تجميعة أسئلة مراجعة وفق الهيكل الوزاري منهج بريدج

Haykal

Grade 12 General

1st term (2024-2025)

Mr. Karam Asaad (0505308082)

State the domain of each function.

39. $f(x) = \dfrac{8x + 12}{x^2 + 5x + 4}$

40. $g(x) = \dfrac{x + 1}{x^2 - 3x - 40}$

41. $g(a) = \sqrt{1 + a^2}$

42. $h(x) = \sqrt{6 - x^2}$

43. $f(a) = \dfrac{5a}{\sqrt{4a - 1}}$

44. $g(x) = \dfrac{3}{\sqrt{x^2 - 16}}$

45. $f(x) = \dfrac{2}{x} + \dfrac{4}{x + 1}$

46. $g(x) = \dfrac{6}{x + 3} + \dfrac{2}{x - 4}$

Use the graph of each function to find its y-intercept and zero(s). Then find these values algebraically. (Examples 3 and 4)

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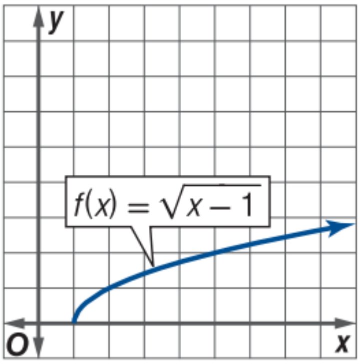

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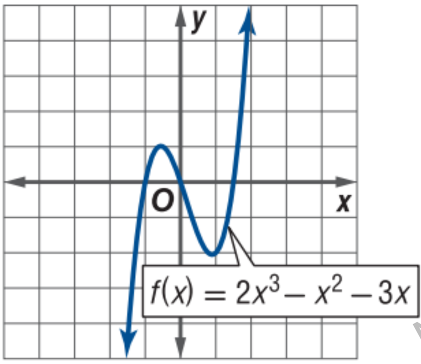

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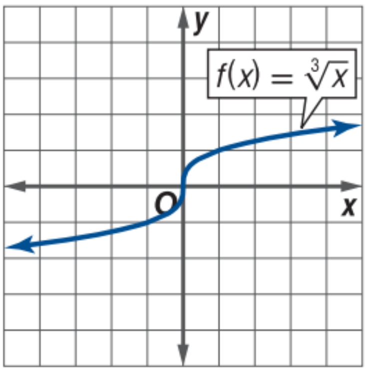

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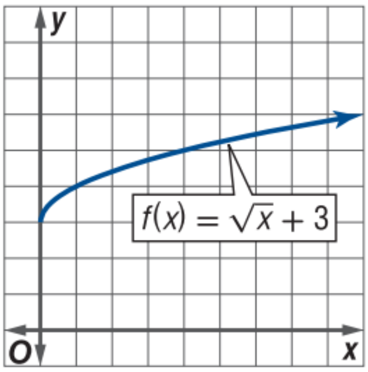

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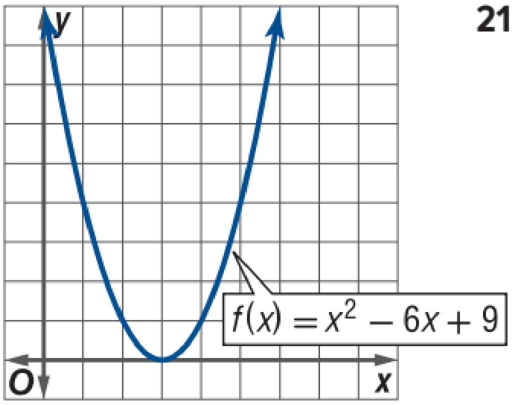

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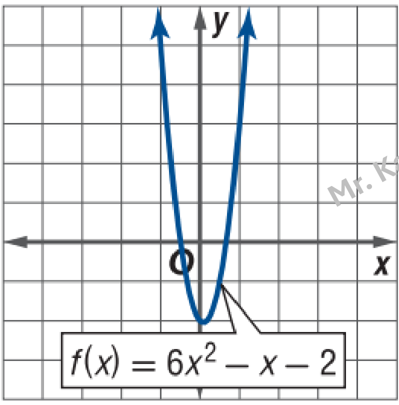

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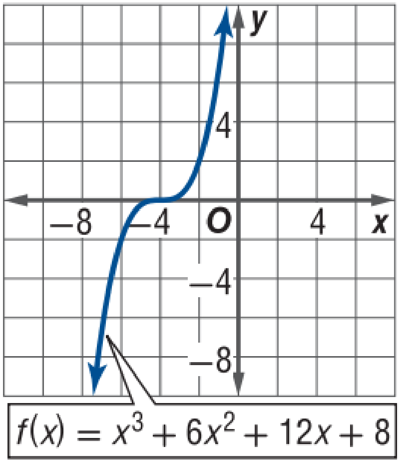

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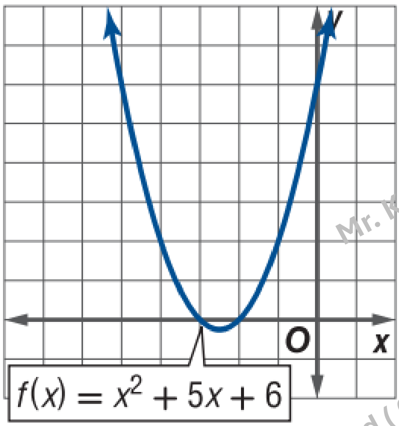
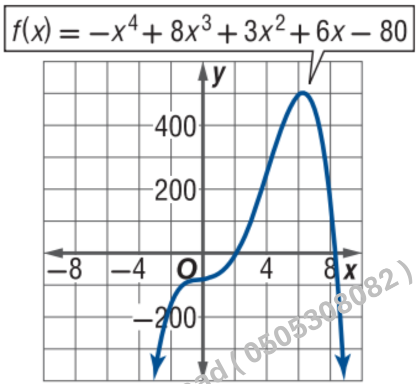


Use the graph of $f(x) = -x^4 + 8x^3 + 3x^2 + 6x - 80$ to describe its end behavior. Support the conjecture numerically.

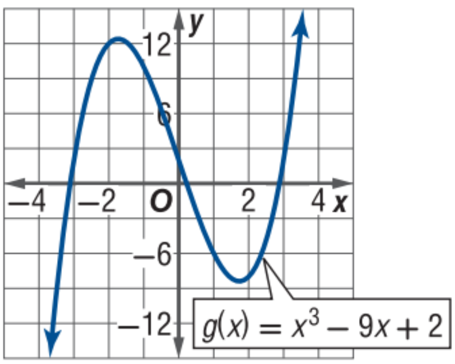


Analyze Graphically

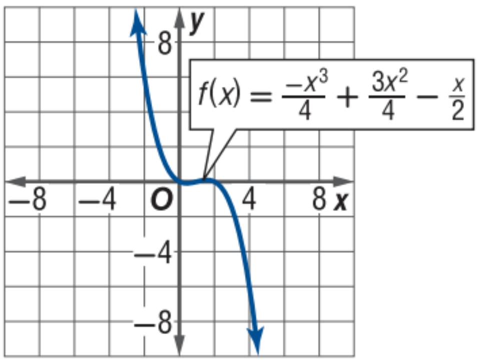
In the graph of $f(x)$, it appears that $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$.

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

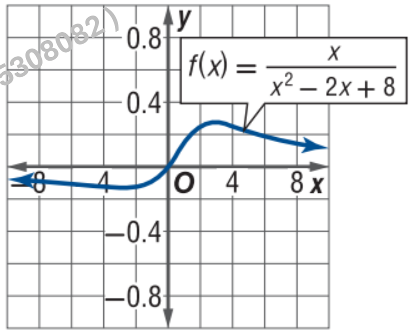
4A.



4B.



Use the graph of $f(x) = \dfrac{x}{x^2 - 2x + 8}$ to describe its end behavior. Support the conjecture numerically.



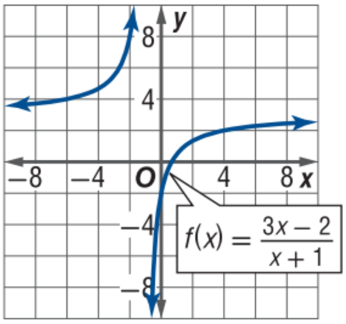
Analyze Graphically

In the graph of $f(x)$, it appears that $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$.

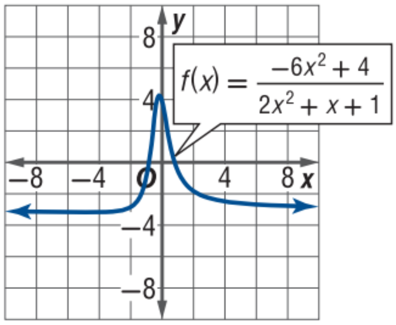
Use the graph of each function to describe its end behavior. Support the conjecture numerically.

Mr. Karam Asaad (0505308082)

5A.



5B.



Use the graph of each function to describe its end behavior. Support the conjecture numerically. (Examples 4 and 5)

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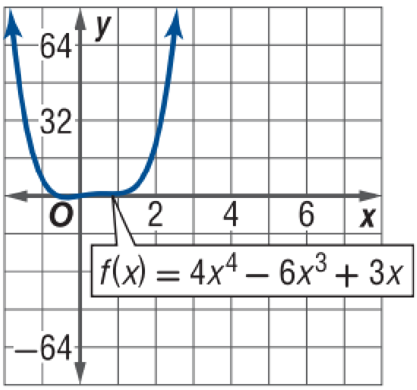
22.



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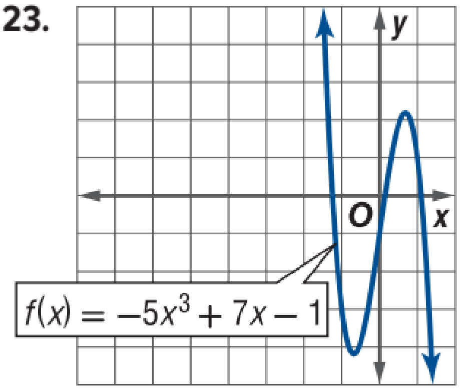

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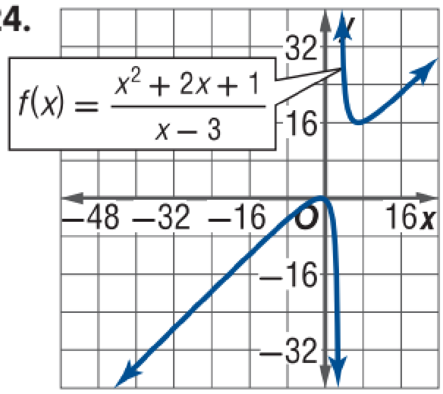

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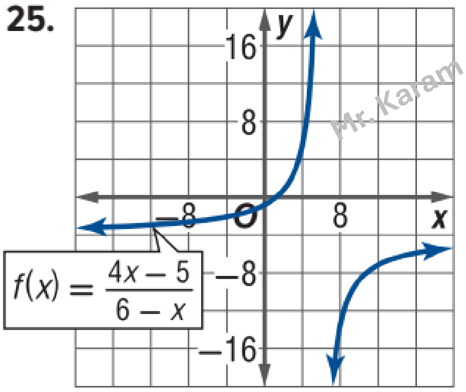

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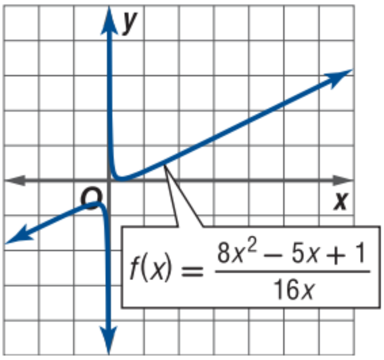
26.



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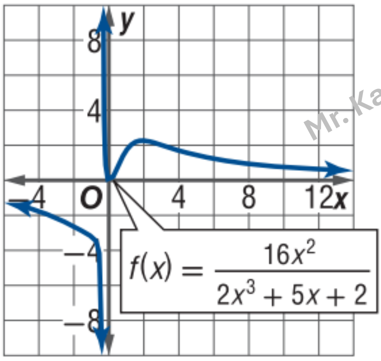

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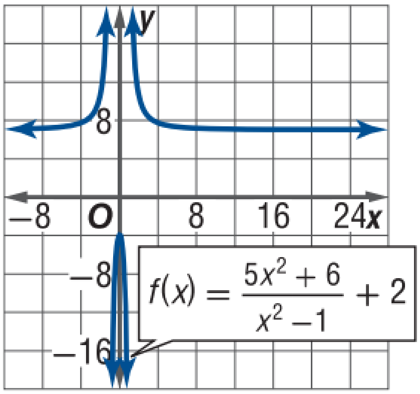

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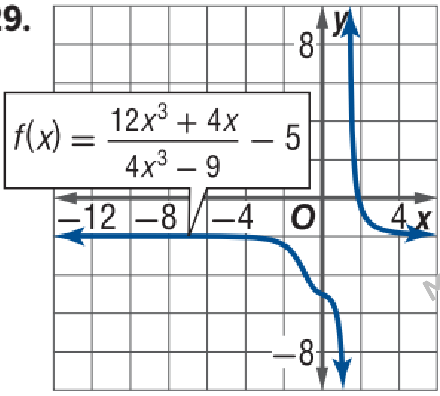
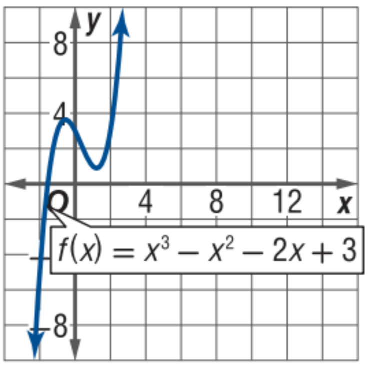


Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.
(Example 1)

1.



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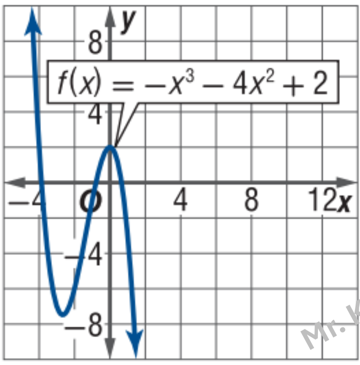

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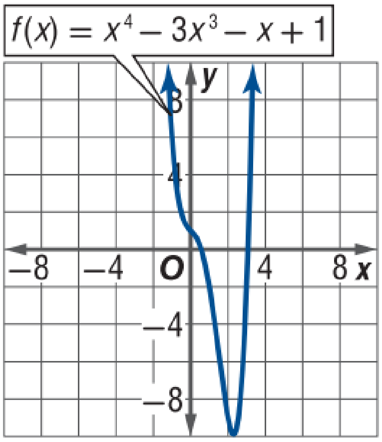

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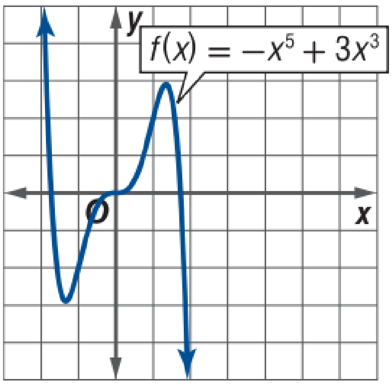

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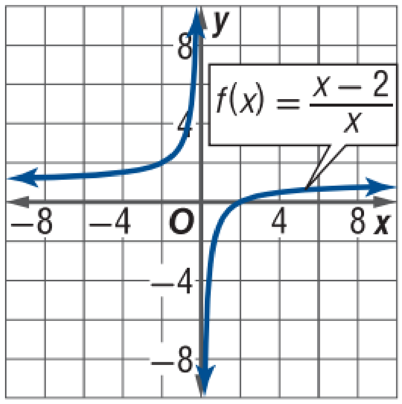
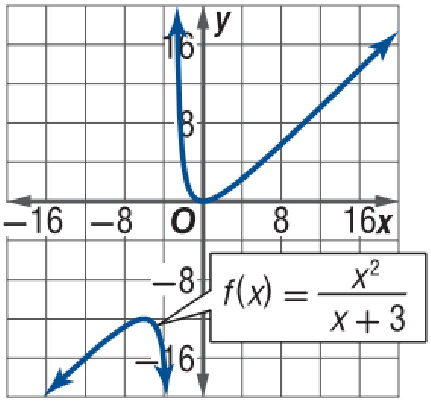


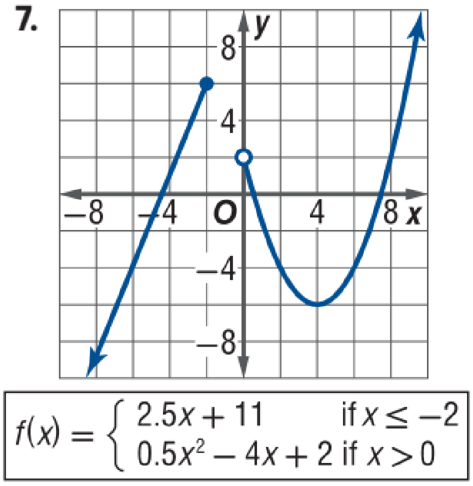
y
x
8
4
−4
−8
O
$f(x) = \frac{x-2}{x}$

6.



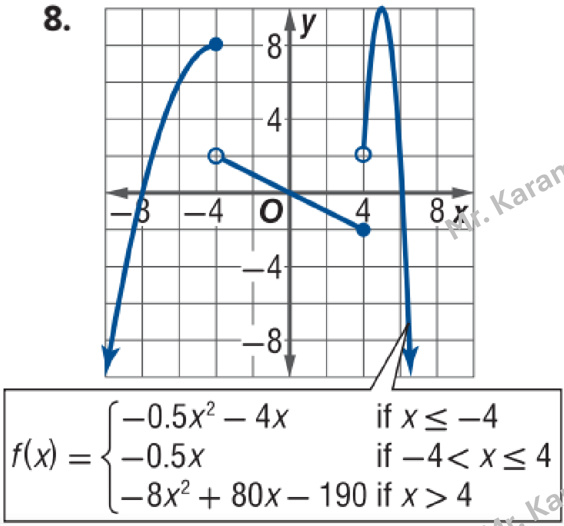
y
x
16
8
−8
−16
O
$f(x) = \frac{x^2}{x+3}$

7.



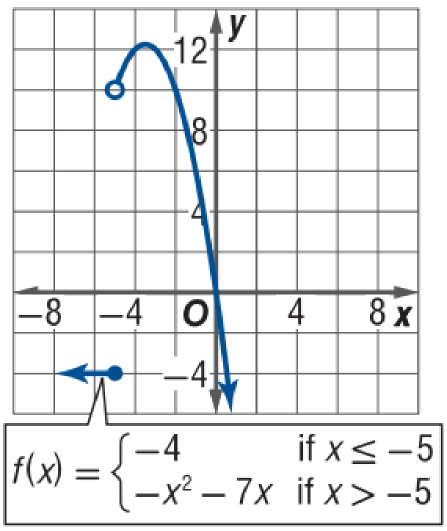
y
x
8
4
−4
−8
O
$f(x) = \begin{cases} 2.5x + 11 & \text{if } x \le -2 \\ 0.5x^2 - 4x + 2 & \text{if } x > 0 \end{cases}$

8.



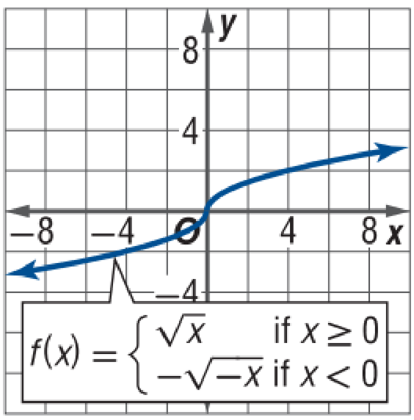
y
x
8
4
−4
−8
O
$f(x) = \begin{cases} -0.5x^2 - 4x & \text{if } x \le -4 \\ -0.5x & \text{if } -4 < x \le 4 \\ -8x^2 + 80x - 190 & \text{if } x > 4 \end{cases}$

9.



y
x
12
8
4
−4
−8
O
$f(x) = \begin{cases} -4 & \text{if } x \le -5 \\ -x^2 - 7x & \text{if } x > -5 \end{cases}$

10.



y
x
8
4
−4
−8
O
$f(x) = \begin{cases} \sqrt{x} & \text{if } x \ge 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$

Solve each equation.

a. $2x = \sqrt{100 - 12x} - 2$

$2x = \sqrt{100 - 12x} - 2$	Original equation
$2x + 2 = \sqrt{100 - 12x}$	Isolate the radical.
$4x^2 + 8x + 4 = 100 - 12x$	Square each side to eliminate the radical.
$4x^2 + 20x - 96 = 0$	Subtract 100 − 12x from each side.
$4(x^2 + 5x - 24) = 0$	Factor.
$4(x + 8)(x - 3) = 0$	Factor.
$x + 8 = 0$ or $x - 3 = 0$	Zero Product Property
$x = -8$ $\quad x = 3$	Solve.

CHECK $x = -8$

$2x = \sqrt{100 - 12x} - 2$

$-16 \stackrel{?}{=} \sqrt{100 - 12(-8)} - 2$

$-16 \stackrel{?}{=} \sqrt{196} - 2$

$-16 \neq 12$ ✗

CHECK $x = 3$

$2x = \sqrt{100 - 12x} - 2$

$6 \stackrel{?}{=} \sqrt{100 - 12(3)} - 2$

$6 \stackrel{?}{=} \sqrt{64} - 2$

$6 = 6$ ✓

One solution checks and the other solution does not. Therefore, the solution is 3.

b. $\sqrt[3]{(x-5)^2} + 14 = 50$

$\sqrt[3]{(x-5)^2} + 14 = 50$	Original equation
$\sqrt[3]{(x-5)^2} = 36$	Isolate the radical.
$(x-5)^2 = 46{,}656$	Raise each side to the third power. (The index is 3.)
$x - 5 = \pm 216$	Take the square root of each side.
$x = 221$ or -211	Add 5 to each side.

A check of the solutions in the original equation confirms that the solutions are valid.

c. $\sqrt{x-2} = 5 - \sqrt{15 - x}$

$\sqrt{x-2} = 5 - \sqrt{15 - x}$	Original equation
$x - 2 = 25 - 10\sqrt{15 - x} + (15 - x)$	Square each side.
$2x - 42 = -10\sqrt{15 - x}$	Isolate the radical.
$4x^2 - 168x + 1764 = 100(15 - x)$	Square each side.
$4x^2 - 168x + 1764 = 1500 - 100x$	Distributive Property
$4x^2 - 68x + 264 = 0$	Combine like terms.
$4(x^2 - 17x + 66) = 0$	Factor.
$4(x - 6)(x - 11) = 0$	Factor.
$x - 6 = 0$ or $x - 11 = 0$	Zero Product Property
$x = 6$ $\quad x = 11$	Solve.

A check of the solutions in the original equation confirms that both solutions are valid.

6A. $3x = 3 + \sqrt{18x - 18}$ **6B.** $\sqrt[3]{4x + 8} + 3 = 7$ **6C.** $\sqrt{x + 7} = 3 + \sqrt{2 - x}$

Solve each equation. (Example 6)

44. $4 = \sqrt{-6 - 2x} + \sqrt{31 - 3x}$

45. $0.5x = \sqrt{4 - 3x} + 2$

46. $-3 = \sqrt{22 - x} - \sqrt{3x - 3}$

47. $\sqrt{(2x - 5)^3} - 10 = 17$

48. $\sqrt[4]{(4x + 164)^3} + 36 = 100$

49. $x = \sqrt{2x - 4} + 2$

50. $7 + \sqrt{(-36 - 5x)^5} = 250$

51. $x = 5 + \sqrt{x + 1}$

52. $\sqrt{6x - 11} + 4 = \sqrt{12x + 1}$

53. $\sqrt{4x - 40} = -20$

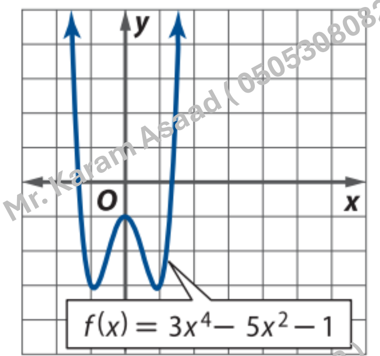
54. $\sqrt{x + 2} - 1 = \sqrt{-2 - 2x}$

55. $7 + \sqrt[5]{1054 - 3x} = 11$

Describe the end behavior of the graph of each polynomial function using limits. Explain your reasoning using the leading term test.

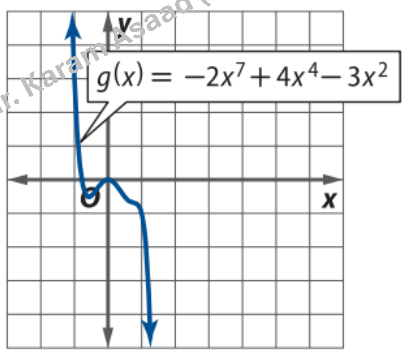
a. $f(x) = 3x^4 - 5x^2 - 1$

The degree is 4, and the leading coefficient is 3. Because the degree is even and the leading coefficient is positive, $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$.



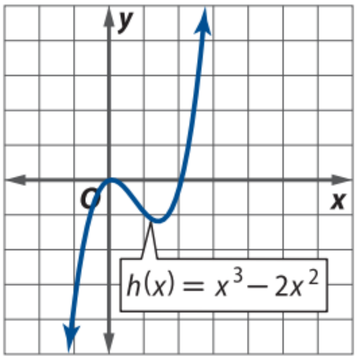
b. $g(x) = -3x^2 - 2x^7 + 4x^4$

Write in standard form as $g(x) = -2x^7 + 4x^4 - 3x^2$. The degree is 7, and the leading coefficient is -2. Because the degree is odd and the leading coefficient is negative, $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$.



c. $h(x) = x^3 - 2x^2$

The degree is 3, and the leading coefficient is 1. Because the degree is odd and the leading coefficient is positive, $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$.

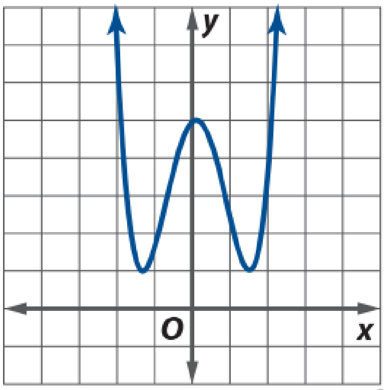


2A. $g(x) = 4x^5 - 8x^3 + 20$

2B. $h(x) = -2x^6 + 11x^4 + 2x^2$

Determine whether the degree n of the polynomial for each graph is *even* or *odd* and whether its leading coefficient a_n is *positive* or *negative*.

64.



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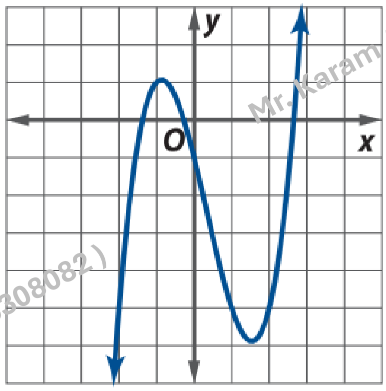

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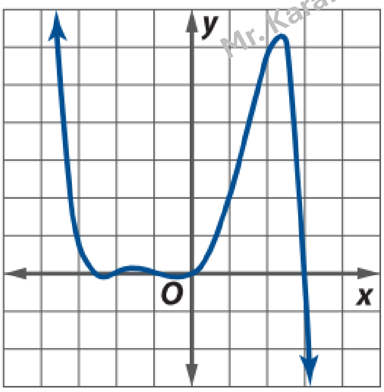

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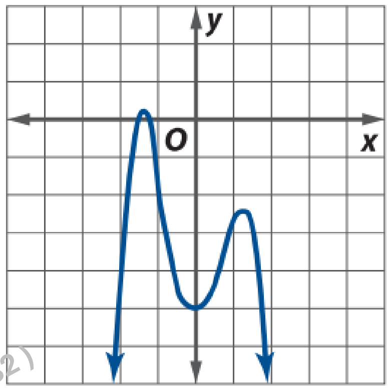


Divide using long division. (Examples 2 and 3)

9. $(5x^4 - 3x^3 + 6x^2 - x + 12) \div (x - 4)$

10. $(x^6 - 2x^5 + x^4 - x^3 + 3x^2 - x + 24) \div (x + 2)$

11. $(4x^4 - 8x^3 + 12x^2 - 6x + 12) \div (2x + 4)$

12. $(2x^4 - 7x^3 - 38x^2 + 103x + 60) \div (x - 3)$

13. $(6x^6 - 3x^5 + 6x^4 - 15x^3 + 2x^2 + 10x - 6) \div (2x - 1)$

14. $(108x^5 - 36x^4 + 75x^2 + 36x + 24) \div (3x + 2)$

15. $(x^4 + x^3 + 6x^2 + 18x - 216) \div (x^3 - 3x^2 + 18x - 54)$

16. $(4x^4 - 14x^3 - 14x^2 + 110x - 84) \div (2x^2 + x - 12)$

17. $\dfrac{6x^5 - 12x^4 + 10x^3 - 2x^2 - 8x + 8}{3x^3 + 2x + 3}$

18. $\dfrac{12x^5 + 5x^4 - 15x^3 + 19x^2 - 4x - 28}{3x^3 + 2x^2 - x + 6}$

Divide using synthetic division. (Example 4)

19. $(x^4 - x^3 + 3x^2 - 6x - 6) \div (x - 2)$

20. $(2x^4 + 4x^3 - 2x^2 + 8x - 4) \div (x + 3)$

21. $(3x^4 - 9x^3 - 24x - 48) \div (x - 4)$

22. $(x^5 - 3x^3 + 6x^2 + 9x + 6) \div (x + 2)$

23. $(12x^5 + 10x^4 - 18x^3 - 12x^2 - 8) \div (2x - 3)$

24. $(36x^4 - 6x^3 + 12x^2 - 30x - 12) \div (3x + 1)$

25. $(45x^5 + 6x^4 + 3x^3 + 8x + 12) \div (3x - 2)$

26. $(48x^5 + 28x^4 + 68x^3 + 11x + 6) \div (4x + 1)$

27. $(60x^6 + 78x^5 + 9x^4 - 12x^3 - 25x - 20) \div (5x + 4)$

28. $\dfrac{16x^6 - 56x^5 - 24x^4 + 96x^3 - 42x^2 - 30x + 105}{2x - 7}$

Solve $\frac{4}{x-6} + \frac{2}{x+1} > 0$.

$\frac{4}{x-6} + \frac{2}{x+1} > 0$ — Original inequality

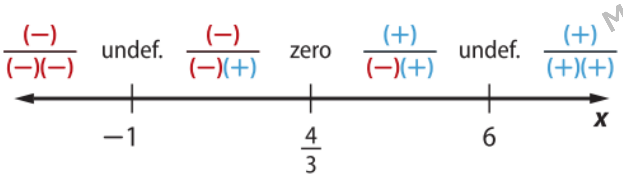
$\frac{4x + 4 + 2x - 12}{(x-6)(x+1)} > 0$ — Use the LCD, $(x - 6)(x + 1)$, to rewrite each fraction. Then add.

$\frac{6x-8}{(x-6)(x+1)} > 0$ — Simplify.

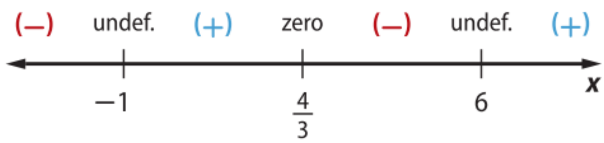
Let $f(x) = \frac{6x-8}{(x-6)(x+1)}$. The zeros and undefined points of the inequality are the zeros of the numerator, $\frac{4}{3}$, and denominator, 6 and -1. Create a sign chart using these numbers. Then choose and test x-values in each interval to determine if $f(x)$ is positive or negative.

$f(x) = \frac{6x-8}{(x-6)(x+1)}$

Test $x = -2$. Test $x = 0$. Test $x = 2$. Test $x = 7$.



$f(x) = \frac{6x-8}{(x-6)(x+1)}$



The solution set of the original inequality is the union of those intervals for which $f(x)$ is positive, $\left(-1, \frac{4}{3}\right) \cup (6, \infty)$. The graph of $f(x) = \frac{4}{x-6} + \frac{2}{x+1}$ in Figure 1.6.1 supports this conclusion.

Solve each inequality. (Example 4)

18. $\frac{x-3}{x+4} > 3$

19. $\frac{x+6}{x-5} \leq 1$

20. $\frac{2x+1}{x-6} \geq 4$

21. $\frac{3x-2}{x+3} < 6$

22. $\frac{3-2x}{5x+2} < 5$

23. $\frac{4x+1}{3x-5} \geq -3$

24. $\frac{(x+2)(2x-3)}{(x-3)(x+1)} \leq 6$

25. $\frac{(4x+1)(x-2)}{(x+3)(x-1)} \leq 4$

26. $\frac{12x+65}{(x+4)^2} \geq 5$

27. $\frac{2x+4}{(x-3)^2} < 12$

Use the graph of $f(x) = \log x$ to describe the transformation that results in each function. Then sketch the graphs of the functions.

a. $k(x) = \log(x + 4)$

This function is of the form $k(x) = f(x + 4)$. Therefore, the graph of $k(x)$ is the graph of $f(x)$ translated 4 units to the left (Figure 3.2.1).

b. $m(x) = -\log x - 5$

The function is of the form $m(x) = -f(x) - 5$. Therefore, the graph of $m(x)$ is the graph of $f(x)$ reflected in the x-axis and then translated 5 units down (Figure 3.2.2).

c. $p(x) = 3\log(x + 2)$

The function is of the form $p(x) = 3f(x + 2)$. Therefore, the graph of $p(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3 and then translated 2 units to the left (Figure 3.2.3).

Use the graph of $f(x) = \ln x$ to describe the transformation that results in each function. Then sketch the graphs of the functions.

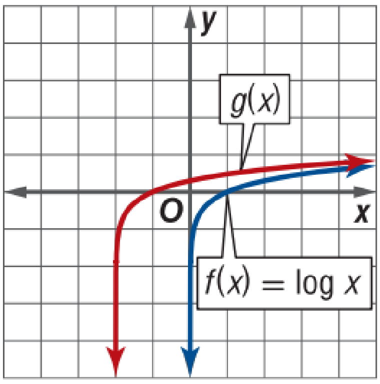
6A. $a(x) = \ln(x - 6)$

6B. $b(x) = 0.5 \ln x - 2$

6C. $c(x) = \ln(x + 4) + 3$

Use the parent graph of $f(x) = \log x$ to find the equation of each function.

60.



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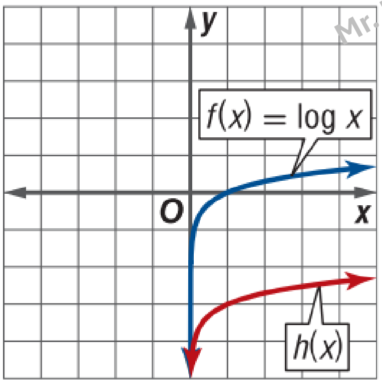

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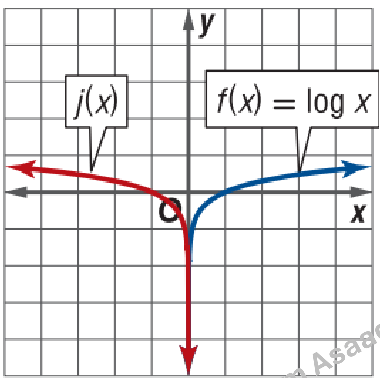

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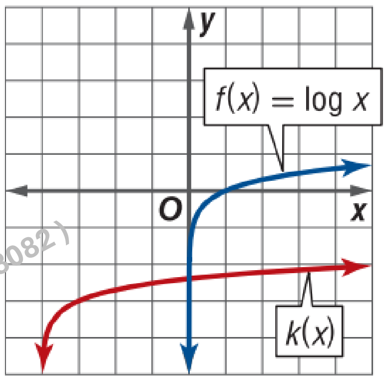


Condense each expression.

a. $4\log_3 x - \frac{1}{3}\log_3(x+6)$

$$4\log_3 x - \frac{1}{3}\log_3(x+6) = \log_3 x^4 - \log_3 (x+6)^{\frac{1}{3}}$$

$$= \log_3 x^4 - \log_3 \sqrt[3]{x+6}$$

$$= \log_3 \frac{x^4}{\sqrt[3]{x+6}}$$

$$= \log_3 \frac{x^4\sqrt[3]{(x+6)^2}}{x+6}$$

b. $6\ln(x-4) + 3\ln x$

$$6\ln(x-4) + 3\ln x = \ln(x-4)^6 + \ln x^3$$

$$= \ln x^3(x-4)^6$$

4A. $-5\log_2(x+1) + 3\log_2(6x)$

4B. $\ln(3x+5) - 4\ln x - \ln(x-1)$

Condense each expression. (Example 4)

39. $3\log_5 x - \frac{1}{2}\log_5(6-x)$

40. $5\log_7(2x) - \frac{1}{3}\log_7(5x+1)$

41. $7\log_3 a + \log_3 b - 2\log_3(8c)$

42. $4\ln(x+3) - \frac{1}{5}\ln(4x+7)$

43. $2\log_8(9x) - \log_8(2x-5)$

44. $\ln 13 + 7\ln a - 11\ln b + \ln c$

45. $2\log_6(5a) + \log_6 b + 7\log_6 c$

46. $\log_2 x - \log_2 y - 3\log_2 z$

47. $\frac{1}{4}\ln(2a-b) - \frac{1}{5}\ln(3b+c)$

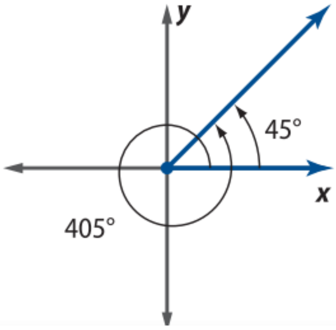
48. $\log_3 4 - \frac{1}{2}\log_3(6x-5)$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

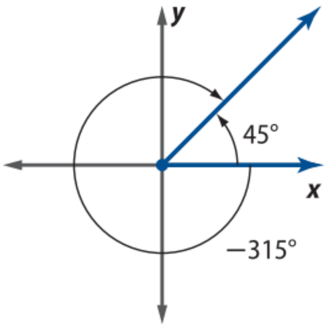
a. **45°**

All angles measuring $45^\circ + 360n^\circ$ are coterminal with a 45° angle. Let $n = 1$ and -1.

$45^\circ + 360(\mathbf{1})^\circ = 45^\circ + 360^\circ$ or 405°



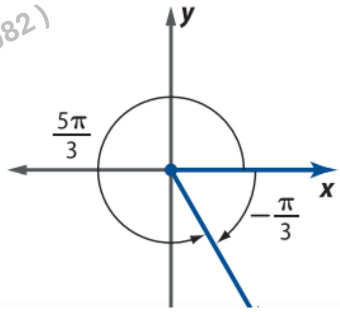
$45^\circ + 360(\mathbf{-1})^\circ = 45^\circ - 360^\circ$ or -315°



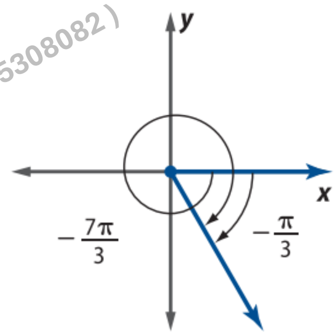
b. $-\frac{\pi}{3}$

All angles measuring $-\frac{\pi}{3} + 2n\pi$ are coterminal with a $-\frac{\pi}{3}$ angle. Let $n = 1$ and -1.

$-\frac{\pi}{3} + 2(\mathbf{1})\pi = -\frac{\pi}{3} + 2\pi$ or $\frac{5\pi}{3}$



$-\frac{\pi}{3} + 2(\mathbf{-1})\pi = -\frac{\pi}{3} - 2\pi$ or $-\frac{7\pi}{3}$



3A. -30°

3B. $\frac{3\pi}{4}$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. (Example 3)

18. 120°

19. -75°

20. 225°

21. -150°

22. $\frac{\pi}{3}$

23. $-\frac{3\pi}{4}$

24. $-\frac{\pi}{12}$

25. $\frac{3\pi}{2}$

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ. (Example 1)

1. $(3, 4)$ **2.** $(-6, 6)$

3. $(-4, -3)$ **4.** $(2, 0)$

5. $(1, -8)$ **6.** $(5, -3)$

7. $(-8, 15)$ **8.** $(-1, -2)$

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*. (Example 2)

9. $\sin \frac{\pi}{2}$ **10.** $\tan 2\pi$

11. $\cot(-180°)$ **12.** $\csc 270°$

13. $\cos(-270°)$ **14.** $\sec 180°$

15. $\tan \pi$ **16.** $\sec\left(-\frac{\pi}{2}\right)$

Sketch each angle. Then find its reference angle. (Example 3)

17. $135°$ **18.** $210°$

19. $\frac{7\pi}{12}$ **20.** $\frac{11\pi}{3}$

21. $-405°$ **22.** $-75°$

23. $\frac{5\pi}{6}$ **24.** $\frac{13\pi}{6}$

Find the exact value of each expression. (Example 4)

25. $\cos \frac{4\pi}{3}$ **26.** $\tan \frac{7\pi}{6}$

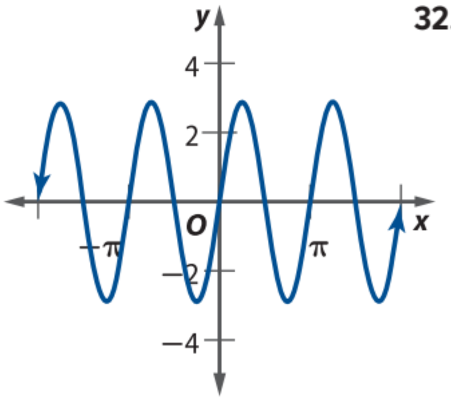
27. $\sin \frac{3\pi}{4}$ **28.** $\cot(-45°)$

29. $\csc 390°$ **30.** $\sec(-150°)$

31. $\tan \frac{11\pi}{6}$ **32.** $\sin 300°$

Write an equation that corresponds to each graph.

31.



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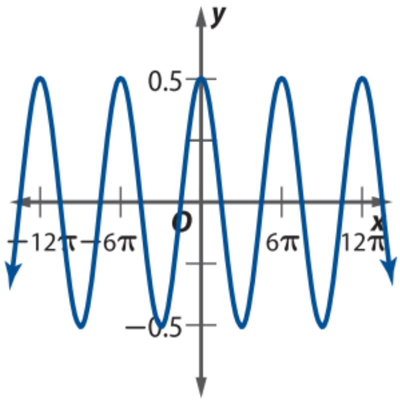

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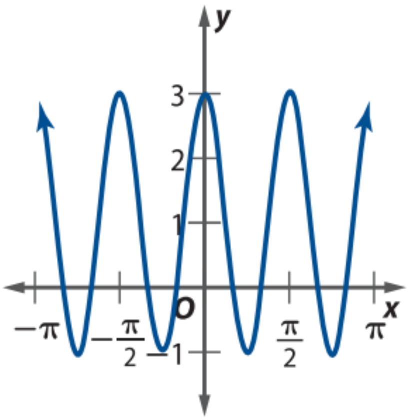

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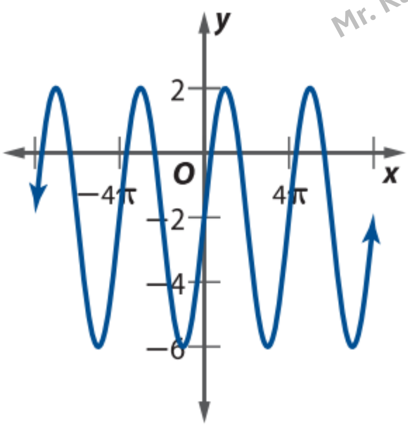


Find the exact value of each expression, if it exists.

a. $\sin\left[\sin^{-1}\left(-\frac{1}{4}\right)\right]$

The inverse property applies because $-\frac{1}{4}$ lies on the interval $[-1, 1]$.

Therefore, $\sin\left[\sin^{-1}\left(-\frac{1}{4}\right)\right] = -\frac{1}{4}$.

b. $\arctan\left(\tan\frac{\pi}{2}\right)$

Because $\tan x$ is not defined when $x = \frac{\pi}{2}$, $\arctan\left(\tan\frac{\pi}{2}\right)$ does not exist.

c. $\arcsin\left(\sin\frac{7\pi}{4}\right)$

Notice that the angle $\frac{7\pi}{4}$ does not lie on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. However, $\frac{7\pi}{4}$ is coterminal with $\frac{7\pi}{4} - 2\pi$ or $-\frac{\pi}{4}$, which is on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\arcsin\left(\sin\frac{7\pi}{4}\right) = \arcsin\left[\sin\left(-\frac{\pi}{4}\right)\right]$ — $\sin\frac{7\pi}{4} = \sin\left(-\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$ — Since $-\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}$, $\arcsin(\sin x) = x$.

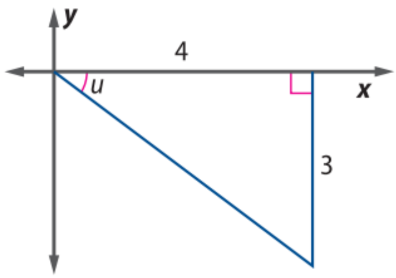
Therefore, $\arcsin\left(\sin\frac{7\pi}{4}\right) = -\frac{\pi}{4}$.

6A. $\tan\left(\tan^{-1}\frac{\pi}{3}\right)$ **6B.** $\cos^{-1}\left(\cos\frac{3\pi}{4}\right)$ **6C.** $\arcsin\left(\sin\frac{2\pi}{3}\right)$

Find the exact value of $\cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right]$.

To simplify the expression, let $u = \tan^{-1}\left(-\frac{3}{4}\right)$, so $\tan u = -\frac{3}{4}$.

Because the tangent function is negative in Quadrants II and IV, and the domain of the inverse tangent function is restricted to Quadrants I and IV, u must lie in Quadrant IV.



Using the Pythagorean Theorem, you can find that the length of the hypotenuse is 5. Now, solve for $\cos u$.

$\cos u = \frac{\text{adj}}{\text{hyp}}$ — Cosine function

$= \frac{4}{5}$ — adj = 4 and hyp = 5

Find the exact value of each expression.

7A. $\cos^{-1}\left(\sin\frac{\pi}{3}\right)$

7B. $\sin\left(\arctan\frac{5}{12}\right)$

Find the exact value of each expression, if it exists.

(Examples 6 and 7)

29. $\sin\left(\sin^{-1}\frac{3}{4}\right)$

30. $\sin^{-1}\left(\sin\frac{\pi}{2}\right)$

31. $\cos\left(\cos^{-1}\frac{2}{9}\right)$

32. $\cos^{-1}(\cos\pi)$

33. $\tan\left(\tan^{-1}\frac{\pi}{4}\right)$

34. $\tan^{-1}\left(\tan\frac{\pi}{3}\right)$

35. $\cos(\tan^{-1} 1)$

36. $\sin^{-1}\left(\cos\frac{\pi}{2}\right)$

37. $\sin\left(2\cos^{-1}\frac{\sqrt{2}}{2}\right)$

38. $\sin(\tan^{-1} 1 - \sin^{-1} 1)$

39. $\cos(\tan^{-1} 1 - \sin^{-1} 1)$

40. $\cos\left(\cos^{-1} 0 + \sin^{-1}\frac{1}{2}\right)$

Rewrite $\frac{1}{1+\cos x}$ as an expression that does not involve a fraction.

$$\frac{1}{1+\cos x} = \frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}$$ Multiply numerator and denominator by the conjugate of $1+\cos x$, which is $1-\cos x$.

$$= \frac{1-\cos x}{1-\cos^2 x}$$ Multiply.

$$= \frac{1-\cos x}{\sin^2 x}$$ Pythagorean Identity

$$= \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$$ Write as the difference of two fractions.

$$= \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$$ Factor.

$$= \csc^2 x - \cot x\csc x$$ Reciprocal and Quotient Identities

Rewrite as an expression that does not involve a fraction.

7A. $\frac{\cos^2 x}{1 - \sin x}$

7B. $\frac{4}{\sec x + \tan x}$

Rewrite as an expression that does not involve a fraction.
(Example 7)

38. $\frac{\sin x}{\csc x - \cot x}$

39. $\frac{\csc x}{1 - \sin x}$

40. $\frac{\cot x}{\sec x - \tan x}$

41. $\frac{\cot x}{1 + \sin x}$

42. $\frac{3 \tan x}{1 - \cos x}$

43. $\frac{2 \sin x}{\cot x + \csc x}$

44. $\frac{\sin x}{1 - \sec x}$

45. $\frac{\cot^2 x \cos x}{\csc x - 1}$

46. $\frac{5}{\sec x + 1}$

47. $\frac{\sin x \tan x}{\cos x + 1}$

FRQ (كتابي) اسئلة مقالية

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Given $f(x) = x^2 + 4x$, $g(x) = \sqrt{x+2}$, and $h(x) = 3x - 5$, find each function and its domain.

a. $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$
$$= (x^2 + 4x) + \left(\sqrt{x+2}\right)$$
$$= x^2 + 4x + \sqrt{x+2}$$

The domain of f is $(-\infty, \infty)$, and the domain of g is $[-2, \infty)$. So, the domain of $(f + g)$ is the intersection of these domains or $[-2, \infty)$.

b. $(f - h)(x)$

$$(f - h)(x) = f(x) - h(x)$$
$$= (x^2 + 4x) - (3x - 5)$$
$$= x^2 + 4x - 3x + 5$$
$$= x^2 + x + 5$$

The domains of f and h are both $(-\infty, \infty)$, so the domain of $(f - h)$ is $(-\infty, \infty)$.

c. $(f \cdot h)(x)$

$$(f \cdot h)(x) = f(x) \cdot h(x)$$
$$= (x^2 + 4x)(3x - 5)$$
$$= 3x^3 - 5x^2 + 12x^2 - 20x$$
$$= 3x^3 + 7x^2 - 20x$$

The domains of f and h are both $(-\infty, \infty)$, so the domain of $(f \cdot h)$ is $(-\infty, \infty)$.

d. $\left(\frac{h}{f}\right)(x)$

$$\left(\frac{h}{f}\right)(x) = \frac{h(x)}{f(x)} \text{ or } \frac{3x - 5}{x^2 + 4x}$$

The domain of h and f are both $(-\infty, \infty)$, but $x = 0$ or $x = -4$ yields a zero in the denominator of $\left(\frac{h}{f}\right)$. So, the domain of $\left(\frac{h}{f}\right)$ is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function. (Example 1)

1. $f(x) = x^2 + 4$
$g(x) = \sqrt{x}$

2. $f(x) = 8 - x^3$
$g(x) = x - 3$

3. $f(x) = x^2 + 5x + 6$
$g(x) = x + 2$

4. $f(x) = x - 9$
$g(x) = x + 5$

5. $f(x) = x^2 + x$
$g(x) = 9x$

6. $f(x) = x - 7$
$g(x) = x + 7$

7. $f(x) = \frac{6}{x}$
$g(x) = x^3 + x$

8. $f(x) = \frac{x}{4}$
$g(x) = \frac{3}{x}$

9. $f(x) = \frac{1}{\sqrt{x}}$
$g(x) = 4\sqrt{x}$

10. $f(x) = \frac{3}{x}$
$g(x) = x^4$

11. $f(x) = \sqrt{x+8}$
$g(x) = \sqrt{x+5} - 3$

12. $f(x) = \sqrt{x+6}$
$g(x) = \sqrt{x-4}$

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$.

a. $h(x) = \sqrt{x^3 - 4}$

Observe that h is defined using the square root of $x^3 - 4$. So one way to write h as a composition of two functions is to let $g(x) = x^3 - 4$ and $f(x) = \sqrt{x}$. Then

$$h(x) = \sqrt{x^3 - 4} = \sqrt{g(x)} = f[g(x)] \text{ or } [f \circ g](x).$$

b. $h(x) = 2x^2 + 20x + 50$

$h(x) = 2x^2 + 20x + 50$ — Notice that $h(x)$ is factorable.

$= 2(x^2 + 10x + 25)$ or $2(x+5)^2$ — Factor.

One way to write $h(x)$ as a composition is to let $f(x) = 2x^2$ and $g(x) = x + 5$.

$$h(x) = 2(x+5)^2 = 2[g(x)]^2 = f[g(x)] \text{ or } [f \circ g](x).$$

4A. $h(x) = x^2 - 2x + 1$ **4B.** $h(x) = \dfrac{1}{x+7}$

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$. (Example 4)

30. $h(x) = \sqrt{4x+2} + 7$

31. $h(x) = \dfrac{6}{x+5} - 8$

32. $h(x) = |4x + 8| - 9$

33. $h(x) = [\![-3(x-9)]\!]$

34. $h(x) = \sqrt{\dfrac{5-x}{x+2}}$

35. $h(x) = (\sqrt{x} + 4)^3$

36. $h(x) = \dfrac{6}{(x+2)^2}$

37. $h(x) = \dfrac{8}{(x-5)^2}$

38. $h(x) = \dfrac{\sqrt{4+x}}{x-2}$

39. $h(x) = \dfrac{x+5}{\sqrt{x-1}}$

Use the graph of $f(x)$ in Figure 1.7.3 to graph $f^{-1}(x)$.

Graph the line $y = x$. Locate a few points on the graph of $f(x)$. Reflect these points in $y = x$. Then connect them with a smooth curve that mirrors the curvature of $f(x)$ in line $y = x$ (Figure 1.7.4).

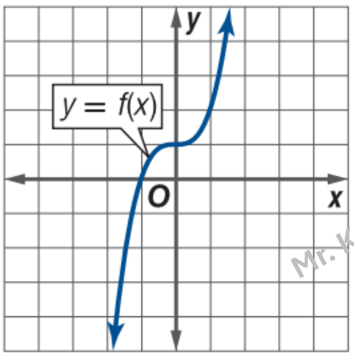


Figure 1.7.3

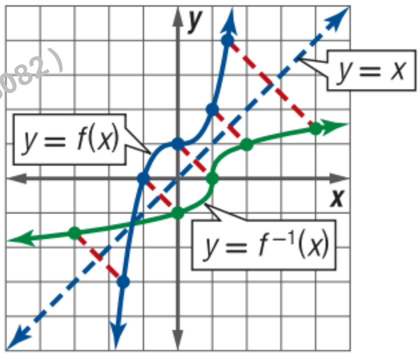
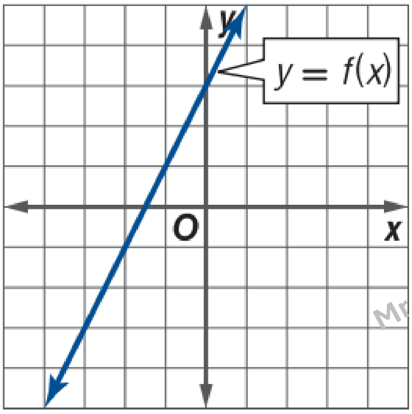


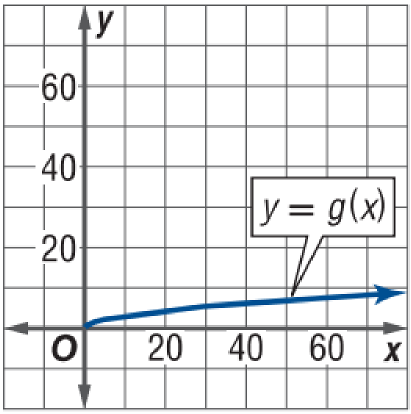
Figure 1.7.4

Use the graph of each function to graph its inverse function.

4A.

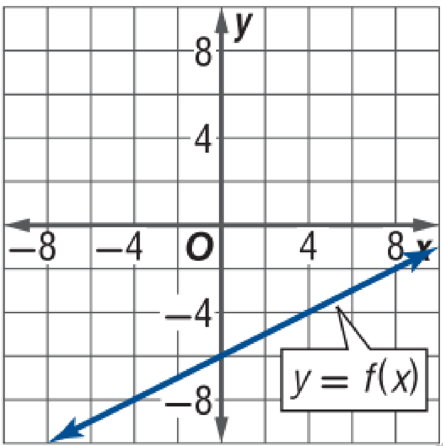


4B.



Use the graph of each function to graph its inverse function. (Example 4)

38.



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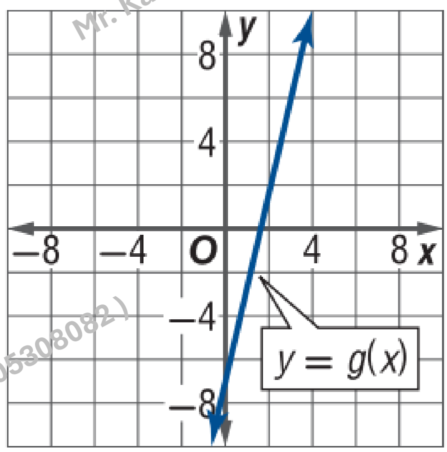
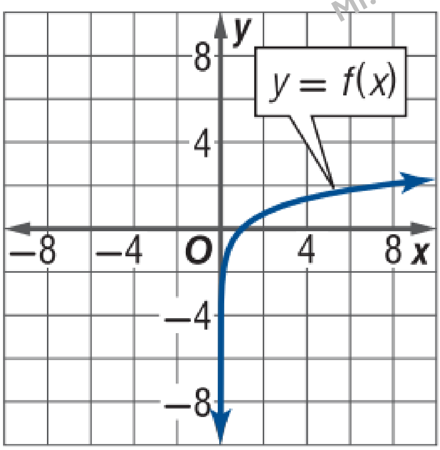


40.



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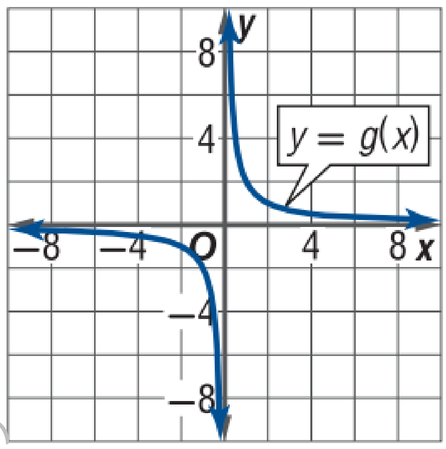

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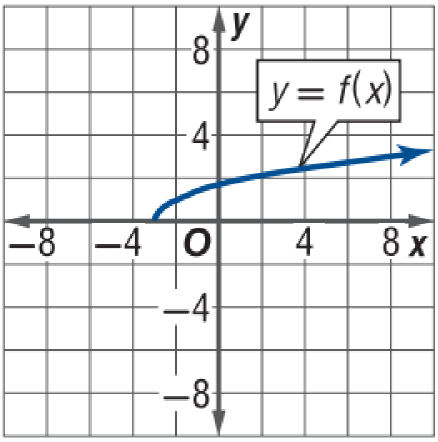

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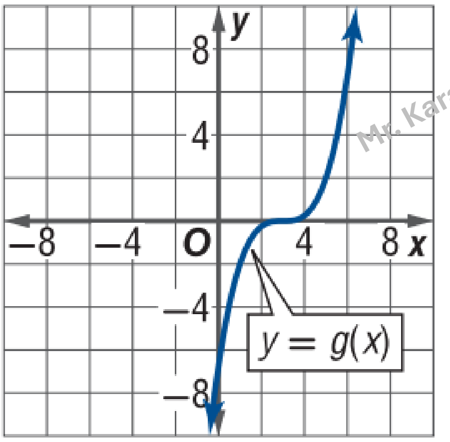


Write a polynomial function of least degree with real coefficients in standard form that has -2, 4, and $3 - i$ as zeros.

Because $3 - i$ is a zero and the polynomial is to have real coefficients, you know that $3 + i$ must also be a zero. Using the Linear Factorization Theorem and the zeros -2, 4, $3 - i$, and $3 + i$, you can write $f(x)$ as follows.

$$f(x) = a[x - (-2)](x - 4)[x - (3 - i)][x - (3 + i)]$$

While a can be any nonzero real number, it is simplest to let $a = 1$. Then write the function in standard form.

$$f(x) = (1)(x + 2)(x - 4)[x - (3 - i)][x - (3 + i)] \quad \text{Let } a = 1.$$

$$= (x^2 - 2x - 8)(x^2 - 6x + 10) \quad \text{Multiply.}$$

$$= x^4 - 8x^3 + 14x^2 + 28x - 80 \quad \text{Multiply.}$$

Therefore, a function of least degree that has -2, 4, $3 - i$, and $3 + i$ as zeros is $f(x) = x^4 - 8x^3 + 14x^2 + 28x - 80$ or any nonzero multiple of $f(x)$.

Write a polynomial function of least degree with real coefficients in standard form with the given zeros.

6A. -3, 1 (multiplicity: 2), $4i$

6B. $2\sqrt{3}, -2\sqrt{3}, 1 + i$

Write a polynomial function of least degree with real coefficients in standard form that has the given zeros.
(Example 6)

32. $3, -4, 6, -1$

33. $-2, -4, -3, 5$

34. $-5, 3, 4 + i$

35. $-1, 8, 6 - i$

36. $2\sqrt{5}, -2\sqrt{5}, -3, 7$

37. $-5, 2, 4 - \sqrt{3}, 4 + \sqrt{3}$

38. $\sqrt{7}, -\sqrt{7}, 4i$

39. $\sqrt{6}, -\sqrt{6}, 3 - 4i$

40. $2 + \sqrt{3}, 2 - \sqrt{3}, 4 + 5i$

41. $6 - \sqrt{5}, 6 + \sqrt{5}, 8 - 3i$

FINANCIAL LITERACY **Suppose Mariam finds an account that will allow her to invest her AED 300 at a 6% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Mariam's account balance be after 20 years?**

$A = Pe^{rt}$ — Continuous Compound Interest Formula

$= 300e^{(0.06)(20)}$ — $P = 300$, $r = 0.06$, and $t = 20$

≈ 996.04 — Simplify.

With continuous compounding, Mariam's account balance after 20 years will be AED 996.04.

ONLINE BANKING If AED 1000 is invested in an online savings account earning 8% per year compounded continuously, how much will be in the account at the end of 10 years if there are no other deposits or withdrawals?

FINANCIAL LITERACY **Copy and complete the table below to find the value of an investment A for the given principal P, rate r, and time t if the interest is compounded n times annually.** (Examples 4 and 5)

n	1	4	12	365	continuously
A					

21. $P = \$500, r = 3\%, t = 5$ years

22. $P = \$1000, r = 4.5\%, t = 10$ years

23. $P = \$1000, r = 5\%, t = 20$ years

24. $P = \$5000, r = 6\%, t = 30$ years

25 **FINANCIAL LITERACY** Ahmed acquired an inheritance of AED 20,000 at age 8, but he will not have access to it until he turns 18. (Examples 4 and 5)

a. If his inheritance is placed in a savings account earning 4.6% interest compounded monthly, how much will Ahmed's inheritance be worth on his 18th birthday?

b. How much will Ahmed's inheritance be worth if it is placed in an account earning 4.2% interest compounded continuously?

26. **FINANCIAL LITERACY** Eman invests AED 1200 in a certificate of deposit (CD). The table shows the interest rates offered by the bank on 3- and 5-year CDs. (Examples 4 and 5)

CD Offers		
Years	3	5
Interest	3.45%	4.75%
Compounded	continuously	monthly

a. How much would her investment be worth with each option?

b. How much would her investment be worth if the 5-year CD was compounded continuously?

Solve each equation.

a. $36^{x+1} = 6^{x+6}$

$36^{x+1} = 6^{x+6}$

$(6^2)^{x+1} = 6^{x+6}$

$6^{2x+2} = 6^{x+6}$

$2x + 2 = x + 6$

$x + 2 = 6$

$x = 4$

b. $\left(\frac{1}{2}\right)^c = 64^{\frac{1}{2}}$

$\left(\frac{1}{2}\right)^c = 64^{\frac{1}{2}}$

$2^{-c} = (2^6)^{\frac{1}{2}}$

$2^{-c} = 2^3$

$-c = 3$

$c = -3$

1A. $16^{x+3} = 4^{4x+7}$

1B. $\left(\frac{2}{3}\right)^{x-5} = \left(\frac{9}{4}\right)^{\frac{3x}{4}}$

Solve each equation. (Example 1)

1. $4^{x+7} = 8^{x+3}$

2. $8^{x+4} = 32^{3x}$

3. $49^{x+4} = 7^{18-x}$

4. $32^{x-1} = 4^{x+5}$

5. $\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$

6. $12^{3x+11} = 144^{2x+7}$

7. $25^{\frac{x}{3}} = 5^{x-4}$

8. $\left(\frac{5}{6}\right)^{4x} = \left(\frac{36}{25}\right)^{9-x}$

9. **INTERNET** The number of people P in millions using two different search engines to surf the Internet t weeks after the creation of the search engine can be modeled by $P_1(t) = 1.5^{t+4}$ and $P_2(t) = 2.25^{t-3.5}$, respectively. During which week did the same number of people use each search engine? (Example 1)

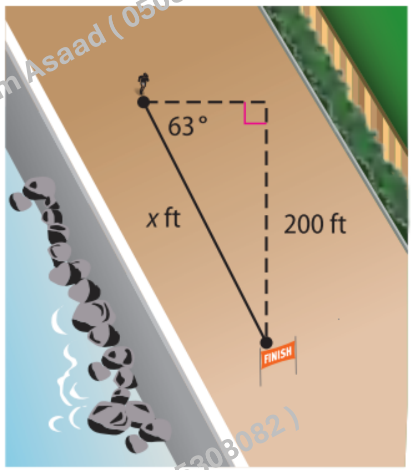
10. **FINANCIAL LITERACY** Essam is planning on investing AED 5000 and is considering two savings accounts. The first account is continuously compounded and offers a 3% interest rate. The second account is annually compounded and also offers a 3% interest rate, but the bank will match 4% of the initial investment. (Example 1)

a. Write an equation for the balance of each savings account at time t years.

b. How many years will it take for the continuously compounded account to catch up with the annually compounded savings account?

c. If Essam plans on leaving the money in the account for 30 years, which account should he choose?

TRIATHLONS **A competitor in a triathlon is running along the course shown. Determine the length in feet that the runner must cover to reach the finish line.**



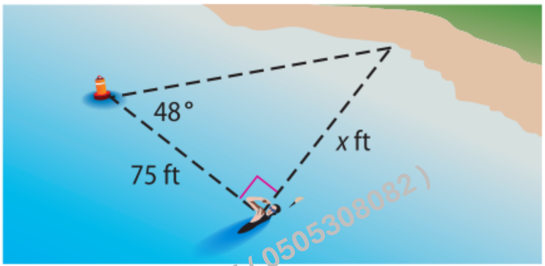
An acute angle measure and the opposite side length are given, so the sine function can be used to find the hypotenuse.

$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	Sine function
$\sin 63^\circ = \frac{200}{x}$	$\theta = 63^\circ$, opp = 200, and hyp = x
$x \sin 63^\circ = 200$	Multiply each side by x.
$x = \frac{200}{\sin 63^\circ}$ or about 224.47	Divide each side by sin 63°.

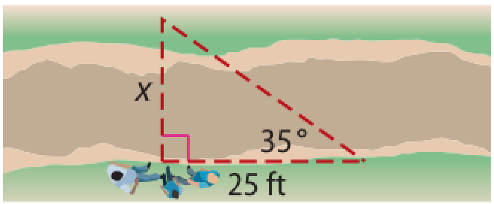
So, the competitor must run about 224.5 feet to finish the triathlon.

GuidedPractice

4. **TRIATHLONS** Suppose a competitor in the swimming portion of the race is swimming along the course shown. Find the distance the competitor must swim to reach the shore.



27 **MOUNTAIN CLIMBING** A team of climbers must determine the width of a ravine in order to set up equipment to cross it. If the climbers walk 25 feet along the ravine from their crossing point, and sight the crossing point on the far side of the ravine to be at a 35° angle, how wide is the ravine? (Example 4)



28. **SNOWBOARDING** Ahmed built a snowboarding ramp with a height of 3.5 feet and an 18° incline. (Example 4)

a. Draw a diagram to represent the situation.

b. Determine the length of the ramp.

29. **DETOUR** Traffic is detoured from Nasser Ave., left 0.8 kilometer on Etihad Street, and then right on Hessa Street, which intersects Nasser Ave. at a 32° angle. (Example 4)

a. Draw a diagram to represent the situation.

b. Determine the length of Nasser Ave. that is detoured.

30. **PARACHUTING** A paratrooper encounters stronger winds than anticipated while parachuting from 411.5 meters, causing him to drift at an 8° angle. How far from the drop zone will the paratrooper land? (Example 4)

Verify each identity. (Examples 1–3)

1. $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

2. $\sec^2\theta(1 - \cos^2\theta) = \tan^2\theta$

3. $\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$

4. $\csc\theta - \cos\theta\cot\theta = \sin\theta$

5. $\cot^2\theta\csc^2\theta - \cot^2\theta = \cot^4\theta$

6. $\tan\theta\csc^2\theta - \tan\theta = \cot\theta$

7. $\dfrac{\sec\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta} = \cot\theta$

8. $\dfrac{\sin\theta}{1 - \cos\theta} + \dfrac{1 - \cos\theta}{\sin\theta} = 2\csc\theta$

9. $\dfrac{\cos\theta}{1 + \sin\theta} + \tan\theta = \sec\theta$

10. $\dfrac{\sin\theta}{1 - \cot\theta} + \dfrac{\cos\theta}{1 - \tan\theta} = \sin\theta + \cos\theta$

11. $\dfrac{1}{1 - \tan^2\theta} + \dfrac{1}{1 - \cot^2\theta} = 1$

12. $\dfrac{1}{\csc\theta + 1} + \dfrac{1}{\csc\theta - 1} = 2\sec^2\theta\sin\theta$

13. $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$

14. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

15. $\dfrac{1}{1 - \sin\theta} + \dfrac{1}{1 + \sin\theta} = 2\sec^2\theta$

16. $\dfrac{\cos\theta}{1 + \sin\theta} + \dfrac{\cos\theta}{1 - \sin\theta} = 2\sec\theta$

17. $\csc^4\theta - \cot^4\theta = 2\cot^2\theta + 1$

18. $\dfrac{\csc^2\theta + 2\csc\theta - 3}{\csc^2\theta - 1} = \dfrac{\csc\theta + 3}{\csc\theta + 1}$