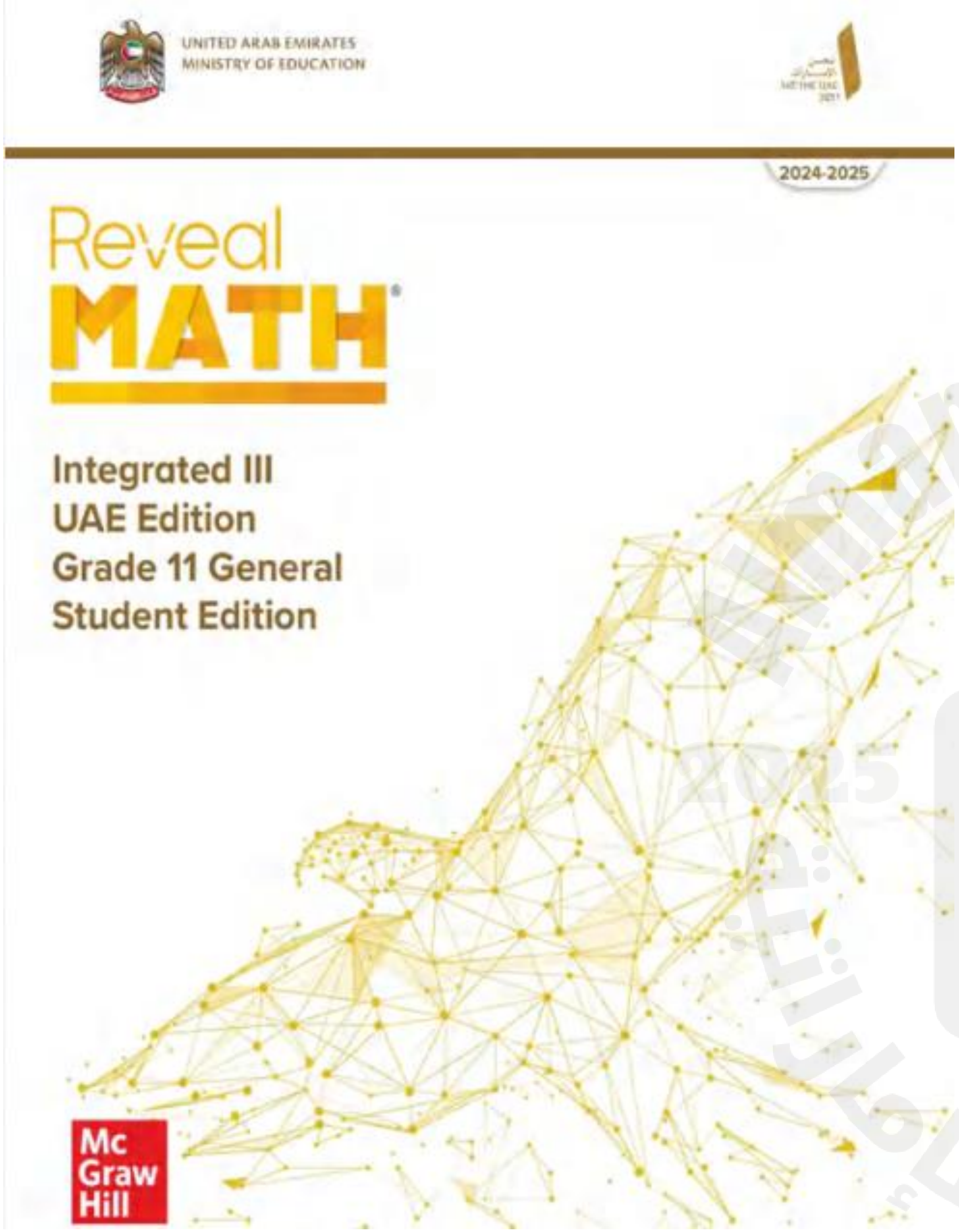
حل ملزمة أسئلة مراجعة وفق الهيكل الوزاري منهج ريفيل

إعداد: AlSabhi Abdulaziz Saif



11-general-Math-EOT ✿

Compiled by student: Saif Abdulaziz AlSabhi

Academic Year العام الدراسي	2024/2025
Term الفصل	1
Subject المادة	Mathematics/Reveal الرياضيات/ريفيل
Grade الصف	11
Stream المسار	General العام
Number of MCQ عدد الأسئلة الموضوعية	15
Marks of MCQ درجة الأسئلة الموضوعية	4
Number of FRQ عدد الأسئلة المقالية	5
Marks per FRQ الدرجات للأسئلة المقالية	(6-10)
Type of All Questions نوع كافة الأسئلة	MCQ/ الأسئلة الموضوعية FRQ/ الأسئلة المقالية
Maximum Overall Grade الدرجة القصوى الممكنة	100

	Question* السؤال*	Learning Outcome/Performance Criteria** ناتج التعلم/ معاييرالأداء**	rence(s) in the Student Book (English Vers المرجع في كتاب الطالب (النسخة الانجليزية) Example/Exercise مثال/تمرين	Page الصفحة
MCQ - الأسئلة الموضوعية	1	Find and interpret the average rate of change of quadratic functions given symbolically, in tables, and in graphs	Exercises (13-21)	P10
	2	Solve quadratic equations by graphing	Exercises (1-10)	P17
	3	Solve quadratic equations by graphing	Exercises (50-53)	P19
	4	Perform operations with complex numbers	Exercises (1-12)	P25
	5	Perform operations with complex numbers	Exercises (25-37)	P25
	6	Solve quadratic equations by factoring	Exercises(15-32)	P41
	7	Complete the square in the case of a trinomial that is not a perfect square	Exercises (19-24)	P39
	8	Solve quadratic equations by completing the square	Exercises (44-49)	P40
	9	Solve quadratic inequalities in one variable	Exercises (21-29)	P55

Exam Duration - مدة الامتحان	150 minutes
de of Implementation -ة التطبيق	SwiftAssess & Paper-Based
Calculator	Allowed
الآلة الحاسبة	مسموحة

	10	Graph polynomial functions and locate their zeros	Example5	P77
	11	Find the relative maxima and minima of polynomial functions	Example2	P84
	12	Add, subtract, and multiply polynomials	Exercises (30-39)	P98
	13	Divide polynomials using synthetic division	Exercises (11-16)	P105
	14	Determine whether a binomial is a factor of a polynomial by using synthetic substitution	Exercises (23-30)	P139
	15	Graph vertical translations of trigonometric functions.	Exercises(1-10)	P127
FRQ - الأسئلة المقالية	16	Graph quadratic functions	Exercises (27-32)	P11
	17	Solve quadratic equations by using the Quadratic Formula	Exercises(8-23)	P47
	18	Solve quadratic equations by factoring	Exercises (31-34)	P107
	19	Use Pascal’s Triangle to write binomial expansions	Exercises (1-12)	P111
	20	Factorize polynomials	Example2	P120

1	Find and interpret the average rate of change of quadratic functions given symbolically, in tables, and in graphs	Exercises (13-21)	P10

Example 4

Determine the average rate of change of $f(x)$ over the specified interval.

13. $f(x) = x^2 - 10x + 5$; interval $[-4, 4]$

14. $f(x) = 2x^2 + 4x - 6$; interval $[-3, 3]$

15. $f(x) = 3x^2 - 3x + 1$; interval $[-5, 5]$

16. $f(x) = 4x^2 + x + 3$; interval $[-2, 2]$

17. $f(x) = 2x^2 - 11$; interval $[-3, 3]$

18. $f(x) = -2x^2 + 8x + 7$; interval $[-4, 4]$

1	Find and interpret the average rate of change of quadratic functions given symbolically, in tables, and in graphs	Exercises (13-21)	P10

Example 5

Determine the average rate of change of $f(x)$ over the specified interval.

19. interval $[-3, 3]$

x	$f(x)$
−3	0
−2	3
−1	−4
0	−3
1	0
2	5
3	12

20. interval $[-4, 4]$

x	$f(x)$
−4	−27
−2	−3
0	5
2	−3
4	−27

21. interval $[-2, 2]$

x	$f(x)$
−2	−3
−1	−3
0	−1
1	3
2	9

solution method

Learn Finding and Interpreting Average Rate of Change

A function's **rate of change** is how a quantity is changing with respect to a change in another quantity. For nonlinear functions, the rate of change is not the same over the entire function. You can calculate the **average rate of change** of a nonlinear function over an interval.

Key Concept • Average Rate of Change

The average rate of change of a function $f(x)$ is equal to the change in the value of the dependent variable $f(b) - f(a)$ divided by the change in the value of the independent variable $b - a$ over the interval $[a, b]$.

$$\frac{f(b) - f(a)}{b - a}$$

Example 4 Find Average Rate of Change from an Equation

Example 5 Find Average Rate of Change from a Table

Example 4 Find Average Rate of Change from an Equation

Example 4

Determine the average rate of change of $f(x)$ over the specified interval.

13. $f(x) = x^2 - 10x + 5$; interval $[-4, 4]$

The average rate of change is equal to $\frac{f(4)-f(-4)}{4-(-4)}$

First find $f(4)$ and $f(-4)$.

$f(4) = (4)^2 - 10(4) + 5$ or -19

$f(-4) = (-4)^2 - 10(-4) + 5$ or 61

Then substitute to find the average rate of change.

$$\frac{f(4)-f(-4)}{4-(-4)} = \frac{-19-61}{4-(-4)} = -10$$

14. $f(x) = 2x^2 + 4x - 6$; interval $[-3, 3]$

The average rate of change is equal to $\frac{f(3)-f(-3)}{3-(-3)}$.

First find $f(3)$ and $f(-3)$.

$f(3) = 2(3)^2 + 4(3) - 6$ or 24

$f(-3) = 2(-3)^2 + 4(-3) - 6$ or 0

Then substitute to find the average rate of change.

$$\frac{f(3)-f(-3)}{3-(-3)} = \frac{24-0}{3-(-3)} = 4$$

15. $f(x) = 3x^2 - 3x + 1$; interval $[-5, 5]$

The average rate of change is equal to $\frac{f(5) - f(-5)}{5 - (-5)}$.

First find $f(5)$ and $f(-5)$.

$f(5) = 3(5)^2 - 3(5) + 1$ or 61

$f(-5) = 3(-5)^2 - 3(-5) + 1$ or 91

Then substitute to find the average rate of change.

$$\frac{f(5) - f(-5)}{5 - (-5)} = \frac{61 - 91}{5 - (-5)} = -3$$

16. $f(x) = 4x^2 + x + 3$; interval $[-2, 2]$

The average rate of change is equal to $\frac{f(2) - f(-2)}{2 - (-2)}$.

First find $f(2)$ and $f(-2)$.

$f(2) = 4(2)^2 + (2) + 3$ or 21

$f(-2) = 4(-2)^2 + (-2) + 3$ or 17

Then substitute to find the average rate of change.

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{21 - 17}{2 - (-2)} = 1$$

17. $f(x) = 2x^2 - 11$; interval $[-3, 3]$

The average rate of change is equal to $\frac{f(3) - f(-3)}{3 - (-3)}$.

First find $f(3)$ and $f(-3)$.

$f(3) = 2(3)^2 - 11$ or 7

$f(-3) = 2(-3)^2 - 11$ or 7

Then substitute to find the average rate of change.

$$\frac{f(3) - f(-3)}{3 - (-3)} = \frac{7 - 7}{3 - (-3)} = 0$$

18. $f(x) = -2x^2 + 8x + 7$; interval $[-4, 4]$

The average rate of change is equal to $\frac{f(4) - f(-4)}{4 - (-4)}$.

First find $f(4)$ and $f(-4)$.

$f(4) = -2(4)^2 + 8(4) + 7$ or 7

$f(-4) = -2(-4)^2 + 8(-4) + 7$ or -57

Then substitute to find the average rate of change.

$$\frac{f(4) - f(-4)}{4 - (-4)} = \frac{7 - (-57)}{4 - (-4)} = 8$$

Example 5

Determine the average rate of change of $f(x)$ over the specified interval.

19. interval $[-3, 3]$

x	$f(x)$
−3	0
−2	3
−1	−4
0	−3
1	0
2	5
3	12

$$\frac{12 - 0}{3 - -3} = \frac{12}{6} = 2$$

20. interval $[-4, 4]$

x	$f(x)$
−4	−27
−2	−3
0	5
2	−3
4	−27

average rate of change

$$\frac{f(b) - f(a)}{b - a} = \frac{-27 - -27}{4 - -4}$$

$$= \frac{0}{0}$$

21. interval $[-2, 2]$

x	$f(x)$
−2	−3
−1	−3
0	−1
1	3
2	9

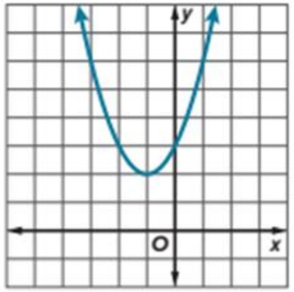
average rate of change

$$\frac{9 - -3}{2 - -2} = 3$$

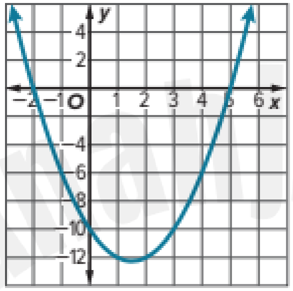
Example 1

Use the related graph of each equation to determine its solutions.

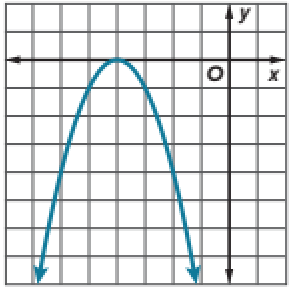
1. $x^2 + 2x + 3 = 0$



2. $x^2 - 3x - 10 = 0$



3. $-x^2 - 8x - 16 = 0$



Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

5. $4x^2 + 4x + 1 = 0$

6. $x^2 + x - 6 = 0$

2	Solve quadratic equations by graphing	Exercises (1-10)	P17

7. $x^2 + 2x - 3 = 0$

8. $-x^2 - 6x - 9 = 0$

9. $x^2 - 6x + 5 = 0$

10. $x^2 + 2x + 3 = 0$

Solution

solution method

Learn Solving Quadratic Equations by Graphing

A **quadratic equation** is an equation that includes a quadratic expression.

Key Concept • Standard Form of a Quadratic Equation

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, and c are integers.

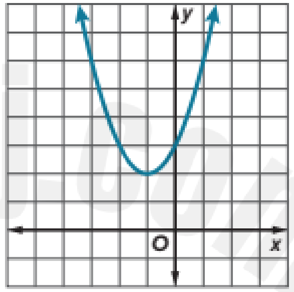
One method for finding the roots of a quadratic equation is to find the zeros of a related quadratic function. You can identify the solutions or roots of an equation by finding the x-intercepts of the graph of a related function. Often, exact roots cannot be found by graphing. You can estimate the solutions by finding the integers between which the zeros are located on the graph of the related function.

Example 1 One Real Solution

Example 1

Use the related graph of each equation to determine its solutions.

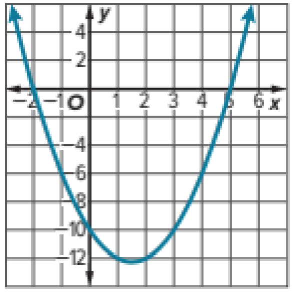
1. $x^2 + 2x + 3 = 0$



There are no zeros of the function.

Therefore, there is no real solution of the equation.

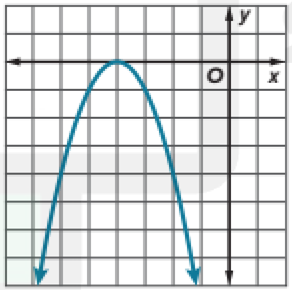
2. $x^2 - 3x - 10 = 0$



The zeros of the function are −2 and 5.

Therefore, the solutions of the equation are −2 and 5.

3. $-x^2 - 8x - 16 = 0$



The zero of the function is −4.

Therefore, the solution of the equation is −4.

Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

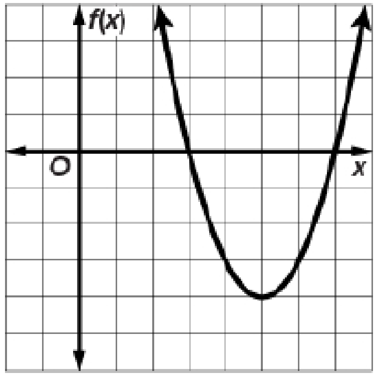
$x = -\frac{-10}{2(1)}$ $a = 1, b = -10$

$x = 5$ Simplify.

Make a table of values.

x	y
3	0
4	–3
5	–4
6	–3
7	0

Plot the points and connect them with a curve.



The zeros of the function are 3 and 7.

5. $4x^2 + 4x + 1 = 0$

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

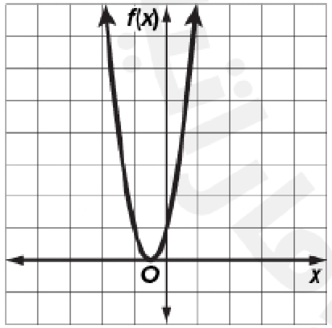
$x = -\frac{4}{2(4)}$ $a = 4, b = 4$

$x = -\frac{1}{2}$ Simplify.

Make a table of values.

x	y
$-1\frac{1}{2}$	4
–1	1
$-\frac{1}{2}$	0
0	1
$\frac{1}{2}$	4

Plot the points and connect them with a curve.



The zero of the function is $-\frac{1}{2}$.

6. $x^2 + x - 6 = 0$

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

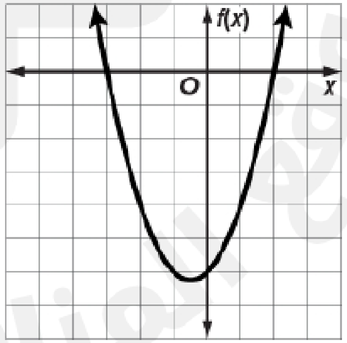
$x = -\frac{1}{2(1)}$ $a = 1, b = 1$

$x = -\frac{1}{2}$ Simplify.

Make a table of values.

x	y
–3	0
–2	–4
–1	–6
$-\frac{1}{2}$	–6.25
0	–6
1	–4
2	0

Plot the points and connect them with a curve.



The zeros of the function are –3 and 2.

7. $x^2 + 2x - 3 = 0$

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

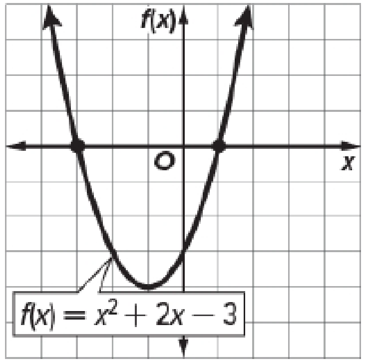
$x = -\frac{2}{2(1)}$ $a = 1, b = 2$

$x = -1$ Simplify.

Make a table of values.

x	y
–3	0
–2	–3
–1	–4
0	–3
1	0

Plot the points and connect them with a curve.



The zeros of the function are –3 and 1.

Solve each equation by graphing.

8. $-x^2 - 6x - 9 = 0$

Find the axis of symmetry.

$x = -\dfrac{b}{2a}$ Equation of the axis of symmetry

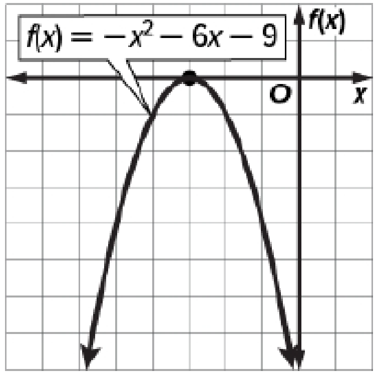
$x = -\dfrac{-6}{2(-1)}$ $a = -1, b = -6$

$x = -3$ Simplify.

Make a table of values.

x	y
–5	–4
–4	–1
–3	0
–2	–1
–1	–4

Plot the points and connect them with a curve.



The zero of the function is –3.

9. $x^2 - 6x + 5 = 0$

Find the axis of symmetry.

$x = -\dfrac{b}{2a}$ Equation of the axis of symmetry

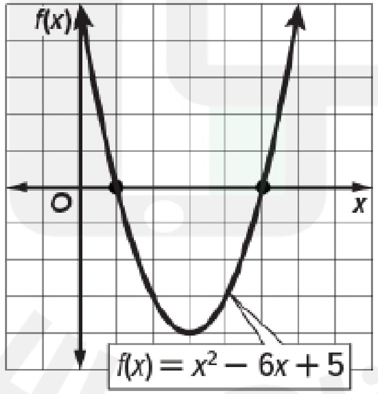
$x = -\dfrac{-6}{2(1)}$ $a = 1, b = -6$

$x = 3$ Simplify.

Make a table of values.

x	y
1	0
2	–3
3	–4
4	–3
5	0

Plot the points and connect them with a curve.



The zeros of the function are 1 and 5.

10. $x^2 + 2x + 3 = 0$

Find the axis of symmetry.

$x = -\dfrac{b}{2a}$ Equation of the axis of symmetry

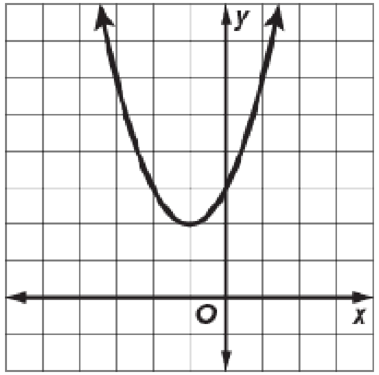
$x = -\dfrac{2}{2(1)}$ $a = 1, b = 2$

$x = -1$ Simplify.

Make a table of values.

x	y
–3	6
–2	3
–1	2
0	3
1	6

Plot the points and connect them with a curve.



There are no zeros of the function.

3	Solve quadratic equations by graphing	Exercises (50-53)	P19

REGULARITY **Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.**

50. Their sum is 4, and their product is -117.

51. Their sum is 12, and their product is -85.

3	Solve quadratic equations by graphing	Exercises (50-53)	P19

REGULARITY **Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.**

52. Their sum is -13, and their product is 42.

53. Their sum is -8, and their product is -209.

Example 2 Two Real Solutions

50. Their sum is 4, and their product is −117.

Let x represent one of the numbers. Then $4 - x$ will represent the other number. So $x(4 - x) = -117$.

Solve the equation for 0.

$x(4-x) = -117$	Original equation
$4x - x^2 = -117$	Distributive Property
$-x^2 = -4x - 117$	Subtract $4x$ from each side.
$0 = x^2 - 4x - 117$	Add x^2 to each side.

Find the axis of symmetry.

$x = -\frac{b}{2a}$	Equation of the axis of symmetry
$x = -\frac{-4}{2(1)}$	$a = 1, b = -4$
$x = 2$	Simplify.

Make a table of values.

x	y
–9	0
–7	–40
–4	–85
–1	–112
2	–121
5	–112
8	–85
11	–40
13	0

The zeros of the function are –9 and 13.

$x = -9$ and $x = 13$, so $4 - x = 13$ or $4 - x = -9$. Thus, the two numbers with a sum of 4 and a product of –117 are –9 and 13.

51. Their sum is 12, and their product is −85.

Let x represent one of the numbers. Then $12 - x$ will represent the other number. So $x(12 - x) = -85$.

Solve the equation for 0.

$x(12-x) = -85$	Original equation
$12x - x^2 = -85$	Distributive Property
$-x^2 = -12x - 85$	Subtract $12x$ from each side.
$0 = x^2 - 12x - 85$	Add x^2 to each side.

Find the axis of symmetry.

$x = -\frac{b}{2a}$	Equation of the axis of symmetry
$x = -\frac{-12}{2(1)}$	$a = 1, b = -12$
$x = 6$	Simplify.

Make a table of values.

x	y
–5	0
–3	–40
–1	–72
1	–96
3	–112
5	–120
6	–121
7	–120
9	–112
11	–96
13	–72
15	–40
17	0

The zeros of the function are –5 and 17.

$x = -5$ and $x = 17$, so $12 - x = 17$ or $12 - x = -5$. Thus, the two numbers with a sum of 12 and a product of –85 are –5 and 17.

52. Their sum is −13, and their product is 42.

Let x represent one of the numbers. Then $-13 - x$ will represent the other number. So $x(-13 - x) = 42$.

Solve the equation for 0.

$x(-13-x)=42$	Original equation
$-13x-x^2=42$	Distributive Property
$0=x^2+13x+42$	Add $13x$ and x^2 to each side.

Find the axis of symmetry.

$x=-\frac{b}{2a}$	Equation of the axis of symmetry
$x=-\frac{13}{2(1)}$	$a=1, b=13$
$x=-6.5$	Simplify.

Make a table of values.

x	y
–7	0
–6.5	–0.25
–6	0

The zeros of the function are –7 and –6.

$x = -7$ and $x = -6$, so $-13 - x = -6$ or $-13 - x = -7$. Thus, the two numbers with a sum of –13 and a product of 42 are –7 and –6.

53. Their sum is −8, and their product is −209.

Let x represent one of the numbers. Then $-8 - x$ will represent the other number. So $x(-8 - x) = -209$.

Solve the equation for 0.

$x(-8-x)=-209$	Original equation
$-8x-x^2=-209$	Distributive Property
$0=x^2+8x-209$	Add x^2 and $8x$ to each side.

Find the axis of symmetry.

$x=-\frac{b}{2a}$	Equation of the axis of symmetry
$x=-\frac{8}{2(1)}$	$a=1, b=8$
$x=-4$	Simplify.

Make a table of values.

x	y
–19	0
–15	–104
–11	–176
–7	–216
–4	–225
–1	–216
3	–176
7	–104
11	0

The zeros of the function are –19 and 11.

$x = -19$ and $x = 11$, so $-8 - x = 11$ or $-8 - x = -19$. Thus, the two numbers with a sum of –8 and a product of –209 are –19 and 11.

4	Perform operations with complex numbers	Exercises (1-12)	P25

Examples 1 and 2

Simplify.

1. $\sqrt{-48}$

2. $\sqrt{-63}$

3. $\sqrt{-72}$

4. $\sqrt{-24}$

5. $\sqrt{-84}$

6. $\sqrt{-99}$

7. $\sqrt{-23} \cdot \sqrt{-46}$

8. $\sqrt{-6} \cdot \sqrt{-3}$

9. $\sqrt{-5} \cdot \sqrt{-10}$

10. $(3i)(-2i)(5i)$

11. i^{11}

12. $4i(-6i)^2$

Solution

solution method

Learn Pure Imaginary Numbers

In your math studies so far, you have worked with real numbers. However, some equations such as $x^2 + x + 1 = 0$ do not have real solutions. This led mathematicians to define imaginary numbers. The **imaginary unit** i is the principal square root of -1. Thus, $i = \sqrt{-1}$ and $i^2 = -1$.

Numbers of the form $6i$, $-2i$, and $i\sqrt{3}$ are called pure imaginary numbers. A **pure imaginary number** is a number of the form bi, where b is a real number and $i = \sqrt{-1}$. For any positive real number $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers. The first few powers of i are shown.

$i^1 = i \qquad i^2 = -1 \qquad i^3 = i^2 \cdot i$ or $-i$

Example 1 Square Roots of Negative Numbers

Example 2 Products of Pure Imaginary Numbers

Examples 1 and 2

Simplify.

1. $\sqrt{-48}$

$$\sqrt{-48} = \sqrt{-1 \cdot 4^2 \cdot 3} = \sqrt{-1} \cdot \sqrt{4^2} \cdot \sqrt{3} = i \cdot 4 \cdot \sqrt{3} \text{ or } 4i\sqrt{3}$$

2. $\sqrt{-63}$

$$\sqrt{-63} = \sqrt{-1 \cdot 3^2 \cdot 7} = \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{7} = i \cdot 3 \cdot \sqrt{7} \text{ or } 3i\sqrt{7}$$

3. $\sqrt{-72}$

$$\sqrt{-72} = \sqrt{-1 \cdot 6^2 \cdot 2} = \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{2} = i \cdot 6 \cdot \sqrt{2} \text{ or } 6i\sqrt{2}$$

4. $\sqrt{-24}$

$$\sqrt{-24} = \sqrt{-1 \cdot 2^2 \cdot 6} = \sqrt{-1} \cdot \sqrt{2^2} \cdot \sqrt{6} = i \cdot 2 \cdot \sqrt{6} \text{ or } 2i\sqrt{6}$$

5. $\sqrt{-84}$

$$\sqrt{-84} = \sqrt{-1 \cdot 2^2 \cdot 21} = \sqrt{-1} \cdot \sqrt{2^2} \cdot \sqrt{21} = i \cdot 2 \cdot \sqrt{21} \text{ or } 2i\sqrt{21}$$

6. $\sqrt{-99}$

$$\sqrt{-5} \cdot \sqrt{-10} = i\sqrt{5} \cdot i\sqrt{10} = i^2 \cdot \sqrt{50} = -1 \cdot \sqrt{5^2} \cdot \sqrt{2} = -5\sqrt{2}$$

10. $(3i)(-2i)(5i)$

$$(3i)(-2i)(5i) = (3 \cdot -2 \cdot 5) \cdot i^3 = -30i^3 = -30 \cdot i^2 \cdot i = -30 \cdot -1 \cdot i = 30i$$

11. i^{11}

$$i^{11} = i^{10} \cdot i = (i^2)^5 \cdot i = (-1)^5 \cdot i = -i$$

12. $4i(-6i)^2$

$$(4i)(-6i)^2 = 4i \cdot 36i^2 = 144i^3 = 144 \cdot i^2 \cdot i = 144 \cdot -1 \cdot i = -144i$$

Examples 5 and 6

Simplify.

25. $(6 + i) + (4 - 5i)$

26. $(8 + 3i) - (6 - 2i)$

27. $(5 - i) - (3 - 2i)$

28. $(-4 + 2i) + (6 - 3i)$

29. $(6 - 3i) + (4 - 2i)$

30. $(-11 + 4i) - (1 - 5i)$

31. $(2 + i)(3 - i)$

32. $(5 - 2i)(4 - i)$

33. $(4 - 2i)(1 - 2i)$

34. **ELECTRICITY** Using the formula $V = CI$, find the voltage V in a circuit when the current $C = 3 - j$ amps and the impedance $I = 3 + 2j$ ohms.

Example 7

Simplify.

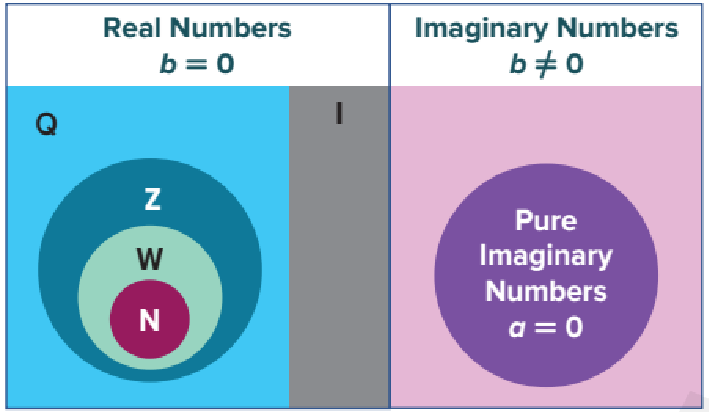
35. $\frac{5}{3+i}$

36. $\frac{7-13i}{2i}$

37. $\frac{6-5i}{3i}$

The Venn diagram shows the set of complex numbers. Notice that all of the real numbers are part of the set of complex numbers.

Complex Numbers ($a + bi$)



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. The Commutative and Associative Properties of Multiplication and Addition and the Distributive Property hold true for complex numbers. To add or subtract complex numbers, combine like terms. That is, combine the real parts, and combine the imaginary parts.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number.

A radical expression is in simplest form if no radicands contain fractions and no radicals appear in the denominator of a fraction. Similarly, a complex number is in simplest form if no imaginary numbers appear in the denominator of a fraction. You can use complex conjugates to simplify a fraction with a complex number in the denominator. This process is called **rationalizing the denominator**.

Example 5 Add or Subtract Complex Numbers

Example 6 Multiply Complex Numbers

Example 7 Divide Complex Numbers

Examples 5 and 6

Simplify.

25. $(6 + i) + (4 - 5i)$

$(6+i)+(4-5i)=(6+4)+(i-5i)$
$=10-4i$

26. $(8 + 3i) - (6 - 2i)$

$(8+3i)-(6-2i)=(8-6)+(3i-(-2i))$
$=2+5i$

27. $(5 - i) - (3 - 2i)$

$(5-i)-(3-2i)=(5-3)+(-i-(-2i))$
$=2+i$

28. $(-4 + 2i) + (6 - 3i)$

$(-4+2i)+(6-3i)=(-4+6)+(2i-3i)$
$=2-i$

29. $(6 - 3i) + (4 - 2i)$

$(6-3i)+(4-2i)=(6+4)+(-3i-2i)$
$=10-5i$

30. $(-11 + 4i) - (1 - 5i)$

$(-11+4i)-(1-5i)=(-11-1)+(4i-(-5i))$
$=-12+9i$

31. $(2 + i)(3 - i)$

$(2+i)(3-i)=2(3)+2(-i)+i(3)+i(-i)$
$=6-2i+3i-i^2$
$=6+i-(-1)$
$=7+i$

32. $(5 - 2i)(4 - i)$

$(5-2i)(4-i)=5(4)+5(-i)-2i(4)-2i(-i)$
$=20-5i-8i+2i^2$
$=20-13i+2(-1)$
$=18-13i$

33. $(4 - 2i)(1 - 2i)$

$(4-2i)(1-2i)=4(1)+4(-2i)-2i(1)-2i(-2i)$
$=4-8i-2i+4i^2$
$=4-10i+4(-1)$
$=-10i$

34. ELECTRICITY Using the formula $V = CI$, find the voltage V in a circuit when the current $C = 3 - j$ amps and the impedance $I = 3 + 2j$ ohms.

$V=CI$
$=(3-j)(3+2j)$
$=3(3)+3(2j)-j(3)-j(2j)$
$=9+6j-3j-2j^2$
$=9+3j-2(-1)$
$=11+3j$

$11+3j$ volts

Example 7

Simplify.

35. $\frac{5}{3+i}$

$$\frac{5}{3+i}=\frac{5}{3+i}\cdot\frac{3-i}{3-i}$$
$$=\frac{15-5i}{9-i^2}$$
$$=\frac{15-5i}{9-(-1)}$$
$$=\frac{15-5i}{10}$$
$$=\frac{15}{10}-\frac{5}{10}i$$
$$=\frac{3}{2}-\frac{1}{2}i$$

36. $\frac{7-13i}{2i}$

$$\frac{7-13i}{2i}=-\frac{13}{2}+\frac{7}{2}i$$

37. $\frac{6-5i}{3i}$

$$\frac{6-5i}{3i}=-\frac{5}{3}-2i$$

6	Solve quadratic equations by factoring	Exercises(15-32)	P31

Examples 5-7

Solve each equation by factoring. Check your solution.

15. $x^2 = 64$

16. $x^2 - 100 = 0$

17. $289 = x^2$

18. $x^2 + 14 = 50$

19. $x^2 - 169 = 0$

20. $124 = x^2 + 3$

21. $4x^2 - 28x + 49 = 0$

22. $9x^2 + 6x = -1$

23. $16x^2 - 24x + 13 = 4$

24. $81x^2 + 36x = -4$

25. $25x^2 + 80x + 64 = 0$

6	Solve quadratic equations by factoring	Exercises(15-32)	P31

26. $9x^2 + 60x + 95 = -5$

27. $x^2 + 12 = -13$

28. $x^2 + 100 = 0$

29. $x^2 = -225$

30. $x^2 + 4 = 0$

31. $36x^2 = -25$

32. $64x^2 = -49$

solution method

Solution

Learn Solving Quadratic Equations by Factoring Special Products

Key Concept • Factoring Differences of Squares

Words: To factor $a^2 - b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Symbols: $a^2 - b^2 = (a + b)(a - b)$

Key Concept • Factoring Perfect Square Trinomials

Words: To factor $a^2 + 2ab + b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Symbols: $a^2 + 2ab + b^2 = (a + b)^2$

Not all quadratic equations have solutions that are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$. For example, you know that the solution of $x^2 = -4$ must be complex because there is no real number for which its square is -4. If you take the square root of each side, $x = 2i$ or $-2i$.

15. $x^2 = 64$

$x^2 = 64$
$x^2 - 64 = 0$
$x^2 - 8^2 = 0$
$(x+8)(x-8) = 0$
$x + 8 = 0$ or $x - 8 = 0$
$x = -8 \quad x = 8$

16. $x^2 - 100 = 0$

$x^2 - 100 = 0$
$x^2 - 10^2 = 0$
$(x+10)(x-10) = 0$
$x + 10 = 0$ or $x - 10 = 0$
$x = -10 \quad x = 10$

17. $289 = x^2$

$289 = x^2$
$289 - x^2 = 0$
$17^2 - x^2 = 0$
$(17 + x)(17 - x) = 0$
$17 + x = 0$ or $17 - x = 0$
$x = -17 \quad x = 17$

18. $x^2 + 14 = 50$

$x^2 + 14 = 50$
$x^2 - 36 = 0$
$x^2 - 6^2 = 0$
$(x+6)(x-6) = 0$
$x + 6 = 0$ or $x - 6 = 0$
$x = -6 \quad x = 6$

19. $x^2 - 169 = 0$

$x^2 - 169 = 0$
$x^2 - 13^2 = 0$
$(x+13)(x-13) = 0$
$x + 13 = 0$ or $x - 13 = 0$
$x = -13 \quad x = 13$

20. $124 = x^2 + 3$

$124 = x^2 + 3$
$0 = x^2 - 121$
$0 = x^2 - 11^2$
$0 = (x+11)(x-11)$
$0 = x + 11$ or $x - 11 = 0$
$-11 = x$ or $\quad x = 11$

21. $4x^2 - 28x + 49 = 0$

$4x^2 - 28x + 49 = 0$
$(2x)^2 - 2(2x)(7) + 7^2 = 0$
$(2x - 7)^2 = 0$
$x = \frac{7}{2}$

22. $9x^2 + 6x = -1$

$9x^2 + 6x = -1$
$9x^2 + 6x + 1 = 0$
$(3x)^2 + 2(3x)(1) + 1^2 = 0$
$(3x + 1)^2 = 0$
$x = -\frac{1}{3}$

23. $16x^2 - 24x + 13 = 4$

$16x^2 - 24x + 13 = 4$
$16x^2 - 24x + 9 = 0$
$(4x)^2 - 2(4x)(3) + 3^2 = 0$
$(4x - 3)^2 = 0$
$x = \frac{3}{4}$

24. $81x^2 + 36x = -4$

$81x^2 + 36x = -4$
$81x^2 + 36x + 4 = 0$
$(9x)^2 + 2(9x)(2) + 2^2 = 0$
$(9x + 2)^2 = 0$
$x = -\frac{2}{9}$

25. $25x^2 + 80x + 64 = 0$

$25x^2 + 80x + 64 = 0$
$(5x)^2 + 2(5x)(8) + 8^2 = 0$
$(5x + 8)^2 = 0$
$x = -\frac{8}{5}$

26. $9x^2 + 60x + 95 = -5$

$9x^2 + 60x + 95 = -5$
$9x^2 + 60x + 100 = 0$
$(3x)^2 + 2(3x)(10) + 10^2 = 0$
$(3x + 10)^2 = 0$
$x = -\frac{10}{3}$

27. $x^2 + 12 = -13$

$x^2 + 12 = -13$
$x^2 + 25 = 0$
$x^2 - (-25) = 0$
$x^2 - (5i)^2 = 0$
$(x + 5i)(x - 5i) = 0$
$x + 5i = 0$ or $x - 5i = 0$
$x = -5i \quad x = 5i$

28. $x^2 + 100 = 0$

$x^2 + 100 = 0$
$x^2 - (-100) = 0$
$x^2 - (10i)^2 = 0$
$(x + 10i)(x - 10i) = 0$
$x + 10i = 0$ or $x - 10i = 0$
$x = -10i \quad x = 10i$

7	Complete the square in the case of a trinomial that is not a perfect square	Exercises (19-24)	P39

Example 4

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square trinomial.

19. $x^2 + 10x + c$

20. $x^2 - 14x + c$

21. $x^2 + 24x + c$

22. $x^2 + 5x + c$

23. $x^2 - 9x + c$

24. $x^2 - x + c$

7	Complete the square in the case of a trinomial that is not a perfect square	Exercises (19-24)	P39

Solution

solution method

Learn Solving Quadratic Equations by Completing the Square

All quadratic equations can be solved by using the properties of equality to manipulate the equation until one side is a perfect square. This process is called **completing the square**.

Key Concept • Completing the Square

Words: To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.
Step 2 Square the result in Step 1.
Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

To solve an equation of the form $x^2 + bx + c = 0$ by completing the square, first subtract c from each side of the equation. Then add $\left(\frac{b}{2}\right)^2$ to each side of the equation and solve for x.

Example 4 Complete the Square

Example 4

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square trinomial.

19. $x^2 + 10x + c$

Step 1 Find one half of 10. $\frac{10}{2} = 5$

Step 2 Square the result from Step 1. $5^2 = 25$

Step 3 Add the result from Step 2 to $x^2 + 10x$. $x^2 + 10x + 25$

The expression $x^2 + 10x + 25$ can be written as $(x + 5)^2$.

20. $x^2 - 14x + c$

Step 1 Find one half of -14. $\frac{-14}{2} = -7$

Step 2 Square the result from Step 1. $(-7)^2 = 49$

Step 3 Add the result from Step 2 to $x^2 - 14x$. $x^2 - 14x + 49$

The expression $x^2 - 14x + 49$ can be written as $(x - 7)^2$.

21. $x^2 + 24x + c$

Step 1 Find one half of 24. $\frac{24}{2} = 12$

Step 2 Square the result from Step 1. $12^2 = 144$

Step 3 Add the result from Step 2 to $x^2 + 24x$. $x^2 + 24x + 144$

22. $x^2 + 5x + c$

Step 1 Find one half of 5. $\frac{5}{2}$

Step 2 Square the result from Step 1. $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

Step 3 Add the result from Step 2 to $x^2 + 5x$. $x^2 + 5x + \frac{25}{4}$

The expression $x^2 + 10x + \frac{25}{4}$ can be written as $\left(x + \frac{5}{2}\right)^2$.

23. $x^2 - 9x + c$

Step 1 Find one half of -9. $\frac{-9}{2} = -\frac{9}{2}$

Step 2 Square the result from Step 1. $\left(-\frac{9}{2}\right)^2 = \frac{81}{4}$

Step 3 Add the result from Step 2 to $x^2 - 9x$. $x^2 - 9x + \frac{81}{4}$

The expression $x^2 - 9x + \frac{81}{4}$ can be written as $\left(x - \frac{9}{2}\right)^2$.

24. $x^2 - x + c$

Step 1 Find one half of -1. $\frac{-1}{2} = -\frac{1}{2}$

Step 2 Square the result from Step 1. $\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

Step 3 Add the result from Step 2 to $x^2 - x$. $x^2 - x + \frac{1}{4}$

The expression $x^2 - x + \frac{1}{4}$ can be written as $\left(x - \frac{1}{2}\right)^2$.

Examples 9 and 10

Write each function in vertex form. Find the axis of symmetry. Then find the vertex, and determine if it is a *maximum* or *minimum*.

44. $y = x^2 + 2x - 5$

45. $y = x^2 + 6x + 1$

46. $y = -x^2 + 4x + 2$

47. $y = -x^2 - 8x - 5$

48. $y = 2x^2 + 4x + 3$

49. $y = 3x^2 + 6x - 1$

Learn Quadratic Functions in Vertex Form

When a function is given in standard form, $y = ax^2 + bx + c$, you can complete the square to write it in vertex form.

Key Concept • Vertex Form of a Quadratic Function	
Words: The vertex form of a quadratic function is $y = a(x - h)^2 + k$.	
Symbols: Standard Form $y = ax^2 + bx + c$	Vertex Form $y = a(x - h)^2 + k$ The vertex is (h, k).
Example: Standard Form $y = 2x^2 + 12x + 16$	Vertex Form $y = 2(x + 3)^2 - 2$ The vertex is $(-3, -2)$.

After completing the square and writing a quadratic function in vertex form, you can analyze key features of the function. The vertex is (h, k) and $x = h$ is the equation of the axis of symmetry. The shape of the parabola and the direction that it opens are determined by a. The value of k is a minimum value if $a > 0$ or a maximum value if $a < 0$.

The path that an object travels when influenced by gravity is called a *trajectory*, and trajectories can be modeled by quadratic functions. The formula below relates the height of the object $h(t)$ and time t, where g is acceleration due to gravity, v is the initial velocity of the object, and h_0 is the initial height of the object.

$$h(t) = -\frac{1}{2}gt^2 + vt + h_0$$

The acceleration due to gravity g is 9.8 meters per second squared or 32 feet per second squared. Problems that involve objects being thrown or dropped are called **projectile motion problems**.

Example 9 Write Functions in Vertex Form

Example 10 Determine the Vertex and Axis of Symmetry

44. $y = x^2 + 2x - 5$

$y = x^2 + 2x - 5$	Original equation
$y = (x^2 + 2x) - 5$	Group $ax^2 + bx$.
$y = (x^2 + 2x + 1) - 5 - 1$	Complete the square.
$y = (x + 1)^2 - 6$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = (x + 1)^2 - 6$ is $(-1, -6)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 1$, which is greater than 0, the value of k is a minimum value.

45. $y = x^2 + 6x + 1$

$y = x^2 + 6x + 1$	Original equation
$y = (x^2 + 6x) + 1$	Group $ax^2 + bx$.
$y = (x^2 + 6x + 9) + 1 - 9$	Complete the square.
$y = (x + 3)^2 - 8$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = (x + 3)^2 - 8$ is $(-3, -8)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -3$.
Since the value of $a = 1$, which is greater than 0, the value of k is a minimum value.

46. $y = -x^2 + 4x + 2$

$y = -x^2 + 4x + 2$	Original equation
$y = (-x^2 + 4x) + 2$	Group $ax^2 + bx$.
$y = -(x^2 - 4x) + 2$	Factor out -1.
$y = -(x^2 - 4x + 4) + 2 - (-1)(4)$	Complete the square.
$y = -(x - 2)^2 + 6$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = -(x - 2)^2 + 6$ is $(2, 6)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = 2$.
Since the value of $a = -1$, which is less than 0, the value of k is a maximum value.

47. $y = -x^2 - 8x - 5$

$y = -x^2 - 8x - 5$	Original equation
$y = (-x^2 - 8x) - 5$	Group $ax^2 + bx$.
$y = -(x^2 + 8x) - 5$	Factor out -1.
$y = -(x^2 + 8x + 16) - 5 - (-1)16$	Complete the square.
$y = -(x + 4)^2 + 11$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = -(x + 4)^2 + 11$ is $(-4, 11)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -4$.
Since the value of $a = -1$, which is less than 0, the value of k is a maximum value.

48. $y = 2x^2 + 4x + 3$

$y = 2x^2 + 4x + 3$	Original equation
$y = (2x^2 + 4x) + 3$	Group $ax^2 + bx$.
$y = 2(x^2 + 2x) + 3$	Factor.
$y = 2(x^2 + 2x + 1) + 3 - 2(1)$	Complete the square.
$y = 2(x + 1)^2 + 1$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = 2(x + 1)^2 + 1$ is $(-1, 1)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 2$, which is greater than 0, the value of k is a minimum value.

49. $y = 3x^2 + 6x - 1$

$y = 3x^2 + 6x - 1$	Original equation
$y = (3x^2 + 6x) - 1$	Group $ax^2 + bx$.
$y = 3(x^2 + 2x) - 1$	Factor.
$y = 3(x^2 + 2x + 1) - 1 - 3(1)$	Complete the square.
$y = 3(x + 1)^2 - 4$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = 3(x + 1)^2 - 4$ is $(-1, -4)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 3$, which is greater than 0, the value of k is a minimum value.

9	Solve quadratic inequalities in one variable	Exercises (21-29)	P55

Mixed Exercises

Solve each quadratic inequality by using a graph, a table, or algebraically.

21. $-2x^2 + 12x < -15$

22. $5x^2 + x + 3 \geq 0$

23. $11 \leq 4x^2 + 7x$

24. $x^2 - 4x \leq -7$

25. $-3x^2 + 10x < 5$

26. $-1 \geq -x^2 - 5x$

9	Solve quadratic inequalities in one variable	Exercises (21-29)	P55

27. $x^2 + 2x + 1 > 0$

28. $x^2 - 3x + 2 \leq 0$

29. $x^2 + 10x + 7 \geq 0$

solution method

9	Solve quadratic inequalities in one variable	Exercises (21-29)	P55

Solution

Learn Graphing Quadratic Inequalities

You can graph quadratic inequalities in two variables by using the same techniques used to graph linear inequalities in two variables. A **quadratic inequality** is an inequality of the form $y > ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y < ax^2 + bx + c$, or $y \leq ax^2 + bx + c$.

Key Concept • Graphing Quadratic Inequalities

Step 1 Graph the related function.
Step 2 Test a point not on the parabola.
Step 3 Shade accordingly.

Example 1 Graph a Quadratic Inequality ($<$ or $\leq$)

Example 2 Graph a Quadratic Inequality ($>$ or $\geq$)

Example 3 Solve a Quadratic Inequality ($<$ or $\leq$) by Graphing

Example 4 Solve a Quadratic Inequality ($>$ or $\geq$) by Graphing

21. $-2x^2 + 12x < -15$

$-2x^2 + 12x = -15$ Related equation
$-2x^2 + 12x - 15 = 0$ Add 15 to each side.
$x \approx -1.06$ or $x \approx 7.06$ Quadratic Formula

Plot the solutions on a number line and test a value from each interval.

Test $x = -2$, $x = 0$, and $x = 8$. The values that satisfy the original inequality are $x = -2$ and $x = 8$, so the solution set is $\{x \mid x < -1.06 \text{ or } x > 7.06\}$.

22. $5x^2 + x + 3 \geq 0$

$5x^2 + x + 3 = 0$ Related equation
$x = -\frac{1}{10} \pm \frac{i\sqrt{59}}{10}$ Quadratic Formula

There are no real solutions, therefore the graph never intersections the x-axis. The graph is always above the x-axis, which means $5x^2 + x + 3$ is always greater than 0. So, the original inequality is true for all real numbers. The solution set is {all real numbers}.

23. $11 \leq 4x^2 + 7x$

$11 = 4x^2 + 7x$ Related equation
$0 = 4x^2 + 7x - 11$ Subtract 11 from each side.
$0 = (x - 1)(4x + 11)$ Factor.
$1 = x$ or $-2.75 = x$ Zero Product Property

Plot the solutions on a number line and test a value from each interval.

Test $x = -3$, $x = 0$, and $x = 2$. The values that satisfies the original inequality are $x = -3$ and $x = 2$, so the solution set is $\{x \mid x \leq -2.75 \text{ or } x \geq 1\}$.

24. $x^2 - 4x \le -7$

$x^2 - 4x = -7$ Related equation
$x^2 - 4x + 7 = 0$ Add 7 to each side.
$x = 2 \pm 3i$ Quadratic Formula

There are no real solutions, therefore the graph never intersections the x-axis. The graph is always above the x-axis, which means $x^2 - 4x + 7$ is always greater than 0, so it cannot be less than or equal to 0. So, the original inequality is never true for any real number. The solution set is $\varnothing$.

25. $-3x^2 + 10x < 5$

$-3x^2 + 10x = 5$ Related equation
$-3x^2 + 10x - 5 = 0$ Subtract 5 from each side.
$x \approx 0.61$ or $x \approx 2.72$ Quadratic Formula

Plot the solutions on a number line and test a value from each interval.

Test $x = 0$, $x = 1$, and $x = 3$. The values that satisfies the original inequality are $x = 0$ and $x = 3$, so the solution set is $\{x \mid x < 0.61 \text{ or } x > 2.72\}$.

26. $-1 \ge -x^2 - 5x$

$-1 = -x^2 - 5x$ Related equation
$0 = -x^2 - 5x + 1$ Add 1 to each side.
$-5.19 \approx x$ or $0.19 \approx x$ Quadratic Formula

Plot the solutions on a number line and test a value from each interval.

Test $x = -6$, $x = 0$, and $x = 1$. The values that satisfies the original inequality are $x = -6$ and $x = 1$, so the solution set is $\{x \mid x \le -5.19 \text{ or } x \ge 0.19\}$.

27. $x^2 + 2x + 1 > 0$

$x^2 + 2x + 1 = 0$ Related equation
$(x + 1)(x + 1) = 0$ Factor.
$x = -1$ Zero Product Property

Plot the solution on a number line and test a value from each interval.

Test $x = -2$ and $x = 0$. The values that satisfies the original inequality are $x = -2$ and $x = 0$. When $x = -1$, the original inequality is equal to 0, so the solution set is {all real numbers}.

28. $x^2 - 3x + 2 \le 0$

$x^2 - 3x + 2 = 0$ Related equation
$(x - 1)(x - 2) = 0$ Factor.
$x = 1$ or $x = 2$ Zero Product Property

Plot the solutions on a number line and test a value from each interval.

Test $x = 0$, $x = 1.5$, and $x = 3$. The only value that satisfies the original inequality is $x = 1.5$, so the solution set is $\{x \mid 1 \le x \le 2\}$.

29. $x^2 + 10x + 7 \ge 0$

$x^2 + 10x + 7 = 0$ Related equation
$x \approx -9.24$ or $x \approx -0.76$ Quadratic Formula

Plot the solutions on a number line and test a value from each interval.

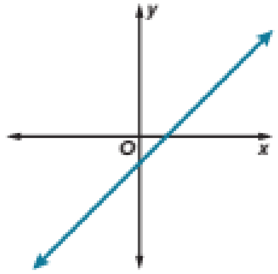
Test $x = -10$, $x = -5$, and $x = 0$. The values that satisfies the original inequality are $x = -10$ and $x = 0$, so the solution set is $\{x \mid x \le -9.24 \text{ or } x \ge -0.76\}$.

10	Graph polynomial functions and locate their zeros	Example5	P77

Example 5

Use the graph to state the number of real zeros of the function.

15.



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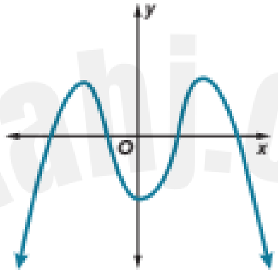

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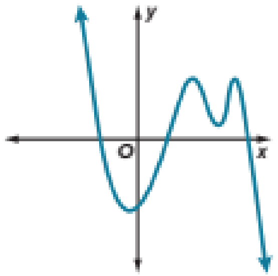

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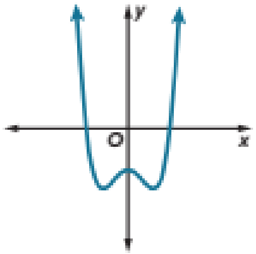

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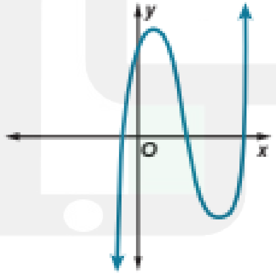

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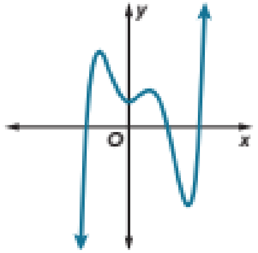


10	Graph polynomial functions and locate their zeros	Example5	P77

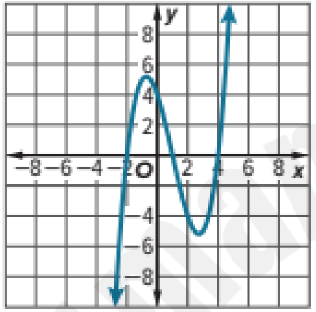
Solution

solution method

Example 5 Zeros of a Polynomial Function

Use the graph to state the number of real zeros of the function.

The real zeros occur at $x = -2$, 1, and 4, so there are three real zeros.



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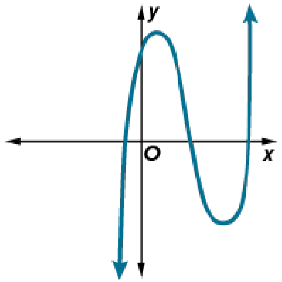
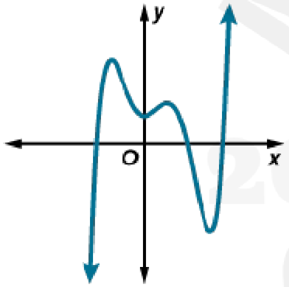


SOLUTION:

The graph crosses the x-axis three times, so there are 3

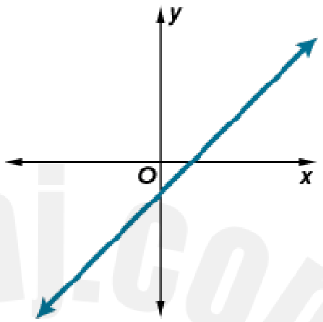
20.



SOLUTION:

The graph crosses the x-axis three times, so there are 3 real zeroes.

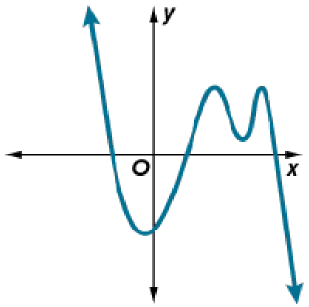
15.



SOLUTION:

The graph crosses the x-axis one time, so there is 1 real zero.

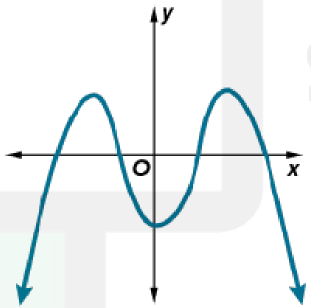
17.



SOLUTION:

The graph crosses the x-axis three times, so there are 3 real zeroes

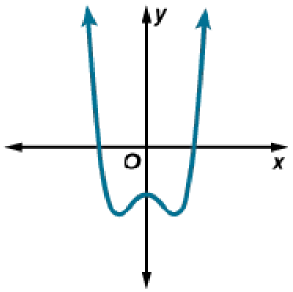
16.



SOLUTION:

The graph crosses the x-axis four times, so there are 4 real zeroes.

18.



SOLUTION:

The graph crosses the x-axis two times, so there are 2 real zeroes.

Example 2

Use a table to graph each function. Then estimate the *x*-coordinates at which relative maxima and relative minima occur.

5. $f(x) = -2x^3 + 12x^2 - 8x$

x	$f(x)$

6. $f(x) = 2x^3 - 4x^2 - 3x + 4$

x	$f(x)$

11	Find the relative maxima and minima of polynomial functions	Example2	P84

7. $f(x) = x^4 + 2x - 1$

x	$f(x)$

8. $f(x) = x^4 + 8x^2 - 12$

x	$f(x)$

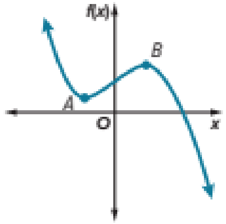
solution method

Solution

Learn Extrema of Polynomials

Extrema occur at relative maxima or minima of the function.

Point A is a relative minimum, and point B is a relative maximum. Both points A and B are extrema. The graph of a polynomial of degree n has at most $n - 1$ extrema.

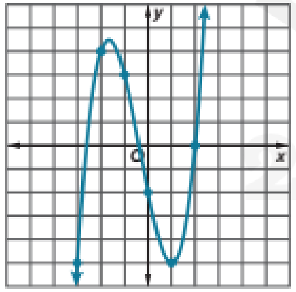


Example 2 Identify Extrema

Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the x-coordinates at which the relative maxima and relative minima occur.

Step 1 Make a table of values and graph the function.

x	$f(x)$
−4	−30
−3	−5
−2	4
−1	3
0	−2
1	−5
2	0
3	19



Step 2 Estimate the locations of the extrema.

The value of $f(x)$ at $x = -2$ is greater than the surrounding points indicating a maximum *near* $x = -2$.

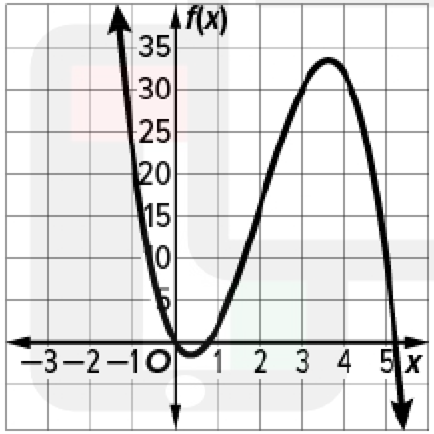
The value of $f(x)$ at $x = 1$ is less than the surrounding points indicating a minimum *near* $x = 1$.

You can use a graphing calculator to find the extrema of a function and confirm your estimates.

5. $f(x) = -2x^3 + 12x^2 - 8x$

Make a table of values and graph the function.

x	$f(x)$
−2	80
−1	22
0	0
1	2
2	16
3	30
4	32
5	10



The value of $f(x)$ between $x = 0$ and $x = 1$ is less than the surrounding points indicating a minimum between $x = 0$ and $x = 1$.

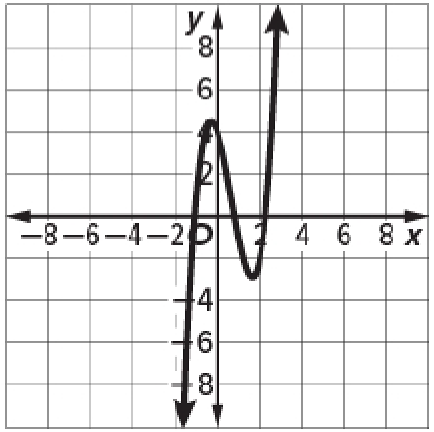
The value of $f(x)$ at $x = 4$ is greater than the surrounding points indicating a maximum near $x = 4$.

11	Find the relative maxima and minima of polynomial functions	Example2	P84

6. $f(x) = 2x^3 - 4x^2 - 3x + 4$

Make a table of values and graph the function.

x	$f(x)$
–4	–176
–3	–77
–2	–22
–1	1
0	4
1	–1
2	–2
3	13
4	56

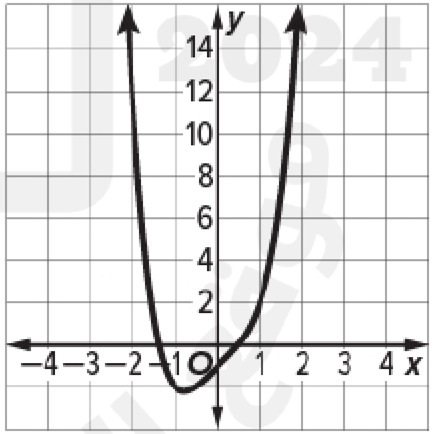


The value of $f(x)$ at $x = –0.3$ is greater than the surrounding points indicating a maximum near $x = –0.3$. The value of $f(x)$ at $x = 1.6$ is less than the surrounding points indicating a minimum near $x = 1.6$.

7. $f(x) = x^4 + 2x - 1$

Make a table of values and graph the function.

x	$f(x)$
–4	247
–3	74
–2	11
–1	–2
0	–1
1	2
2	19
3	86
4	263



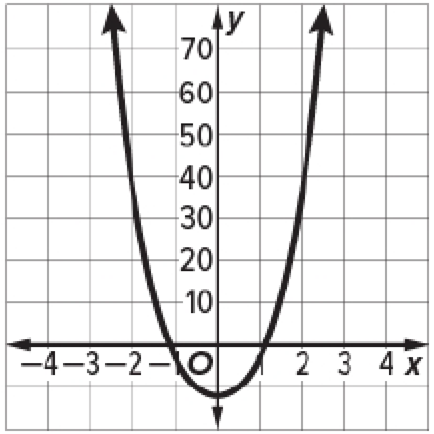
The value of $f(x)$ at $x = –1$ is less than the surrounding points indicating a minimum near $x = –1$. There is no relative maximum.

11	Find the relative maxima and minima of polynomial functions	Example2	P84

8. $f(x) = x^4 + 8x^2 - 12$

Make a table of values and graph the function.

x	$f(x)$
–4	372
–3	141
–2	36
–1	–3
0	–12
1	–3
2	36
3	141
4	372



The value of $f(x)$ at $x = 0$ is less than the surrounding points indicating a minimum near $x = 0$.
There is no relative maximum.

Mixed Exercises

Simplify.

30. $5xy(2x - y) + 6y^2(x^2 + 6)$

31. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$

32. $\frac{1}{4}g^2(8g + 12h - 16gh^2)$

33. $\frac{1}{3}n^3(6n - 9p + 18np^4)$

34. $(g^3 - h)(g^3 + h)$

35. $(n^2 - 7)(2n^3 + 4)$

36. $(2x - 2y)^3$

37. $(4n - 5)^3$

38. $(3z - 2)^3$

39. $\frac{1}{4}(16x - 12y) + \frac{1}{3}(9x + 3y)$

Learn Adding and Subtracting Polynomials

A polynomial is a monomial or the sum of two or more monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. The degree of a polynomial is the greatest degree of any term in the polynomial.

Polynomials can be added or subtracted by performing the operations indicated and combining like terms. You can subtract a polynomial by adding its additive inverse.

The sum or difference of polynomials will have the same variables and exponents as the original polynomials, but possibly different coefficients. Thus, the sum or difference of two polynomials is also a polynomial.

A set is **closed** if and only if an operation on any two elements of the set produces another element of the same set. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is closed under the operations of addition and subtraction.

Example 2 Add Polynomials

Example 3 Subtract Polynomials

Example 4 Simplify by Using the Distributive Property

Example 5 Multiply Binomials

35. $(n^2 - 7)(2n^3 + 4)$

$$(n^2-7)(2n^3+4) = n^2(2n^3) + n^2(4) + (-7)(2n^3) + (-7)(4)$$
$$= 2n^5 + 4n^2 - 14n^3 - 28$$
$$= 2n^5 - 14n^3 + 4n^2 - 28$$

30. $5xy(2x - y) + 6y^2(x^2 + 6)$

$$5xy(2x-y) + 6y^2(x^2+6) = 5xy(2x) + 5xy(-y) + 6y^2(x^2) + 6y^2(6)$$
$$= 10x^2y - 5xy^2 + 6x^2y^2 + 36y^2$$

31. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$

$$3ab(4a-5b) + 4b^2(2a^2+1) = 3ab(4a) + 3ab(-5b) + 4b^2(2a^2) + 4b^2(1)$$
$$= 12a^2b - 15ab^2 + 8a^2b^2 + 4b^2$$
$$= 12a^2b + 8a^2b^2 - 15ab^2 + 4b^2$$

32. $\frac{1}{4}g^2(8g + 12h - 16gh^2)$

$$\frac{1}{4}g^2(8g+12h-15gh^2) = \frac{1}{4}g^2(8g) + \frac{1}{4}g^2(12h) + \frac{1}{4}g^2(-16gh^2)$$
$$= 2g^3 + 3g^2h - 4g^3h^2$$

33. $\frac{1}{3}n^3(6n - 9p + 18np^4)$

$$\frac{1}{3}n^3(6n-9p+18np^4) = \frac{1}{3}n^3(6n) + \frac{1}{3}n^3(-9p) + \frac{1}{3}n^3(18np^4)$$
$$= 2n^4 - 3n^3p + 6n^4p^4$$

34. $(g^3 - h)(g^3 + h)$

$$(g^3-h)(g^3+h) = g^3(g^3) + g^3(h) + (-h)(g^3) + (-h)(h)$$
$$= g^6 + g^3h - g^3h - h^2$$
$$= g^6 - h^2$$

36. $(2x-2y)^3$

$$(2x-2y)(2x-2y)=2x(2x)+2x(-2y)+(-2y)(2x)+(-2y)(-2y)$$
$$=4x^2-4xy-4xy+4y^2$$
$$=4x^2-8xy+4y^2$$
$$(4x^2-8xy+4y^2)(2x-2y)=4x^2(2x)+4x^2(-2y)+(-8xy)(2x)+(-8xy)(-2y)+4y^2(2x)+4y^2(-2y)$$
$$=8x^3-8x^2y-16x^2y+16x^2y+8xy^2-8y^3$$
$$=8x^3+(-8x^2y-16x^2y)+(16xy^2+8xy^2)-8y^3$$
$$=8x^3-24x^2y+24xy^2-8y^3$$

37. $(4n-5)^3$

$$(4n-5)(4n-5)=4n(4n)+4n(-5)+(-5)4n+(-5)(-5)$$
$$=16n^2-20n-20n+25$$
$$=16n^2-40n+25$$
$$(16n^2-40n+25)(4n-5)=16n^2(4n)+16n^2(-5)+(-40n)(4n)+(-40n)(-5)+25(4n)+25(-5)$$
$$=64n^3-160n^2-80n^2+200n+100n-125$$
$$=64n^3+(-160n^2-80n^2)+(200n+100n)-125$$
$$=64n^3-240n^2+300n-125$$

38. $(3z-2)^3$

$$(3z-2)(3z-2)=3z(3z)+3z(-2)+(-2)3z+(-2)(-2)$$
$$=9z^2-6z-6z+4$$
$$=9z^2-12z+4$$
$$(9z^2-12z+4)(3z-2)=9z^2(3z)+9z^2(-2)+(-12z)(3z)+(-12z)(-2)+4(3z)+4(-2)$$
$$=27z^3-18z^2-36z^2+24z+12z-8$$
$$=27z^3+(-18z^2-36z^2)+(24z+12z)-8$$
$$=27z^3-54z^2+36z-8$$

39. $\frac{1}{4}(16x-12y)+\frac{1}{3}(9x+3y)$

$$\frac{1}{4}(16x-12y)+\frac{1}{3}(9x+3y)=\frac{1}{4}(16x)+\frac{1}{4}(-12y)+\frac{1}{3}(9x)+\frac{1}{3}(3y)$$
$$=4x-3y+3x+y$$
$$=(4x+3x)+(-3y+y)$$
$$=7x-2y$$

13	Divide polynomials using synthetic division	Exercises (11-16)	P105

Examples 4 and 5

Simplify using synthetic division.

11. $(3v^2 - 7v - 10)(v - 4)^{-1}$

12. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

13	Divide polynomials using synthetic division	Exercises (11-16)	P105

13. $\frac{y^3 + 6}{y + 2}$

14. $\frac{2x^3 - x^2 - 18x + 32}{2x - 6}$

13	Divide polynomials using synthetic division	Exercises (11-16)	P105

15. $(4p^3 - p^2 + 2p) \div (3p - 1)$

16. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

13	Divide polynomials using synthetic division	Exercises (11-16)	P105

Solution

solution method

Learn Dividing Polynomials by Using Synthetic Division

Synthetic division is an alternate method used to divide a polynomial by a binomial of degree 1. You may find this to be a quicker, simpler method.

Key Concept • Synthetic Division

Step 1 After writing a polynomial in standard form, write the coefficients of the dividend. If the dividend is missing a term, use 0 as a placeholder. Write the constant a of the divisor $x - a$ in the box. Bring the first coefficient down.

Step 2 Multiply the number just written in the bottom row by a, and write the product under the next coefficient.

Step 3 Add the product and the coefficient above it.

Step 4 Repeat **Steps 2** and **3** until you reach a sum in the last column.

Step 5 Write the quotient. The numbers along the bottom row are the coefficients of the quotient. The power of the first term is one less than the degree of the dividend. The final number is the remainder.

Example 4 Use Synthetic Division

Example 5 Divisor with a Coefficient Other Than 1

11. $(3v^2 - 7v - 10)(v - 4)^{-1}$

$(3v^2 - 7v - 10)(v - 4)^{-1} = (3v^2 - 7v - 10) \div (v - 4)$

Write the coefficients of the dividend and write the constant a in the box. Then bring the first coefficient down.

$$\begin{array}{r|rrr} +4 & 3 & -7 & -10 \\ & \downarrow & & \\ \hline & 3 & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $3(4) = 12$.

$$\begin{array}{r|rrr} +4 & 3 & -7 & -10 \\ & & 12 & \\ \hline & 3 & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrr} +4 & 3 & -7 & -10 \\ & & 12 & \\ \hline & 3 & 5 & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $3(5) = 20$.

$$\begin{array}{r|rrr} +4 & 3 & -7 & -10 \\ & & 12 & 20 \\ \hline & 3 & 5 & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rr|r} +4 & 3 & -7 & -10 \\ & & 12 & 20 \\ \hline & 3 & 5 & 10 \end{array}$$

Write the quotient. Because the degree of the dividend is 2 and the degree of the divisor is 1, the degree of the quotient is 1.
The final sum in the synthetic division is 10, so the remainder is 10.

The quotient is $3v + 5 + \dfrac{10}{v-4}$.

12. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

$(3t^4+4t^3-32t^2-5t-20)(t+4)^{-1} = (3t^4+4t^3-32t^2-5t-20) \div (t+4)$

Write the coefficients of the dividend and write the constant a in the box.
Because $t + 4 = t - (-4)$, $a = -4$.
Then bring the first coefficient down.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & & & & \\ \hline & 3 & & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-4(3) = -12$.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & & & \\ \hline & 3 & & & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & & & \\ \hline & 3 & -8 & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-4(-8) = 32$.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & 32 & & \\ \hline & 3 & -8 & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-4(0) = 0$.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & 32 & 0 & \\ \hline & 3 & -8 & 0 & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & 32 & 0 & \\ \hline & 3 & -8 & 0 & -5 & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-4(-5) = 20$.

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & 32 & 0 & 20 \\ \hline & 3 & -8 & 0 & -5 & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr|r} -4 & 3 & 4 & -32 & -5 & -20 \\ & \downarrow & -12 & 32 & 0 & 20 \\ \hline & 3 & -8 & 0 & -5 & 0 \end{array}$$

Write the quotient. Because the degree of the dividend is 4 and the degree of the divisor is 1, the degree of the quotient is 3.
The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is $3t^3 - 8t^2 - 5$.

13. $\dfrac{y^3+6}{y+2}$

$$\frac{y^3+6}{y+2}=(y^3+6)\div(y+2)$$

Use placeholder zeroes in the dividend.

$$(y^3+6)\div(y+2)=(y^3+0y^2+0y+6)\div(y+2)$$

Write the coefficients of the dividend and write the constant a in the box.
Then bring the first coefficient down.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(1)=-2$.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & & \\ \hline & 1 & & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & & \\ \hline & 1 & -2 & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(-2)=4$.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & 4 & \\ \hline & 1 & -2 & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & 4 & \\ \hline & 1 & -2 & 4 & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(4)=-8$.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrr|r} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & -2 \end{array}$$

Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2.
The final sum in the synthetic division is -2, so the remainder is -2.

The quotient is $y^2-2y+4-\dfrac{2}{y+2}$.

Solution

14. $\dfrac{2x^3 - x^2 - 18x + 32}{2x - 6}$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 6 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & |-2 \end{array}$$

Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2.
The final sum in the synthetic division is –2, so the remainder is –2.

The quotient is $y^2 - 2y + 4 - \dfrac{2}{y+2}$.

4. $\dfrac{2x^3 - x^2 - 18x + 32}{2x - 6}$

SOLUTION:

To use synthetic division, the lead coefficient of the divisor must be 1.

$$\frac{(2x^3 - x^2 - 18x + 32) \div 2}{(2x - 6) \div 2}$$ Divide the numerator and denominator by 2.

$$= \frac{x^3 - \frac{1}{2}x^2 - 9x + 16}{x - 3}$$ Simplify the numerator and denominator.

$$\frac{x^3 - \frac{1}{2}x^2 - 9x + 16}{x - 3} = \left(x^3 - \frac{1}{2}x^2 - 9x + 16\right) \div (x - 3)$$

Write the coefficients of the dividend and write the constant a in the box.
Then bring the first coefficient down.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & & & \\ \hline & 1 & & & | \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $3(1) = 3$.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & & \\ \hline & 1 & & & | \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & & \\ \hline & 1 & \frac{5}{2} & & | \end{array}$$

Multiply by a and write the product.

The product of the coefficient and a is $3\left(\frac{5}{2}\right) = \frac{15}{2}$.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & \frac{15}{2} & \\ \hline & 1 & \frac{5}{2} & & | \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & \frac{15}{2} & \\ \hline & 1 & \frac{5}{2} & -\frac{3}{2} & | \end{array}$$

Multiply by a and write the product.

The product of the coefficient and a is $3\left(-\frac{3}{2}\right) = -\frac{9}{2}$.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & \frac{15}{2} & -\frac{9}{2} \\ \hline & 1 & \frac{5}{2} & -\frac{3}{2} & | \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr} 3 & 1 & -\frac{1}{2} & -9 & 16 \\ & \downarrow & 3 & \frac{15}{2} & -\frac{9}{2} \\ \hline & 1 & \frac{5}{2} & -\frac{3}{2} & |\frac{23}{2} \end{array}$$

Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2.

The final sum in the synthetic division is $\frac{23}{2}$, so the remainder is $\frac{23}{2}$.

The quotient is $x^2 + \frac{5}{2}x - \frac{3}{2} + \dfrac{\frac{23}{2}}{x-3}$, or $x^2 + \frac{5}{2}x - \frac{3}{2} + \dfrac{23}{2x-6}$.

15. $(4p^3 - p^2 + 2p) \div (3p - 1)$

To use synthetic division, the lead coefficient of the divisor must be 1.

$$\frac{(4p^3 - p^2 + 2p) \div 3}{(3p-1) \div 3}$$ Divide the numerator an

$$= \frac{\frac{4}{3}p^3 - \frac{1}{3}p^2 + \frac{2}{3}p}{p - \frac{1}{3}}$$ Simplify the numerator .

Use a placeholder zero in the dividend.

$$\frac{\frac{4}{3}p^3 - \frac{1}{3}p^2 + \frac{2}{3}p}{p - \frac{1}{3}} = \left(\frac{4}{3}p^3 - \frac{1}{3}p^2 + \frac{2}{3}p + 0\right) \div \left(p - \frac{1}{3}\right)$$

Write the coefficients of the dividend and write the constant a in the box. Then bring the first coefficient down.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & & & \\ \hline & \frac{4}{3} & & & \end{array}$$

Multiply by a and write the product.

The product of the coefficient and a is $\frac{1}{3}\left(\frac{4}{3}\right) = \frac{4}{9}$.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & & \\ \hline & \frac{4}{3} & & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & & \\ \hline & \frac{4}{3} & \frac{1}{9} & & \end{array}$$

Multiply by a and write the product.

The product of the coefficient and a is $\frac{1}{3}\left(\frac{1}{9}\right) = \frac{1}{27}$.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & \frac{1}{27} & \\ \hline & \frac{4}{3} & \frac{1}{9} & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & \frac{1}{27} & \\ \hline & \frac{4}{3} & \frac{1}{9} & \frac{19}{27} & \end{array}$$

Multiply by a and write the product.

The product of the coefficient and a is $\frac{1}{3}\left(\frac{19}{27}\right) = \frac{19}{81}$.

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & \frac{1}{27} & \frac{19}{81} \\ \hline & \frac{4}{3} & \frac{1}{9} & \frac{19}{27} & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{c|ccc|c} \frac{1}{3} & \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ & \downarrow & \frac{4}{9} & \frac{1}{27} & \frac{19}{27} \\ \hline & \frac{4}{3} & \frac{1}{9} & \frac{19}{27} & \frac{19}{27} \end{array}$$

Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2.

The final sum in the synthetic division is $\frac{19}{27}$, so the remainder is $\frac{19}{27}$.

16. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

Write the coefficients of the dividend and write the constant a in the box.
Because $c + 2 = c - (-2)$, $a = -2$.
Then bring the first coefficient down.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & & & & \\ \hline & 3 & & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(3) = -6$.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & & & \\ \hline & 3 & & & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & & & \\ \hline & 3 & 0 & & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(0) = 0$.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & & \\ \hline & 3 & 0 & & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & & \\ \hline & 3 & 0 & 0 & & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(0) = 0$.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & 0 & \\ \hline & 3 & 0 & 0 & & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & 0 & \\ \hline & 3 & 0 & 0 & -2 & \end{array}$$

Multiply by a and write the product.
The product of the coefficient and a is $-2(-2) = 4$.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & 0 & 4 \\ \hline & 3 & 0 & 0 & -2 & \end{array}$$

Add the product and the coefficient.

$$\begin{array}{r|rrrr|r} -2 & 3 & 6 & 0 & -2 & 4 \\ & \downarrow & -6 & 0 & 0 & 4 \\ \hline & 3 & 0 & 0 & -2 & 8 \end{array}$$

Write the quotient. Because the degree of the dividend is 4 and the degree of the divisor is 1, the degree of the quotient is 3.

The final sum in the synthetic division is 8, so the remainder is $\frac{8}{c+2}$.

The quotient is $3c^3 - 2 + \frac{8}{c+2}$.

14	Determine whether a binomial is a factor of a polynomial by using synthetic substitution	Exercises (23-30)	P139

Example 3

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

23. $x^3 - 3x + 2; x + 2$

24. $x^4 + 2x^3 - 8x - 16; x + 2$

25. $x^3 - x^2 - 10x - 8; x + 2$

26. $x^3 - x^2 - 5x - 3; x - 3$

27. $2x^3 + 17x^2 + 23x - 42; x - 1$

28. $2x^3 + 7x^2 - 53x - 28; x - 4$

14	Determine whether a binomial is a factor of a polynomial by using synthetic substitution	Exercises (23-30)	P139

29. $x^4 + 2x^3 + 2x^2 - 2x - 3;\ x - 1$

30. $x^3 + 2x^2 - x - 2;\ x + 2$

solution method

Learn The Factor Theorem

When a binomial evenly divides a polynomial, the binomial is a factor of the polynomial. The quotient of this division is called a depressed polynomial. The **depressed polynomial** has a degree that is one less than the original polynomial.

A special case of the Remainder Theorem is called the Factor Theorem.

Key Concept • Factor Theorem

Words: The binomial $x - a$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.

Example:

$$\underbrace{x^3 - x^2 - 30x + 72}_{\text{dividend}} = \underbrace{(x^2 - 7x + 12)}_{\text{quotient}} \cdot \underbrace{(x + 6)}_{\text{divisor}} + \underbrace{0}_{\text{remainder}}$$

$x + 6$ is a factor of $x^3 - x^2 - 30x + 72$.

Example 3 Use the Factor Theorem

23. $x^3 - 3x + 2$; $x + 2$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & \mid 0 \end{array}$$

Because the remainder is 0, $x + 2$ is a factor of the polynomial by the Factor Theorem. So $x^3 - 3x + 2$ can be factored as $(x + 2)(x^2 - 2x + 1)$. The depressed polynomial is $x^2 - 2x + 1$.

Check to see if this polynomial can be factored.

$x^2 - 2x + 1 = (x - 1)(x - 1)$ or $(x - 1)^2$ Factor the polynomial.

Therefore, $x^3 - 3x + 2 = (x + 2)(x - 1)^2$.

24. $x^4 + 2x^3 - 8x - 16$; $x + 2$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 0 & -8 & -16 \\ & & -2 & 0 & 0 & 16 \\ \hline & 1 & 0 & 0 & -8 & \mid 0 \end{array}$$

Because the remainder is 0, $x + 2$ is a factor of the polynomial by the Factor Theorem. So $x^4 + 2x^3 - 8x - 16$ can be factored as $(x + 2)(x^3 - 8)$. The depressed polynomial is $x^3 - 8$.

Check to see if this polynomial can be factored.

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ Factor the polynomial.

Therefore, $x^4 + 2x^3 - 8x - 16 = (x + 2)(x - 2)(x^2 + 2x + 4)$.

25. $x^3 - x^2 - 10x - 8; x + 2$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 2 & -8 \\ & & -2 & -6 & 8 \\ \hline & 1 & 3 & -4 & |0 \end{array}$$

Because the remainder is 0, $x + 2$ is a factor of the polynomial by the Factor Theorem. So $x^3 + 5x^2 + 2x - 8$ can be factored as $(x + 2)(x^2 + 3x - 4)$. The depressed polynomial is $x^2 + 3x - 4$.

Check to see if this polynomial can be factored.

$x^2 + 3x - 4 = (x + 4)(x - 1)$ Factor the polynomial.

Therefore, $x^3 + 5x^2 + 2x - 8 = (x + 2)(x + 4)(x - 1)$.

26. $x^3 - x^2 - 5x - 3; x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -5 & -3 \\ & & 3 & 6 & 3 \\ \hline & 1 & 2 & 1 & |0 \end{array}$$

Because the remainder is 0, $x - 3$ is a factor of the polynomial by the Factor Theorem. So $x^3 - x^2 - 5x - 3$ can be factored as $(x - 3)(x^2 + 2x + 1)$. The depressed polynomial is $x^2 + 2x + 1$.

Check to see if this polynomial can be factored.

$x^2 + 2x + 1 = (x + 1)(x + 1)$ or $(x + 1)^2$ Factor the polynomial.

Therefore, $x^3 - x^2 - 5x - 3 = (x - 3)(x + 1)^2$.

27. $2x^3 + 17x^2 + 23x - 42; x - 1$

$$\begin{array}{r|rrrr} 1 & 2 & 17 & 23 & -42 \\ & & 2 & 19 & 42 \\ \hline & 2 & 19 & 42 & |0 \end{array}$$

Because the remainder is 0, $x - 1$ is a factor of the polynomial by the Factor Theorem. So $2x^3 + 17x^2 + 23x - 42$ can be factored as $(x - 1)(2x^2 + 19x + 42)$. The depressed polynomial is $2x^2 + 19x + 42$.

Check to see if this polynomial can be factored.

$2x^2 + 19x + 42 = (x + 6)(2x + 7)$ Factor the polynomial.

Therefore, $2x^3 + 17x^2 + 23x - 42 = (x - 1)(x + 6)(2x + 7)$.

28. $2x^3 + 7x^2 - 53x - 28; x - 4$

$$\begin{array}{r|rrrr} 4 & 2 & 7 & -53 & -28 \\ & & 8 & 60 & 28 \\ \hline & 2 & 15 & 7 & |0 \end{array}$$

Because the remainder is 0, $x - 4$ is a factor of the polynomial by the Factor Theorem. So $2x^3 + 7x^2 - 53x - 28$ can be factored as $(x - 4)(2x^2 + 15x + 7)$. The depressed polynomial is $2x^2 + 15x + 7$.

Check to see if this polynomial can be factored.

$2x^2 + 15x + 7 = (x + 7)(2x + 1)$ Factor the polynomial.

Therefore, $2x^3 + 7x^2 - 53x - 28 = (x - 4)(x + 7)(2x + 1)$.

29. $x^4 + 2x^3 + 2x^2 - 2x - 3;\ x - 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & 2 & -2 & -3 \\ & & 1 & 3 & 5 & 3 \\ \hline & 1 & 3 & 5 & 3 & |0 \end{array}$$

Because the remainder is 0, $x - 1$ is a factor of the polynomial by the Factor Theorem. So $x^4 + 2x^3 + 2x^2 - 2x - 3$ can be factored as $(x - 1)(x^3 + 3x^2 + 5x + 3)$. The depressed polynomial is $x^3 + 3x^2 + 5x + 3$.

Because $(x - 1)$ is a factor, determine if $(x + 1)$ is a factor.

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 5 & 3 \\ & & -1 & -2 & -3 \\ \hline & 1 & 2 & 3 & |0 \end{array}$$

Because the remainder is 0, $x + 1$ is a factor of the polynomial by the Factor Theorem. So $x^4 + 2x^3 + 2x^2 - 2x - 3$ can be factored as $(x - 1)(x + 1)(x^2 + 2x + 3)$. The depressed polynomial is $x^2 + 2x + 3$. The depressed polynomial cannot be factored.

Therefore, $x^4 + 2x^3 + 2x^2 - 2x - 3 = (x - 1)(x + 1)(x^2 + 2x + 3)$.

30. $x^3 + 2x^2 - x - 2;\ x + 2$

$$\begin{array}{r|rrrr} 7 & 3 & -19 & -15 & 7 \\ & & 21 & 14 & -7 \\ \hline & 3 & 2 & -1 & |0 \end{array}$$

Because the remainder is 0, $x - 7$ is a factor of the polynomial by the Factor Theorem. So $3x^3 - 19x^2 - 15x + 7$ can be factored as $(x - 7)(3x^2 + 2x - 1)$. The depressed polynomial is $3x^2 + 2x - 1$.

Check to see if this polynomial can be factored.

$3x^2 + 2x - 1 = (x + 1)(3x - 1)$ Factor the polynomial.

Therefore, $3x^3 - 19x^2 - 15x + 7 = (x - 7)(x + 1)(3x - 1)$.

15	Graph vertical translations of trigonometric functions.	Exercises(1-10)	P127

Examples 1-3

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

2. $64x^4 + xy^3$

3. $a^8 - a^2b^6$

4. $x^6y^3 + y^9$

5. $18x^6 + 5y^6$

6. $w^3 - 2y^3$

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

9. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

Learn Solving Polynomial Equations by Factoring

Like quadratics, some polynomials of higher degrees can be factored. A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**. Like a prime real number, the only factors of a prime polynomial are 1 and itself.

Similar to quadratics, some cubic polynomials can be factored by using polynomial identities.

Key Concept • Sum and Difference of Cubes

Factoring Technique	General Case
Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Polynomials can be factored by using a variety of methods, the most common of which are summarized in the table below. When factoring a polynomial, always look for a common factor first to simplify the expression. Then, determine whether the resulting polynomial factors can be factored using one or more methods.

Concept Summary • Factoring Techniques

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$2a^4b^3 + 6ab = 2ab(a^3b^2 + 6)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
	Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd$ $= (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$

1. $8c^3 - 27d^3$

$$8c^3 - 27d^3$$
$$= (2c)^3 - (3d)^3$$
$$= (2c - 3d)[(2c)^2 + 2c(3d) + (3d)^2]$$
$$= (2c - 3d)(4c^2 + 6cd + 9d^2)$$

2. $64x^4 + xy^3$

$$64x^4 + xy^3$$
$$= x(64x^3 + y^3)$$
$$= x[(4x)^3 + (y)^3]$$
$$= x(4x + y)[(4x)^2 - 4x(y) + (y)^2]$$
$$= x(4x + y)(16x^2 - 4xy + y^2)$$

3. $a^8 - a^2b^6$

$$a^8 - a^2b^6$$
$$= a^2(a^6 - b^6)$$
$$= a^2[(a^3)^2 - (b^3)^2]$$
$$= a^2(a^3 - b^3)(a^3 + b^3)$$
$$= a^2[(a)^3 - (b)^3][(a)^3 + (b)^3]$$
$$= a^2(a - b)[(a)^2 + a(b) + (b)^2](a + b)[(a)^2 - a(b) + (b)^2]$$
$$= a^2(a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)$$

4. $x^6y^3 + y^9$

$$x^6y^3 + y^9$$
$$= y^3(x^6 + y^6)$$
$$= y^3[(x^2)^3 + (y^2)^3]$$
$$= y^3(x^2 + y^2)[(x^2)^2 - x^2(y^2) + (y^2)^2]$$
$$= y^3(x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

5. $18x^6 + 5y^6$

There is not an integer, that when raised to the third power is equal to 18. There is not an integer, that when raised to the third power is equal to 5. Therefore, the polynomial $18x^6 + 5y^6$ is prime.

6. $w^3 - 2y^3$

There is not an integer, that when raised to the third power is equal to 2. Therefore, the polynomial $w^3 - 2y^3$ is prime.

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

$$\begin{aligned}
&gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2 \\
&= (6fx^2 + gx^2 - 3hx^2) + (-6fy^2 - gy^2 + 3hy^2) \\
&= x^2(6f + g - 3h) - y^2(6f + g - 3h) \\
&= (x^2 - y^2)(6f + g - 3h) \\
&= [(x)^2 - (y)^2](6f + g - 3h) \\
&= (x + y)(x - y)(6f + g - 3h)
\end{aligned}$$

8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

$$12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$$
$$= (12ax^2 - 18bx^2 + 24cx^2) + (-10ay^2 + 15by^2 - 20cy^2)$$
$$= 6x^2(2a - 3b + 4c) - 5y^2(2a - 3b + 4c)$$
$$= (6x^2 - 5y^2)(2a - 3b + 4c)$$

9. $a^3x^2 - 16\,a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

$$a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$$
$$= (a^3x^2 - 16a^3x + 64a^3) + (-b^3x^2 + 16b^3x - 64b^3)$$
$$= a^3(x^2 - 16x + 64) - b^3(x^2 - 16x + 64)$$
$$= (a^3 - b^3)(x^2 - 16x + 64)$$
$$= [(a)^3 - (b)^3](x^2 - 16x + 64)$$
$$= (a - b)[(a)^2 + a(b) + (b)^2](x^2 - 16x + 64)$$
$$= (a - b)(a^2 + ab + b^2)(x^2 - 16x + 64)$$
$$= (a - b)(a^2 + ab + b^2)(x - 8)^2$$

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

$$8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$$
$$= (8x^5 + 80x^4 + 200x^3) + (-x^2y^3 - 10xy^3 - 25y^3)$$
$$= 8x^3(x^2 + 10x + 25) - y^3(x^2 + 10x + 25)$$
$$= (8x^3 - y^3)(x^2 + 10x + 25)$$
$$= [(2x)^3 - (y)^3](x^2 + 10x + 25)$$
$$= (2x - y)[(2x)^2 + 2x(y) + (y)^2](x^2 + 10x + 25)$$
$$= (2x - y)(4x^2 + 2xy + y^2)(x^2 + 10x + 25)$$
$$= (2x - y)(4x^2 + 2xy + y^2)(x + 5)^2$$

الأسئلة الكتابية للامتحان

FRQ - الأسئلة المقالية	16	Graph quadratic functions	Exercises (27-32)	P11
	17	Solve quadratic equations by using the Quadratic Formula	Exercises(8-23)	P47
	18	Solve quadratic equations by factoring	Exercises (31-34)	P107
	19	Use Pascal's Triangle to write binomial expansions	Exercises (1-12)	P111
	20	Factorize polynomials	Example2	P120

Mixed Exercises

Complete parts a-c for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

27. $f(x) = 2x^2 - 6x - 9$

28. $f(x) = -3x^2 - 9x + 2$

29. $f(x) = -4x^2 + 5x$

30. $f(x) = 2x^2 + 11x$

31. $f(x) = 0.25x^2 + 3x + 4$

32. $f(x) = -0.75x^2 + 4x + 6$

27. $f(x) = 2x^2 - 6x - 9$

x	$2x^2 - 6x - 9$	$(x, f(x))$

28. $f(x) = -3x^2 - 9x + 2$

x	$-3x^2 - 9x + 2$	$(x, f(x))$

29. $f(x) = -4x^2 + 5x$

x	$-4x^2 + 5x$	$(x, f(x))$

30. $f(x) = 2x^2 + 11x$

x	$2x^2 + 11x$	$(x, f(x))$

31. $f(x) = 0.25x^2 + 3x + 4$

x	$0.25x^2 + 3x + 4$	$(x, f(x))$

32. $f(x) = -0.75x^2 + 4x + 6$

x	$0.75x^2 + 4x + 6$	$(x, f(x))$

27. $f(x) = 2x^2 - 6x - 9$

a. For $f(x) = 2x^2 - 6x - 9$, $a = 2$, $b = -6$, and $c = -9$. c is the y-intercept, so the y-intercept is -9.

Find the axis of symmetry.

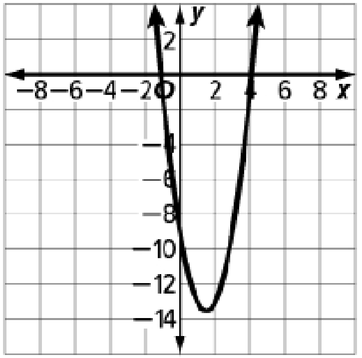
$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry.}$$
$$= -\frac{-6}{2(2)} \quad a = 2, b = -6$$
$$= \frac{3}{2} \text{ or } 1.5 \quad \text{Simplify.}$$

The equation of the axis of symmetry is $x = 1.5$, so the x-coordinate of the vertex is 1.5.

b. Complete the table.

x	$2x^2 - 6x - 9$	$(x, f(x))$
0	$2(0)^2 - 6(0) - 9$	-9
1	$2(1)^2 - 6(1) - 9$	-13
1.5	$2(1.5)^2 - 6(1.5) - 9$	-13.5
2	$2(2)^2 - 6(2) - 9$	-13
3	$2(3)^2 - 6(3) - 9$	-9

c. Graph the function.



28. $f(x) = -3x^2 - 9x + 2$

a. For $f(x) = -3x^2 - 9x + 2$, $a = -3$, $b = -9$, and $c = 2$. c is the y-intercept, so the y-intercept is 2.

Find the axis of symmetry.

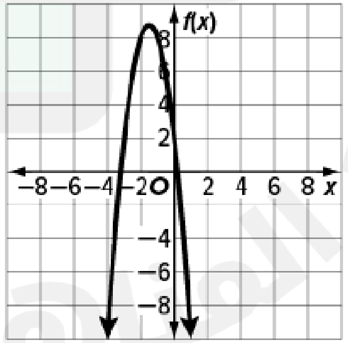
$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry.}$$
$$= -\frac{-9}{2(-3)} \quad a = -3, b = -9$$
$$= -\frac{3}{2} \text{ or } -1.5 \quad \text{Simplify.}$$

The equation of the axis of symmetry is $x = -1.5$, so the x-coordinate of the vertex is -1.5.

b. Complete the table.

x	$-3x^2 - 9x + 2$	$(x, f(x))$
-3	$-3(-3)^2 - 9(-3) + 2$	2
-2	$-3(-2)^2 - 9(-2) + 2$	8
-1.5	$-3(-1.5)^2 - 9(-1.5) + 2$	8.75
-1	$-3(-1)^2 - 9(-1) + 2$	8
0	$-3(0)^2 - 9(0) + 2$	2

c. Graph the function.



29. $f(x) = -4x^2 + 5x$

a. For $f(x) = -4x^2 + 5x$, $a = -4$, $b = 5$, and $c = 0$. c is the y-intercept, so the y-intercept is 0.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

$= -\frac{5}{2(-4)}$ $a = -4, b = 5$

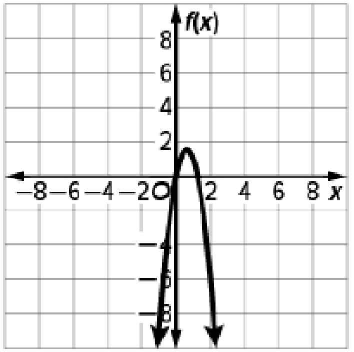
$= \frac{5}{8}$ Simplify.

The equation of the axis of symmetry is $x = \frac{5}{8}$, so the x-coordinate of the vertex is $\frac{5}{8}$.

b. Complete the table.

x	$-4x^2 + 5x$	$(x, f(x))$
$-\frac{3}{4}$	$-4\left(-\frac{3}{4}\right)^2 + 5\left(-\frac{3}{4}\right)$	-6
$\frac{1}{4}$	$-4\left(\frac{1}{4}\right)^2 + 5\left(\frac{1}{4}\right)$	1
$\frac{5}{8}$	$-4\left(\frac{5}{8}\right)^2 + 5\left(\frac{5}{8}\right)$	1.5625
1	$-4(1)^2 + 5(1)$	1
2	$-4(2)^2 + 5(2)$	-6

c. Graph the function.



30. $f(x) = 2x^2 + 11x$

a. For $f(x) = 2x^2 + 11x$, $a = 2$, $b = 11$, and $c = 0$. c is the y-intercept, so the y-intercept is 0.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

$= -\frac{11}{2(2)}$ $a = 2, b = 11$

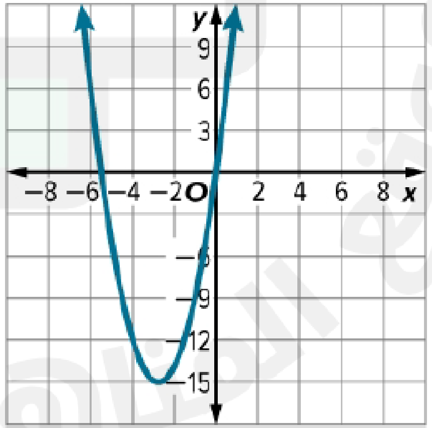
$= -2.75$ Simplify.

The equation of the axis of symmetry is $x = -2.75$, so the x-coordinate of the vertex is -2.75.

b. Complete the table.

x	$2x^2 + 11x$	$(x, f(x))$
-4	$2(-4)^2 + 11(-4)$	-12
-3	$2(-3)^2 + 11(-3)$	-15
-2.75	$2(-2.75)^2 + 11(-2.75)$	-15.125
-2.5	$2(-2.5)^2 + 11(-2.5)$	-15
-1.5	$2(-1.5)^2 + 11(-1.5)$	-9

c. Graph the function.



31. $f(x) = 0.25x^2 + 3x + 4$

a. For $f(x) = 0.25x^2 + 3x + 4$, $a = 0.25$, $b = 3$, and $c = 4$. c is the y-intercept, so the y-intercept is 4.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

$= -\frac{3}{2(0.25)}$ $a = 0.25$, $b = 3$

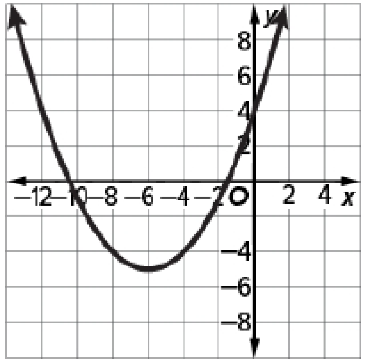
$= 6$ Simplify.

The equation of the axis of symmetry is $x = 6$, so the x-coordinate of the vertex is 6.

b. Complete the table.

x	$0.25x^2 + 3x + 4$	$(x, f(x))$
–10	$0.25(0)^2 + 3(0) + 4$	–1
–8	$0.25(0)^2 + 3(0) + 4$	–4
–6	$0.25(0)^2 + 3(0) + 4$	–5
–4	$0.25(0)^2 + 3(0) + 4$	–4
–2	$0.25(0)^2 + 3(0) + 4$	–1

c. Graph the function.



32. $f(x) = -0.75x^2 + 4x + 6$

a. For $f(x) = -0.75x^2 + 4x + 6$, $a = -0.75$, $b = 4$, and $c = 6$. c is the y-intercept, so the y-intercept is 6.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

$= -\frac{4}{2(-0.75)}$ $a = -0.75$, $b = 4$

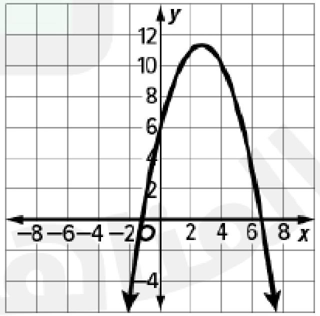
$= \frac{8}{3}$ Simplify.

The equation of the axis of symmetry is $x = \frac{8}{3}$, so the x-coordinate of the vertex is $\frac{8}{3}$.

b. Complete the table.

x	$0.75x^2 + 4x + 6$	$(x, f(x))$
$\frac{4}{3}$	$-0.75\left(\frac{4}{3}\right)^2 + 4\left(\frac{4}{3}\right) + 6$	10
$\frac{7}{3}$	$-0.75\left(\frac{7}{3}\right)^2 + 4\left(\frac{7}{3}\right) + 6$	11.25
$\frac{8}{3}$	$-0.75\left(\frac{8}{3}\right)^2 + 4\left(\frac{8}{3}\right) + 6$	$11\frac{1}{3}$
3	$-0.75(3)^2 + 4(3) + 6$	11.25
4	$-0.75(4)^2 + 4(4) + 6$	10

c. Graph the function.



17	Solve quadratic equations by using the Quadratic Formula	Exercises(8-23)	P47

Examples 2 and 3

Solve each equation by using the Quadratic Formula.

8. $x^2 + 2x - 35 = 0$

9. $4x^2 + 19x - 5 = 0$

10. $2x^2 - x - 15 = 0$

11. $3x^2 + 5x = 2$

12. $x^2 + x - 8 = 0$

13. $8x^2 + 5x - 1 = 0$

14. $x^2 - x - 5 = 0$

15. $16x^2 - 24x - 25 = 0$

Examples 2 and 3

Solve each equation by using the Quadratic Formula.

16. $x^2 - 6x + 21 = 0$

17. $x^2 + 25 = 0$

18. $3x^2 + 36 = 0$

19. $8x^2 - 4x + 1 = 0$

20. $2x^2 + 2x + 3 = 0$

21. $x^2 - 14x + 53 = 0$

22. $4x^2 + 2x + 9 = 0$

23. $3x^2 - 6x + 11 = 0$

Solution

solution method

Learn Using the Quadratic Formula

To solve any quadratic equation, you can use the Quadratic Formula.

Key Concept • Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2 Real Roots, c Is Negative

Example 3 Complex Roots

8. $x^2 + 2x - 35 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-35)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{144}}{2}$$
$$= \frac{-2 \pm 12}{2}$$
$$= 5 \text{ or } -7$$

9. $4x^2 + 19x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-19 \pm \sqrt{(19)^2 - 4(4)(-5)}}{2(4)}$$
$$= \frac{-19 \pm \sqrt{441}}{8}$$
$$= \frac{-19 \pm 21}{8}$$
$$= \frac{1}{4} \text{ or } -5$$

10. $2x^2 - x - 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-15)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{121}}{4}$$
$$= \frac{1 \pm 11}{4}$$
$$= 3 \text{ or } -\frac{5}{2}$$

11. $3x^2 + 5x = 2$

$3x^2 + 5x = 2$ Original eq

$3x^2 + 5x - 2 = 0$ Subtract 2 f

Use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{-5 \pm \sqrt{49}}{6}$$
$$= \frac{-5 \pm 7}{6}$$
$$= \frac{1}{3} \text{ or } -2$$

Solution

12. $x^2 + x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-8)}}{2(1)} = \frac{-1 \pm \sqrt{33}}{2}$$

13. $8x^2 + 5x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{(5)^2 - 4(8)(-1)}}{2(8)} = \frac{-1 \pm \sqrt{57}}{16}$$

14. $x^2 - x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2(1)} = \frac{1 \pm \sqrt{21}}{2}$$

15. $16x^2 - 24x - 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(-25)}}{2(16)} = \frac{24 \pm \sqrt{2176}}{32} = \frac{24 \pm \sqrt{64} \cdot \sqrt{34}}{32} = \frac{24 \pm 8\sqrt{34}}{32} = \frac{3 \pm \sqrt{34}}{4}$$

16. $x^2 - 6x + 21 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(21)}}{2(1)} = \frac{6 \pm \sqrt{-48}}{2} = \frac{6 \pm \sqrt{-1} \cdot \sqrt{16} \cdot \sqrt{3}}{2} = \frac{6 \pm 4i\sqrt{3}}{2} = 3 \pm 2i\sqrt{3}$$

17. $x^2 + 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(25)}}{2(1)} = \frac{\pm\sqrt{-100}}{2} = \frac{\pm\sqrt{-1} \cdot \sqrt{100}}{2} = \frac{\pm 10i}{2} = \pm 5i$$

18. $3x^2 + 36 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{(0)^2 - 4(3)(36)}}{2(3)} = \frac{\pm\sqrt{-432}}{6} = \frac{\pm\sqrt{-1} \cdot \sqrt{144} \cdot \sqrt{3}}{6} = \frac{\pm 12i\sqrt{3}}{6} = \pm 2i\sqrt{3}$$

19. $8x^2 - 4x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(8)(1)}}{2(8)} = \frac{4 \pm \sqrt{-16}}{16} = \frac{4 \pm \sqrt{-1} \cdot \sqrt{16}}{16} = \frac{4 \pm 4i}{16} = \frac{1 \pm i}{4}$$

Solution

20. $2x^2 + 2x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{-20}}{4}$$
$$= \frac{-2 \pm \sqrt{-1} \bullet \sqrt{4} \bullet \sqrt{5}}{4}$$
$$= \frac{-2 \pm 2i\sqrt{5}}{4}$$
$$= \frac{-1 \pm i\sqrt{5}}{2}$$

21. $x^2 - 14x + 53 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(53)}}{2(1)}$$
$$= \frac{14 \pm \sqrt{-16}}{2}$$
$$= \frac{14 \pm \sqrt{-1} \bullet \sqrt{16}}{2}$$
$$= \frac{14 \pm 4i}{2}$$
$$= 7 \pm 2i$$

22. $4x^2 + 2x + 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(4)(9)}}{2(4)}$$
$$= \frac{-2 \pm \sqrt{-140}}{8}$$
$$= \frac{-2 \pm \sqrt{-1} \bullet \sqrt{4} \bullet \sqrt{35}}{8}$$
$$= \frac{-2 \pm 2i\sqrt{35}}{8}$$
$$= \frac{-1 \pm i\sqrt{35}}{4}$$

23. $3x^2 - 6x + 11 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(11)}}{2(3)}$$
$$= \frac{6 \pm \sqrt{-96}}{6}$$
$$= \frac{6 \pm \sqrt{-1} \bullet \sqrt{16} \bullet \sqrt{6}}{6}$$
$$= \frac{6 \pm 4i\sqrt{6}}{6}$$
$$= \frac{3 \pm 2i\sqrt{6}}{3}$$

18	Solve quadratic equations by factoring	Exercises (31-34)	P107

31. **VOLUME** The volume of a cylinder is $\pi(x^3 + 32x^2 - 304x + 640)$. If the height of the cylinder is $x + 40$ feet, find the area of its base in terms of x and π.

32. **REASONING** Rewrite $\frac{6x^4 + 2x^3 - 16x^2 + 24x + 32}{2x + 4}$ as $q(x) + \frac{r(x)}{d(x)}$ using long division.

What does the remainder indicate in this problem?

33. **CONSTRUCT ARGUMENTS** Determine whether you have enough information to fill in the missing pieces of the long division exercise shown. If so, copy and complete the long division. Justify your response.

$$
\begin{array}{r}
3x - \square \\
\square \overline{\big) 9x^2 + \square} \\
\underline{9x^2 + 3x} \\
-3x + 5
\end{array}
$$

34. **REGULARITY** Rewrite $\frac{2x^5 - 7x^4 - 15x^3 + 2x^2 + 3x + 6}{2x + 3}$ as $q(x) + \frac{r(x)}{g(x)}$ using long division.

a. Identify $q(x)$, $r(x)$, and $g(x)$.

b. How can you check your work using the expressions of $q(x)$, $g(x)$, and $r(x)$?

31. **VOLUME** The volume of a cylinder is $\pi(x^3 + 32x^2 - 304x + 640)$. If the height of the cylinder is $x + 40$ feet, find the area of its base in terms of x and π.

The formula for the volume of a cylinder is $V = \pi r^2 h$.

To find the value that represents the area of the base, divide the volume by the height, $\frac{V}{h} = \pi r^2$.

$$\frac{V}{h} = \frac{\pi(x^3 + 32x^2 - 304x + 640)}{x+40}$$

$$\begin{array}{r|rrrr} -40 & 1 & 32 & -304 & 640 \\ & \downarrow & -40 & 320 & -640 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2.
The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is $(x^2 - 8x + 16)$, which represents r^2 in the volume formula.
However, the area of the base is πr^2, so the area of the base in terms of π and x is $\pi(x^2 - 8x + 16)$.

32. REASONING Rewrite $\dfrac{6x^4 + 2x^3 - 16x^2 + 24x + 32}{2x + 4}$ as $q(x) + \dfrac{r(x)}{d(x)}$ using long division.

What does the remainder indicate in this problem?

$$
\begin{array}{r l}
 & 3x^3 - 5x^2 + 2x + 8 \\
2x+4 \,\big) & 6x^4 + 2x^3 - 16x^2 + 24x + 32 \\
 & \underline{(-)6x^4 + 12x^3} \\
 & -10x^3 - 16x^2 \\
 & \underline{(-)-10x^3 - 20x^2} \\
 & 4x^2 + 24x \\
 & \underline{(-)4x^2 + 8x} \\
 & -16x + 32 \\
 & \underline{(-)-16x + 32} \\
 & 0
\end{array}
$$

$\dfrac{6x^4 + 2x^3 - 16x^2 + 24x + 32}{2x + 4} = 3x^3 - 5x^2 + 2x + 8$, remainder 0.

Because the remainder is 0, $2x + 4$ is a factor of $6x^4 + 2x^3 - 16x^2 + 24x + 32$.

33. **CONSTRUCT ARGUMENTS** Determine whether you have enough information to fill in the missing pieces of the long division exercise shown. If so, copy and complete the long division. Justify your response.

$$\begin{array}{r} 3x - \square \\ \square \overline{)\, 9x^2 + \square} \\ \underline{9x^2 + 3x} \\ -3x + 5 \end{array}$$

$$\begin{array}{r} 3x - \boxed{1} \\ \boxed{3x+1} \overline{)\, 9x^2 + \boxed{0x+5}} \\ \underline{9x^2 + 3x} \\ -3x + 5 \end{array}$$

Because $3x$ times the divisor is $9x^2 + 3x$, the divisor must be $3x + 1$.

The second and third terms of the dividend must be $0x + 5$ because the first difference is $-3x + 5$.

Yes, there is enough information to fill in the missing pieces of the long division exercise.

34. **REGULARITY** Rewrite $\frac{2x^5 - 7x^4 - 15x^3 + 2x^2 + 3x + 6}{2x + 3}$ as $q(x) + \frac{r(x)}{g(x)}$ using long division.

a. Identify $q(x)$, $r(x)$, and $g(x)$.

b. How can you check your work using the expressions of $q(x)$, $g(x)$, and $r(x)$?

a.

$$\begin{array}{r}
x^4 - 5x^3 + x \\
2x+3 \overline{\smash{)}\, 2x^5 - 7x^4 - 15x^3 + 2x^2 + 3x + 6} \\
\underline{(-)2x^5 + 3x^4} \qquad\qquad\qquad\qquad \\
-10x^4 - 15x^3 \qquad\qquad\quad \\
\underline{(-)-10x^4 - 15x^3} \qquad\qquad\quad \\
0 + 2x^2 + 3x \quad\;\; \\
\underline{(-)2x^2 + 3x} \quad\;\; \\
0 + 6 \\
6
\end{array}$$

$$\frac{2x^5 - 7x^4 - 15x^3 + 2x^2 + 3x + 6}{2x + 3} = x^4 - 5x^3 + x + \frac{6}{2x + 3}$$

So, $q(x) = x^4 - 5x^3 + x$, $r(x) = 6$, and $g(x) = 2x + 3$.

b. Multiply $q(x)$ and $g(x)$ and then add $r(x)$. The result will equal the original dividend.

19	Use Pascal’s Triangle to write binomial expansions	Exercises (1-12)	P111

Example 1

Use Pascal’s triangle to expand each binomial.

1. $(x - y)^3$

2. $(a + b)^4$

3. $(g - h)^4$

19	Use Pascal's Triangle to write binomial expansions	Exercises (1-12)	P111

Example 1

Use Pascal's triangle to expand each binomial.

4. $(m + 1)^4$

5. $(y - z)^6$

6. $(d + 2)^8$

19	Use Pascal's Triangle to write binomial expansions	Exercises (1-12)	P111

Example 2

7. BAND A school band went to 4 competitions during the year and received a superior rating 2 times. If the band is as likely to receive a superior rating as to not receive a superior rating, find the probability of this outcome by expanding $(s + n)^4$. Round to the nearest percent if necessary.

8. BASKETBALL Oliver shot 8 free throws at practice, making 6 free throws and missing 2 free throws. If Oliver is equally likely to make a free throw as he is to miss a free throw, find the probability of this outcome by expanding $(m + n)^8$. Round to the nearest percent if necessary.

Example 3

Expand each binomial.

9. $(3x + 4y)^5$

10. $(2c - 2d)^7$

11. $(8h - 3j)^4$

12. $(4a + 3b)^6$

Learn Powers of Binomials

You can expand binomials by following a set of rules and using patterns.

Key Concept • Binomial Expansion

In the binomial expansion of $(a + b)^n$,

- there are $n + 1$ terms.
- n is the exponent of a in the first term and b in the last term.
- in successive terms, the exponent of a decreases by 1, and the exponent of b increases by 1.
- the sum of the exponents in each term is n.
- the coefficients are symmetric.

Pascal's triangle is a triangle of numbers in which a row represents the coefficients of an expanded binomial $(a + b)^n$. Each row begins and ends with 1. Each coefficient can be found by adding the two coefficients above it in the previous row.

Instead of writing out the rows of Pascal's triangle, you can use the Binomial Theorem to expand a binomial. The Binomial Theorem uses combinations to calculate the coefficients of the binomial expansion.

Key Concept • Binomial Theorem

If n is a natural number, then $(a + b)^n =$

$$_nC_0a^nb^0 + {_nC_1}a^{n-1}b^1 + {_nC_2}a^{n-2}b^2 + {_nC_3}a^{n-3}b^3 + \ldots + {_nC_n}a^0b^n$$

or

$$1a^nb^0 + \frac{n!}{1!(n-1)!}a^{n-1}b^1 + \frac{n!}{2!(n-2)!}a^{n-2}b^2 + \frac{n!}{3!(n-3)!}a^{n-3}b^3 + \ldots + 1a^0b^n$$

Example 1 Use Pascal's Triangle

Use Pascal's triangle to expand $(x + y)^7$.

1	$(x+y)^0$
1 1	$(x+y)^1$
1 2 1	$(x+y)^2$
1 3 3 1	$(x+y)^3$
1 4 6 4 1	$(x+y)^4$
1 5 10 10 5 1	$(x+y)^5$
1 6 15 20 15 6 1	
1 7 21 35 35 21 7 1	

1. $(x - y)^3$

In the binomial expansion of $(x - y)^3$, since the exponent n is 3, use the fourth row of Pascal's Triangle, 1 3 3 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 3 is the exponent of x in the first term and y in the last term.
- in successive terms, the exponent of x decreases by 1, and the exponent of y increases by 1.
- the sum of the exponents in each term is 3.

The expansion is $x^3 - 3x^2y + 3xy^2 - y^3$.
Note: the even numbered terms are preceded by subtraction symbols since this is the expansion of a difference, rather than a sum.

2. $(a + b)^4$

In the binomial expansion of $(a + b)^4$, since the exponent n is 4, use the fifth row of Pascal's Triangle, 1 4 6 4 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 4 is the exponent of a in the first term and b in the last term.
- in successive terms, the exponent of a decreases by 1, and the exponent of b increases by 1.
- the sum of the exponents in each term is 4.

The expansion is $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

3. $(g - h)^4$

In the binomial expansion of $(g - h)^4$, since the exponent n is 4, use the fifth row of Pascal's Triangle, 1 4 6 4 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 4 is the exponent of g in the first term and h in the last term.
- in successive terms, the exponent of g decreases by 1, and the exponent of h increases by 1.
- the sum of the exponents in each term is 4.

The expansion is $g^4 - 4g^3h + 6g^2h^2 - 4gh^3 + h^4$.
Note: the even numbered terms are preceded by subtraction symbols since this is the expansion of a difference, rather than a sum.

4. $(m + 1)^4$

In the binomial expansion of $(m + 1)^4$, since the exponent n is 4, use the fifth row of Pascal's Triangle, 1 4 6 4 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 4 is the exponent of m in the first term and 1 in the last term.
- in successive terms, the exponent of m decreases by 1, and the exponent of 1 increases by 1.
- the sum of the exponents in each term is 4.

The expansion is $m^4 + 4m^3(1) + 6m^2(1)^2 + 4m(1)^3 + 1^4 = m^4 + 4m^3 + 6m^2 + 4m + 1$.

5. $(y - z)^6$

In the binomial expansion of $(y - z)^6$, since the exponent n is 6, use the seventh row of Pascal's Triangle, 1 6 15 20 15 6 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 6 is the exponent of y in the first term and z in the last term.
- in successive terms, the exponent of y decreases by 1, and the exponent of z increases by 1.
- the sum of the exponents in each term is 6.

The expansion is $y^6 - 6y^5z + 15y^4z^2 - 20y^3z^3 + 15y^2z^4 - 6yz^5 + z^6$.
Note: the even numbered terms are preceded by subtraction symbols since this is the expansion of a difference, rather than a sum.

6. $(d + 2)^8$

In the binomial expansion of $(d + 2)^8$, since the exponent n is 8, use the ninth row of Pascal's Triangle, 1 8 28 56 70 56 28 8 1, to represent the number of terms of the expansion, as well as the symmetric coefficients of the terms. Also apply the following key elements:

- 8 is the exponent of d in the first term and 2 in the last term.
- in successive terms, the exponent of d decreases by 1, and the exponent of 2 increases by 1.
- the sum of the exponents in each term is 8.

The expansion is $d^8 + 8d^7(2) + 28d^6(2)^2 + 56d^5(2)^3 + 70d^4(2)^4 + 56d^3(2)^5 + 28d^2(2)^6 + 8d(2)^7 + 2^8 = d^8 + 16d^7 + 112d^6 + 448d^5 + 1120d^4 + 1792d^3 + 1792d^2 + 1024d + 256$.

Example 2

7. BAND A school band went to 4 competitions during the year and received a superior rating 2 times. If the band is as likely to receive a superior rating as to not receive a superior rating, find the probability of this outcome by expanding $(s + n)^4$. Round to the nearest percent if necessary.

$(s+n)^4$

$= {}_4C_0s^4 + {}_4C_1s^3n + {}_4C_2s^2n^2 + {}_4C_3sn^3 + {}_4C_4n^4$

$= s^4 + \frac{4!}{3!}s^3n + \frac{4!}{2!2!}s^2n^2 + \frac{4!}{3!}sn^3 + n^4$

$= s^4 + 4s^3n + 6s^2n^2 + 4sn^3 + n^4$

By adding the coefficients, you can determine that there were 16 combinations of superior and not superior ratings that could have occurred.

$6s^2n^2$ represents the number of combinations of 2 superior and 2 not superior ratings. Therefore, there was a $\frac{6}{16}$ or about a 38% chance of the school band receiving 2 superior and 2 not superior ratings.

8. BASKETBALL Oliver shot 8 free throws at practice, making 6 free throws and missing 2 free throws. If Oliver is equally likely to make a free throw as he is to miss a free throw, find the probability of this outcome by expanding $(m + n)^8$. Round to the nearest percent if necessary.

$(m+n)^8$

$= {}_8C_0m^8 + {}_8C_1m^7n + {}_8C_2m^6n^2 + {}_8C_3m^5n^3 + {}_8C_4m^4n^4 + {}_8C_5m^3n^5 + {}_8C_6m^2n^6 + {}_8C_7mn^7 + {}_8C_8n^8$

$= m^8 + \frac{8!}{7!}m^7n + \frac{8!}{6!2!}m^6n^2 + \frac{8!}{5!3!}m^5n^3 + \frac{8!}{4!4!}m^4n^4 + \frac{8!}{3!5!}m^3n^5 + \frac{8!}{2!6!}m^2n^6 + \frac{8!}{7!}mn^7 + n^8$

$= m^8 + 8m^7n + 28m^6n^2 + 56m^5n^3 + 70m^4n^4 + 56m^3n^5 + 28m^2n^6 + 8mn^7 + n^8$

By adding the coefficients, you can determine that there were 256 combinations of made and not made free throws that could have occurred.

$28m^6n^2$ represents the number of combinations of 6 made and 2 not made free throws. Therefore, there was a $\frac{28}{256}$ or about a 11% chance of Oliver making 6 and not making 2 free throws.

9. $(3x + 4y)^5$

$(3x+4y)^5$

$= {}_5C_0(3x)^5 + {}_5C_1(3x)^4(4y) + {}_5C_2(3x)^3(4y)^2 + {}_5C_3(3x)^2(4y)^3 + {}_5C_4(3x)(4y)^4 + {}_5C_5(4y)^5$

$= 243x^5 + \frac{5!}{4!}(81x^4)(4y) + \frac{5!}{3!2!}(27x^3)(16y^2) + \frac{5!}{2!3!}(9x^2)(64y^3) + \frac{5!}{4!}(3x)(256y^4) + 1024y^5$

$= 243x^5 + 1620x^4y + 4320x^3y^2 + 5760x^2y^3 + 3840xy^4 + 1024y^5$

10. $(2c - 2d)^7$

$(2c-2d)^7$

$= {}_7C_0(2c)^7 - {}_7C_1(2c)^6(2d) + {}_7C_2(2c)^5(2d)^2 - {}_7C_3(2c)^4(2d)^3 + {}_7C_4(2c)^3(2d)^4 - {}_7C_5(2c)^2(2d)^5 + {}_7C_6(2c)(2d)^6 - {}_7C_7(2d)^7$

$= 128c^7 - \frac{7!}{6!}(64c^6)(2d) + \frac{7!}{5!2!}(32c^5)(4d^2) - \frac{7!}{4!3!}(16c^4)(8d^3) + \frac{7!}{3!4!}(8c^3)(16d^4) - \frac{7!}{2!5!}(4c^2)(32d^5) + \frac{7!}{6!}(2c)(64d^6) + 128d^7$

$= 128c^7 - 896c^6d + 2688c^5d^2 - 4480c^4d^3 + 4480c^3d^4 - 2688c^2d^5 + 896cd^6 - 128d^7$

11. $(8h - 3j)^4$

$(8h-3j)^4$

$= {}_4C_0(8h)^4 - {}_4C_1(8h)^3(3j) + {}_4C_2(8h)^2(3j)^2 - {}_4C_3(8h)(3j)^3 + {}_4C_4(3j)^4$

$= 4096h^4 - \frac{4!}{3!}(512h^3)(3j) + \frac{4!}{2!2!}(64h^2)(9j^2) - \frac{4!}{3!}(8h)(27j^3) + 81j^4$

$= 4096h^4 - 6144h^3j + 3456h^2j^2 - 864hj^3 + 81j^4$

12. $(4a + 3b)^6$

$(4a+3b)^6$

$= {}_6C_0(4a)^6 + {}_6C_1(4a)^5(3b) + {}_6C_2(4a)^4(3b)^2 + {}_6C_3(4a)^3(3b)^3 + {}_6C_4(4a)^2(3b)^4 + {}_6C_5(4a)(3b)^5 + {}_6C_6(3b)^6$

$= 4096a^6 + \frac{6!}{5!}(1024a^5)(3b) + \frac{6!}{4!2!}(256a^4)(9b^2) + \frac{6!}{3!3!}(64a^3)(27b^3) + \frac{6!}{2!4!}(16a^2)(81b^4) + \frac{6!}{5!}(4a)(243b^5) + 729b^6$

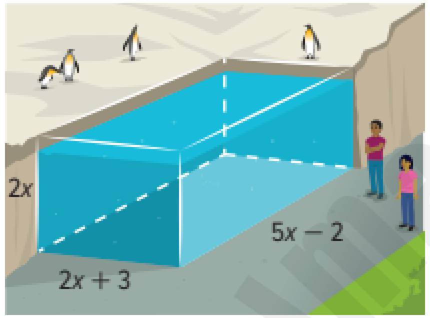
$= 4096a^6 + 18{,}432a^5b + 34{,}560a^4b^2 + 34{,}560a^3b^3 + 19{,}440a^2b^4 + 5832ab^5 + 729b^6$

20	Factorize polynomials	Example2	P120

solution method

Example 2 Solve a Polynomial Equation by Using a System

ANIMALS For an exhibit with six or fewer Emperor penguins, the pool must have a depth of at least 4 feet and a volume of at least 1620 gallons, or about 217 ft^3, per bird. If a zoo has five Emperor penguins, what should the dimensions of the pool shown at the right be to meet the minimum requirements?



Part A Write a polynomial equation.

Use the formula for the volume of a rectangular prism, $V = \ell wh$, to write a polynomial equation that represents the volume of the pool. Let h represent the depth of the pool.

Since the minimum required volume for the pool is 217 ft^3 per penguin, or $217 \cdot 5 = 1085$ ft^3, the equation that represents the volume of the pool is $(2x + 3)(5x - 2)2x = 1085$. Simplify the equation.

$(2x + 3)(5x - 2)2x = 1085$	Volume of pool
$[2x(5x) + 2x(-2) + 3(5x) + 3(-2)]2x = 1085$	FOIL
$(10x^2 - 4x + 15x - 6)2x = 1085$	Simplify.
$(10x^2 + 11x - 6)2x = 1085$	Combine like terms.
$20x^3 + 22x^2 - 12x = 1085$	Distributive Property

So, the volume of the pool is $20x^3 + 22x^2 - 12x = 1085$.

So, the volume of the pool is $20x^3 + 22x^2 - 12x = 1085$.

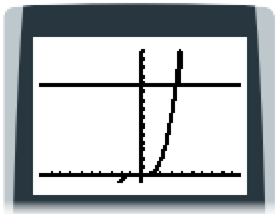
Part B Write and solve a system of equations.

Set each side equal to y to create a system of equations.

$y = 20x^3 + 22x^2 - 12x$	First equation
$y = 1085$	Second equation

Enter the equations in the **Y =** list and graph.

Use the **intersect** feature on the **CALC** menu to find the coordinates of the point of intersection.



The real solution is the x-coordinate of the intersection, which is 3.5.

Part C Find the dimensions.

Substitute 3.5 feet for x in the length, width, and depth of the pool.

Length: $2x + 3 = 10$ ft Width: $5x - 2 = 15.5$ ft

Depth: $2x = 7$ ft