes all possible outcomes of tossing a fair coin.

   A fair coin can land on heads or tails.

   $S = \{H, T\}$

2. A numbered spinner with six equal parts is spun once.

a. What is the sample space of the experiment?

b. What is the sample space for the event of landing on a prime number?

SOLUTION:

a. The sample space includes all possible outcomes of spinning the spinner.

The spinner can land on 1, 2, 3, 4, 5, or 6.

$S = \{1, 2, 3, 4, 5, 6\}$

b. The sample space for the event of landing on a prime number is all outcomes from the sample space that are prime numbers.

The prime numbers from the sample space are 2, 3, and 5.

$S(\text{prime number}) = \{2, 3, 5\}$

3. **DODECAGON** A regular, 12-sided dodecagon is rolled once.

a. What is the sample space of the experiment?

b. What is the sample space for the event of rolling an even number?

SOLUTION:

a. The sample space includes all possible outcomes of rolling the dodecagon.

The dodecagon can land on 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

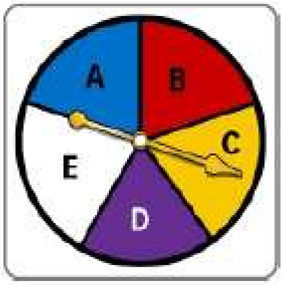
$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

b. The sample space for the event of rolling an even number is all outcomes from the sample space that are even numbers.

The even numbers from the sample space are 2, 4, 6, 8, 10, and 12.

S(even number) = {2, 4, 6, 8, 10, 12}

4. **SPINNERS** A numbered spinner with six equal parts is spun once.



a. What is the sample space of the experiment?

b. What is the sample space for landing on a vowel?

SOLUTION:

a. The sample space includes all possible outcomes of spinning the spinner.

The spinner can land on A, B, C, D, or E.

S = {A, B, C, D, E}

b. The sample space for the event of landing on a vowel is all outcomes from the sample space that are vowels.

The vowels from the sample space are A and E.

S(vowel) = {A, E}

5. **UNIFORMS** For away games, the baseball team can wear blue or white shirts with blue or white pants. Represent the sample space for each experiment by completing the table and tree diagram, and by making an organized list.

SOLUTION:

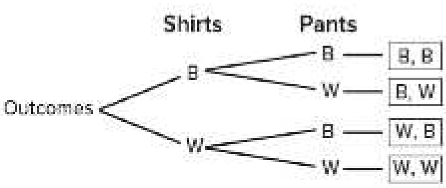
Organized List
Pair each possible outcome from the pants color choices with the possible outcomes from the shirts color choices using letters to represent the colors.

S = {BB, BW, WB, WW}

Table
Shirt color choices are represented vertically, and pants colored choices are represented horizontally.

	Blue Pants	White Pants
Blue Shirts	B, B	B, W
White Shirts	W, B	W, W

Tree Diagram
Each event is represented by a different stage of the tree diagram.



6. **CHILDCARE** Khalid's baby sister can drink either apple juice or milk from a bottle or a toddler cup. Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

SOLUTION:

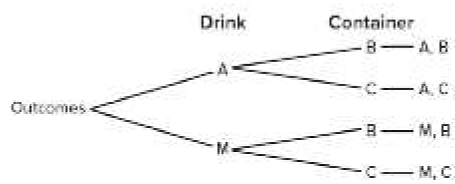
Organized List
Pair each possible outcome from the drink choices with the possible outcomes from the container choices using letters to represent the choices.

S = {AB, AC, MB, MC}

Table
Drink choices are represented vertically, and container colored choices are represented horizontally.

	Bottle	Cup
Apple Juice	A, B	A, C
Milk	M, B	M, C

Tree Diagram
Each event is represented by a different stage of the tree diagram.



8. A numbered spinner with eight equal parts is spun until it lands on 2.

SOLUTION:

There are an infinite number of possible outcomes of this experiment, so its sample space is infinite.

Because the experiment ends after a certain number of spins when the spinner lands on 2, the sample space is discrete.

10. A letter is randomly chosen from the alphabet.

SOLUTION:

There are only 26 possible outcomes of this experiment: each letter of the alphabet.

The sample space *S* is finite.

S = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}

Classify each sample space as *finite* or *infinite*. If it is finite, write the sample space. If it is infinite, classify whether it is *discrete* or *continuous*.

7. A color tile is drawn from a cup that contains 1 yellow, 2 blue, 3 green, and 4 red color tiles.

SOLUTION:

There are only four possible outcomes of this experiment: selecting a yellow, blue, green, or red color tile.

The sample space S is finite.

S = {yellow, blue, green, red}

9. An angler casts a fishing line into a body of water and its distance is recorded in centimeters.

SOLUTION:

There are an infinite number of possible outcomes of this experiment, so its sample space is infinite.

The angler can continue to record distances of the casts indefinitely, so the sample space is continuous.

10. A letter is randomly chosen from the alphabet.

SOLUTION:

There are only 26 possible outcomes of this experiment: each letter of the alphabet.

The sample space S is finite.

S = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}

Find the number of possible outcomes for each situation.

11. A video game lets you decorate a bedroom using one choice from each category.

Bedroom Décor	Number of Choices
Paint color	8
Comforter set	6
Sheet set	8
Throw rug	5
Lamp	3
Wall hanging	5

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$8 \times 6 \times 8 \times 5 \times 3 \times 5 = 28{,}800$

There are 28,800 ways to decorate the bedroom.

12. A cafeteria meal at Angela's work includes one choice from each category.

Cafeteria Meal	Number of Choices
Main dish	3
Side dish	4
Vegetable	2
Salad	2
Salad Dressing	3
Dessert	2
Drink	3

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$3 \times 4 \times 2 \times 2 \times 3 \times 2 \times 3 = 864$

There are 864 meal combinations.

13. **SHOPPING** On a website showcasing outdoor patio plans, there are 4 types of stone, 3 types of edging, 5 dining sets, and 6 grills. Kamar plans to order one item from each category. How many different patio sets can Kamar order?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$4 \times 3 \times 5 \times 6 = 360$

14. **AUDITIONS** The drama club held tryouts for 6 roles in a one-act play. Five people auditioned for lead female, 3 for lead male, 8 for the best friend, 4 for the mother, 2 for the father, and 3 for the humorous aunt. How many different casts can be created from those who auditioned?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$5 \times 3 \times 8 \times 4 \times 2 \times 3 = 2880$

There are 2880 different casts that can be created.

15. **BOARD GAMES** The spinner shown is used in a board game. If the spinner is spun 4 times, how many different possible outcomes are there?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$6 \times 6 \times 6 \times 6 = 1296$

There are 1296 different possible outcomes.

16. **BASKETBALL** In a city basketball league there must be a minimum of 14 players on a team's roster. One 14-player team has three centers, four power forwards, two small forwards, three shooting guards, and the rest of the players are point guards. How many different 5-player teams are possible if one player is selected from each position?

SOLUTION:

Find the number of point guards on the team:
$14 - 3 - 4 - 2 - 3 = 2$

Find the number of possible outcomes by using the Fundamental Counting Principle.

$3 \times 4 \times 2 \times 3 \times 2 = 144$

There are 144 different 5-player teams if one player is selected from each position.

17. **VACATION RENTAL** Angelica is comparing vacation prices in Boulder, Colorado, and Sarasota, Florida. In Boulder, she can choose a 1- or 2-week stay in a 1- or 2-bedroom suite. In Sarasota, she can choose a 1-, 2-, or 3-week stay in a 2- or 3-bedroom suite, on the beach or not?

a. How many outcomes are available in Boulder?

b. How many outcomes are available in Sarasota?

c. How many total outcomes are available?

SOLUTION:

a. There are 2 choices for weeks and 2 choices for the number of bedrooms.

Find the number of possible outcomes by using the Fundamental Counting Principle.
$2 \times 2 = 4$

b. There are 3 choices for weeks, 2 choices for the number of bedrooms, and 2 choices for beach proximity.

Find the number of possible outcomes by using the Fundamental Counting Principle.
$3 \times 2 \times 2 = 12$

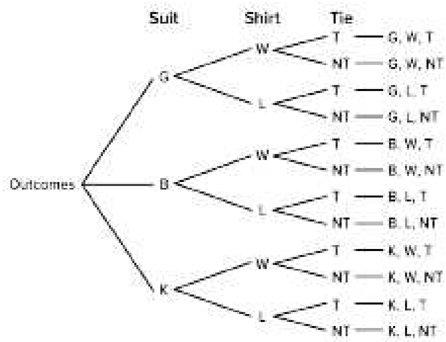
c. The total outcomes is the sum of the outcomes for Boulder and the outcomes for Sarasota.
$4 + 12 = 16$

18. **TRAVEL** Maurice packs suits, shirts, and ties that can be mixed and matched. Use his packing list to draw a tree diagram to represent the sample space for possible suit combinations using one article from each category.

Maurice's Packing List
1. Suits: Gray, black, khaki
2. Shirts: White, light blue
3. Ties: Striped (But optional)

SOLUTION:

Each event is represented by a different stage of the tree diagram.



Find the number of possible outcomes for each situation.

19. **SCHOOL** Tala wears a school uniform that consists of a skirt or pants, a white shirt, a blue jacket or sweater, white socks, and black shoes. She has 3 pairs of pants, 3 skirts, 6 white shirts, 2 jackets, 2 sweaters, 6 pairs of white socks, and 3 pairs of black shoes.

SOLUTION:

Skirt or pants: $3 + 3 = 6$
Blue jacket or sweater: $2 + 2 = 4$

Find the number of possible outcomes by using the Fundamental Counting Principle.
(skirt or pants) × (white shirts) × (blue jacket or sweater) × (white socks) × (black shoes)
$6 \times 6 \times 4 \times 6 \times 3 = 2592$

There are 2592 different school uniforms Tala can wear.

20. **FOOD** A sandwich shop provides its customers with a number of choices for bread, meats, and cheeses. Provided one item from each category is selected, how many different sandwiches can be made?

Bread	Meats	Cheeses
White	Turkey	American
Wheat	Ham	Swiss
Whole Grain	Roast Beef	Provolone
	Chicken	Colby-Jack
		Muenster

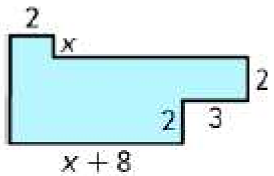
SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$3 \times 4 \times 5 = 60$

There are 60 different sandwiches.

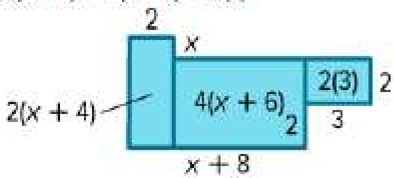
21. List six different expressions that could be used to evaluate the area of the composite figure.



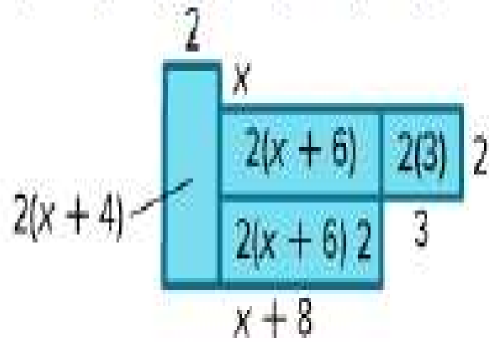
SOLUTION:

There are 6 different ways:

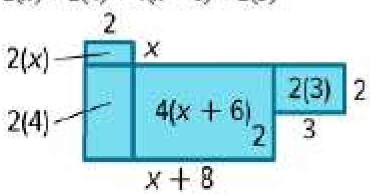
$2(x + 4) + 4(x + 6) + 2(3)$



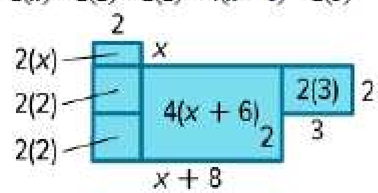
$2(x + 4) + 2(x + 6) + 2(x + 6) + 2(3)$



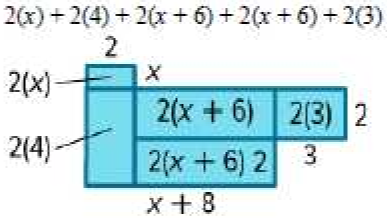
$2(x) + 2(4) + 4(x + 6) + 2(3)$



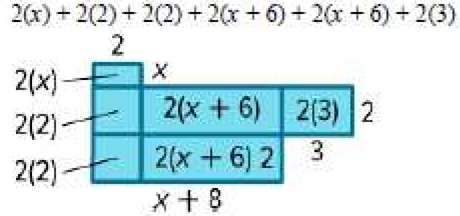
$2(x) + 2(2) + 2(2) + 4(x + 6) + 2(3)$



$2(x) + 2(4) + 2(x + 6) + 2(x + 6) + 2(3)$



$2(x) + 2(2) + 2(2) + 2(x + 6) + 2(x + 6) + 2(3)$



22. **LICENSE PLATES** One state requires license plates to consist of three letters followed by three numbers. The letter "O" and the number "0" may not be used, but any other combination of letters or numbers is allowed. How many different license plates can be created?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$25 \times 25 \times 25 \times 9 \times 9 \times 9 = 11{,}390{,}625$

There are 11,390,625 different license plates.

23. **COLLEGE** Jack has been offered a number of internships that could occur in 3 different months, in 4 different departments, and for 3 different companies. Jack is only available to complete his internship in July. How many different outcomes are there for his internship?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

(July only) × (departments) × (companies)

$1 \times 3 \times 4 = 12$

There are 12 different outcomes.

1. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling a number greater than 4. Find $A \cap B$.

SOLUTION:

The possible outcomes for event A are all of the numbers on a die that are even, or {2, 4, 6}.

The possible outcomes for event B are all of the numbers on a die that are greater than 4, or {5, 6}.

$A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \{6\}$

2. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling an odd number. Find $A \cap B$.

SOLUTION:

The possible outcomes for event A are all of the numbers on a die that are even, or {2, 4, 6}.

The possible outcomes for event B are all of the numbers on a die that are odd, or {1, 3, 5}.

$A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \emptyset$

Use the spinner.

3. Let A be the event of the spinner landing on 4 or 10, and let B be the event of the spinner landing on a section with a number divisible by 4. What are the possible outcomes of each event?

a. A = {___}

b. B = {___}

c. $A \cap B$ = {___}

SOLUTION:

a. The possible outcomes for event A are all of the numbers on the spinner that are 4 or 10, or {4, 10}.

b. The possible outcomes for event B are all of the numbers on the spinner that are divisible by 4, or {4, 8, 12}.

c. $A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \{4\}$

4. Let P be the event of the spinner landing on a section with a prime number, and let Q be the event of the spinner landing on a section with a number that is a multiple of 3. What are the possible outcomes of each event?

a. P = {___}

b. Q = {___}

c. $P \cap Q$ = {___}

SOLUTION:

a. The possible outcomes for event P are all of the numbers on the spinner that are prime, or {2, 3, 5, 7, 11}.

b. The possible outcomes for event Q are all of the numbers on the spinner that are a multiple of 3, or {3, 6, 9, 12}.

c. $P \cap Q$ contains all of the outcomes that are in both sample space P and Q.

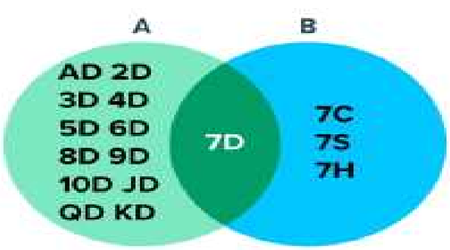
$P \cap Q = \{3\}$

5. A card is selected from a standard deck of cards. What is the probability that the card is a diamond and is a seven?

SOLUTION:

Let A be the event of choosing a diamond, and let B be the event of choosing a 7. The total number of outcomes is the total number of cards in a deck, or 52.

Write the corresponding number of each card in its correct place in the Venn diagram. In the diagram, D stands for diamonds, H stands for hearts, C stands for clubs, and S stands for spades.



There are 13 cards that are diamonds in a deck of cards and there is only 1 diamond that is also a 7.

$$P(A \cap B) = \frac{\text{number of outcomes in } A \text{ and } B}{\text{total number of possible outcomes}} \quad \text{Probability Rule for Intersections}$$

$$= \frac{1}{52} \quad \text{Substitution}$$

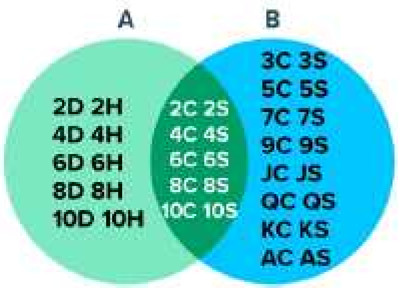
The probability that the card is both a diamond and a 7 is $\frac{1}{52}$, or about 1.92%.

6. A card is selected from a standard deck of cards. What is the probability that the card has a number on it that is divisible by 2 and is black?

SOLUTION:

Let *A* be the event of choosing a card that has a number on it that is divisible by 2, and let *B* be the event of choosing a card that is black. The total number of outcomes is the total number of cards in a deck, or 52.

Write the corresponding number of each card in its correct place in the Venn diagram. In the diagram, D stands for diamonds, H stands for hearts, C stands for clubs, and S stands for spades.



There are 20 cards that have a number on it that is divisible by 2 and there are only 10 that is also black cards.

$P(A \cap B) = \frac{\text{number of outcomes in } A \text{ and } B}{\text{total number of possible outcomes}}$ Probability Rule for Intersections

$= \frac{10}{52}$ Substitution

$= \frac{5}{26}$ Simplify

The probability that the card has a number on it that is both divisible by 2 and is black is $\frac{10}{52} = \frac{5}{26}$, or about 19.23%.

Use the spinner.

7. Let *A* be the event that the spinner lands on a vowel. Let *B* be the event that it lands on the letter J. What are the possible outcomes of each event?

a. A = {___}

b. B = {___}

c. $A \cup B$ = {___}

SOLUTION:

a. The possible outcomes for event *A* are all of the letters on the spinner that are vowels, or {A, E, O, U}.

b. The possible outcomes for event *B* are all of the letters on the spinner that are J, or {J}.

c. $A \cup B$ contains all of the outcomes that are in either sample space(s) *A* or *B*.

$A \cup B$ = {A, E, O, U, J}

8. Let *X* be the event that the spinner lands on a consonant. Let *Y* be the event that it lands on the letter K. What are the possible outcomes of each event?

a. X = {___}

b. Y = {___}

c. $X \cup Y$ = {___}

SOLUTION:

a. The possible outcomes for event *X* are all of the letters on the spinner that are a consonant, or {K, H, S, J}.

b. The possible outcomes for event *Y* are all of the letters on the spinner that are K, or {K}.

c. $X \cup Y$ contains all of the outcomes that are in either sample space(s) *X* or *Y*.

$X \cup Y$ = {K, H, S, J}

9. A random number generator is used to generate one integer between 1 and 20. Let *C* be the event of generating a multiple of 5, and let *D* be the event of generating a number less than 12. What are the possible outcomes of each event?

a. C = {___}

b. D = {___}

c. $C \cup D$ = {___}

SOLUTION:

a. The possible outcomes for event *C* are all of the numbers that are a multiple of 5, or {5, 10, 15, 20}.

b. The possible outcomes for event *D* are all of the numbers that are less than 12, or {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.

c. $C \cup D$ contains all of the outcomes that are in either sample space(s) *C* or *D*.

$C \cup D$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 20}

10. A random number generator is used to generate one integer between 1 and 100. Let A be the event of generating a multiple of 10, and let B be the event of generating a factor of 30. What are the possible outcomes of each event?

a. $A = \{\_\_\_\}$

b. $B = \{\_\_\_\}$

c. $A \cup B = \{\_\_\_\}$

SOLUTION:

a. The possible outcomes for event A are all of the numbers that are a multiple of 10, or {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}.

b. The possible outcomes for event B are all of the numbers that are a factor of 30, or {1, 2, 3, 5, 6, 10, 15, 30}.

c. $A \cup B$ contains all of the outcomes that are in either sample space(s) A or B.

$A \cup B = \{1, 2, 3, 5, 6, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

11. What is the probability of drawing a card from a standard deck and not getting a spade?

SOLUTION:

Let A be the event of choosing a card that is a spade. Then find the probability of the complement of A.

There are 13 cards that are spades.

There are 52 cards in a deck.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{13}{52} && \text{Substitution} \\ &= \frac{39}{52} \text{ or } \frac{3}{4} && \text{Subtract and simplify.} \end{aligned}$$

The probability of not getting a spade is $\frac{39}{52}$ or $\frac{3}{4}$ or 0.75.

12. What is the probability of flipping a coin and not landing on tails?

SOLUTION:

Let A be the event of a coin landing on tails. Then find the probability of the complement of A.

There is 1 side of a coin that is a tail.

There are 2 sides of a coin.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{1}{2} && \text{Substitution} \\ &= \frac{1}{2} && \text{Subtract and simplify.} \end{aligned}$$

The probability of flipping a coin and not landing on tails is $\frac{1}{2}$ or 0.5.

13. Carmela purchased 10 raffle tickets. If 250 were sold, what is the probability that one of Carmela's tickets will not be drawn?

SOLUTION:

Let A be the event of one of Carmela's tickets being drawn. Then find the probability of the complement of A.

There are 10 raffle tickets that are Carmela's.

There were 250 raffle tickets sold.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{10}{250} && \text{Substitution} \\ &= \frac{240}{250} && \text{Subtract and simplify.} \end{aligned}$$

The probability that one of Carmela's tickets will not be drawn is $\frac{240}{250}$ or 0.96.

14. What is the probability of spinning a spinner numbered 1 to 6 and not landing on 5?

SOLUTION:

Let A be the event of spinning a spinner numbered 1 to 6 and landing on 5. Then find the probability of the complement of A.

There is 1 section on the spinner labeled 5.

There are 6 sections on the spinner.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{1}{6} && \text{Substitution} \\ &= \frac{5}{6} && \text{Subtract and simplify.} \end{aligned}$$

The probability spinning a spinner numbered 1 to 6 and not landing on 5 is $\frac{5}{6}$ or about 0.83.

15. **STATISTICS** A survey found that about 90% of the junior class is right-handed. If 1 junior is chosen at random out of 100 juniors, what is the probability that he or she is left-handed?

SOLUTION:

Let A be the event of choosing a right-handed junior. Then find the probability of the complement of A.

There are 90 juniors that are right-handed because 90% of 100 is $0.9 \times 100 = 90$.

There are 100 juniors.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{90}{100} && \text{Substitution} \\ &= \frac{10}{100} \text{ or } \frac{1}{10} && \text{Subtract and simplify.} \end{aligned}$$

The probability of choosing a junior that is not right-handed, or that is left-handed, is $\frac{10}{100}$ or $\frac{1}{10}$ or 0.10.

16. **RAFFLE** Raul bought 24 raffle tickets out of 1545 tickets sold. What is the probability that Raul will not win the grand prize of the raffle?

SOLUTION:

Let A be the event of one of Raul will win the grand prize of the raffle. Then find the probability of the complement of A.

Raul bought 24 raffle tickets.

There were 1545 raffle tickets sold.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{24}{1545} && \text{Substitution} \\ &= \frac{1521}{1545} \text{ or } \frac{507}{515} && \text{Subtract and simplify.} \end{aligned}$$

The probability that Raul will not win the grand prize of the raffle is $\frac{507}{515}$ or 0.98.

17. **MASCOT** At Riverview High School, 120 students were asked whether they prefer a lion or a timber wolf as the new school mascot. What is the probability that a randomly selected student will have voted for a lion as the new school mascot?

	Votes
Lion	78
Timber Wolf	42
Total	120

SOLUTION:

Let A be the event of a student preferring a lion. The total number of outcomes is the total number of votes, or 120.

There were 78 students that prefer a lion.

$$\begin{aligned} P(A \cap B) &= \frac{\text{number of outcomes in } A}{\text{total number of possible outcomes}} && \text{Probability Rule} \\ &= \frac{78}{120} && \text{Substitution} \\ &= \frac{13}{20} && \text{Simplify.} \end{aligned}$$

The probability that a randomly selected student will have voted for a lion as the new school mascot is $\frac{13}{20}$, or 0.65.

18. **COLLEGE** In Evan's senior class of 240 students, 85% are planning to attend college after graduation. What is the probability that a senior chosen at random is not planning to attend college after graduation?

SOLUTION:

Let A be the event of a senior planning to attend college after graduation. Then find the probability of the complement of A.

There are 204 seniors planning to attend college after graduation because 85% of 240 is 0.85 × 240 = 204.

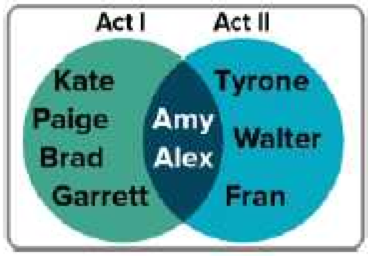
There are 240 seniors.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{204}{240} && \text{Substitution} \\ &= \frac{36}{240} \text{ or } \frac{3}{20} && \text{Subtract and simplify.} \end{aligned}$$

The probability that a senior chosen at random is not planning to attend college after graduation is $\frac{36}{240} = \frac{3}{20}$ or 0.15.

19. **DRAMA CLUB** The Venn diagram shows the cast members who are in Acts I and II of a school play. One of the students will be chosen at random to attend a statewide performing arts conference. Let A be the event that a cast member is in Act I of the play and let B be the event that a cast member is in Act II of the play.



a. Find $A \cap B$.

b. What is the probability that the student who is chosen to attend the conference is a cast member in only one of the two Acts of the play.

SOLUTION:

a. The possible outcomes for event A are all of the cast members in Act I of the play, or {Kate, Paige, Brad, Garrett, Amy, Alex}.

The possible outcomes for event B are all of the cast members in Act II of the play, or {Tyrone, Walter, Fran, Amy, Alex}.

$A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B$ = {Amy, Alex}

b. Let C be the event of a cast member in both Acts of the play. Then find the probability of the complement of A.

There are 2 cast members in both Acts of the play.

There are 9 cast members.

The probability of the complement of C is $P(C') = 1 - P(C)$.

20. **GAMES** LaRae is playing a game that uses a spinner. What is the probability that the spinner will land on a prime number on her next spin?

Let A be the event of spinning a prime number, and let B be the event of spinning a composite number. The total number of outcomes is the total number of sections on the spinner, or 7.

There are 7 sections on the spinner that are prime numbers. None of the sections on the spinner are composite numbers.

$$\begin{aligned} P(A \cap B) &= \frac{\text{number of outcomes in } A \text{ and } B}{\text{total number of possible outcomes}} && \text{Probability Rule for Intersections} \\ &= \frac{7}{7} && \text{Substitution} \\ &= 1 && \text{Simplify.} \end{aligned}$$

The probability that the spinner will land on a prime number on her next spin is 1.

1. **CHEERLEADING** The cheerleading squad is made up of 12 girls. A captain and a co-captain are selected at random. What is the probability that Chantel and Clover are chosen as leaders?

SOLUTION:

The number of possible outcomes in the sample space is the number of permutations of the 12 cheerleaders' order, or 12!

The number of favorable out comes is the number of permutations of Chantel and Clover times the other cheerleaders' order given that Chantel and Clover are the leaders.

$$P(\text{Chantel and Clover lead}) = \frac{2(10!)}{12!} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{\cancel{2}(\cancel{10!})}{\cancel{12}_6(11)(\cancel{10!})} \quad \text{Expand 12! and divide out common factors.}$$

$$= \frac{1}{66} \quad \text{Simplify.}$$

2. **BOOKS** You have a textbook for each of the following subjects: Spanish, English, Chemistry, Geometry, History, and Psychology. If you choose 4 of these books at random to arrange on a shelf, what is the probability that the Geometry textbook will be first from the left and the Chemistry textbook will be second from the left?

SOLUTION:

The number of possible outcomes in the sample space is the number of permutations of the 6 books' order, or 6!

The number of favorable out comes is the number of permutations of the 4 remaining books order.

$$P(\text{Geometry 1st, Chemistry 2nd}) = \frac{4!}{6!} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{(\cancel{4!})}{6(5)(\cancel{4!})} \quad \text{Expand 6! and divide out common factors.}$$

$$= \frac{1}{30} \quad \text{Simplify.}$$

3. **RAFFLE** Alfonso and Cordell each bought one raffle ticket at the state fair. If 50 tickets were randomly sold, what is the probability that Alfonso got ticket 14 and Cordell got ticket 23?

SOLUTION:

The number of possible outcomes in the sample space is the number of permutations of the 50 raffle tickets, or 50!

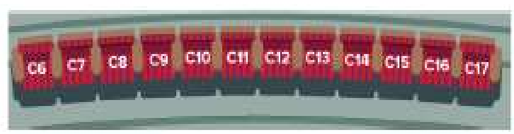
The number of favorable out comes is the number of permutations of the 48 remaining raffle tickets.

$$P(\text{Alfonso 14, Cordell 23}) = \frac{48!}{50!} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{(\cancel{48!})}{50(49)(\cancel{48!})} \quad \text{Expand 50! and divide out common factors.}$$

$$= \frac{1}{2450} \quad \text{Simplify.}$$

4. **CONCERT** Nia and Ciro are going to a concert with their high school's key club. If they choose a seat in the row below at random, what is the probability that Ciro will be in seat C11 and Nia will be in C12?



SOLUTION:

The number of possible outcomes in the sample space is the number of permutations of the 12 seats, or 12!

The number of favorable out comes is the number of permutations of the 10 remaining seats.

$$P(\text{Ciro C11, Nia C12}) = \frac{10!}{12!} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{(\cancel{10!})}{12(11)(\cancel{10!})} \quad \text{Expand 12! and divide out common factors.}$$

$$= \frac{1}{132} \quad \text{Simplify.}$$

5. **PHONE NUMBERS** What is the probability that a 7-digit telephone number generated using the digits 2, 3, 2, 5, 2, 7, and 3 is the number 222-3357?

SOLUTION:

There is a total of seven numbers. Of these numbers, 2 occurs 3 times, 3 occurs 2 times, and 5 and 7 occur 1 time each. So, the number of distinguishable permutations of these numbers is $\frac{7!}{3! \cdot 2!} = 420$.

There is only one favorable arrangement, 222–3357.

The probability that a permutation of these numbers selected at random results in the correct phone number, 222–3357, is $\frac{1}{420}$.

6. **IDENTIFICATION** A store randomly assigns their employees work identification numbers to track productivity. Each number consists of 5 digits ranging from 1–9. If the digits cannot repeat, find the probability that a randomly generated number is 25938.

SOLUTION:

Because the digits cannot repeat, order in this situation is important. The number of possible outcomes in the sample space is the number of permutations of 9 digits taken 5 at a time, ${}_9P_5$.

$${}_9P_5 = \frac{9!}{(9-5)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 15{,}120.$$

There is only one favorable arrangement, 25938.

The probability that a permutation of these numbers selected at random results in the correct identification number, 25938, is $\frac{1}{15{,}120}$.

7. **STUDENT COUNCIL** The table shows the finalists for class president. The order in which they will give their speeches will be chosen randomly.

Class President Finalists are Alan Shepherd, Chaminade Hudson, Denny Murano, Kelli Baker, Tanika Johnson, Jerome Murdock, Marlene Lindeman.

a. What is the probability that Denny, Kelli, and Chaminade are the first 3 speakers, in any order?

b. What is the probability that Denny is first, Kelli is second, and Chaminade is third?

SOLUTION:

a. The number of possible outcomes in the sample space is the number of permutations of 7 finalists taken 3 at a time, ${}_7P_3$.

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 210$$

The number of favorable outcomes is the number of permutations of Denni, Kelly, and Chaminade, or 3!.

The probability that Denni, Kelly, and Chaminade are selected at random in any order, is $\frac{3!}{210}$ or $\frac{1}{35}$.

b. The probability that Denny is first, Kelli is second, and Chaminade is third has the same number of possible outcomes as part **a**, 210. However, now the favorable outcome is 1 because the order is restricted. The probability is $\frac{1}{210}$.

1. **CLOTHING** Omari has two pairs of red socks and two pairs of white socks in a drawer. He has a drawer with 2 red T-shirts and 1 white T-shirt. If he randomly chooses a pair of socks from the sock drawer and a T-shirt from the T-shirt drawer, what is the probability that he gets a pair of red socks and a white T-shirt?

SOLUTION:

These events are independent because Omari is selecting a pair of socks, and then selecting a T-shirt. Let A represent a pair of red socks and B represent a white T-shirt.

Complete the equation to determine the probability of independent events.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) && \text{Probability of independent events} \\ &= \frac{2}{4} \cdot \frac{1}{3} && P(A) = \frac{2}{4} \text{ and } P(B) = \frac{1}{3} \\ &= \frac{2}{12} \text{ or } \frac{1}{6} && \text{Simplify.} \end{aligned}$$

2. Phyllis drops a penny in a pond, and then she drops a nickel in the pond. What is the probability that both coins land with tails showing?

SOLUTION:

These events are independent because Phyllis drops a penny in a pond, and then drops a nickel in the pond. Let A represent the penny landing with tails showing and B represent the nickel landing with tails showing.

Complete the equation to determine the probability of independent events.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) && \text{Probability of independent events} \\ &= \frac{1}{2} \cdot \frac{1}{2} && P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{2} \\ &= \frac{1}{4} && \text{Simplify.} \end{aligned}$$

So, the probability that both coins land with tails showing is $\frac{1}{4}$ or 25%.

3. A die is rolled and a penny is flipped. Find the probability of rolling a two and landing on a tail.

SOLUTION:

These events are independent because a die is rolled, and then a penny is flipped. Let A represent rolling a 2 on the die and B represent the penny landing with tails showing.

Complete the equation to determine the probability of independent events.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) && \text{Probability of independent events} \\ &= \frac{1}{6} \cdot \frac{1}{2} && P(A) = \frac{1}{6} \text{ and } P(B) = \frac{1}{2} \\ &= \frac{1}{12} && \text{Simplify.} \end{aligned}$$

So, the probability of rolling a two and landing on a tail is $\frac{1}{12}$ or about 8%.

4. A bag contains 3 red marbles, 2 green marbles, and 4 blue marbles. A marble is drawn randomly from the bag and replaced before a second marble is chosen. Find the probability that both marbles are blue.

SOLUTION:

These events are independent because the first marble is replaced before a second marble is chosen. Let A represent drawing a blue marble.

Complete the equation to determine the probability of independent events.

$$\begin{aligned} P(A \text{ and } A) &= P(A) \cdot P(A) && \text{Probability of independent events} \\ &= \frac{4}{9} \cdot \frac{4}{9} && P(A) = \frac{4}{9} \\ &= \frac{16}{81} && \text{Simplify.} \end{aligned}$$

So, the probability that both marbles are blue is $\frac{16}{81}$ or about 20%.

5. The forecast predicts a 40% chance of rain on Tuesday and a 60% chance on Wednesday. If these probabilities are independent, what is the chance that it will rain on both days?

SOLUTION:

It is stated that the events are independent. Let *A* represent the chance of rain on Tuesday and *B* represent the chance of rain on Wednesday.

Complete the equation to determine the probability of independent events.

$P(A \text{ and } B) = P(A) \cdot P(B)$	Probability of independent events
$= \frac{2}{5} \cdot \frac{3}{5}$	$P(A) = 40\% = \frac{2}{5}$ and $P(B) = 60\% = \frac{3}{5}$
$= \frac{6}{25}$	Simplify.

So, the chance that it will rain on both days is $\frac{6}{25}$ or 24%.

7. An ace is drawn from a standard deck of 52 cards, and is not replaced. Then, a second ace is drawn.

SOLUTION:

After the first ace is drawn, the card is removed and cannot be chosen again. This affects the probability of the second ace being drawn because the sample space is reduced by one card.

Because the first ace drawn was not replaced, the probability of drawing the second card is affected. So, the events are dependent.

9. You roll two fair dice and roll a 5 on each.

SOLUTION:

The outcome of rolling the first fair die in no way changes the probability of the outcome of rolling the second fair die.

These two rolls have no bearing on each other. So, the events are independent.

Determine whether the events are *independent* or *dependent*. Explain your reasoning.

6. You roll an even number on a fair die, and then spin a spinner numbered 1 through 5 and it lands on an odd number.

SOLUTION:

The outcome of rolling the fair die in no way changes the probability of the outcome of spinning the spinner.

These two events have no bearing on each other. So, the events are independent.

8. In a bag of 3 green and 4 blue marbles, a blue marble is drawn and not replaced. Then, a second blue marble is drawn.

SOLUTION:

After the first blue marble is drawn, the marble is removed and cannot be chosen again. This affects the probability of a second blue marble being drawn because the sample space is reduced by one marble.

Because the first blue marble drawn was not replaced, the probability of the drawing the second marble is affected. So, the events are dependent.

10. **LOTTERY** Mr. Hanes places the names of four of his students, Joe, Sofia, Hayden, and Bonita, on slips of paper. From these, he intends to randomly select two students to represent his class at the robotics convention. He draws the name of the first student, sets it aside, then draws the name of the second student. What is the probability he draws Sofia, then Joe?

SOLUTION:

These events are dependent because Mr Hanes does not replace the slip of paper he selected. Let *A* represent selecting Sofia and *B* represent selecting Joe.

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$	[illegible]
$= \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$	[illegible]

So, the probability that he draws Sofia, then Joe is $\frac{1}{12}$ or about 8%.

1. **CLUBS** The Spanish Club is having a potluck lunch where each student brings in a cultural dish. The 10 students randomly draw cards numbered with consecutive integers from 1 to 10. Students who draw odd numbers will bring main dishes. Students who draw even numbers will bring desserts. If Cynthia is bringing a dessert, what is the probability that she drew the number 10?

SOLUTION:

Let $P(A)$ = the probability that the number is even.
Let $P(A \text{ and } B)$ = the probability that the number is both 10 and even.
There are 10 available integers.
The sample space for event A contains 5 outcomes: {2, 4, 6, 8, 10}.
So, $P(A) = \frac{5}{10}$ or $\frac{1}{2}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {10}.
So, $P(A \text{ and } B) = \frac{1}{10}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability
$$= \frac{\frac{1}{10}}{\frac{1}{2}}$$ Substitute.
$$= \frac{1}{5} \text{ or } 20\%$$ Simplify.

2. A card is randomly drawn from a standard deck of 52 cards. What is the probability that the card is a king of diamonds, given that the card drawn is a king?

SOLUTION:

Let $P(A)$ = the probability that the card is a king.
Let $P(A \text{ and } B)$ = the probability that the card is a king and a diamond.
There are 52 possible cards.
The sample space for event A contains 4 outcomes: {king of hearts, king of diamonds, king of spades, king of clubs}
So, $P(A) = \frac{4}{52}$ or $\frac{1}{13}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {king of diamonds}.
So, $P(A \text{ and } B) = \frac{1}{52}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability
$$= \frac{\frac{1}{52}}{\frac{1}{13}}$$ Substitute.
$$= \frac{1}{4} \text{ or } 25\%$$ Simplify.

3. **GAME** In a game, a spinner with the 7 colors of the rainbow is spun. Find the probability that the color spun is blue, given the color is one of the three primary colors: red, yellow, or blue.

SOLUTION:

Let $P(A)$ = the probability that the color spun is a primary color.
Let $P(A \text{ and } B)$ = the probability that the color spun is blue.
There are 7 possible colors.
The sample space for event A contains 4 outcomes: {red, orange, yellow, green, blue, indigo, violet}
So, $P(A) = \frac{3}{7}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {blue}.
So, $P(A \text{ and } B) = \frac{1}{7}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability
$$= \frac{\frac{1}{7}}{\frac{3}{7}}$$ Substitute.
$$= \frac{1}{3}$$ Simplify.

4. Fifteen cards numbered 1–15 are placed in a hat. What is the probability that the card has a multiple of 3 on it, given that the card picked is an odd number?

SOLUTION:

Let $P(A)$ = the probability that the number is odd.
Let $P(A \text{ and } B)$ = the probability that the number is odd and a multiple of three.
There are 15 possible numbers.
The sample space for event A contains 8 outcomes: {1, 3, 5, 7, 9, 11, 13, 15}
So, $P(A) = \frac{8}{15}$.

The sample space for $P(A \text{ and } B)$ contains 3 outcomes: {3, 9, 15}.
So, $P(A \text{ and } B) = \frac{3}{15}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability
$$= \frac{\frac{3}{15}}{\frac{8}{15}}$$ Substitute.
$$= \frac{3}{8}$$ Simplify.

5. A blue marble is selected at random from a bag of 3 red and 9 blue marbles and not replaced. What is the probability that a second marble selected will be blue?

SOLUTION:

Let $P(A)$ = the probability that the first marble selected is blue.
Let $P(B)$ = the probability that the second marble selected is blue.
Let $P(A \text{ and } B)$ = the probability that the first and second marbles are both blue.
Since there are a total of 12 marbles to begin with, 9 of which are blue, $P(A) = \frac{9}{12}$ or $\frac{3}{4}$.

Since the first marble was blue, now there are 3 red and 8 blue marbles in the sample space, $P(B) = \frac{8}{11}$, or about 73%.

Another way to look at this is $P(A \text{ and } B) = \frac{3}{4} \cdot \frac{8}{11} = \frac{6}{11}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$
$$= \frac{\frac{6}{11}}{\frac{3}{4}} \quad \text{Substitute.}$$
$$= \frac{8}{11} \quad \text{Simplify.}$$

So, the probability that a second marble selected will be blue is $\frac{8}{11}$ or about 73%.

6. A die is rolled. If the number rolled is less than 5, what is the probability that it is the number 2?

SOLUTION:

Let $P(A)$ = the probability that the number is less than 5.
Let $P(A \text{ and } B)$ = the probability that the number is less than 5 and 2.
There are 6 possible numbers.
The sample space for event A contains 4 outcomes: $\{1, 2, 3, 4\}$
So, $P(A) = \frac{4}{6}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: $\{2\}$.
So, $P(A \text{ and } B) = \frac{1}{6}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$
$$= \frac{\frac{1}{6}}{\frac{4}{6}} \quad \text{Substitute.}$$
$$= \frac{1}{4} \quad \text{Simplify.}$$

So, the probability that the number is 2 if the number rolled is less than 5 is $\frac{1}{4}$ or 25%.

7. If two dice are rolled, what is the probability that the sum of the faces is 4, given that the first die rolled is odd?

SOLUTION:

Let $P(A)$ = the probability that the first die rolled is odd.
Let $P(A \text{ and } B)$ = the probability that the first die rolled is odd and the sum of the faces is 4.
There are 36 possible combinations of dice rolls.
The sample space for event A contains 18 outcomes: {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}.
So, $P(A) = \frac{18}{36} = \frac{1}{2}$.

The sample space for $P(A \text{ and } B)$ contains 2 outcomes: {(1, 3), (3, 1)}.
So, $P(A \text{ and } B) = \frac{2}{36} = \frac{1}{18}$.

$$
\begin{aligned}
P(B|A) &= \frac{P(A \text{ and } B)}{P(A)} && \text{Formula for conditional probability} \\
&= \frac{\frac{1}{18}}{\frac{1}{2}} && \text{Substitute.} \\
&= \frac{1}{9} && \text{Simplify.}
\end{aligned}
$$

So, the probability that the sum of the faces is 4, given that the first die rolled is odd is $\frac{1}{9}$ or about 11%.

8. A spinner numbered 1 through 12 is spun. Find the probability that the number spun is an 11 given that the number spun was an odd number.

SOLUTION:

Let $P(A)$ = the probability that the number spun is odd.
Let $P(A \text{ and } B)$ = the probability that the number spun is odd and 11.
There are 12 possible numbers.
The sample space for event A contains 6 outcomes: {1, 3, 5, 7, 9, 11}
So, $P(A) = \frac{6}{12}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {11}.
So, $P(A \text{ and } B) = \frac{1}{12}$.

$$
\begin{aligned}
P(B|A) &= \frac{P(A \text{ and } B)}{P(A)} && \text{Formula for conditional probability} \\
&= \frac{\frac{1}{12}}{\frac{6}{12}} && \text{Substitute.} \\
&= \frac{1}{6} && \text{Simplify.}
\end{aligned}
$$

So, the probability of spinning a number that is 11 given that it is an odd numbers is $\frac{1}{6}$ or about 17%.

Solve each inequality. Graph the solution set on a number line.

11. $|2x + 2| - 7 \leq -5$

SOLUTION:

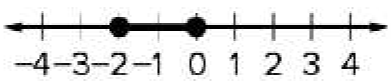
$|2x + 2| - 7 \leq -5$ Original inequality
$|2x + 2| \leq 2$ Add 7 to each side.

Since the inequality uses $\leq$, rewrite it as a compound inequality joined by the word *and*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$2x + 2 \leq 2$ and $2x + 2 \geq -2$
$2x \leq 0$ $2x \geq -4$
$x \leq 0$ $x \geq -2$

So, $x \leq 0$ and $x \geq -2$. The solution set is $\{x \mid -2 \leq x \leq 0\}$. All values of x between, and including, –2 and 0 satisfy the original inequality.

The solution set represents the interval between two numbers. Since the $\leq$ and $\geq$ symbols indicate –2 and 0 are solutions, graph the endpoints of the interval on a number line using dots. Then, shade the interval from –2 to 0.



12. $\left|\frac{x}{2} - 5\right| + 2 > 10$

SOLUTION:

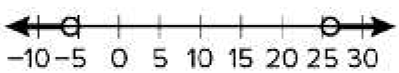
$\left|\frac{x}{2} - 5\right| + 2 > 10$ Original inequality
$\left|\frac{x}{2} - 5\right| > 8$ Subtract 2 from each side.

Since the inequality uses >, rewrite it as a compound inequality joined by the word *or*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$\frac{x}{2} - 5 > 8$ or $\frac{x}{2} - 5 < -8$
$\frac{x}{2} > 13$ $\frac{x}{2} < -3$
$x > 26$ $x < -6$

So, $x > 26$ or $x < -6$. The solution set is $\{x \mid x < -6 \text{ or } x > 26\}$. All values of x less than –6 as well as values of x greater than 26 satisfy the constraints of the original inequality.

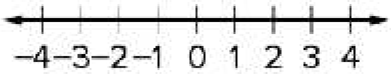
The solution set represents the union of two intervals. Since the > and < symbols indicate –6 and 26 are not solutions, graph the endpoints of the interval on a number line using circles. Then, shade all points less than –6 and all points greater than 26.



13. $|3b + 5| \leq -2$

SOLUTION:

Because the absolute value of a number is always positive or zero, this sentence is never true. The solution is ø.



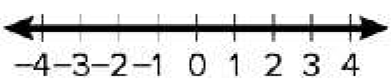
14. $|x| > x - 1$

SOLUTION:

Since the inequality uses >, rewrite it as a compound inequality joined by the word *or*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$$x > x - 1 \qquad \text{or} \qquad x < -(x-1)$$
$$0 > -1 \qquad\qquad x < -x + 1$$
$$2x < 1$$
$$x < \frac{1}{2}$$

Because the statement 0 > −1 is always true, the solution set is all real numbers.



15. $|4 - 5x| < 13$

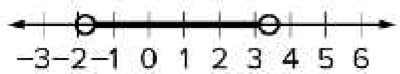
SOLUTION:

Since the inequality uses <, rewrite it as a compound inequality joined by the word *and*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$$4 - 5x < 13 \qquad \text{and} \qquad 4 - 5x > -13$$
$$-9 < 5x \qquad\qquad 17 > 5x$$
$$-1.8 < x \qquad\qquad 3.4 > x$$

So, $x > -1.8$ and $x < 3.4$. The solution set is $\{x \mid -1.8 < x < 3.4\}$. All values of x between −1.8 and 3.4 satisfy the original inequality.

The solution set represents the interval between two numbers. Since the < and > symbols indicate −1.8 and 3.4 are not solutions, graph the endpoints of the interval on a number line using circles. Then, shade the interval from −1.8 to 3.4.



16. $|3n-2|-2<1$

SOLUTION:

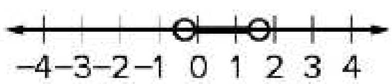
$|3n-2|-2<1$ Original inequality
$|3n-2|<3$ Add 2 to each side.

Since the inequality uses <, rewrite it as a compound inequality joined by the word *and*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$3n-2<3$ and $3n-2>-3$
$3n<5$ $\quad 3n>-1$
$n<\frac{5}{3}$ $\quad n>-\frac{1}{3}$

So, $n<\frac{5}{3}$ and $n>-\frac{1}{3}$. The solution set is $\left\{n \,\middle|\, -\frac{1}{3}<n<\frac{5}{3}\right\}$. All values of n between $-\frac{1}{3}$ and $\frac{5}{3}$ satisfy the original inequality.

The solution set represents the interval between two numbers. Since the < and > symbols indicate $-\frac{1}{3}$ and $\frac{5}{3}$ are not solutions, graph the endpoints of the interval on a number line using circles. Then, shade the interval from $-\frac{1}{3}$ to $\frac{5}{3}$.



17. $|3x+1|>2$

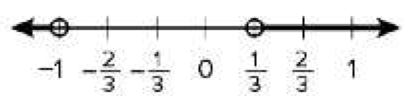
SOLUTION:

Since the inequality uses >, rewrite it as a compound inequality joined by the word *or*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$3x+1>2$ or $3x+1<-2$
$3x>1$ $\quad 3x<-3$
$x>\frac{1}{3}$ $\quad x<-1$

So, $x>\frac{1}{3}$ or $x<-1$. The solution set is $\left\{x \,\middle|\, x<-1 \text{ or } x>\frac{1}{3}\right\}$. All values of x less than -1 as well as values of x greater than $\frac{1}{3}$ satisfy the constraints of the original inequality.

The solution set represents the union of two intervals. Since the > and < symbols indicate -1 and $\frac{1}{3}$ are not solutions, graph the endpoints of the interval on a number line using circles. Then, shade all points less than -1 and al points greater than $\frac{1}{3}$.



18. $|2x-1|<5+0.5x$

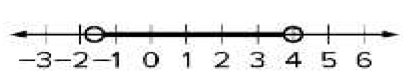
SOLUTION:

Since the inequality uses <, rewrite it as a compound inequality joined by the word *and*. For the case where the expression inside the absolute value symbols is negative, revere the inequality symbol.

$2x-1<5+0.5x$ and $2x-1>-(5+0.5x)$
$1.5x<6$ $\quad 2x-1>-5-0.5x$
$x<4$ $\quad 2.5x>-4$
$\quad x>-1.6$

So, $x<4$ and $x>-1.6$. The solution set is $\{x \mid -1.6<x<4\}$. All values of x between -1.6 and 4 satisfy the original inequality.

The solution set represents the interval between two numbers. Since the < and > symbols indicate -1.6 and 4 are not solutions, graph the endpoints of the interval on a number line using circles. Then, shade the interval from -1.6 to 4.



Write each equation in standard form. Identify A, B, and C.

1. $-7x - 5y = 35$

SOLUTION:

$-7x - 5y = 35$	Original equation
$7x + 5y = -35$	Multiply each side by -1.

$A = 7, B = 5, C = -35$

2. $8x + 3y + 6 = 0$

SOLUTION:

$8x + 3y + 6 = 0$	Original equation
$8x + 3y = -6$	Subtract 6 from each side.

$A = 8, B = 3, C = -6$

5. $\frac{4}{5}y + \frac{1}{8}x = 4$

SOLUTION:

$\frac{4}{5}y + \frac{1}{8}x = 4$	Original equation
$5x + 32y = 160$	Multiply each side by 40.

$A = 5, B = 32, C = 160$

6. $-0.08x = 1.24y - 3.12$

SOLUTION:

$-0.08x = 1.24y - 3.12$	Original equation
$-0.08x - 1.24y = -3.12$	Subtract 1.24y from each side.
$8x + 124y = 312$	Multiply each side by -100.
$2x + 31y = 78$	Divide each side by 4.

$A = 2, B = 31, C = 78$

3. $10y - 3x + 6 = 11$

SOLUTION:

$10y - 3x + 6 = 11$	Original equation
$-3x + 10y = 5$	Subtract 6 from each side.
$3x - 10y = -5$	Multiply each side by -1.

$A = 3, B = -10, C = -5$

4. $\frac{2}{3}y - \frac{3}{4}x + \frac{1}{6} = 0$

SOLUTION:

$\frac{2}{3}y - \frac{3}{4}x + \frac{1}{6} = 0$	Original equation
$9x - 8y - 2 = 0$	Multiply each side by -12.
$9x - 8y = 2$	Add 2 toeach side.

$A = 9, B = -8, C = 2$

Write each equation in slope-intercept form. Identify the slope m and the y-intercept b.

7. $6x + 3y = 12$

SOLUTION:

$6x + 3y = 12$	Original equation
$3y = -6x + 12$	Subtract 6x from each side.
$y = -2x + 4$	Divide each side by 3.

$m = -2, b = 4$

8. $14x - 7y = 21$

SOLUTION:

$14x - 7y = 21$	Original equation
$-7y = -14x + 21$	Subtract 14x from each side.
$y = 2x - 3$	Divide each side by -7.

$m = 2, b = -3$

9. $\frac{2}{3}x + \frac{1}{6}y = 2$

SOLUTION:

$\frac{2}{3}x + \frac{1}{6}y = 2$	Original equation
$\frac{1}{6}y = -\frac{2}{3}x + 2$	Subtract $\frac{2}{3}x$ from each side.
$y = -4x + 12$	Mulitply each side by 6.

$m = -4, b = 12$

10. $5x + 10y = 20$

SOLUTION:

$5x + 10y = 20$	Original equation
$10y = -5x + 20$	Subtract $5x$ from each side.
$y = -\frac{1}{2}x + 2$	Divide each side by 10.

$m = -\frac{1}{2}, b = 2$

11. $6x + 9y = 15$

SOLUTION:

$6x + 9y = 15$	Original equation
$9y = -6x + 15$	Subtract $6x$ from each side.
$y = -\frac{2}{3}x + \frac{5}{3}$	Divide each side by 9.

$m = -\frac{2}{3}, b = \frac{5}{3}$

12. $\frac{1}{5}x + \frac{1}{2}y = 4$

SOLUTION:

$\frac{1}{5}x + \frac{1}{2}y = 4$	Original equation
$\frac{1}{2}y = -\frac{1}{5}x + 4$	Subtract $\frac{1}{5}x$ from each side.
$y = -\frac{2}{5}x + 8$	Muliply each side by 2.

$m = -\frac{2}{5}, b = 8$

13. **CHARITY** The linear equation $y - 20x = 83$ relates the number of shirts collected during a charity clothing drive, where x is the number of hours since noon and y is the total number of shirts collected. Write the equation in slope-intercept form and interpret the parameters of the equation in the context of the situation.

SOLUTION:

Part A Write the equation in slope-intercept form.

$y - 20x = 83$	Original equation
$y = 20x + 83$	Add $20x$ to each side.

Part B Interpret the parameters in the context of the situation.

20 represents the number of shirts collect each hour after noon.

83 represents the number of shirts collected before noon.

14. **GROWTH** Suppose the body length y in inches of a baby snake is given by $4x - 2y = -3$, where x is the age of the snake in months until it becomes 1 year old. Write the equation in slope-intercept form and interpret the parameters of the equation in the context of the situation.

SOLUTION:

Part A Write the equation in slope-intercept form.

$4x - 2y = -3$	Original equation
$-2y = -4x - 3$	Subtract $4x$ from each side.
$y = 2x + 1.5$	Divide each side by -2.

Part B Interpret the parameters in the context of the situation.

1.5 represents the number of inches in length the snake was when it hatched.

2 represents the number of inches the snake grew each month for the first 12 months.

15. **PLUMBER** Two neighbors, Camila and Conner, hire the same plumber for household repairs. The plumber worked at Camila's house for 3 hours and charged her \$191. The plumber worked at Conner's house for 1 hour and charged him \$107.

a. Define the variables to represent the situation.
b. Find the slope and y-intercept. Then, write an equation.
c. How much would it cost to hire the plumber for 5 hours of work?

SOLUTION:

a. Define the variables. Because you want to find the cost to hire the plumber for 5 hours of work, write an equation that represents the total cost y after x hours. Let x represent the number of hours the plumber spends working at a job site, and let y represent the total cost for the services.

b. Find the slope. Since x is the number of hours after 0 hours, (1, 107) and (3, 191) represent the cost to hire the plumber for 1 and 3 hours, respectively.

$$m = \frac{191 - 107}{3 - 1} = \frac{84}{2} = 42$$

So, the total cost to hire the plumber is increasing at a rate of \$42 per hour.

Find the y-intercept. The y-intercept represents the cost to hire the plumber when $x = 0$. Use the slope-intercept form of the equation and substitute (1, 107) for x and y, and 42 for m to solve for b.

$$y = mx + b$$
$$107 = 42(1) + b$$
$$65 = b$$

Write an equation. Use $m = 42$ and $b = 65$ to write $y = 42x + 65$.

c. Find cost for 5 hours. Substitute 5 for x into $y = 42x + 65$ to get $y = 42(5) + 65$; $y = 210 + 65$; $y = 275$

So, the cost to hire the plumber for 5 hours of work is \$275.

Write an equation in point-slope form for the line that satisfies each set of conditions.

17. slope of –5, passes through (–3, –8)

SOLUTION:

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-8) &= -5(x-(-3)) && m=-5;\ (x_1,y_1)=(-3,-8) \\ y+8 &= -5(x+3) && \text{Simplify.} \end{aligned}$$

18. slope of $\frac{4}{5}$, passes through (10, –3)

SOLUTION:

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-3) &= \frac{4}{5}(x-10) && m=\frac{4}{5};\ (x_1,y_1)=(10,-3) \\ y &= \frac{4}{5}(x-10) && \text{Simplify.} \end{aligned}$$

19. slope of $-\frac{2}{3}$, passes through (6, –8)

SOLUTION:

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-8) &= -\frac{2}{3}(x-6) && m=-\frac{2}{5};\ (x_1,y_1)=(6,-8) \\ y+8 &= -\frac{2}{3}(x-6) && \text{Simplify.} \end{aligned}$$

20. slope of 0, passes through (0, –10)

SOLUTION:

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-10) &= 0(x-0) && m=0;\ (x_1,y_1)=(0,-10) \\ y+10 &= 0(x-0) && \text{Simplify.} \end{aligned}$$

Write an equation in point-slope form for a line that passes through each set of points.

21. (2, –3) and (1, 5)

SOLUTION:

Step 1 Find the slope.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} && \text{Slope formula} \\ &= \frac{5-(-3)}{1-2} && (x_1,y_1)=(2,-3);\ (x_2,y_2)=(1,5) \\ &= \frac{8}{-1} && \text{Simplify.} \\ &= -8 && \text{Simplify.} \end{aligned}$$

Step 2 Write an equation.

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-3) &= -8(x-2) && m=-8;\ (x_1,y_1)=(0,-10) \\ y+3 &= -8(x-2) && \text{Simplify.} \end{aligned}$$

or

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-5 &= -8(x-1) && m=-8;\ (x_1,y_1)=(1,5) \end{aligned}$$

22. (3, 5) and (–6, –4)

SOLUTION:

Step 1 Find the slope.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} && \text{Slope formula} \\ &= \frac{-4-5}{-6-3} && (x_1,y_1)=(3,5);\ (x_2,y_2)=(-6,-4) \\ &= \frac{-9}{-9} && \text{Simplify.} \\ &= 1 && \text{Simplify.} \end{aligned}$$

Step 2 Write an equation.

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-5 &= 1(x-3) && m=1;\ (x_1,y_1)=(3,5) \end{aligned}$$

or

$$\begin{aligned} y-y_1 &= m(x-x_1) && \text{Point – slope form} \\ y-(-4) &= 1(x-(-6)) && m=1;\ (x_1,y_1)=(-6,-4) \\ y+4 &= 1(x+6) && \text{Simplify.} \end{aligned}$$

23. (–1, –2) and (–3, 1)

SOLUTION:

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{1-(-2)}{-3-(-1)}$ $(x_1, y_1) = (-1, -2)$; $(x_2, y_2) = (-3, 1)$

$= \frac{3}{-2}$ Simplify.

Step 2 Write an equation.

$y - y_1 = m(x - x_1)$ Point – slope form

$y-(-2) = -\frac{3}{2}(x-(-1))$ $m = -\frac{3}{2}$; $(x_1, y_1) = (-1, -2)$

$y + 2 = -\frac{3}{2}(x+1)$ Simplify.

or

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 1 = -\frac{3}{2}(x-(-3))$ $m = -\frac{3}{2}$; $(x_1, y_1) = (-3, 1)$

$y - 1 = -\frac{3}{2}(x+3)$ Simplify.

24. (–2, –4) and (1, 8)

SOLUTION:

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{8-(-4)}{1-(-2)}$ $(x_1, y_1) = (-2, -4)$; $(x_2, y_2) = (1, 8)$

$= \frac{12}{3}$ Simplify.

$= 4$ Simplify.

Step 2 Write an equation.

$y - y_1 = m(x - x_1)$ Point – slope form

$y-(-4) = 4(x-(-2))$ $m = 4$; $(x_1, y_1) = (-2, -4)$

$y + 4 = 4(x+2)$ Simplify.

or

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 8 = 4(x - 1)$ $m = 4$; $(x_1, y_1) = (1, 8)$

25. **SALES** Light truck is a vehicle classification for trucks weighing up to 8500 pounds. In 2011, 5.919 million light trucks were sold in the U.S. In 2017, 11.055 million light trucks were sold. Write an equation in point-slope form that represents the number of light trucks y sold x years after 2010.

SOLUTION:

Step 1 Find the slope.

There were 5.919 million light trucks sold in 2011, 1 year after 2010.

There were 11.055 million light trucks sold in 2017, 7 years after 2010.

$m = \frac{11.055 - 5.919}{7 - 1} = 0.856$ The number of light trucks sold increased at a rate of 0.856 trucks per year.

Step 2 Write an equation.

Substitute the slope m and the coordinates of either of the given points for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 5.919 = 0.856(x - 1)$ $m = 0.856$; $(x_1, y_1) = (1, 5.919)$

or

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 11.055 = 0.856(x - 7)$ $m = 0.856$; $(x_1, y_1) = (7, 11.055)$

26. **RESTAURANTS** In 2012, a popular pizza franchise had 2483 restaurants. In 2017, there were 2606 franchised restaurants. Write an equation in point-slope form that represents the number of restaurants y that are franchised x years after 2010.

SOLUTION:

Step 1 Find the slope.

There were 2483 restaurants in 2012, 2 years after 2010.

There were 2606 restaurants in 2017, 7 years after 2010.

$m = \frac{2606 - 2483}{7 - 2} = 24.6$ The number of restaurants increased at a rate of 24.6 restaurants per year.

Step 2 Write an equation.

Substitute the slope m and the coordinates of either of the given points for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 2483 = 24.6(x - 2)$ $m = 24.6$; $(x_1, y_1) = (2, 2483)$

or

$y - y_1 = m(x - x_1)$ Point – slope form

$y - 2606 = 24.6(x - 7)$ $m = 24.6$; $(x_1, y_1) = (7, 2606)$

Use substitution to solve each system of equations.

1. $2x - y = 9$
$x + 3y = -6$

SOLUTION:

Because the coefficient of x in Equation 2 is 1, solve for x in that equation.

$$x + 3y = -6 \quad \text{Equation 2}$$
$$x = -3y - 6 \quad \text{Subtract } 3y \text{ from each side.}$$

Substitute the expression. Substitute for x. Then solve for y.

$$2x - y = 9 \quad \text{Equation 1}$$
$$2(-3y - 6) - y = 9 \quad x = -3y - 6$$
$$-6y - 12 - y = 9 \quad \text{Distributive Property}$$
$$-7y - 12 = 9 \quad \text{Simplify.}$$
$$-7y = 21 \quad \text{Add 12 to each side.}$$
$$y = -3 \quad \text{Divide each side by } -7.$$

Substitute the value of y into one of the original equations to solve for x.

$$x + 3y = -6 \quad \text{Equation 2}$$
$$x + 3(-3) = -6 \quad y = -3$$
$$x - 9 = -6 \quad \text{Multiply.}$$
$$x = 3 \quad \text{Add.}$$

The solution is (3, –3).

2. $2x - y = 7$
$6x - 3y = 14$

SOLUTION:

Because the coefficient of y in Equation 1 is –1, solve for y in that equation.

$$2x - y = 7 \quad \text{Equation 1}$$
$$-y = -2x + 7 \quad \text{Subtract } 2x \text{ from each side.}$$
$$y = 2x - 7 \quad \text{Divide each side by } -1.$$

Substitute the expression. Substitute for y. Then solve for x.

$$6x - 3y = 14 \quad \text{Equation 2}$$
$$6x - 3(2x - 7) = 14 \quad y = 2x - 7$$
$$6x - 6x + 21 = 14 \quad \text{Distributive Property}$$
$$21 = 14 \quad \text{False}$$

This system has no solution because 21 = 14 is not true.

3. $2x + y = 5$
$3x - 3y = 3$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$$2x + y = 5 \quad \text{Equation 1}$$
$$y = -2x + 5 \quad \text{Subtract } 2x \text{ from each side.}$$

Substitute the expression. Substitute for y. Then solve for x.

$$3x - 3y = 3 \quad \text{Equation 2}$$
$$3x - 3(-2x + 5) = 3 \quad y = -2x + 5$$
$$3x + 6x - 15 = 3 \quad \text{Distributive Property}$$
$$9x - 15 = 3 \quad \text{Simplify.}$$
$$9x = 18 \quad \text{Add 15 to each side.}$$
$$x = 2 \quad \text{Divide each side by 9.}$$

Substitute the value of x into one of the original equations to solve for y.

$$2x + y = 5 \quad \text{Equation 1}$$
$$2(2) + y = 5 \quad x = 2$$
$$4 + y = 5 \quad \text{Multiply.}$$
$$y = 1 \quad \text{Subtract.}$$

The solution is (2, 1).

4. $3x + y = 7$
$4x + 2y = 16$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$$3x + y = 7 \quad \text{Equation 1}$$
$$y = -3x + 7 \quad \text{Subtract } 3x \text{ from each side.}$$

Substitute the expression. Substitute for y. Then solve for x.

$$4x + 2y = 16 \quad \text{Equation 2}$$
$$4x + 2(-3x + 7) = 16 \quad y = -3x + 7$$
$$4x - 6x + 14 = 16 \quad \text{Distributive Property}$$
$$-2x + 14 = 16 \quad \text{Simplify.}$$
$$-2x = 2 \quad \text{Subtract 14 from each side.}$$
$$x = -1 \quad \text{Divide each side by } -2.$$

Substitute the value of x into one of the original equations to solve for y.

$$3x + y = 7 \quad \text{Equation 1}$$
$$3(-1) + y = 7 \quad x = -1$$
$$-3 + y = 7 \quad \text{Multiply.}$$
$$y = 10 \quad \text{Add.}$$

The solution is (–1, 10).

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

SOLUTION:

Because the coefficient of y in Equation 1 is –1, solve for y in that equation.

$4x - y = 6$	Equation 1
$-y = -4x + 6$	Subtract $4x$ from each side.
$y = 4x - 6$	Divide each side by -1.

Substitute the expression. Substitute for y. Then solve for x.

$2x - \frac{y}{2} = 4$	Equation 2
$2x - \frac{4x-6}{2} = 4$	$y = 4x - 6$
$2x - (2x - 3) = 4$	Simplify fraction.
$2x - 2x + 3 = 4$	Distributive Property
$3 = 4$	False

This system has no solution because 3= 4 is not true.

6. $2x + y = 8$
$3x + \frac{3}{2}y = 12$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$2x + y = 8$	Equation 1
$y = -2x + 8$	Subtract $2x$ from each side.

Substitute the expression. Substitute for y. Then solve for x.

$3x + \frac{3}{2}y = 12$	Equation 2
$3x + \frac{3}{2}(-2x + 8) = 12$	$y = -2x + 8$
$3x - 3x + 12 = 12$	Distributive Property
$12 = 12$	True

This system has infinitely many solutions because 12= 12 is true.

Solve each problem.

7. **BAKE SALE** Cassandra and Alberto are selling pies for a fundraiser. Cassandra sold 3 small pies and 14 large pies for a total of \$203. Alberto sold 11 small pies and 11 large pies for a total of \$220. Determine the cost of each pie.

a. Write a system of equations and solve by using substitution.

b. What does the solution represent in terms of this situation?

c. How can you verify that the solution is correct?

SOLUTION:

a. Write two equations in two variables. Let x be the number of small pies and y be the number of large pies.

Cassandra: $3x + 14y = 203$
Alberto: $11x + 11y = 220$

Because Equation 2 is divisible by 11, divide then solve for x in that equation.

$11x + 11y = 220$	Alberto
$x + y = 20$	Divide each side by 11.
$y = -x + 20$	Subtract x from each side.

Substitute the expression. Substitute for y. Then solve for x.

$3x + 14y = 203$	*Cassandra*
$3x + 14(-x + 20) = 203$	$y = -x + 20$
$3x - 14x + 280 = 203$	Distributive Property
$-11x + 280 = 203$	Simplify.
$-11x = -77$	Subtract 280 from each side.
$x = 7$	Divide each side by -11.

Substitute the value of x into one of the original equations to solve for y.

$11(7) + 11y = 220$	Substitute 7 for x in Alberto's equation.
$77 + 11y = 220$	Multiply.
$11y = 143$	Subtract 77 from each side.
$y = 13$	Divide each side by 11.

The solution of the system is $x = 7$, $y = 13$.

b. Since y represents the cost of a small pie, the cost of each small pie is $7. Since x represents the cost of a large pie, the cost of each large pie is $13.

c. Sample answer: By substituting the solution into each equation in the system, you can verify that it is correct. $3(7) + 14(13) = 203$, and $11(7) + 11(13) = 220$.

8. **STOCKS** Ms. Patel invested a total of $825 in two stocks. At the time of her investment, one share of Stock A was valued at $12.41 and a share of Stock B was valued at $8.62. She purchased a total of 79 shares.

a. Write a system of equations and solve by substitution.

b. How many shares of each stock did Ms. Patel buy? How much did she invest in each of the two stocks?

SOLUTION:

a. Write two equations in two variables. Let a be the number of shares of Stock A and b be the number of shares of Stock B.
Number of shares: $a + b = 79$
Value of shares: $12.41a + 8.62b = 825$

Because Equation 1 has coefficients of 1, solve for a in that equation.

$a + b = 79$ Number of shares
$a = -b + 79$ Subtract b from each side.

Substitute the expression. Substitute for a. Then solve for b.

$12.41a + 8.62b = 825$ Value of shares
$12.41(-b + 79) + 8.62b = 825$ $a = -b + 79$
$-12.41b + 980.39 + 8.62b = 825$ Distributive Property
$-3.79b + 980.39 = 825$ Simplify.
$-3.79b = -155.39$ Subtract 980.39 from each side.
$b = 41$ Divide each side by -3.79.

Substitute the value of b into one of the original equations to solve for a.

$a + b = 79$ Number of shares.
$a + 41 = 79$ Substitution.
$a = 38$ Subtract 41 from each side.

$a + b = 79$; $12.41a + 8.62b = 825$; $a = 38$, $b = 41$

b. How many shares of each stock did Ms. Patel buy? How much did she invest in each of the two stocks?

SOLUTION:

a. Write two equations in two variables. Let a be the number of shares of Stock A and b be the number of shares of Stock B.
Number of shares: $a + b = 79$
Value of shares: $12.41a + 8.62b = 825$

Because Equation 1 has coefficients of 1, solve for a in that equation.

$a + b = 79$ Number of shares
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Substitute the expression. Substitute for a. Then solve for b.

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$-12.41b + 980.39 + 8.62b = 825$ Distributive Property
$-3.79b + 980.39 = 825$ Simplify.
$-3.79b = -155.39$ Subtract 980.39 from each side.
$b = 41$ Divide each side by -3.79.

Substitute the value of b into one of the original equations to solve for a.

$a + b = 79$ Number of shares.
$a + 41 = 79$ Substitution.
$a = 38$ Subtract 41 from each side.

$a + b = 79$; $12.41a + 8.62b = 825$; $a = 38$, $b = 41$

8. **TROPHIES** Taryn has 15 soccer trophies but she only has room to display 9 of them on a shelf. If she chooses them at random, what is the probability that each of the trophies from the school invitational from the 1st through 9th grades will be chosen?

SOLUTION:

Because the order in which the trophies are selected does not matter, the number of possible outcomes in the sample space is the number of combinations of 15 trophies taken 9 at a time, ${}_{15}C_9$.

$${}_{15}C_9 = \frac{15!}{(15-9)!9!} = \frac{15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9!}{6!\cdot9!} = \text{[illegible]} = 5005$$

There is only one favorable arrangement of the trophies from the school invitational from the 1st through 9th grades, so the probability is $\frac{1}{5005}$.

9. **FROZEN YOGURT** Kali has a choice of 20 flavors for her triple scoop cone. If she chooses the flavors at random, what is the probability that the 3 flavors she chooses will be vanilla, chocolate, and strawberry?

SOLUTION:

Because the order in which the ice cream is selected does not matter, the number of possible outcomes in the sample space is the number of combinations of 20 types of ice cream taken 3 at a time, ${}_{10}C_3$.

$${}_{20}C_3 = \frac{20!}{(20-3)!3!} = \frac{20\cdot19\cdot18\cdot17!}{17!\cdot3!} = \frac{20\cdot19\cdot18^3}{3\cdot2} = 1140$$

There is only one favorable arrangement of the ice cream, vanilla, chocolate, and strawberry, so the probability is $\frac{1}{1140}$.

10. **BUSINESS** Kaja has a dog walking business that serves 9 dogs. If she chooses 4 of the dogs at random to take an extra trip to the dog park, what is the probability that Cherish, Taffy, Haunter, and Maverick are chosen?

SOLUTION:

Because the order in which the dogs are selected does not matter, the number of possible outcomes in the sample space is the number of combinations of 9 dogs taken 4 at a time, ${}_9C_4$.

$${}_9C_4 = \frac{9!}{(9-4)!4!} = \frac{9\cdot8\cdot7\cdot6\cdot5!}{5!\cdot4!} = \frac{9\cdot8^2\cdot7\cdot6}{4\cdot3\cdot2} = 126$$

There is only one favorable arrangement of the dogs, Cherish, Taffy, Haunter, and Maverick, so the probability is $\frac{1}{126}$.

11. **FOOD TRUCKS** A restaurant critic has 10 new food trucks to try. If she tries half of them this week, what is the probability that she will choose Nacho Best Tacos, Creme Bruleezin, Fre Sha Vaca Do's, You Can't Get Naan, and Grillarious?

SOLUTION:

Because the order in which the food trucks are visited does not matter, the number of possible outcomes in the sample space is the number of combinations of 10 food trucks taken 5 at a time, ${}_{10}C_5$.

$${}_{10}C_5 = \frac{10!}{(10-5)!5!} = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot5!}{5!\cdot5!} = \frac{10^2\cdot9\cdot8^2\cdot7\cdot6}{5\cdot4\cdot3\cdot2} = 252$$

12. **DONATIONS** Emily has 20 collectible dolls from different countries that she will donate. If she selects 10 of them at random, what is the probability that she chooses the dolls from Ecuador, Paraguay, Chile, France, Spain, Sweden, Switzerland, Germany, Greece, and Italy?

SOLUTION:

Because the order in which the dolls are chosen does not matter, the number of possible outcomes in the sample space is the number of combinations of 20 dolls taken 10 at a time, ${}_{20}C_{10}$.

$${}_{20}C_{10} = \frac{20!}{(20-10)!10!} = \text{[illegible]} = 184756$$

There is only one favorable arrangement of the dolls, Ecuador, Paraguay, Chile, France, Spain, Sweden, Switzerland, Germany, Greece, and Italy so the probability is $\frac{1}{184,756}$.

13. **AMUSEMENT PARK** An amusement park has 12 major attractions: four roller coasters, two carousels, two drop towers, two gravity rides, and two dark rides. The park's app will randomly select attractions for you to visit in order. What is the probability that the four roller coasters are the first four suggested attractions?

SOLUTION:

Because the order in which the attractions are chosen does not matter, the number of possible outcomes in the sample space is the number of combinations of 12 attractions taken 4 at a time, ${}_{12}C_4$.

There is only one favorable arrangement of the food trucks, Nacho Best Tacos, Creme Bruleezin, Fre Sha Vaca Do's, You Can't Get Naan, and Grillarious, so the probability is $\frac{1}{252}$.

$${}_{12}C_4 = \frac{12!}{(12-4)!4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!} \cdot 4!} = \frac{\cancel{12} \cdot 11 \cdot \cancel{10}^5 \cdot 9}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}} = 495$$

There is only one favorable arrangement of the attractions, 4 rollercoasters, so the probability is $\frac{1}{495}$.

Use substitution or elimination to solve each system of equations.

15. $0.5x + 2y = 5$
$x - 2y = -8$

SOLUTION:

Since the y-coefficients are opposites, add the equations to eliminate y. Then solve for x.

$0.5x + 2y = 5$	Equation 1
$x - 2y = -8$	Equation 2
$1.5x = -3$	Equation after addition.
$x = -2$	Divide each side by 1.5.

Substitute the value of x into one of the original equations to solve for y.

$x - 2y = -8$	Equation 2
$(-2) - 2y = -8$	$x = -2$
$-2y = -6$	Add 2 to each side.
$y = 3$	Divide each side by -2.

The solution of the system is (–2, 3).

16. $h - z = 3$
$-3h + 3z = 6$

SOLUTION:

Because the coefficient of h in Equation 1 is 1, solve for h in that equation.

$h - z = 3$	Equation 1
$h = z + 3$	Add z to each side.

Substitute the expression. Substitute for h. Then solve for z.

$-3h + 3z = 6$	Equation 2
$-3(z+3) + 3z = 6$	$h = z + 3$
$-3z - 9 + 3z = 6$	Distributive Property
$-9 = 6$	Simplify. False statement.

Since $-9 \neq 6$, there is no solution.

17. $-r + t = 5$
$-2r + t = 4$

SOLUTION:

Because the coefficient of t in Equation 1 is 1, solve for t in that equation.

$-r + t = 5$	Equation 1
$t = r + 5$	Add r to each side.

Substitute the expression. Substitute for t. Then solve for r.

$-2r + t = 4$	Equation 2
$-2r + (r+5) = 4$	$t = r + 5$
$-2r + r + 5 = 4$	Distributive Property
$-r + 5 = 4$	Simplify.
$-r = -1$	Subtract 5 from each side.
$r = 1$	Divide each side by -1.

Substitute the value of r into one of the original equations to solve for t.

$-r + t = 5$	Equation 1
$-(1) + t = 5$	$r = 1$
$t = 6$	Add 1 to each side.

The solution is (1, 6).

18. $\begin{aligned} 3r - 2t &= 1 \\ 2r - 3t &= 9 \end{aligned}$

SOLUTION:

Multiply Equation 1 by 2 and Equation 2 by –3.

$$\begin{aligned} 3r - 2t &= 1 && \text{Original Equation 1} \\ 2(3r - 2t) &= 2(1) && \text{Multiply each side by 2.} \\ 6r - 4t &= 2 && \text{Multiply.} \end{aligned}$$

$$\begin{aligned} 2r - 3t &= 9 && \text{Original Equation 2} \\ -3(2r - 3t) &= -3(9) && \text{Multiply each side by } -3. \\ -6r + 9t &= -27 && \text{Multiply.} \end{aligned}$$

Add to eliminate r. Then solve for t.

$$\begin{aligned} 6r - 4t &= 2 && \text{Equation 1} \\ -6r + 9t &= -27 && \text{Equation 2} \\ 5t &= -25 && \text{Equation after addition.} \\ t &= -5 && \text{Divide each side by 5.} \end{aligned}$$

Substitute the value of t into one of the original equations to solve for r.

$$\begin{aligned} 3r - 2t &= 1 && \text{Equation 1} \\ 3r - 2(-5) &= 1 && t = -5 \\ 3r + 10 &= 1 && \text{Multiply.} \\ 3r &= -9 && \text{Subtract 10 from each side of the equation.} \\ r &= -3 && \text{Divide each side by 3.} \end{aligned}$$

The solution of the system is (–3, –5).

19. $\begin{aligned} 5g + 4k &= 10 \\ -3g - 5k &= 7 \end{aligned}$

SOLUTION:

Multiply Equation 1 by 3 and Equation 2 by 5.

$$\begin{aligned} 5g + 4k &= 10 && \text{Original Equation 1} \\ 3(5g + 4k) &= 3(10) && \text{Multiply each side by 3.} \\ 15g + 12k &= 30 && \text{Multiply.} \end{aligned}$$

$$\begin{aligned} -3g - 5k &= 7 && \text{Original Equation 2} \\ 5(-3g - 5k) &= 5(7) && \text{Multiply each side by 5.} \\ -15g - 25k &= 35 && \text{Multiply.} \end{aligned}$$

Add to eliminate g. Then solve for k.

$$\begin{aligned} 15g + 12k &= 30 && \text{Equation 1} \\ -15g - 25k &= 35 && \text{Equation 2} \\ -13k &= 65 && \text{Equation after addition.} \\ k &= -5 && \text{Divide each side by } -13. \end{aligned}$$

Substitute the value of k into one of the original equations to solve for g.

$$\begin{aligned} 5g + 4k &= 10 && \text{Equation 1} \\ 5g + 4(-5) &= 10 && k = -5 \\ 5g - 20 &= 10 && \text{Multiply.} \\ 5g &= 30 && \text{Add 20 to each side of the equation.} \\ g &= 6 && \text{Divide each side by 5.} \end{aligned}$$

The solution of the system is (6, –5).

20. $\begin{aligned} 4m - 2p &= 0 \\ -3m + 9p &= 5 \end{aligned}$

SOLUTION:

Multiply Equation 1 by 3 and Equation 2 by 4.

$4m - 2p = 0$	Original Equation 1
$3(4m - 2p) = 3(0)$	Multiply each side by 3.
$12m - 6p = 0$	Multiply.

$-3m + 9p = 5$	Original Equation 2
$4(-3m + 9p) = 4(5)$	Multiply each side by 4.
$-12m + 36p = 20$	Multiply.

Add to eliminate m. Then solve for p.

$12m - 6p = 0$	Equation 1
$-12m + 36p = 20$	Equation 2
$30p = 20$	Equation after addition.
$p = \frac{2}{3}$	Divide each side by 30.

Substitute the value of p into one of the original equations to solve for m.

$4m - 2p = 0$	Equation 1
$4m - 2\left(\frac{2}{3}\right) = 0$	$p = \frac{2}{3}$
$4m - \frac{4}{3} = 0$	Multiply.
$4m = \frac{4}{3}$	Add $\frac{4}{3}$ to each side of the equation.
$m = \frac{1}{3}$	Divide each side by 4.

The solution of the system is $\left(\frac{1}{3}, \frac{2}{3}\right)$.

21. The sum of two numbers is 12. The difference of the same two numbers is –4. Find the two numbers.

SOLUTION:

Write two equations in two variables. Let a be the first number and b be the second number.
Sum: $a + b = 12$
Difference: $a - b = -4$

Because each equation has coefficients of 1, solve for a in either equation.

$a + b = 12$	Sum Equation
$a = -b + 12$	Subtract b from each side.

Substitute the expression. Substitute for a. Then solve for b.

$a - b = -4$	Difference equation.
$(-b + 12) - b = -4$	$a = -b + 12$
$-b + 12 - b = -4$	Distributive Property
$-2b + 12 = -4$	Simplify.
$-2b = -16$	Subtract 12 from each side.
$b = 8$	Divide each side by -2.

Substitute the value of b into one of the original equations to solve for a.

$a + b = 12$	Sum Equation.
$a + 8 = 12$	Substitution.
$a = 4$	Subtract 8 from each side.

The numbers are 4 and 8.

22. Twice a number minus a second number is –1. Twice the second number added to three times the first number is 9. Find the two numbers.

SOLUTION:

Write two equations in two variables. Let a be the first number and b be the second number.
Equation 1: $2a - b = -1$
Equation 2: $3a + 2b = 9$

Because Equation 1 has a coefficients of 1, solve for b in that equation.

$2a - b = -1$	Equation 1
$-b = -2a - 1$	Subtract $2a$ from each side.
$b = 2a + 1$	Divide each side by -1.

Substitute the expression. Substitute for b. Then solve for a.

$3a + 2b = 9$	Equation 2.
$3a + 2(2a + 1) = 9$	$b = 2a + 1$
$3a + 4a + 2 = 9$	Distributive Property
$7a + 2 = 9$	Simplify.
$7a = 7$	Subtract 2 from each side.
$a = 1$	Divide each side by 7.

Substitute the value of a into one of the original equations to solve for b.

$2a - b = -1$	Equation 1.
$2(1) - b = -1$	Substitution.
$2 - b = -1$	Multiply.
$-b = -3$	Subtract 2 from each side.
$b = 3$	Divide each side by -1.

The numbers are 1 and 3.

23. **REASONING** Mr. Janson paid for admission to the high school football game for his family. He purchased 3 adult tickets and 2 student tickets for a total of $22. Ms. Pham purchased 5 adult tickets and 3 student tickets for a total of $35.75. What is the cost of each adult ticket and each student ticket?

SOLUTION:

Write two equations in two variables. Let a be the cost of an adult ticket and s be the cost of a student ticket.
Mr. Janson: $3a + 2s = 22$
Ms. Pham: $5a + 3s = 35.75$

Multiply Mr. Janson's Equation by 5 and Ms. Pham's Equation by –3.

$3a + 2s = 22$	Mr. Janson's Equation
$5(3a + 2s) = 5(22)$	Multiply each side by 5.
$15a + 10s = 110$	Multiply.

$5a + 3s = 35.75$	Ms. Pham's Equation
$-3(5a + 3s) = -3(35.75)$	Multiply each side by -3.
$-15a - 9s = 107.25$	Multiply.

Add to eliminate a. Then solve for s.

$15a + 10s = 110$	Mr. Janson's Equation
$-15a - 9s = 107.25$	Ms. Pham's Equation
$s = 2.75$	Equation after addition.

Substitute the value of s into one of the original equations to solve for a.

$3a + 2s = 22$	Mr. Janson's Equation
$3a + 2(2.75) = 22$	$s = 2.75$
$3a + 5.50 = 22$	Multiply.
$3a = 16.50$	Subtract5.50 from each side of the equation.
$a = 5.50$	Divide each side by 3.

So, an adult ticket costs $5.50 and a student ticket costs $2.75.

24. **USE A MODEL** The Newton City Park has 11 basketball courts, which are all in use. There are 54

people playing basketball. Some are playing one-on-one, and some are playing in teams. A one-on-one game requires 2 players, and a team game requires 10 players.

a. Write a system of equations that represents the number of one-on-one and team games being played.

b. Solve the system of equations and interpret your results.

SOLUTION:

a. Write two equations in two variables. Let x be one-on-one games and y be team games.
Equation 1: $x + y = 11$
Equation 2: $2x + 10y = 54$

b. Because Equation 1 has coefficients of 1, solve for x in that equation.

$x + y = 11$	Equation 1
$x = -y + 11$	Subtract y from each side.

Substitute the expression. Substitute for x. Then solve for y.

$2x + 10y = 54$	Equation 2.
$2(-y + 11) + 10y = 54$	$x = -y + 13$
$-2y + 22 + 10y = 54$	Distributive Property
$8y + 22 = 54$	Simplify.
$8y = 32$	Subtract 22 from each side.
$y = 4$	Divide each side by 8.

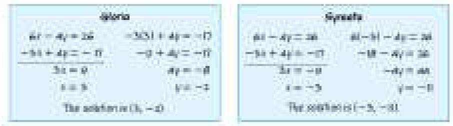
Substitute the value of y into one of the original equations to solve for x.

27. **WRITE** Why is substitution sometimes more helpful than elimination?

SOLUTION:

Sample answer: It is more helpful to use substitution when one of the variables has a coefficient of 1 or if a coefficient can easily be reduced to 1.

25. **FIND THE ERROR** Gloria and Syreeta are solving the system $6x - 4y = 26$ and $-3x + 4y = -17$. Is either of them correct? Explain your reasoning.



SOLUTION:

Gloria is correct.; Sample answer: Syreeta subtracted 26 from 17 instead of 17 from 26 and got $3x = -9$ instead of $3x = 9$. She proceeded to get a value of -11 for y. She would have found her error if she had substituted the solution into the original equations.

26. **CREATE** Write a system of equations in which one equation should be multiplied by 3 and the other should be multiplied by 4 in order to solve the system with elimination. Then solve your system.

SOLUTION:

Multiply Equation 1 by 3 and Equation 2 by 4.

$4x + 5y = 21$	Original Equation 1
$3(4x + 5y) = 3(21)$	Multiply each side by 3.
$12x + 15y = 63$	Multiply.

$3x - 2y = 10$	Original Equation 2
$4(3x - 2y) = 4(10)$	Multiply each side by 4.
$12x - 8y = 40$	Multiply.

Subtract to eliminate x. Then solve for y.

$12x + 15y = 63$	Equation 1
$12x - 8y = 40$	Equation 2
$23y = 23$	Equation after subtraction.
$y = 1$	Divide each side by 23.

Substitute the value of y into one of the original equations to solve for x.

$4x + 5y = 21$	Equation 1
$4x + 5(1) = 21$	$y = 1$
$4x + 5 = 21$	Multiply.
$4x = 16$	Subtract 5 from each side of the equation.
$x = 4$	Divide each side by 4.

The solution of the system is (4, 1).

Solve each system of inequalities.

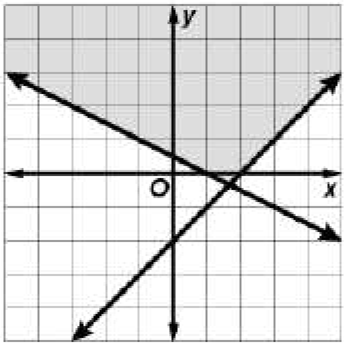
1. $x - y \le 2$
$x + 2y \ge 1$

SOLUTION:

Use a solid line to graph the first boundary $x - y \le 2$. This is the line drawn with y-intercept of −2, and the upper half of the plane would be shaded.

Use a solid line to graph the second boundary $x + 2y \ge 1$. This is the line drawn with y-intercept of $\frac{1}{2}$, and the upper half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is bounded.



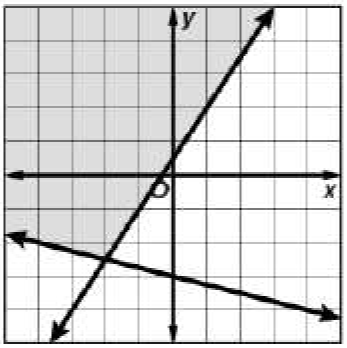
2. $3x - 2y \le -1$
$x + 4y \ge -12$

SOLUTION:

Use a solid line to graph the first boundary $3x - 2y \le -1$. This is the line drawn with y-intercept of $\frac{1}{2}$, and the upper half of the plane would be shaded.

Use a solid line to graph the second boundary $x + 4y \ge -12$. This is the line drawn with y-intercept of −3, and the upper half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is bounded.



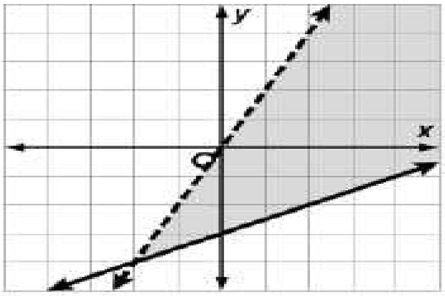
3. $y \ge \frac{x}{2} - 3$
$y < 2x$

SOLUTION:

Use a solid line to graph the first boundary $y \ge \frac{x}{2} - 3$. This is the line drawn with y-intercept of −3, and the upper half of the plane would be shaded.

Use a dashed line to graph the second boundary $y < 2x$. This is the line drawn with y-intercept of 0, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



$y \ge \frac{x}{2} - 3$	Original inequality	$y < 2x$
$0 \overset{?}{\ge} \frac{1}{2} - 3$	$x = 1$ and $y = 0$	$0 \overset{?}{<} 2(1)$
$0 \ge -2.5$	True	$0 < 2$

4. $y < \frac{x}{3} + 2$

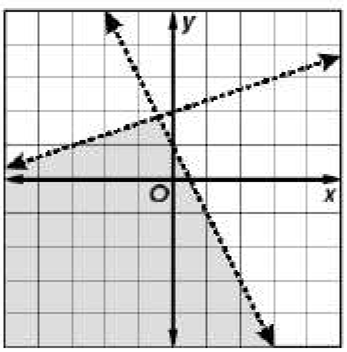
$y < 2x + 1$

SOLUTION:

Use a dashed line to graph the first boundary $y < \frac{x}{3} + 2$. This is the line drawn with y-intercept of 2, and the lower half of the plane would be shaded.

Use a dashed line to graph the second boundary $y < 2x + 1$. This is the line drawn with y-intercept of 1, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



5. $x + y \geq 4$

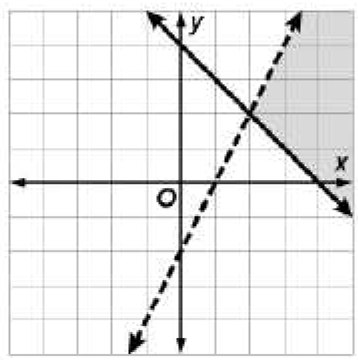
$2x - y > 2$

SOLUTION:

Use a solid line to graph the first boundary $x + y \geq 4$. This is the line drawn with y-intercept of 4, and the upper half of the plane would be shaded.

Use a dashed line to graph the second boundary $2x - y > 2$. This is the line drawn with y-intercept of -2, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



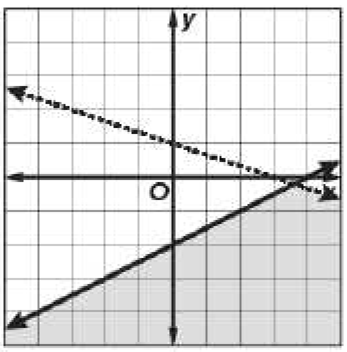
6. $x + 3y < 3$
$x - 2y \geq 4$

SOLUTION:

Use a dashed line to graph the first boundary $x + 3y < 3$. This is the line drawn with y-intercept of 1, and the lower half of the plane would be shaded.

Use a solid line to graph the second boundary $x - 2y \geq 4$. This is the line drawn with y-intercept of –2, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



CHECK
Test the solution by substituting the coordinates of a point in the unbounded region, such as (0, –3), into the system of inequalities. If the point is viable for both inequalities, it is a solution of the system.

$x + 3y < 3$	Original inequality	$x - 2y \geq 4$
$0 + 3(-3) \overset{?}{<} 3$	$x = 0$ and $y = -3$	$x - 2(-3) \overset{?}{\geq} 4$
$-9 < 3$	True	$6 \geq 4$

Solve each system of inequalities.

7. $y \geq -3x + 7$
$y > \frac{1}{2}x$
$y < 2$

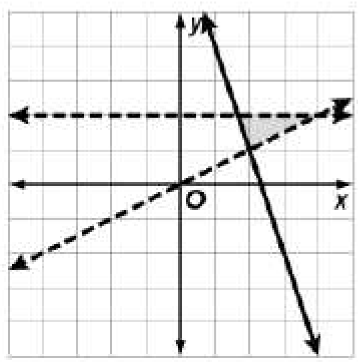
SOLUTION:

Use a solid line to graph the first boundary $y \geq -3x + 7$. This is the line drawn with y-intercept of 7, and the upper half of the plane would be shaded.

Use a dashed line to graph the second boundary $y > \frac{1}{2}x$. This is the line drawn with y-intercept of 0, and the upper half of the plane would be shaded.

Use a dashed line to graph the third boundary $y < 2$. This is the line drawn with y-intercept of 2, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



8. $x > -3$

$y < -\frac{1}{3}x + 3$

$y > x - 1$

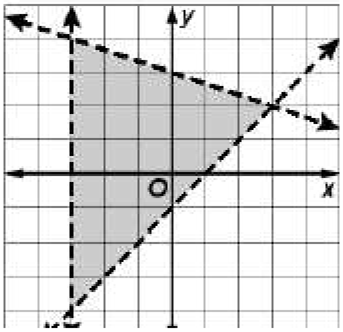
SOLUTION:

Use a dashed line to graph the first boundary $x > -3$. This is the line drawn with no y-intercept, and the upper half of the plane would be shaded.

Use a dashed line to graph the second boundary $y < -\frac{1}{3}x + 3$. This is the line drawn with y-intercept of 3, and the lower half of the plane would be shaded.

Use a dashed line to graph the third boundary $y > x - 1$. This is the line drawn with y-intercept of -1, and the upper half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



9. $y < -\frac{1}{2}x + 3$

$y > \frac{1}{2}x + 1$

$y < 3x + 10$

SOLUTION:

Use a dashed line to graph the first boundary $y < -\frac{1}{2}x + 3$. This is the line drawn with y-intercept of 3, and the lower half of the plane would be shaded.

Use a dashed line to graph the second boundary $y > \frac{1}{2}x + 1$. This is the line drawn with y-intercept of 1, and the upper half of the plane would be shaded.

Use a dashed line to graph the third boundary $y < 3x + 10$. This is the line drawn with y-intercept of 10, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.

10. $y \leq 0$
$x \leq 0$
$y \geq -x - 1$

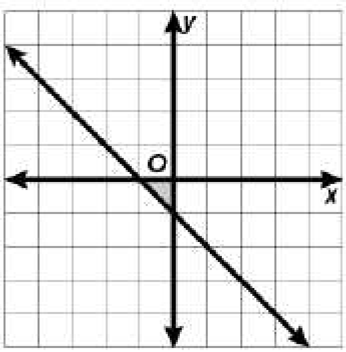
SOLUTION:

Use a solid line to graph the first boundary $y \leq 0$. This is the line drawn with y-intercept of 0, and the lower half of the plane would be shaded.

Use a solid line to graph the second boundary $x \leq 0$. This is the line drawn with y-intercept of 0, and the lower half of the plane would be shaded.

Use a solid line to graph the third boundary $y \geq -x - 1$. This is the line drawn with y-intercept of −1, and the upper half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



11. $y \leq 3 - x$
$y \geq 3$
$x \geq -5$

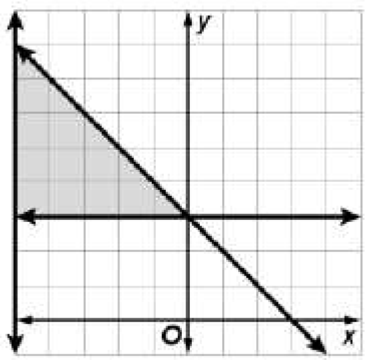
SOLUTION:

Use a solid line to graph the first boundary $y \leq 3 - x$. This is the line drawn with y-intercept of 3, and the lower half of the plane would be shaded.

Use a solid line to graph the second boundary $y \geq 3$. This is the line drawn with y-intercept of 3, and the upper half of the plane would be shaded.

Use a solid line to graph the third boundary $x \geq -5$. This is the line drawn with no y-intercept, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



12. $x \geq -2$
$y \geq x - 2$
$x + y \leq 2$

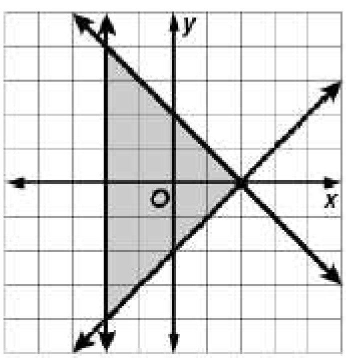
SOLUTION:

Use a solid line to graph the first boundary $x \geq -2$. This is the line drawn with no y-intercept, and the upper half of the plane would be shaded.

Use a solid line to graph the second boundary $y \geq x - 2$. This is the line drawn with y-intercept of -2, and the upper half of the plane would be shaded.

Use a solid line to graph the third boundary $x + y \leq 2$. This is the line drawn with y-intercept of 2, and the lower half of the plane would be shaded.

The solution of the system is the set of ordered pairs in the intersection which is the overlapping area shaded in the graph below. The feasible region is unbounded.



Determine whether the events are *mutually exclusive* or *not mutually exclusive*. Explain your reasoning.

1. A die is rolled while a game is being played. The result of the next roll is a 6 or an even number.

SOLUTION:

6 is an outcome that both events have in common. 6 is also an even number.

These are not mutually exclusive events.

2. **SALES** A street vendor is selling T-shirts outside of a concert arena. The colors and sizes of the available T-shirts are shown in the table. The vendor selects a T-shirt that is blue or large.

	Red	Blue	White
Small	1	2	2
Medium	3	2	4
Large	4	5	6
Extra Large	7	6	3

SOLUTION:

There are T-shirts that are both blue and large.

These are not mutually exclusive events.

3. **AWARDS** The student of the month gets to choose one award from 9 gift certificates to area restaurants, 8 T-shirts, 6 water bottles, or 5 gift cards to the mall. What is the probability that the student of the month chooses a T-shirt or a water bottle?

SOLUTION:

These are mutually exclusive events, because the award chosen cannot be a T-shirt and a water bottle.

Let event A represent choosing a T-shirt. Let event B represent choosing a water bottle. There are a total of $9 + 8 + 6 + 5 = 28$ awards.

Because the events are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) && \text{Probability of mutually exclusive events} \\ &= \frac{8}{28} + \frac{6}{28} && P(A) = \frac{8}{28} \text{ and } P(B) = \frac{6}{28} \\ &= \frac{14}{28} \text{ or } \frac{1}{2} && \text{Add.} \end{aligned}$$

So, the probability that the student of the month chooses a T-shirt or a water bottle is $\frac{1}{2}$ or 0.5.

18. **SPORTS CARDS** Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that he randomly selects a baseball or a football card?

SOLUTION:

These are mutually exclusive events, because the Dario cannot select a baseball card and a football card.

There are a total of $145 + 102 + 48 = 295$ cards.

$$\begin{aligned} P(\text{baseball card or football card}) &= P(\text{baseball card}) + P(\text{football card}) \\ &= \frac{145}{295} + \frac{102}{295} \\ &= \frac{247}{295} \end{aligned}$$

So, the probability that he randomly selects a baseball or a football card is $\frac{247}{295}$ or about 84%.

4. **SALES PROMOTIONS** At a grand opening event, a store allows customers to choose an envelope from a bag. Ten of the envelopes contain store coupons, 8 envelopes contain gift cards, and 2 envelopes contain $100. What is the probability that a customer selects an envelope with a gift card or an envelope with $100?

SOLUTION:

These are mutually exclusive events, because the envelope chosen cannot contain a gift card and $100.

Let event A represent choosing an envelope that contains a gift card. Let event B represent choosing an envelope that contains $100. There are a total of $10 + 8 + 2 = 20$ envelopes.

Because the events are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) && \text{Probability of mutually exclusive events} \\ &= \frac{8}{20} + \frac{2}{20} && P(A) = \frac{8}{20} \text{ and } P(B) = \frac{2}{20} \\ &= \frac{10}{20} \text{ or } \frac{1}{2} && \text{Add.} \end{aligned}$$

So, the probability that a customer selects an envelope with a gift card or an envelope with $100 is $\frac{1}{2}$ or 0.5.

5. **TRAFFIC** If the chance of making a green light at a certain intersection is 35%, what is the probability of arriving when the light is yellow or red?

SOLUTION:

These are mutually exclusive events, because the light cannot be green and yellow or red.

Let event A represent a green light. Let event B represent a yellow or red light.

Because the events are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) && \text{Probability of mutually exclusive events} \\ 1 &= 0.35 + P(B) && P(A \text{ or } B) = 1 \text{ and } P(A) = 0.35 \\ 0.65 &= P(B) && \text{Subtract.} \end{aligned}$$

So, the probability of arriving when the light is yellow or red is 0.65.

6. **STUDENTS** In a group of graduate students, 4 out of the 5 females are international students, and 2 out of the 3 men are international students. What is the probability of selecting a graduate student from this group that is a male or an international student?

SOLUTION:

Because some of the students are both male and international students, these events are not mutually exclusive. Use the rule for two events that are not mutually exclusive. There are a total of 5 female + 3 male = 8 students. There are a total of 4 female + 2 male = 6 international students.

$$\begin{aligned} P(\text{male or international}) &= P(\text{male}) + P(\text{international}) - P(\text{male and international}) \\ &= \frac{3}{8} + \frac{6}{8} - \frac{2}{8} \\ &= \frac{7}{8} \end{aligned}$$

So, the probability of selecting a graduate student from this group that is a male or an international student is $\frac{7}{8}$ or about 0.88.

CARDS Suppose you pull a card from a standard 52-card deck. Find the probability of each event.

7. The card is a 4.

SOLUTION:

There are 4 cards that are a 4.

So, the probability that the card is a 4 is $\frac{4}{52} = \frac{1}{13}$ or about 7.7%.

8. The card is red.

SOLUTION:

There are 26 cards that are red.

So, the probability that the card is red is $\frac{26}{52} = \frac{1}{2}$ or 50%.

9. The card is a face card.

SOLUTION:

A face card is a jack, queen, or king. There are 12 face cards.

So, the probability that the card is face card is $\frac{12}{52} = \frac{3}{13}$ or about 23.1%.

10. The card is not a face card.

SOLUTION:

A face card is a jack, queen, or king. There are 12 face cards. So, there are 52 – 12 = 40 cards that are not face cards.

So, the probability that the card is not face card is $\frac{40}{52} = \frac{10}{13}$ or about 76.9%.

11. P(queen or heart)

SOLUTION:

These are not mutually exclusive events because a card can be a queen and also a heart.

There are 4 queens. There are 13 hearts. There is 1 queen that is also a heart.

$$\begin{aligned} P(\text{queen or heart}) &= P(\text{queen}) + P(\text{heart}) - P(\text{queen and heart}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \end{aligned}$$

So, the probability that a card is a queen or a heart is $\frac{4}{13}$ or about 31%.

12. P(jack or spade)

SOLUTION:

These are not mutually exclusive events because a card can be a jack and also a spade.

There are 4 jacks. There are 13 spades. There is 1 jack that is also a spade.

$$\begin{aligned} P(\text{jack or spade}) &= P(\text{jack}) + P(\text{spade}) - P(\text{jack and spade}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \end{aligned}$$

So, the probability that a card is a jack or a spade is $\frac{4}{13}$ or about 31%.

13. P(five or prime number)

SOLUTION:

These are not mutually exclusive events because a card can be a five and also a prime number.

There are 4 fives. The numbers 2, 3, 5, and 7 are prime numbers. There are 4 sets of 4 prime numbers, so there are 16 prime numbers. The 4 fives are also prime numbers.

$$\begin{aligned} P(\text{five or prime number}) &= P(\text{five}) + P(\text{prime number}) - P(\text{five and prime number}) \\ &= \frac{4}{52} + \frac{16}{52} - \frac{4}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \end{aligned}$$

So, the probability that a card is a five or a prime number is $\frac{4}{13}$ or about 31%.

14. P(ace or black)

SOLUTION:

These are not mutually exclusive events because a card can be an ace and also black.

There are 4 aces. There are 26 black cards. There are 2 aces that are also black.

$$\begin{aligned} P(\text{ace or black}) &= P(\text{ace}) + P(\text{black}) - P(\text{ace and black}) \\ &= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} \\ &= \frac{28}{52} \text{ or } \frac{7}{13} \end{aligned}$$

So, the probability that a card is an ace or black is $\frac{7}{13}$ or about 54%.

15. A drawing will take place where one ticket is to be drawn from a set of 80 tickets numbered 1 to 80. If a ticket is drawn at random, what is the probability that the number drawn is a multiple of 4 or a factor of 12?

SOLUTION:

The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, and 80. So, there are 20 multiples of 4.

The factors of 12 are 1, 2, 3, 4, 6, and 12. So, there are 6 factors of 12.

The multiples of 4 that are also factors of 12 are 4 and 12. So, there are 2 multiples of 4 that are also factors of 12.

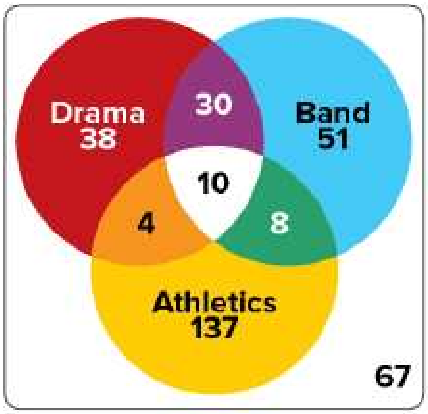
There are 80 total numbers.

These are not mutually exclusive events because a number can be a multiple of 4 and also a factor of 12.

$$\begin{aligned} P(\text{multiple of 4 or factor of 12}) &= P(\text{multiple of 4}) + P(\text{factor of 12}) - P(\text{multiple of 4 and factor of 12}) \\ &= \frac{20}{80} + \frac{6}{80} - \frac{2}{80} \\ &= \frac{24}{80} \text{ or } \frac{3}{10} \end{aligned}$$

So, the probability that a number is a multiple of 4 or factor of 12 is 0.30.

16. **SCHOOL** The Venn diagram shows the extracurricular activities enjoyed by the senior class at Valley View High School.



a. How many students are in the senior class?

b. How many students participate in athletics?

c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?

d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?

SOLUTION:

a. There are 38 + 30 + 51 + 4 + 10 + 8 + 137 + 67 = 345 students in the senior class.

b. There are 4 + 10 + 8 + 137 = 159 students that participate in athletics.

c. There are 38 + 30 + 10 + 4 = 82 students that participate in drama.

participate in drama.

There are 4 + 10 = 14 students that participate in athletics and also participate in drama.

These are not mutually exclusive events because a student can participate in athletics and also participate in drama.

$$\begin{aligned} P(\text{athletics or drama}) &= P(\text{athletics}) + P(\text{drama}) - P(\text{athletics and drama}) \\ &= \frac{159}{345} + \frac{82}{345} - \frac{14}{345} \\ &= \frac{227}{345} \end{aligned}$$

So, the probability that a student participates is athletics or drama is $\frac{227}{345}$ or about 66%.

d. There are 30 students that participate in only drama and band.

$$\begin{aligned} P(\text{only drama or band}) &= \frac{30}{345} \\ &= \frac{2}{23} \end{aligned}$$

So, the probability that a student participates is only drama or band is $\frac{2}{23}$ or about 9%.

17. **BOWLING** Cindy's bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that she will bowl either a spare or a strike for any given frame?

SOLUTION:

These are mutually exclusive events, because the Cindy cannot bowl a spare and a strike.

$$\begin{aligned} P(\text{spare or strike}) &= P(\text{spare}) + P(\text{strike}) \\ &= 45\% + 30\% \\ &= 75\% \end{aligned}$$

So, the probability that Cindy will bowl a spare or a

So, the probability that Cindy will bowl a spare or a strike is 75% or $\frac{3}{4}$.

Solve each equation. Check your solution.

1. $6x - 5 = 7 - 9x$

SOLUTION:

$6x - 5 = 7 - 9x$	Original equation
$15x - 5 = 7$	Add 9x to each side and simplify.
$15x = 12$	Add 5 to each side and simplify.
$x = \frac{12}{15}$ or $\frac{4}{5}$	Divide each side by 15 and simplify.

2. $-1.6r + 5 = -7.8$

SOLUTION:

$-1.6r + 5 = -7.8$	Original equation
$-1.6r = -12.8$	Subtract 5 from each side and simplify.
$r = 8$	Divide each side by -1.6 and simplify.

3. $\frac{3}{4} - \frac{1}{2}n = \frac{5}{8}$

SOLUTION:

$\frac{3}{4} - \frac{1}{2}n = \frac{5}{8}$	Original equation
$-\frac{1}{2}n = -\frac{1}{8}$	Subtract $\frac{3}{4}$ from each side and simplify.
$n = \frac{1}{4}$	Multiply each side by -2 and simplify.

4. $\frac{5}{6}c + \frac{3}{4} = \frac{11}{12}$

SOLUTION:

$\frac{5}{6}c + \frac{3}{4} = \frac{11}{12}$	Original equation
$\frac{5}{6}c = \frac{1}{6}$	Subtract $\frac{3}{4}$ from each side and simplify.
$c = \frac{1}{5}$	Multiply each side by $\frac{6}{5}$ and simplify.

5. $2.2n + 0.8n + 5 = 4n$

SOLUTION:

$2.2n + 0.8n + 5 = 4n$	Original equation
$3n + 5 = 4n$	Combine like terms.
$5 = n$	Subtract $3n$ from each side and simplify.

6. $6y - 5 = -3(2y + 1)$

SOLUTION:

$6y - 5 = -3(2y + 1)$	Original equation
$6y - 5 = -6y - 3$	Distributive Property
$12y - 5 = -3$	Add $6y$ to each side and simplify.
$12y = 2$	Add 5 to each side and simplify.
$y = \frac{1}{6}$	Divide each side by 12 and simplify.

7. $-6(2x + 4) + \frac{1}{2}(8 + 3x) = -20$

SOLUTION:

$-6(2x+4) + \frac{1}{2}(8+3x) = -20$	Original equation
$-12x - 24 + 4 + \frac{3}{2}x = -20$	Distributive Property
$-\frac{21}{2}x - 20 = -20$	Combine like terms.
$-\frac{21}{2}x = 0$	Add 20 to each side and simplify.
$x = 0$	Multiply each side by $-\frac{2}{21}$ and simplify.

8. $7(-1 + 4x) - 12x = 5$

SOLUTION:

$7(-1+4x) - 12x = 5$	Original equation
$-7 + 28x - 12x = 5$	Distributive Property
$-7 + 16x = 5$	Combine like terms.
$16x = 12$	Add 7 to each side and simplify.
$x = \frac{12}{16}$ or $\frac{3}{4}$	Divide each side by 16 and simplify.

9. $-4(10 + 3x) - (x + 8) = -9$

SOLUTION:

$-4(10+3x)-(x+8)=-9$	Original equation
$-40-12x-x-8=-9$	Distributive Property
$-13x-48=-9$	Combine like terms.
$-13x=39$	Add 48 to each side and simplify.
$x=-3$	Divide each side by -13 and simplify.

Solve each problem.

10. **REASONING** The length of a rectangle is twice the width. Find the width if the perimeter is 60 centimeters. Define a variable, write an equation, and solve the problem.

SOLUTION:

Write an equation that represents the situation.

Words	The length of a rectangle is twice the width
Variables	Let w = width
	Let $l = 2w$
Equation	$P = 2l + 2w$

$P=2l+2w$	Formula for perimeter of a rectangle.
$60=2(2w)+2w$	Substitute $P = 60$ and $l = 2w$.
$60=4w+2w$	Multiply.
$60=6w$	Combine like terms.
$10=w$	Divide each side by 6.

w = width; $2(2w) + 2w = 60$; The width is 10 cm.

11. **GOLF** Sergio and three friends went golfing. Two of the friends rented clubs for $6 each. The total cost of the rented clubs and the green fees was $76. What was the cost of the green fees for each person? Define a variable, write an equation, and solve the problem.

SOLUTION:

Words	Two of the friends rented clubs for $6 each. The total cost of the rented clubs and the green fees was $76.
Variables	Let g = green fees per person
Equation	$6(2) + 4g = 76$

$6(2)+4g=76$	Original equation.
$12+4g=76$	Multiply.
$4g=64$	Subtract 12 from each side.
$g=16$	Divide each side by 4.

g = green fees per person; $6(2) + 4g = 76$; The green fees are $16.

Solve each equation or formula for the specified variable.

12. **BANKING** The formula for simple interest I is $I = Prt$, where P is the principal, r is the interest rate, and t is time. Solve for P.

SOLUTION:

$I=Prt$	Original equation.
$\frac{I}{rt}=P$	Divide each side by rt.

13. **MEAN** The mean A of two numbers, x and y, is given by $A = \frac{x+y}{2}$. Solve for y.

SOLUTION:

$A=\frac{x+y}{2}$	Original equation.
$2A=x+y$	Multiply each side by 2 and simplify.
$2A-x=y$	Subtract x from each side and simplify.

14. **SLOPE** The slope m between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Solve for y_2.

SOLUTION:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Original equation.

$$m(x_2 - x_1) = y_2 - y_1$$ Multiply each side by $x_2 - x_1$.

$$m(x_2 - x_1) + y_1 = y_2$$ Add y_1 to each side.

15. **CYLINDERS** The surface area of a cylinder A is given by $A = 2\pi r^2 + 2\pi rh$, where r is radius and h is height. Solve for h.

SOLUTION:

$$A = 2\pi r^2 + 2\pi rh$$ Original equation.

$$A - 2\pi r^2 = 2\pi rh$$ Subtract $2\pi r^2$ from each side.

$$\frac{A - 2\pi r^2}{2\pi r} = h$$ Divide each side by $2\pi r$.

16. **PHYSICS** The height h of a falling object is given by $h = vt - gt^2$, where v is the initial velocity of the object, t is time, and g is the gravitational constant. Solve for v.

SOLUTION:

$$h = vt - gt^2$$ Original equation.

$$h + gt^2 = vt$$ Add gt^2 to each side.

$$\frac{h + gt^2}{t} = v$$ Divide each side by t.

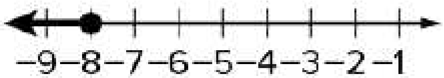
Solve each inequality. Graph the solution set on a number line.

25. $\frac{z}{-4} \geq 2$

SOLUTION:

$\frac{z}{-4} \geq 2$	Original equation
$z \geq -8$	Multiply each side by -4.

The solution set is $\{z \mid z \leq -8\}$.

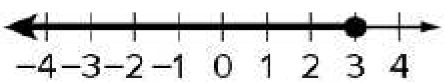


26. $3a + 7 \leq 16$

SOLUTION:

$3a + 7 \leq 16$	Original equation
$3a \leq 9$	Subtract 7 from each side.
$a \leq 3$	Divide each side by 3.

The solution set is $\{a \mid a \leq 3\}$.

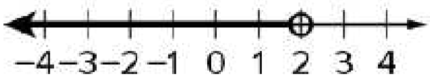


27. $20 - 3n > 7n$

SOLUTION:

$20 - 3n > 7n$	Original equation
$20 > 10n$	Add $3n$ to each side.
$2 > n$	Divide each side by 10.

The solution set is $\{n \mid n < 2\}$.

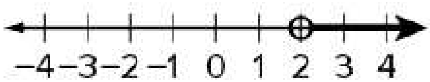


28. $7f - 9 > 3f - 1$

SOLUTION:

$7f - 9 > 3f - 1$	Original equation
$4f - 9 > -1$	Subtract $3f$ from each side.
$4f > 8$	Add 9 to each side.
$f > 2$	Divide each side by 4.

The solution set is $\{f \mid f > 2\}$.



29. $0.7m + 0.3m \geq 2m - 4$

SOLUTION:

$0.7m + 0.3m \geq 2m - 4$	Original equation
$m \geq 2m - 4$	Combine like terms.
$-m \geq -4$	Subtract $2m$ from each side.
$m \leq 4$	Divide each side by -1.

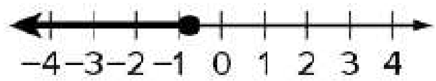
The solution set is $\{m \mid m \leq 4\}$.

30. $4(5x + 7) \leq 13$

SOLUTION:

$4(5x + 7) \leq 13$	Original equation
$20x + 28 \leq 13$	Distributive Property.
$20x \leq -15$	Subtract 28 from each side.
$x \leq -\frac{15}{20}$ or $-\frac{3}{4}$	Divide each side by 20.

The solution set is $\left\{x \mid x \leq -\frac{3}{4}\right\}$.



Solve each problem.

31. **INCOME** Manuel takes a job translating English instruction manuals to Spanish. He will receive \$15 per page plus \$100 per month. Manuel plans to work for 3 months during the summer and wants to make at least \$1500. Write and solve an inequality to find the minimum number of pages Manuel must translate in order to reach his goal. Then, interpret the solution in the context of the situation.

SOLUTION:

Write an inequality that represents the situation.
Let P = the number of pages translated
$15P + 300 \geq 1500$

$15P + 300 \geq 1500$	Original equation.
$15P \geq 1200$	Subtract 300 from each side.
$P \geq 80$	Divide each side by 15.

Manuel must translate at least 80 pages.

32. **STRUCTURE** On a conveyor belt, there can only be two boxes moving at a time. The total weight of the boxes cannot be more than 300 pounds. Let x and y represent the weights of two boxes on the conveyor belt.

a. Write an inequality that describes the weight limitation in terms of x and y.

b. Write an inequality that describes the limit on the average weight a of the two boxes.

c. Two boxes are to be placed on the conveyor belt. The first box weighs 175 pounds. What is the maximum weight of the second box?

SOLUTION:

a.The words "total weight" implies a sum. The sum cannot exceed 300 pound which means it can be less than or equal to 300 pounds

$x + y \leq 300$

b. If the sum cannot be more than 300 pounds, divide by the number of boxes, 2, to determine the average weight. $a \leq 150$.

c. Let the first box equal x, substitute and solve for the second box, y.

$x + y \leq 300$	Original equation
$175 + y \leq 300$	Substitution.
$y \leq 125$	Subtract 175 from each side.

Solve each equation. Check your solutions.

1. $|8 + p| = 2p - 3$

SOLUTION:

Case 1	Case 2
$8 + p = 2p - 3$	$8 + p = -(2p - 3)$
$8 - p = -3$	$8 + p = -2p + 3$
$-p = -11$	$8 + 3p = 3$
$p = 11$	$3p = -5$
	$p = -\frac{5}{3}$

CHECK
Substitute each value in the original equation.

$|8 + 11| \stackrel{?}{=} 2(11) - 3$

$|19| \stackrel{?}{=} 22 - 3$

$19 = 19$ True.

$\left|8 + \left(-\frac{5}{3}\right)\right| \stackrel{?}{=} 2\left(-\frac{5}{3}\right) - 3$

$\left|\frac{19}{3}\right| \stackrel{?}{=} -\frac{10}{3} - 3$

$\frac{19}{3} \neq -\frac{19}{3}$ False.

Only one solution makes the equation true. Thus, the $\{11\}$.

2. $|4w - 1| = 5w + 37$

SOLUTION:

Case 1	Case 2
$4w - 1 = 5w + 37$	$4w - 1 = -(5w + 37)$
$4w - 38 = 5w$	$4w - 1 = -5w - 37$
$-38 = w$	$9w - 1 = -37$
	$9w = -36$
	$w = -4$

CHECK
Substitute each value in the original equation.

$|4(-38) - 1| \stackrel{?}{=} 5(-38) + 37$

$|-153| \stackrel{?}{=} -190 + 37$

$153 \neq -153$ False.

$|4(-4) - 1| \stackrel{?}{=} 5(-4) + 37$

$|-17| \stackrel{?}{=} -20 + 37$

$17 = 17$ True.

Only one solution makes the equation true. Thus, the

3. $4|2y - 7| + 13 = 9$

SOLUTION:

$4|2y - 7| + 13 = 9$ Original equation

$4|2y - 7| = -4$ Subtract 13 from each side.

$|2y - 7| = -1$ Divide each side by 4.

Because the absolute value of a number is always positive or zero, this sentence is never true. The solution is ø.

4. $-2|7 - 3y| - 6 = -14$

SOLUTION:

$-2|7 - 3y| - 6 = -14$ Original equation

$-2|7 - 3y| = -8$ Add 6 to each side.

$|7 - 3y| = 4$ Divide each side by -2.

Case 1	Case 2
$7 - 3y = 4$	$7 - 3y = -4$
$-3y = -3$	$-3y = -11$
$y = 1$	$y = 3\frac{2}{3}$

CHECK
Substitute each value in the original equation.

$-2|7 - 3(1)| - 6 \stackrel{?}{=} -14$

$-2|4| - 6 \stackrel{?}{=} -14$

$-2(4) - 6 \stackrel{?}{=} -14$

$-14 = -14$ True.

$-2\left|7 - 3\left(3\frac{2}{3}\right)\right| - 6 \stackrel{?}{=} -14$

$-2|-4| - 6 \stackrel{?}{=} -14$

$-2(4) - 6 \stackrel{?}{=} -14$

$-14 = -14$ True.

Both solutions make the equation true. Thus, the soh

5. $2|4 - n| = -3n$

SOLUTION:

$2|4 - n| = -3n$ Original equation

$|4 - n| = -\frac{3}{2}n$ Divide each side by 2

Case 1	Case 2
$4 - n = -\frac{3}{2}n$	$4 - n = \frac{3}{2}n$
$4 = -\frac{1}{2}n$	$4 = \frac{5}{2}n$
$-8 = n$	$\frac{8}{5} = n$

CHECK
Substitute each value in the original equation.

$2|4 - (-8)| \stackrel{?}{=} -3(-8)$

$2|12| \stackrel{?}{=} 24$

$2(12) \stackrel{?}{=} 24$

$24 = 24$ True.

$2\left|4 - \left(\frac{8}{5}\right)\right| \stackrel{?}{=} -3\left(\frac{8}{5}\right)$

$2\left|\frac{12}{5}\right| \stackrel{?}{=} -\frac{24}{5}$

$2\left|\frac{12}{5}\right| \stackrel{?}{=} -\frac{24}{5}$

$\frac{24}{5} \neq -\frac{24}{5}$ False.

Only one solution makes the equation true. Thus, the $\{-8\}$.

6. $5 - 3|2 + 2w| = -7$

SOLUTION:

$5 - 3\|2 + 2w\| = -7$	Original equation
$-3\|2 + 2w\| = -12$	Subtract 5 from each side.
$\|2 + 2w\| = 4$	Divide each side by -3.

Case 1

$$2 + 2w = 4$$
$$2w = 2$$
$$w = 1$$

Case 2

$$2 + 2w = -4$$
$$2w = -6$$
$$w = -3$$

CHECK
Substitute each value in the original equation.

$$5 - 3|2 + 2(1)| \stackrel{?}{=} -7$$
$$5 - 3|4| \stackrel{?}{=} -7$$
$$5 - 3(4) \stackrel{?}{=} -7$$
$$-7 = -7 \quad \text{True.}$$

$$5 - 3|2 + 2(-3)| \stackrel{?}{=} -7$$
$$5 - 3|-4| \stackrel{?}{=} -7$$
$$5 - 3(4) \stackrel{?}{=} -7$$
$$-7 = -7 \quad \text{True.}$$

Both solutions make the equation true. Thus, the soh
$\{-3, 1\}$.

7. $5|2r + 3| - 5 = 0$

SOLUTION:

$5\|2r + 3\| - 5 = 0$	Original equation
$5\|2r + 3\| = 5$	Add 5 to each side.
$\|2r + 3\| = 1$	Divide each side by 5.

Case 1

$$2r + 3 = 1$$
$$2r = -2$$
$$r = -1$$

Case 2

$$2r + 3 = -1$$
$$2r = -4$$
$$r = -2$$

CHECK
Substitute each value in the original equation.

$$5|2(-1) + 3| - 5 \stackrel{?}{=} 0$$
$$5|1| - 5 \stackrel{?}{=} 0$$
$$5(1) - 5 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True.}$$

$$5|2(-2) + 3| - 5 \stackrel{?}{=} 0$$
$$5|-1| - 5 \stackrel{?}{=} 0$$
$$5(1) - 5 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True.}$$

Both solutions make the equation true. Thus, the soh
$\{-2, -1\}$.

8. $3 - 5|2d - 3| = 4$

SOLUTION:

$3 - 5\|2d - 3\| = 4$	Original equation
$-5\|2d - 3\| = 1$	Subtract 3 from each side.
$\|2d - 3\| = -\frac{1}{5}$	Divide each side by -5.

Because the absolute value of a number is always positive or zero, this sentence is never true. The solution is ∅.

9. **WEATHER** The packaging of a thermometer claims that the thermometer is accurate within 1.5 degrees of the actual temperature in degrees Fahrenheit. Write and solve an absolute value equation to find the least and greatest possible temperature if the thermometer reads 87.4° F.

SOLUTION:

Let x represent the actual possible temperature. The absolute value of the difference of the actual possible temperature and thermometer temperature, 87.4° F, is equal to the accuracy within 1.5 degrees, so the equation $|x - 87.4| = 1.5$ represents this situation.

Case 1	Case 2
$x - 87.4 = 1.5$	$x - 87.4 = -1.5$
$x = 88.9$	$x = 85.9$

The least possible temperature if the thermometer reads 87.4° F is 85.9° F and the greatest possible temperature is 88.9° F.

10. **OPINION POLLS** Public opinion polls reported in newspapers are usually given with a margin of error. A poll for a local election determined that Candidate Morrison will receive 51% of the votes. The stated margin of error is ±3%. Write and solve an absolute value equation to find the minimum and maximum percent of the vote that Candidate Morrison can expect to receive.

SOLUTION:

Let x represent the actual percent of the vote that Candidate Morrison can expect to receive. The absolute value of the difference of the actual percent of the vote that Candidate Morrison can expect to receive and the percent from the poll, 51%, is equal to the margin of error, ±3%, so the equation $|x - 51| = 3$ represents this situation.

Case 1	Case 2
$x - 51 = 3$	$x - 51 = -3$
$x = 54$	$x = 48$

The minimum percent of the vote that Candidate Morrison can expect to receive is 48% and the maximum percent of the vote is 54%.

Best of luck