EOT1 MATH GUIDE ANSWERS

12Gen

39. $f(x) = \dfrac{8x+12}{x^2+5x+4}$

SOLUTION:

When the denominator of $f(x) = \dfrac{8x+12}{x^2+5x+4}$ is zero, the expression is undefined.

$$x^2+5x+4=0$$
$$(x+4)(x+1)=0$$
$$x=-4 \quad x=-1$$

Therefore, the domain of this function is all real numbers except $x = -4$ and $x = -1$, which can be written as $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$.

ANSWER:

$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

40. $g(x) = \dfrac{x+1}{x^2-3x-40}$

SOLUTION:

When the denominator of $g(x) = \dfrac{x+1}{x^2-3x-40}$ is zero, the expression is undefined.

$$x^2-3x-40=0$$
$$(x-8)(x+5)=0$$
$$x=8 \quad x=-5$$

Therefore, the domain of $g(x)$ is all real numbers except $x = 8$ and $x = -5$, which can be written as $(-\infty\ -5) \cup (-5, 8) \cup (8, \infty)$.

ANSWER:

$(-\infty, -5) \cup (-5, 8) \cup (8, \infty)$

41. $g(a) = \sqrt{1+a^2}$

SOLUTION:

There is no value of a that will make the expression $\sqrt{1+a^2}$ undefined. Any value that is squared will be nonnegative. Any nonnegative number plus one will also always be nonnegative. Therefore, the domain of $g(a)$ includes all real numbers or $(-\infty, \infty)$.

ANSWER:

$(-\infty, \infty)$

42. $h(x) = \sqrt{6 - x^2}$

SOLUTION:

The square root of a negative number cannot be a real number, so $6 - x^2 \geq 0$.

$$6 - x^2 \geq 0$$
$$6 \geq x^2$$
$$\sqrt{6} \geq \pm x$$
$$x \leq \sqrt{6} \text{ or } x \geq -\sqrt{6}$$

If $6 - x^2 \geq 0$, then $x \leq \sqrt{6}$ or $x \geq -\sqrt{6}$.

If x were greater than $\sqrt{6}$ or less than $-\sqrt{6}$, the expression $6 - x^2$ would be negative, and thus would not be a real number.

The domain of $h(x)$ is $[-\sqrt{6}, \sqrt{6}]$.

ANSWER:

$[-\sqrt{6}, \sqrt{6}]$

43. $f(a) = \dfrac{5a}{\sqrt{4a-1}}$

SOLUTION:

This function is defined only when $4a - 1 > 0$.

$$4a - 1 > 0$$
$$4a > 1$$
$$a > 0.25$$

The domain of $f(a)$ is $(0.25, \infty)$.

ANSWER:

$(0.25, \infty)$

44. $g(x) = \dfrac{3}{\sqrt{x^2 - 16}}$

SOLUTION:

This function is defined only when $x^2 - 16 > 0$.

$$x^2 - 16 > 0$$
$$x^2 > 16$$
$$\pm x > 4$$
$$x > 4 \text{ or } x < -4$$

If x were greater than -4 or less than 4, the radicand would be negative, and thus would not be a real number.

The domain of $g(x)$ is $(-\infty, -4) \cup (4, \infty)$.

ANSWER:

$(-\infty, -4) \cup (4, \infty)$

45. $f(x) = \dfrac{2}{x} + \dfrac{4}{x+1}$

SOLUTION:

This function is defined only when $x \neq 0$ and $x + 1 \neq 0$. Therefore, the function is defined for all real numbers except $x = 0$ and $x = -1$. The domain of $f(x)$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

ANSWER:

$(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

46. $g(x) = \dfrac{6}{x+3} + \dfrac{2}{x-4}$

SOLUTION:

This function is defined only when $x + 3 \neq 0$ and $x - 4 \neq 0$. Therefore, the function is defined for all real numbers except $x = -3$ and $x = 4$. The domain of $g(x)$ is $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$.

ANSWER:

$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

30. $g(x) = 2x^2 + 18x - 14$

a. $g(9)$

b. $g(3x)$

c. $g(1 + 5m)$

SOLUTION:

To find each value, replace x in $g(x) = 2x^2 + 18x - 14$.

a.

$$g(x) = 2x^2 + 18x - 14$$
$$g(9) = 2(9)^2 + 18(9) - 14$$
$$= 162 + 162 - 14$$
$$= 310$$

b.

$$g(x) = 2x^2 + 18x - 14$$
$$g(3x) = 2(3x)^2 + 18(3x) - 14$$
$$g(3x) = 2(9x^2) + 54x - 14$$
$$= 18x^2 + 54x - 14$$

c.

$$g(x) = 2x^2 + 18x - 14$$
$$g(1+5m) = 2(1+5m)^2 + 18(1+5m) - 14$$
$$g(1+5m) = 2(1+10m+25m^2) + 18 + 90m - 14$$
$$g(1+5m) = 2 + 20m + 50m^2 + 18 + 90m - 14$$
$$= 50m^2 + 110m + 6$$

31. $h(y) = -3y^3 - 6y + 9$

a. $h(4)$

b. $h(-2y)$

c. $h(5b + 3)$

SOLUTION:

To find each value, replace y in $h(y) = -3y^3 - 6y + 9$.

a.

$$h(y) = -3y^3 - 6y + 9$$
$$h(4) = -3(4)^3 - 6(4) + 9$$
$$= -192 - 24 + 9$$
$$= -207$$

b.

$$h(y) = -3y^3 - 6y + 9$$
$$h(-2y) = -3(-2y)^3 - 6(-2y) + 9$$
$$= 24y^3 + 12y + 9$$

c.

$$h(y) = -3y^3 - 6y + 9$$
$$h(5b+3)$$
$$= -3(5b+3)^3 - 6(5b+3) + 9$$
$$= -3[(5b)^3 + 3(5b)^2(3) + 3(5b)(3)^2 + (3)^3] - 30b - 18 + 9$$
$$= -3[125b^3 + 225b^2 + 135b + 27] - 30b - 9$$
$$= -375b^3 - 675b^2 - 405b - 81 - 30b - 9$$
$$= -375b^3 - 675b^2 - 435b - 90$$

32. $f(t)=\dfrac{4t+11}{3t^2+5t+1}$

a. $f(-6)$
b. $f(4t)$
c. $f(3-2a)$

SOLUTION:

To find each value, replace t in $f(t)=\dfrac{4t+11}{3t^2+5t+1}$.

$$f(t)=\frac{4t+11}{3t^2+5t+1}$$
$$f(-6)=\frac{4(-6)+11}{3(-6)^2+5(-6)+1}=\frac{-24+11}{108-30+1}=-\frac{13}{79}$$

$$f(t)=\frac{4t+11}{3t^2+5t+1}$$
$$f(4t)=\frac{4(4t)+11}{3(4t)^2+5(4t)+1}=\frac{16t+11}{48t^2+20t+1}$$

$$f(t)=\frac{4t+11}{3t^2+5t+1}$$
$$f(3-2a)=\frac{4(3-2a)+11}{3(3-2a)^2+5(3-2a)+1}$$
$$=\frac{12-8a+11}{3\left(9-12a+4a^2\right)+15-10a+1}$$
$$=\frac{-8a+23}{27-36a+12a^2-10a+16}$$
$$=\frac{-8a+23}{12a^2-46a+43}$$

33. $g(x)=\dfrac{3x^3}{x^2+x-4}$

a. $g(-2)$
b. $g(5x)$
c. $g(8-4b)$

SOLUTION:

To find each value, replace x in $g(x)=\dfrac{3x^3}{x^2+x-4}$.

$$g(x)=\frac{3x^3}{x^2+x-4}$$
$$g(-2)=\frac{3(-2)^3}{(-2)^2+(-2)-4}=\frac{3(-8)}{4-2-4}=\frac{-24}{-2}=12$$

$$g(x)=\frac{3x^3}{x^2+x-4}$$
$$g(5x)=\frac{3(5x)^3}{(5x)^2+(5x)-4}=\frac{375x^3}{25x^2+5x-4}$$

$$g(x)=\frac{3x^3}{x^2+x-4}$$
$g(8-4b)$
$$=\frac{3(8-4b)^3}{(8-4b)^2+(8-4b)-4}$$
$$=\frac{3[(8)^3+3(8)^2(-4b)+3(8)(-4b)^2+(-4b)^3]}{16b^2-64b+64+8-4b-4}$$
$$=\frac{3(512-768b+384b^2-64b^3)}{16b^2-68b+68}$$
$$=\frac{3(-64b^3+384b^2-768b+512)}{16b^2-68b+68}$$
$$=\frac{-192b^3+1152b^2-2304b+1536}{16b^2-68b+68}$$
$$=\frac{-48b^3+288b^2-576b+384}{4b^2-17b+17}$$

34. $h(x) = 16 - \dfrac{12}{2x+3}$

a. $h(-3)$
b. $h(6x)$
c. $h(10-2c)$

SOLUTION:

To find each value, replace x in $h(x) = 16 - \dfrac{12}{2x+3}$.

$$h(x) = 16 - \frac{12}{2x+3}$$
$$h(-3) = 16 - \frac{12}{2(-3)+3}$$
$$= 16 - \frac{12}{-6+3}$$
$$= 16 - \frac{12}{-3}$$
$$= 16 + 4$$
$$= 20$$

$$h(x) = 16 - \frac{12}{2x+3}$$
$$h(6x) = 16 - \frac{12}{2(6x)+3}$$
$$= 16 - \frac{12}{12x+3}$$
$$= 16 - \frac{4}{4x+1}$$

$$h(x) = 16 - \frac{12}{2x+3}$$
$$h(10-2c) = 16 - \frac{12}{2(10-2c)+3}$$
$$= 16 - \frac{12}{20-4c+3}$$
$$= 16 - \frac{12}{23-4c}$$

35. $f(x) = -7 + \dfrac{6x+1}{x}$

a. $f(5)$
b. $f(-8x)$
c. $f(6y+4)$

SOLUTION:

To find each value, replace x in $f(x) = -7 + \dfrac{6x+1}{x}$.

$$f(x) = -7 + \frac{6x+1}{x}$$
$$f(5) = -7 + \frac{6(5)+1}{5}$$
$$= -7 + \frac{30+1}{5}$$
$$= -7 + \frac{31}{5}$$
$$= -7 + 6.2$$
$$= -0.8$$

$$f(x) = -7 + \frac{6x+1}{x}$$
$$f(-8x) = -7 + \frac{6(-8x)+1}{-8x}$$
$$= -7 + \frac{-48x+1}{-8x}$$
$$= -7 + \frac{-48x}{-8x} + \frac{1}{-8x}$$
$$= -7 + 6 - \frac{1}{8x}$$
$$= -1 - \frac{1}{8x}$$

$$f(6y+4) = -7 + \frac{6(6y+4)+1}{6y+4}$$
$$= -7 + \frac{36y+24+1}{6y+4}$$
$$= -7 + \frac{36y+25}{6y+4}$$

36. $g(m) = 3 + \sqrt{m^2 - 4}$

a. $g(-2)$

b. $g(3m)$

c. $g(4m - 2)$

SOLUTION:

To find each value, replace m in $g(m) = 3 + \sqrt{m^2 - 4}$.

$$g(m) = 3 + \sqrt{m^2 - 4}$$
$$g(-2) = 3 + \sqrt{(-2)^2 - 4}$$
$$= 3 + \sqrt{4 - 4}$$
$$= 3$$

$$g(m) = 3 + \sqrt{m^2 - 4}$$
$$g(3m) = 3 + \sqrt{(3m)^2 - 4}$$
$$= 3 + \sqrt{9m^2 - 4}$$

$$g(m) = 3 + \sqrt{m^2 - 4}$$
$$g(4m - 2) = 3 + \sqrt{(4m - 2)^2 - 4}$$
$$= 3 + \sqrt{16m^2 - 16m + 4 - 4}$$
$$= 3 + \sqrt{16m^2 - 16m}$$
$$= 3 + \sqrt{16(m^2 - m)}$$
$$= 3 + 4\sqrt{m^2 - m}$$

37. $t(x) = 5\sqrt{6x^2}$

a. $t(-4)$

b. $t(2x)$

c. $t(7 + n)$

SOLUTION:

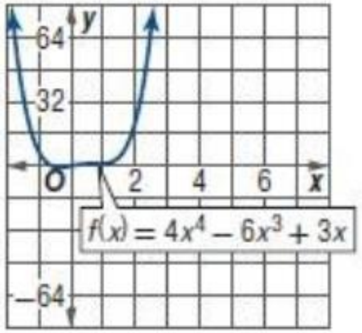
To find each value, replace x in $t(x) = 5\sqrt{6x^2}$.

$$t(x) = 5\sqrt{6x^2}$$
$$t(-4) = 5\sqrt{6(-4)^2}$$
$$= 5\sqrt{96}$$
$$= 5\sqrt{16} \cdot \sqrt{6}$$
$$= 20\sqrt{6}$$

$$t(x) = 5\sqrt{6x^2}$$
$$t(2x) = 5\sqrt{6(2x)^2}$$
$$= 5\sqrt{(2x)^2} \cdot \sqrt{6}$$
$$= 10|x|\sqrt{6}$$

$$t(x) = 5\sqrt{6x^2}$$
$$t(7 + n) = 5\sqrt{6(7 + n)^2}$$
$$= 5\sqrt{(7 + n)^2} \cdot \sqrt{6}$$
$$= 5|7 + n|\sqrt{6}$$

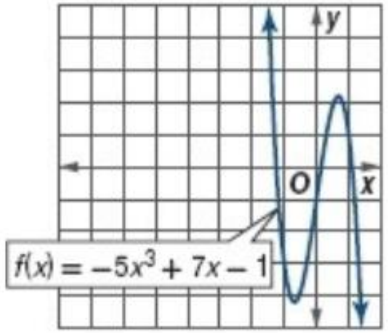
22.



ANSWER:

From the graph, it appears that $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

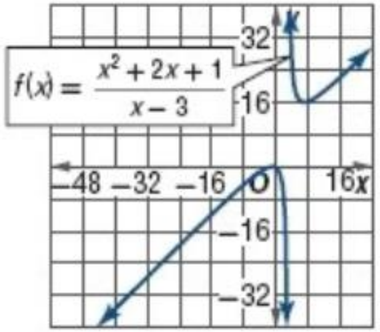
23.



SOLUTION:

From the graph, it appears that $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

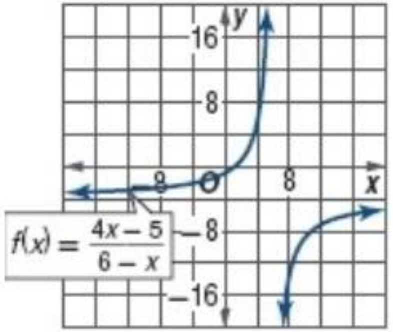
24.



SOLUTION:

From the graph, it appears that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

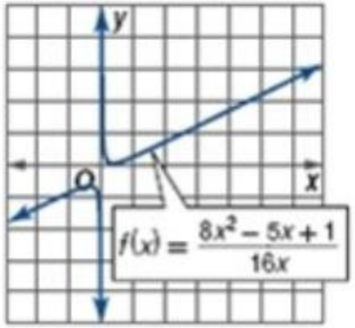
25.



SOLUTION:

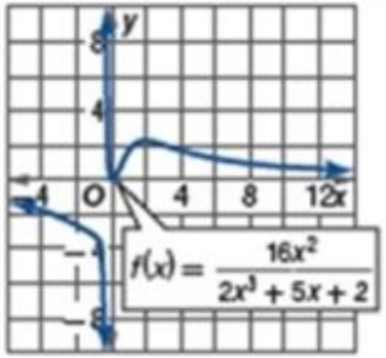
From the graph, it appears that $f(x) \to -4$ as $x \to -\infty$ and $f(x) \to -4$ as $x \to \infty$.

26.



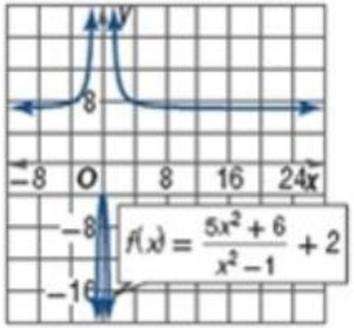
SOLUTION:

From the graph, it appears that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

27. 

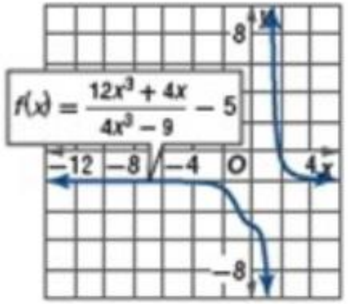
SOLUTION:

From the graph, it appears that $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$.

28. 

SOLUTION:

From the graph, it appears that $f(x) \to 7$ as $x \to -\infty$ and $f(x) \to 7$ as $x \to \infty$.

29. 

SOLUTION:

From the graph, it appears that $f(x) \to -2$ as $x \to -\infty$ and $f(x) \to -2$ as $x \to \infty$.

33. $q(x) = -\frac{24}{x}$

ANSWER:

Sample answer: As $x \to \infty$, the fraction will decrease, and $q(x)$ will approach 0.

34. $f(x) = \frac{0.8}{x^2}$

ANSWER:

Sample answer: As $x \to \infty$, the fraction will decrease, and $f(x)$ will approach 0.

35. $p(x) = \frac{x+1}{x-2}$

ANSWER:

Sample answer: As $x \to \infty$, the fraction will approach $\frac{x}{x}$, so $p(x)$ will approach 1.

36. $m(x) = \frac{4+x}{2x+6}$

ANSWER:

Sample answer: As $x \to \infty$, the fraction will approach $\frac{x}{2x}$, so $m(x)$ will approach $\frac{1}{2}$.

37. $c(x) = \frac{5x^2}{x^3+2x+1}$

Sample answer: As $x \to \infty$, the fraction will decrease, and $c(x)$ will approach 0.

38. $k(x) = \frac{4x^2-3x-1}{11x}$

Sample answer: As $x \to \infty$, the numerator will become very large compared to the denominator, so $k(x)$ will approach ∞.

39. $h(x) = 2x^5 + 7x^3 + 5$

ANSWER:

Sample answer: As $x \to \infty$, the $h(x)$ will increase without bound, or approach ∞.

40. $g(x) = x^4 - 9x^2 + \frac{x}{4}$

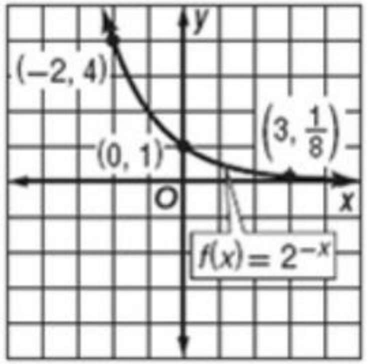
ANSWER:

Sample answer: As $x \to \infty$, the $g(x)$ will increase without bound, or approach ∞.

1. $f(x) = 2^{-x}$

ANSWER:

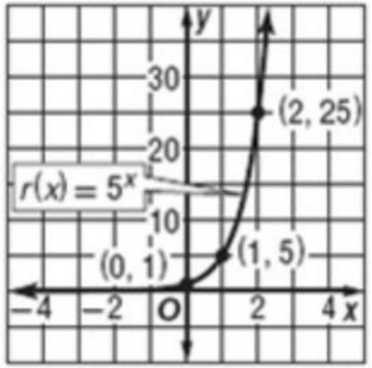
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 0$; decreasing on $(-\infty, \infty)$



2. $r(x) = 5^{x}$

ANSWER:

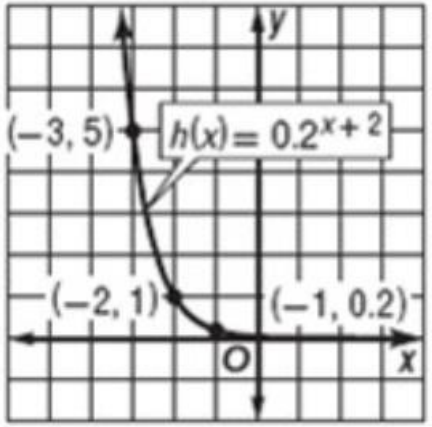
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x \to -\infty} r(x) = 0$, $\lim_{x \to \infty} r(x) = \infty$; increasing on $(-\infty, \infty)$

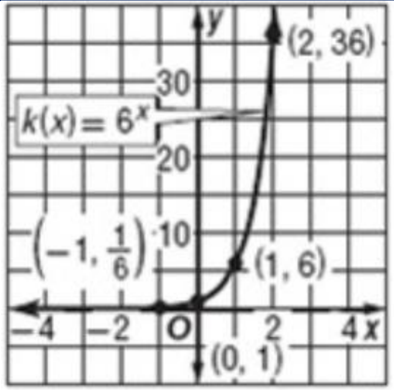


3. $h(x) = 0.2^{x+2}$

ANSWER:

D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 0.04; asymptote: x-axis; $\lim_{x \to -\infty} h(x) = \infty$, $\lim_{x \to \infty} h(x) = 0$; decreasing on $(-\infty, \infty)$

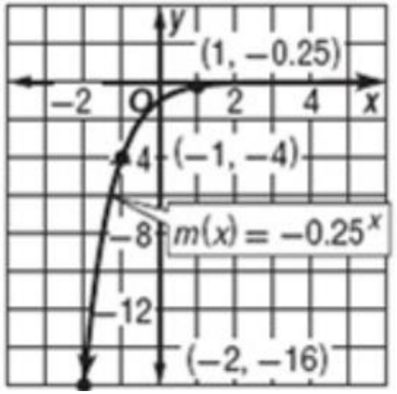




4. $k(x) = 6^x$

ANSWER:

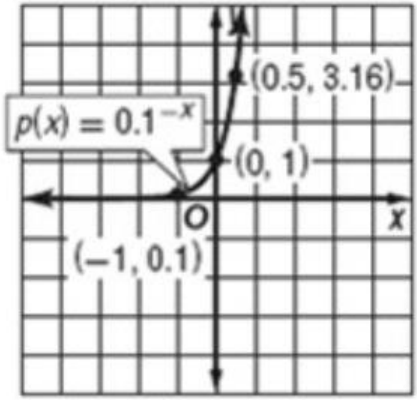
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x \to -\infty} k(x) = 0$, $\lim_{x \to \infty} k(x) = \infty$; increasing on $(-\infty, \infty)$



5. $m(x) = -(0.25)^x$

ANSWER:

D = $(-\infty, \infty)$; R = $(-\infty, 0)$; y-intercept: -1; asymptote: x-axis; $\lim_{x \to -\infty} m(x) = -\infty$, $\lim_{x \to \infty} m(x) = 0$; increasing on $(-\infty, \infty)$



6. $p(x) = 0.1^{-x}$

ANSWER:

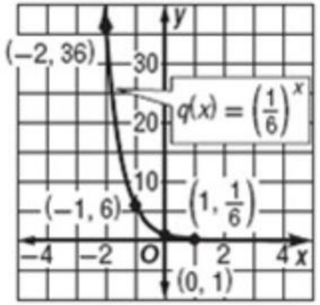
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x \to -\infty} p(x) = 0$, $\lim_{x \to \infty} p(x) = \infty$; increasing for $(-\infty, \infty)$

<table>
<tr><td rowspan="2">5</td><td>إيجاد قيم الدوال الأسية وتمثيلها بيانيًا وتحليلها.</td><td rowspan="2">Exercises (1-10)</td><td rowspan="2">P166</td></tr>
<tr><td>Evaluate, analyze, and graph exponential functions.</td></tr>
</table>

7. $q(x)=\left(\frac{1}{6}\right)^{x}$

ANSWER:

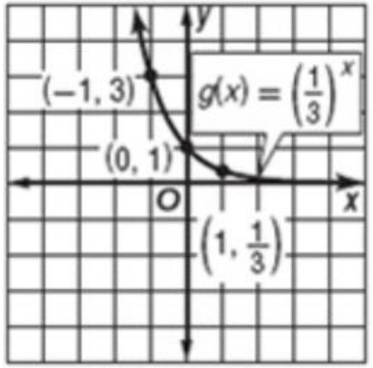
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x\to-\infty} q(x)=\infty, \lim_{x\to\infty} q(x)=0$; decreasing on $(-\infty, \infty)$



8. $g(x)=\left(\frac{1}{3}\right)^{x}$

ANSWER:

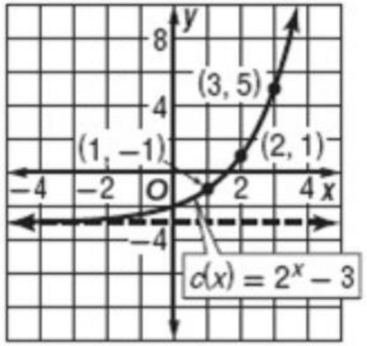
D = $(-\infty, \infty)$; R = $(0, \infty)$; y-intercept: 1; asymptote: x-axis; $\lim_{x\to-\infty} g(x)=\infty, \lim_{x\to\infty} g(x)=0$; decreasing on $(-\infty, \infty)$



9. $c(x)=2^{x}-3$

ANSWER:

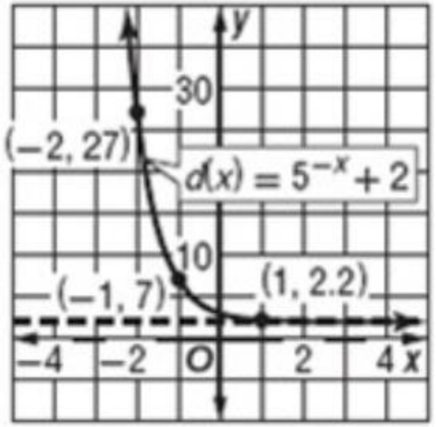
D = $(-\infty, \infty)$; R = $(-3, \infty)$; y- intercept: -2; x-intercept: 1.6; asymptote: $y=-3$; $\lim_{x\to-\infty} c(x)=-3, \lim_{x\to\infty} c(x)=\infty$; increasing on $(-\infty, \infty)$



10. $d(x)=5^{-x}+2$

ANSWER:

D = $(-\infty, \infty)$; R = $(2, \infty)$; y-intercept: 3; asymptote: $y=2$; $\lim_{x\to-\infty} d(x)=\infty, \lim_{x\to\infty} d(x)=2$; decreasing for $(-\infty, \infty)$



6	إيجاد قيم التعابير التي تتضمن لوغاريتمات. Evaluate expressions involving logarithms.	Exercises (1-24)	P178

1. $\log_2 8$

ANSWER: 3

2. $\log_{10} 10$

ANSWER: 1

3. $\log_6 \frac{1}{36}$

ANSWER: -2

4. $4^{\log_4 1}$

ANSWER: 1

5. $\log_{11} 121$

ANSWER: 2

6. $\log_2 2^3$

ANSWER: 3

7. $\log_{\sqrt{9}} 81$

ANSWER: 4

8. $\log 0.01$

ANSWER: -2

9. $\log 42$

ANSWER: ≈ 1.623

10. $\log_x x^2$

ANSWER: 2

11. $\log 5275$

ANSWER: ≈ 3.722

12. $\ln e^{-14}$

ANSWER: -14

13. $3 \ln e^4$

ANSWER: 12

14. $\ln (5 - \sqrt{6})$

ANSWER: ≈ 0.936

15. $\log_{36} \sqrt[5]{6}$

ANSWER: $\frac{1}{10}$

16. $4 \ln (7 - \sqrt{2})$

ANSWER: ≈ 6.88

17. $\log 635$

ANSWER: ≈ 2.803

18. $\frac{\ln 2}{\ln 7}$

ANSWER: ≈ 0.356

19. $\ln (-6)$

ANSWER: $\varnothing$

20. $\ln\left(\frac{1}{e^{12}}\right)$

ANSWER: -12

21. $\ln 8$

ANSWER: ≈ 2.079

22. $\log_{\sqrt[3]{4}} 64$

ANSWER: 21

23. $\frac{7}{\ln e}$

ANSWER: 7

24. $\log 1000$

ANSWER: 3

1. $\ln \frac{4}{5}$

SOLUTION:

$$\ln \frac{4}{5} = \ln 4 - \ln 5$$
$$= \ln 2^2 - \ln 5$$
$$= 2 \ln 2 - \ln 5$$

ANSWER:

$2 \ln 2 - \ln 5$

2. $\ln 200$

SOLUTION:

$$\ln 200 = \ln(8 \cdot 25)$$
$$= \ln 8 + \ln 25$$
$$= \ln 2^3 + \ln 5^2$$
$$= 3 \ln 2 + 2 \ln 5$$

ANSWER:

$3 \ln 2 + 2 \ln 5$

3. $\ln 80$

SOLUTION:

$$\ln 80 = \ln(16 \cdot 5)$$
$$= \ln 16 + \ln 5$$
$$= \ln 2^4 + \ln 5$$
$$= 4 \ln 2 + \ln 5$$

ANSWER:

$4 \ln 2 + \ln 5$

4. $\ln 12.5$

SOLUTION:

$$\ln 12.5 = \ln\left(\frac{25}{2}\right)$$
$$= \ln 25 - \ln 2$$
$$= \ln 5^2 - \ln 2$$
$$= 2 \ln 5 - \ln 2$$

ANSWER:

$2 \ln 5 - \ln 2$

5. $\ln \frac{0.8}{2}$

SOLUTION:

$$\ln \frac{0.8}{2} = \ln \frac{0.4}{1}$$
$$= \ln \frac{4}{10}$$
$$= \ln \frac{2}{5}$$
$$= \ln 2 - \ln 5$$

ANSWER:

$\ln 2 - \ln 5$

6. $\ln \frac{2}{5}$

SOLUTION:

$$\ln \frac{a}{b} = \ln a - \ln b$$
$$\ln \frac{2}{5} = \ln 2 - \ln 5$$

ANSWER:

$\ln 2 - \ln 5$

7. $\ln 2000$

SOLUTION:

$$\ln 2000 = \ln(16 \cdot 125)$$
$$= \ln 16 + \ln 125$$
$$= \ln 2^4 + \ln 5^3$$
$$= 4 \ln 2 + 3 \ln 5$$

ANSWER:

$4 \ln 2 + 3 \ln 5$

8. $\ln 1.6$

SOLUTION:

$$\ln 1.6 = \ln \frac{16}{10}$$
$$= \ln \frac{8}{5}$$
$$= \ln 8 - \ln 5$$
$$= \ln 2^3 - \ln 5$$
$$= 3 \ln 2 - \ln 5$$

ANSWER:

$3 \ln 2 - \ln 5$

9. $\ln 63$

SOLUTION:

$$\ln 63 = \ln(9 \cdot 7)$$
$$= \ln 9 + \ln 7$$
$$= \ln 3^2 + \ln 7$$
$$= 2 \ln 3 + \ln 7$$

ANSWER:

$2 \ln 3 + \ln 7$

10. $\ln \frac{49}{81}$

SOLUTION:

$$\ln \frac{49}{81} = \ln 49 - \ln 81$$
$$= \ln 7^2 - \ln 3^4$$
$$= 2 \ln 7 - 4 \ln 3$$

ANSWER:

$2 \ln 7 - 4 \ln 3$

11. $\ln \frac{7}{9}$

SOLUTION:

$$\ln \frac{7}{9} = \ln 7 - \ln 9$$
$$= \ln 7 - \ln 3^2$$
$$= \ln 7 - 2\ln 3$$

ANSWER:

$\ln 7 - 2 \ln 3$

12. $\ln 147$

SOLUTION:

$$\ln 147 = \ln(49 \cdot 3)$$
$$= \ln 49 + \ln 3$$
$$= \ln 7^2 + \ln 3$$
$$= 2\ln 7 + \ln 3$$

ANSWER:

$\ln 3 + 2 \ln 7$

13. $\ln 1323$

SOLUTION:

$$\ln 1323 = \ln(49 \cdot 27)$$
$$= \ln 49 + \ln 27$$
$$= \ln 7^2 + \ln 3^3$$
$$= 2\ln 7 + 3\ln 3$$

ANSWER:

$3 \ln 3 + 2 \ln 7$

14. $\ln \frac{343}{729}$

SOLUTION:

$$\ln \frac{343}{729} = \ln 343 - \ln 729$$
$$= \ln 7^3 - \ln 3^6$$
$$= 3\ln 7 - 6\ln 3$$

ANSWER:

$3 \ln 7 - 6 \ln 3$

15. $\ln \frac{2401}{81}$

SOLUTION:

$$\ln \frac{2401}{81} = \ln 2401 - \ln 81$$
$$= \ln 7^4 - \ln 3^4$$
$$= 4\ln 7 - 4\ln 3$$

ANSWER:

$4 \ln 7 - 4 \ln 3$

16. $\ln 1701$

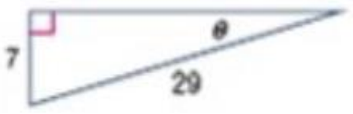
SOLUTION:

$$\ln 1701 = \ln(7 \cdot 243)$$
$$= \ln 7 + \ln 243$$
$$= \ln 7 + \ln 3^5$$
$$= \ln 7 + 5\ln 3$$

ANSWER:

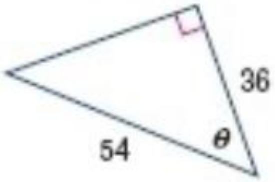
$5 \ln 3 + \ln 7$

8	حل المثلثات القائمة الزاوية. Solve right triangles.	Exercises (31-38)	P229

31. 

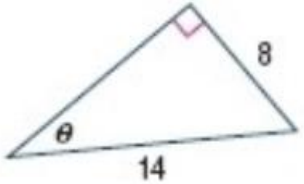
ANSWER:

14°

32. 

ANSWER:

48°

33. 

ANSWER:

35°

34. 

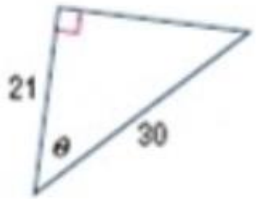
ANSWER:

81°

35. 

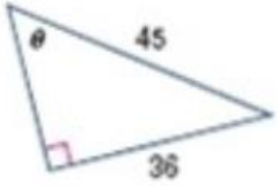
ANSWER:

59°

36. 

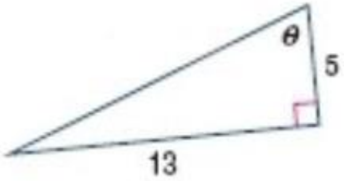
ANSWER:

46°

37. 

ANSWER:

53°

38. 

ANSWER:

69°

27. $\frac{\pi}{6}$, $r = 2.5$ m

SOLUTION:

$s = r\theta$

$= 2.5\left(\frac{\pi}{6}\right)$

$= \frac{5\pi}{12}$ or about 1.3 m

28. $\frac{2\pi}{3}$, $r = 3$ in.

SOLUTION:

$s = r\theta$

$= 3\left(\frac{2\pi}{3}\right)$

$= \frac{6\pi}{3}$ or about 6.3 in.

29. $\frac{5\pi}{12}$, $r = 4$ yd

SOLUTION:

$s = r\theta$

$= 4\left(\frac{5\pi}{12}\right)$

$= \frac{20\pi}{12}$ or about 5.2 yd

30. 105°, $r = 18.2$ cm

SOLUTION:

Method 1

Convert 105° to radian measure, and then use $s = r\theta$ to find the arc length.

$105° = 105°\left(\frac{\pi \text{ radians}}{180°}\right)$

$= \frac{7\pi}{12}$

Substitute $r = 18.2$ and $\theta = \frac{7\pi}{12}$.

$s = r\theta$

$= 18.2\left(\frac{7\pi}{12}\right)$

$= \frac{127.4\pi}{12}$ or about 33.4 cm

31. $45°, r = 5$ mi

SOLUTION:

Method 1

Convert 45° to radian measure, and then use $s = r\theta$ to find the arc length.

$$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \frac{\pi}{4}$$

Substitute $r = 5$ and $\theta = \frac{\pi}{4}$.

$$s = r\theta$$
$$= 5\left(\frac{\pi}{4}\right)$$
$$= \frac{5\pi}{4} \text{ or about } 3.9 \text{ mi}$$

32. $150°, r = 79$ mm

SOLUTION:

Method 1

Convert 150° to radian measure, and then use $s = r\theta$ to find the arc length.

$$150° = 150°\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \frac{5\pi}{6}$$

Substitute $r = 79$ and $\theta = \frac{5\pi}{6}$.

$$s = r\theta$$
$$= 79\left(\frac{5\pi}{6}\right)$$
$$= \frac{395\pi}{6} \text{ or about } 206.8 \text{ mm}$$

10	إيجاد قيم الدوال المثلثية لأي زاوية.	Exercises (1-8)	P253
	Find values of trigonometric functions for any angle.		

1. (3, 4)

ANSWER:

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

2. (−6, 6)

ANSWER:

$\sin\theta = \frac{\sqrt{2}}{2}, \cos\theta = -\frac{\sqrt{2}}{2}, \tan\theta = -1, \csc\theta = \sqrt{2}, \sec\theta = -\sqrt{2}, \cot\theta = -1$

3. (−4, −3)

ANSWER:

$\sin\theta = -\frac{3}{5}, \cos\theta = -\frac{4}{5}, \tan\theta = \frac{3}{4}, \csc\theta = -\frac{5}{3}, \sec\theta = -\frac{5}{4}, \cot\theta = \frac{4}{3}$

4. (2, 0)

ANSWER:

$\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$, $\csc\theta$ is undefined, $\sec\theta = 1$, $\cot\theta$ is undefined.

5. (1, −8)

ANSWER:

$\sin\theta = -\frac{8\sqrt{65}}{65}, \cos\theta = \frac{\sqrt{65}}{65}, \tan\theta = -8, \csc\theta = -\frac{\sqrt{65}}{8}, \sec\theta = \sqrt{65}, \cot\theta = -\frac{1}{8}$

6. (5, −3)

ANSWER:

$\sin\theta = -\frac{3\sqrt{34}}{34}, \cos\theta = \frac{5\sqrt{34}}{34}, \tan\theta = -\frac{3}{5}, \csc\theta = -\frac{\sqrt{34}}{3}, \sec\theta = \frac{\sqrt{34}}{5}, \cot\theta = -\frac{5}{3}$

7. (−8, 15)

ANSWER:

$\sin\theta = \frac{15}{17}, \cos\theta = -\frac{8}{17}, \tan\theta = -\frac{15}{8}, \csc\theta = \frac{17}{15}, \sec\theta = -\frac{17}{8}, \cot\theta = -\frac{8}{15}$

8. (−1, −2)

ANSWER:

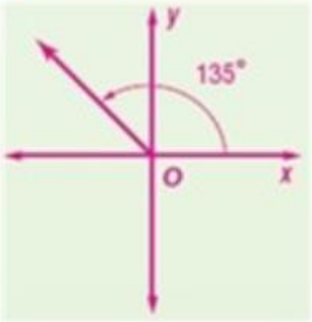
$\sin\theta = -\frac{2\sqrt{5}}{5}, \cos\theta = -\frac{\sqrt{5}}{5}, \tan\theta = 2, \csc\theta = -\frac{\sqrt{5}}{2}, \sec\theta = -\sqrt{5}, \cot\theta = \frac{1}{2}$

11	إيجاد قيم الدوال المثلثية لأي زاوية.	Exercises (17-24)	P253
	Find values of trigonometric functions for any angle.		

17. 135°

ANSWER:

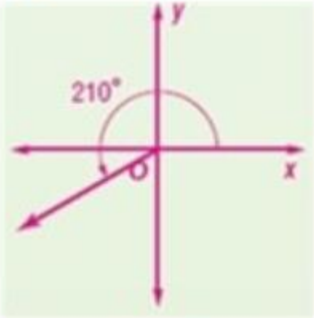
45°



18. 210°

ANSWER:

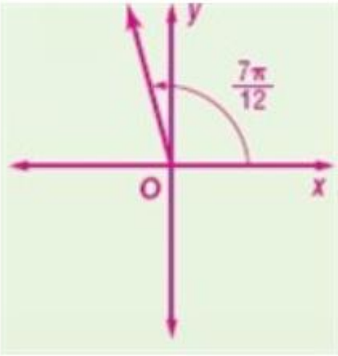
30°



19. $\frac{7\pi}{12}$

ANSWER:

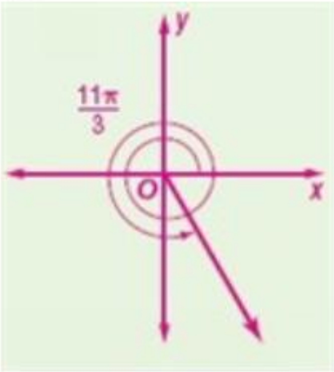
$\frac{5\pi}{12}$



20. $\frac{11\pi}{3}$

ANSWER:

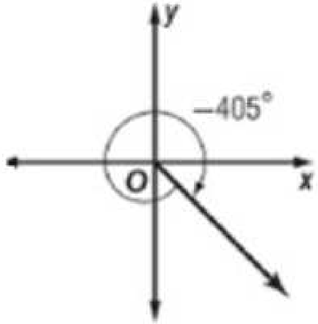
$\frac{\pi}{3}$



21. −405°

ANSWER:

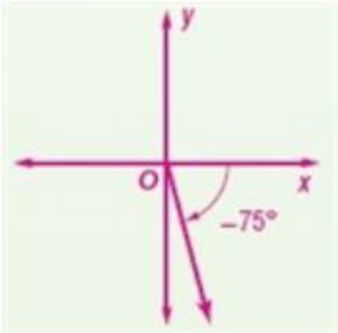
45°



22. −75°

ANSWER:

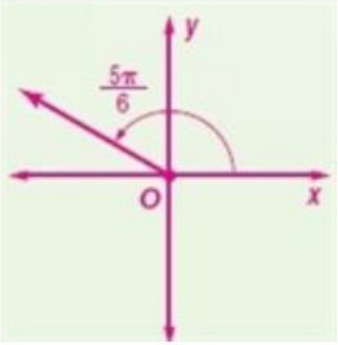
75°



23. $\frac{5\pi}{6}$

ANSWER:

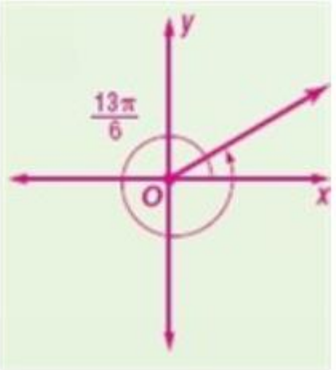
$\frac{\pi}{6}$



24. $\frac{13\pi}{6}$

ANSWER:

$\frac{\pi}{6}$



29. $\sin\left(\sin^{-1}\frac{3}{4}\right)$

SOLUTION:

The inverse property applies, because $\frac{3}{4}$ lies on the interval $[-1, 1]$. Therefore, $\sin\left(\sin^{-1}\frac{3}{4}\right)=\frac{3}{4}$.

30. $\sin^{-1}\left(\sin\frac{\pi}{2}\right)$

SOLUTION:

The inverse property applies, because $\frac{\pi}{2}$ lies on the interval $[-1, 1]$. Therefore, $\sin^{-1}\left(\sin\frac{\pi}{2}\right)=\frac{\pi}{2}$.

31. $\cos\left(\cos^{-1}\frac{2}{9}\right)$

SOLUTION:

The inverse property applies, because $\frac{2}{9}$ lies on the interval $[-1, 1]$. Therefore, $\cos\left(\cos^{-1}\frac{2}{9}\right)=\frac{2}{9}$.

32. $\cos^{-1}(\cos\pi)$

SOLUTION:

The inverse property applies, because π lies on the interval $[0, \pi]$. Therefore, $\cos^{-1}(\cos\pi)=\pi$.

33. $\tan\left(\tan^{-1}\frac{\pi}{4}\right)$

SOLUTION:

The inverse property applies, because $\frac{\pi}{4}$ lies on the interval $[-\infty, \infty]$. Therefore, $\tan\left(\tan^{-1}\frac{\pi}{4}\right)=\frac{\pi}{4}$.

34. $\tan^{-1}\left(\tan\frac{\pi}{3}\right)$

SOLUTION:

The inverse property applies, because $\frac{\pi}{3}$ lies on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Therefore, $\tan^{-1}\left(\tan\frac{\pi}{3}\right)=\frac{\pi}{3}$.

35. $\cos(\tan^{-1} 1)$

SOLUTION:

First, find $\tan^{-1} 1$. The inverse property applies, because 1 is on the interval $[-\infty, \infty]$. Therefore, $\tan^{-1} 1=\frac{\pi}{4}$. Next, find $\cos\frac{\pi}{4}$. On the unit circle, $\frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. So, $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$.

$\cos(\tan^{-1} 1)=\frac{\sqrt{2}}{2}$

36. $\sin^{-1}\left(\cos\frac{\pi}{2}\right)$

SOLUTION:

First, find $\cos\frac{\pi}{2}$. On the unit circle, $\frac{\pi}{2}$ corresponds to $(0, 1)$. So, $\cos\frac{\pi}{2}=0$.

Next, find $\sin^{-1} 0$. The inverse property applies, because 0 is on the interval $[-1, 1]$. Therefore, $\sin^{-1} 0=0$, and $\sin^{-1}\left(\cos\frac{\pi}{2}\right)=0$.

37. $\sin\left(2\cos^{-1}\frac{\sqrt{2}}{2}\right)$

SOLUTION:

First, find $\cos^{-1}\frac{\sqrt{2}}{2}$. To do this, find a point on the unit circle on the interval $[0, 2\pi]$ with an x-coordinate of $\frac{\sqrt{2}}{2}$. When $t = \frac{\pi}{4}$, $\cos t = \frac{\sqrt{2}}{2}$. Therefore, $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

Next, find $\sin\left(2\cdot\frac{\pi}{4}\right)$ or $\sin\frac{\pi}{2}$. On the unit circle, $\frac{\pi}{2}$ corresponds to (0, 1). So, $\sin\frac{\pi}{2} = 1$, and $\sin\left(2\cos^{-1}\frac{\sqrt{2}}{2}\right) = 1$.

38. $\sin(\tan^{-1} 1 - \sin^{-1} 1)$

$\sin(\tan^{-1} 1 - \sin^{-1} 1) = -\frac{\sqrt{2}}{2}$

39. $\cos(\tan^{-1} 1 - \sin^{-1} 1)$

$\cos(\tan^{-1} 1 - \sin^{-1} 1) = \frac{\sqrt{2}}{2}$

40. $\cos\left(\cos^{-1} 0 + \sin^{-1}\frac{1}{2}\right)$

$\cos\left(\cos^{-1} 0 + \sin^{-1}\frac{1}{2}\right) = -\frac{1}{2}.$

22. $\csc x \sec x - \tan x$

SOLUTION:

$$\csc x \sec x - \tan x = \frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$
$$= \frac{1}{\sin x \cos x} - \frac{\sin x}{\cos x}$$
$$= \frac{1}{\sin x \cos x} - \left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x}$$
$$= \frac{1 - \sin^2 x}{\sin x \cos x}$$
$$= \frac{\cos^2 x}{\sin x \cos x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$

23. $\csc x - \cos x \cot x$

SOLUTION:

$$\csc x - \cos x \cot x = \frac{1}{\sin x} - \cos x \frac{\cos x}{\sin x}$$
$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$
$$= \frac{1 - \cos^2 x}{\sin x}$$
$$= \frac{\sin^2 x}{\sin x}$$
$$= \sin x$$

24. $\sec x \cot x - \sin x$

SOLUTION:

$$\sec x \cot x - \sin x = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} - \sin x$$
$$= \frac{1}{\sin x} - \sin x$$
$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$
$$= \frac{1 - \sin^2 x}{\sin x}$$
$$= \frac{\cos^2 x}{\sin x}$$
$$= \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$
$$= \cos x \cdot \cot x$$

25. $\dfrac{\tan x + \sin x \sec x}{\csc x \tan x}$

SOLUTION:

$$\frac{\tan x + \sin x \sec x}{\csc x \tan x} = \frac{\tan x + \sin x \cdot \frac{1}{\cos x}}{\csc x \cdot \tan x}$$
$$= \frac{\tan x + \frac{\sin x}{\cos x}}{\csc x \cdot \tan x}$$
$$= \frac{\tan x + \tan x}{\csc x \cdot \tan x}$$
$$= \frac{2\tan x}{\csc x \cdot \tan x}$$
$$= \frac{2}{\csc x}$$
$$= \frac{2}{\frac{1}{\sin x}}$$
$$= 2\sin x$$

26. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$

SOLUTION:

$$\frac{1 - \sin^2 x}{\csc^2 x - 1} = \frac{\cos^2 x}{\cot^2 x}$$
$$= \frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}}$$
$$= \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\cos^2 x}$$
$$= \sin^2 x$$

27. $\dfrac{\csc x \cos x + \cot x}{\sec x \cot x}$

SOLUTION:

$$\frac{\csc x \cos x + \cot x}{\sec x \cot x} = \frac{\frac{1}{\sin x}\cos x + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}}$$
$$= \frac{\frac{\cos x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$
$$= \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin x}}$$
$$= \frac{2\cos x}{\sin x} \cdot \sin x$$
$$= 2\cos x$$

28. $\dfrac{\sec x \csc x - \tan x}{\sec x \csc x}$

SOLUTION:

$$\frac{\sec x\csc x - \tan x}{\sec x \csc x} = \frac{\frac{1}{\cos x}\cdot\frac{1}{\sin x} - \frac{\sin x}{\cos x}}{\frac{1}{\cos x}\cdot\frac{1}{\sin x}}$$

$$= \frac{\frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x}}{\frac{1}{\cos x \sin x}}$$

$$= \frac{\frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x}}{\frac{1}{\cos x \sin x}} = \frac{\frac{1-\sin^2 x}{\cos x \sin x}}{\frac{1}{\cos x \sin x}}$$

$$= \frac{1-\sin^2 x}{\cos x \sin x}\cdot \cos x \sin x$$

$$= 1 - \sin^2 x$$

$$= \cos^2 x$$

29. $\dfrac{\sec^2 x}{\cot^2 x + 1}$

SOLUTION:

$$\frac{\sec^2 x}{\cot^2 x + 1} = \frac{\sec^2 x}{\csc^2 x}$$

$$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$$

$$= \frac{1}{\cos^2 x}\cdot \sin^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \tan^2 x$$

30. $\cot x - \csc^2 x \cot x$

SOLUTION:

$$\begin{aligned}\cot x - \csc^2 x \cot x &= \cot x(1 - \csc^2 x)\\ &= -\cot x(\csc^2 x - 1)\\ &= -\cot x(\cot^2 x)\\ &= -\cot^3 x\end{aligned}$$

31. $\cot x - \cos^3 x \csc x$

SOLUTION:

$$\begin{aligned}\cot x - \cos^3 x \csc x &= \frac{\cos x}{\sin x} - \cos^3 x \csc x\\ &= \frac{\cos x}{\sin x} - \cos^3 x \cdot \frac{1}{\sin x}\\ &= \frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x}\\ &= \frac{\cos x - \cos^3 x}{\sin x}\\ &= \frac{(1 - \cos^2 x)\cos x}{\sin x}\\ &= \frac{(\sin^2 x)\cos x}{\sin x}\\ &= \sin x \cos x\end{aligned}$$

Verify each identity.

1. $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

SOLUTION:

$(\sec^2\theta - 1)\cos^2\theta$

$= (\tan^2\theta)\cos^2\theta$ Pythagorean Identity

$= \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$ Quotient Identity

$= \sin^2\theta$ Multiply and divide out common factor.

2. $\sec^2\theta\,(1 - \cos^2\theta) = \tan^2\theta$

SOLUTION:

$\sec^2\theta(1 - \cos^2\theta)$

$= \sec^2\theta - \sec^2\theta\cos^2\theta$ Distributive Property

$= \sec^2\theta - \dfrac{1}{\cos^2\theta}\cdot\cos^2\theta$ Reciprocal Identity

$= \sec^2\theta - 1$ Multiply and divide out common factor.

$= \tan^2\theta$ Pythagorean Identity

3. $\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$

SOLUTION:

$\sin\theta - \sin\theta\cos^2\theta$

$= \sin\theta(1 - \cos^2\theta)$ Factor.

$= \sin\theta\sin^2\theta$ Pythagorean Identity

$= \sin^3\theta$ Multiply.

4. $\csc\theta - \cos\theta\cot\theta = \sin\theta$

SOLUTION:

$\csc\theta - \cos\theta\cot\theta$

$= \dfrac{1}{\sin\theta} - \cos\theta\left(\dfrac{\cos\theta}{\sin\theta}\right)$ Reciprocal and Quotient Identities

$= \dfrac{1 - \cos^2\theta}{\sin\theta}$ Write as a fraction with a common denominator.

$= \dfrac{\sin^2\theta}{\sin\theta}$ Pythagorean Identity

$= \sin\theta$ Divide out common factor of $\sin\theta$.

5. $\cot^2\theta\csc^2\theta - \cot^2\theta = \cot^4\theta$

SOLUTION:

$\cot^2\theta\csc^2\theta - \cot^2\theta$

$= \cot^2\theta(\csc^2\theta - 1)$ Factor.

$= \cot^2\theta\cot^2\theta$ Pythagorean Identity

$= \cot^4\theta$ Multiply and add exponents.

6. $\tan\theta\csc^2\theta - \tan\theta = \cot\theta$

SOLUTION:

$\tan\theta\csc^2\theta - \tan\theta$

$= \tan\theta(\csc^2\theta - 1)$ Factor

$= \tan\theta\cot^2\theta$ Pythagorean Identity

$= \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$ Quotient Identities

$= \frac{\cos\theta}{\sin\theta}$ Multiply and divide common factors.

$= \cot\theta$ Quotient Identity

7. $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

SOLUTION:

$\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{1}{\cos\theta}}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$ Reciprocal Identity

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}$ Common denominator

$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$ Write as a fraction with a common denominator.

$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$ Pythagoren Identity

$= \frac{\cos\theta}{\sin\theta}$ Divide out common factor of $\cos\theta$.

$= \cot\theta$ Quotient Identity

8. $\frac{\sin\theta}{1\cos\theta}+\frac{1\cos\theta}{\sin\theta}=2\csc\theta$

SOLUTION:

$\frac{\sin\theta}{1-\cos\theta}+\frac{1-\cos\theta}{\sin\theta}$

$=\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{1-\cos\theta}+\frac{1-\cos\theta}{1-\cos\theta}\cdot\frac{1-\cos\theta}{\sin\theta}$ Rewrite 1 using the common denominator.

$=\frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}+\frac{1-2\cos\theta+\cos^2\theta}{\sin\theta(1-\cos\theta)}$ Mulitply.

$=\frac{\sin^2\theta+\cos^2\theta+1-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Write as a fraction with a common denominator.

$=\frac{1+1-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Pythagorean Identity

$=\frac{2-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Add.

$=\frac{2(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ Factor.

$=\frac{2}{\sin\theta}$ Divide out common factor of $(1-\cos\theta)$.

$=2\csc\theta$ Reciprocal Identity

9. $\frac{\cos\theta}{1+\sin\theta}+\tan\theta=\sec\theta$

SOLUTION:

$\frac{\cos\theta}{1+\sin\theta}+\tan\theta$

$=\frac{\cos\theta}{1+\sin\theta}+\frac{\sin\theta}{\cos\theta}$ — Quotient Identity

$=\frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{1+\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$ — Rewrite 1 using the common denominator.

$=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}+\frac{\sin\theta+\sin^2\theta}{(1+\sin\theta)\cos\theta}$ — Multiply.

$=\frac{\cos^2\theta+\sin\theta+\sin^2\theta}{\cos\theta(1+\sin\theta)}$ — Write as a fraction with a common denominator.

$=\frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$ — Pythagorean Identity

$=\frac{1}{\cos\theta}$ — Divide out common factor of $(1+\sin\theta)$.

$=\sec\theta$ — Reciprocal Identity

10. $\frac{sin\theta}{1cot\theta} + \frac{cos\theta}{1tan\theta} = \sin\theta + \cos\theta$

SOLUTION:

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}$$

$$=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

Quotient Identity

$$=\frac{\sin\theta}{\frac{\sin\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

Rewrite 1 using the common denominator.

$$=\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

Write denominators as fractions with common denominators.

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

Simplify fractions.

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

Factor out -1.

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$

Write as a fraction with common denominator.

$$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}$$

Factor numerator.

$$=\sin\theta+\cos\theta$$

Divide out common factor of $(\sin\theta-\cos\theta)$.

1. $5 \sin x + 2 = \sin x$

SOLUTION:

$$5 \sin x + 2 = \sin x$$
$$4 \sin x + 2 = 0$$
$$2(2 \sin x + 1) = 0$$
$$2 \sin x = -1$$
$$\sin x = -\frac{1}{2}$$

The period of sine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{7\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$.

2. $5 = \sec^2 x + 3$

SOLUTION:

$$5 = \sec^2 x + 3$$
$$2 = \sec^2 x$$
$$\pm\sqrt{2} = \sqrt{\sec^2 x}$$
$$\sec x = \pm\sqrt{2}$$

The period of secant is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{4} + 2n\pi$, $\frac{3\pi}{4} + 2n\pi$, $\frac{5\pi}{4} + 2n\pi$, $\frac{7\pi}{4} + 2n\pi$, $n \in \mathbb{Z}$.

3. $2 = 4 \cos^2 x + 1$

SOLUTION:

$$2 = 4\cos^2 x + 1$$
$$1 = 4\cos^2 x$$
$$\frac{1}{4} = \cos^2 x$$
$$\pm\sqrt{\frac{1}{4}} = \sqrt{\cos^2 x}$$
$$\cos x = \pm\frac{1}{2}$$

The period of cosine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, and $\frac{5\pi}{3}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{3} + 2n\pi$, $\frac{2\pi}{3} + 2n\pi$, $\frac{4\pi}{3} + 2n\pi$, $\frac{5\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

4. $4 \tan x - 7 = 3 \tan x - 6$

SOLUTION:

$$4\tan x - 7 = 3\tan x - 6$$
$$\tan x - 7 = -6$$
$$\tan x = 1$$

The period of tangent is π, so you only need to find solutions on the interval $[0, \pi)$. The only solution on this interval is $\frac{\pi}{4}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π.

Therefore, the general form of the solutions is $\frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}$.

5. $9 + \cot^2 x = 12$

SOLUTION:

$$9 + \cot^2 x = 12$$
$$\cot^2 x = 3$$
$$\sqrt{\cot^2 x} = \pm\sqrt{3}$$
$$\cot x = \pm\sqrt{3}$$

The period of cotangent is π, so you only need to find solutions on the interval $[0, \pi)$. The solutions on this interval are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π.

Therefore, the general form of the solutions is $\frac{\pi}{6} + n\pi,\ \frac{5\pi}{6} + n\pi,\ n \in \mathbb{Z}$.

6. $2 - 10 \sec x = 4 - 9 \sec x$

SOLUTION:

$$2 - 10\sec x = 4 - 9\sec x$$
$$-10\sec x = 2 - 9\sec x$$
$$-\sec x = 2$$
$$\sec x = -2$$

The period of secant is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{2\pi}{3} + 2n\pi,\ \frac{4\pi}{3} + 2n\pi,\ n \in \mathbb{Z}$.

7. $3 \csc x = 2 \csc x + \sqrt{2}$

SOLUTION:

$$3\csc x = 2\csc x + \sqrt{2}$$
$$\csc x = \sqrt{2}$$

The period of cosecant is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{4} + 2n\pi$, $\frac{3\pi}{4} + 2n\pi$, $n \in \mathbb{Z}$.

8. $11 = 3 \csc^2 x + 7$

SOLUTION:

$$11 = 3\csc^2 x + 7$$
$$4 = 3\csc^2 x$$
$$\frac{4}{3} = \csc^2 x$$
$$\pm\sqrt{\frac{4}{3}} = \sqrt{\csc^2 x}$$
$$\csc x = \pm\frac{2}{\sqrt{3}} \text{ or } \pm\frac{2\sqrt{3}}{3}$$

The period of cosecant is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, and $\frac{5\pi}{3}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{3} + 2n\pi$, $\frac{2\pi}{3} + 2n\pi$, $\frac{4\pi}{3} + 2n\pi$, $\frac{5\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

9. $6 \tan^2 x - 2 = 4$

SOLUTION:

$$6\tan^2 x - 2 = 4$$
$$6\tan^2 x = 6$$
$$\tan^2 x = 1$$
$$\sqrt{\tan^2 x} = \pm\sqrt{1}$$
$$\tan x = \pm 1$$

The period of tangent is π, so you only need to find solutions on the interval $[0, \pi)$. The solutions on this interval are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π. Therefore, the general form of the solutions is $\frac{\pi}{4} + n\pi$, $\frac{3\pi}{4} + n\pi$, $n \in \mathbb{Z}$.

10. $9 + \sin^2 x = 10$

SOLUTION:

$$9 + \sin^2 x = 10$$
$$\sin^2 x = 1$$
$$\sqrt{\sin^2 x} = \pm\sqrt{1}$$
$$\sin x = \pm 1$$

The period of sine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{2} + 2n\pi$, $\frac{3\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$.

11. $7 \cot x - \sqrt{3} = 4 \cot x$

SOLUTION:

$$7\cot x - \sqrt{3} = 4\cot x$$
$$7\cot x = 4\cot x + \sqrt{3}$$
$$3\cot x = \sqrt{3}$$
$$\cot x = \frac{\sqrt{3}}{3}$$

The period of cotangent is π, so you only need to find solutions on the interval $[0, \pi)$. The only solution on this interval is $\frac{\pi}{3}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π.

Therefore, the general form of the solutions is $\frac{\pi}{3} + n\pi$, $n \in \mathbb{Z}$.

12. $7 \cos x = 5 \cos x + \sqrt{3}$

SOLUTION:

$$7\cos x = 5\cos x + \sqrt{3}$$
$$2\cos x = \sqrt{3}$$
$$\cos x = \frac{\sqrt{3}}{2}$$

The period of cosine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{6}$ and $\frac{11\pi}{6}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$.

13. $\sin^4 x + 2\sin^2 x - 3 = 0$

SOLUTION:

$$\sin^4 x + 2\sin^2 x - 3 = 0$$
$$(\sin^2 x)^2 + 2\sin^2 x - 3 = 0$$
$$(\sin^2 x - 1)(\sin^2 x + 3) = 0$$
$$\sin^2 x - 1 = 0 \quad \text{or} \quad \sin^2 x + 3 = 0$$
$$\sin^2 x = 1 \qquad \sin^2 x = -3$$
$$\sin x = \pm\sqrt{1} \qquad \sin x = \pm\sqrt{-3}$$
$$\sin x = \pm 1$$

On the interval $[0, 2\pi)$, $\sin x = 1$ when $x = \frac{\pi}{2}$ and $\sin x = -1$ when $x = \frac{3\pi}{2}$. Since $\sqrt{-3}$ is not a real number, the equation $\sin x = \pm\sqrt{-3}$ yields no additional solutions.

14. $-2\sin x = -\sin x \cos x$

SOLUTION:

$$-2\sin x = -\sin x \cos x$$
$$\sin x \cos x - 2\sin x = 0$$
$$\sin x(\cos x - 2) = 0$$
$$\sin x = 0 \quad \text{or} \quad \cos x - 2 = 0$$
$$\cos x = 2$$

The equation $\cos x = 2$ has no real solutions since the maximum value the cosine function can obtain is 1. On the interval $[0, 2\pi)$, the equation $\sin x = 0$ has solutions 0 and π.

15. $4\cot x = \cot x \sin^2 x$

SOLUTION:

$$4\cot x = \cot x \sin^2 x$$
$$4\cot x - \cot x \sin^2 x = 0$$
$$\cot x(4 - \sin^2 x) = 0$$
$$\cot x(2 - \sin x)(2 + \sin x) = 0$$
$$\cot x = 0 \quad \text{or} \quad 2 - \sin x = 0 \quad \text{or} \quad 2 + \sin x = 0$$
$$-\sin x = -2 \qquad \sin x = -2$$
$$\sin x = 2$$

The equations $\sin x = 2$ and $\sin x = -2$ have no real solutions. On the interval $[0, 2\pi)$, the equation $\cot x = 0$ has solutions $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

16. $\csc^2 x - \csc x + 9 = 11$

SOLUTION:

$$\csc^2 x - \csc x + 9 = 11$$
$$\csc^2 x - \csc x - 2 = 0$$
$$(\csc x + 1)(\csc x - 2) = 0$$
$$\csc x + 1 = 0 \quad \text{or} \quad \csc x - 2 = 0$$
$$\csc x = -1 \qquad \csc x = 2$$

On the interval $[0, 2\pi)$, the equation $\csc x = -1$ has a solution of $\frac{3\pi}{2}$ and the equation $\csc x = 2$ has solutions of $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

17. $\cos^3 x + \cos^2 x - \cos x = 1$

SOLUTION:

$$\cos^3 x + \cos^2 x - \cos x = 1$$
$$(\cos^3 x + \cos^2 x) - \cos x - 1 = 0$$
$$\cos^2 x(\cos x + 1) - (\cos x + 1) = 0$$
$$(\cos^2 x - 1)(\cos x + 1) = 0$$
$$(\cos x - 1)(\cos x + 1)^2 = 0$$
$$\cos x - 1 = 0 \quad \text{or} \quad (\cos x + 1)^2 = 0$$
$$\cos x = 1 \qquad \cos x + 1 = 0$$
$$\cos x = -1$$

On the interval $[0, 2\pi)$, the equation $\cos x = 1$ has a solution of 0 and the equation $\cos x = -1$ has a solution of π.

18. $2\sin^2 x = \sin x + 1$

SOLUTION:

$$2\sin^2 x = \sin x + 1$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(\sin x - 1)(2\sin x + 1) = 0$$
$$\sin x - 1 = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$\sin x = 1 \qquad 2\sin x = -1$$
$$\sin x = -\frac{1}{2}$$

On the interval $[0, 2\pi)$, the equation $\sin x = 1$ has a solution of $\frac{\pi}{2}$ and the equation $\sin x = -\frac{1}{2}$ has solutions of $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

1. $f(x) = \sqrt{x^2 - 4}$; at $x = -5$

SOLUTION:

Find $f(-5)$.

$$f(x) = \sqrt{x^2 - 4}$$
$$f(-5) = \sqrt{(-5)^2 - 4}$$
$$= \sqrt{25 - 4}$$
$$= \sqrt{21}$$
$$\approx 4.58$$

The function is defined at $x = -5$.

Find $\lim_{x \to -5} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching -5 from the left and from the right.

x	f(x)
−5.1	4.69
−5.01	4.59
−5.001	4.58
−5	
−4.999	4.58
−4.99	4.57
−4.9	4.47

$\lim_{x \to -5} f(x) \approx 4.58$.

Because $\lim_{x \to -5} f(x) = f(-5)$, $f(x)$ is continuous as $x = -5$.

ANSWER:

Continuous; $f(-5) = \sqrt{21} \approx 4.58$, $\lim_{x \to -5} f(x) \approx 4.58$, and $\lim_{x \to -5} f(x) = f(-5)$.

2. $f(x) = \sqrt{x+5}$; at $x = 8$

SOLUTION:

Find $f(8)$.

$f(x) = \sqrt{x+5}$

$f(8) = \sqrt{(8)+5}$

$= \sqrt{13}$

≈ 3.606

The function is defined at $x = 8$.

Find $\lim_{x\to 8} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 8 from the left and from the right.

x	f(x)
7.9	3.592
7.99	3.604
7.999	3.605
8	
8.001	3.606
8.01	3.607
8.1	3.619

$\lim_{x\to 8} f(x) \approx 3.606.$

Because $\lim_{x\to 8} f(x) = f(8)$, $f(x)$ is continuous as $x =$ 8.

ANSWER:

Continuous; $f(8) = \sqrt{13}$ or about 3.606, $\lim_{x\to 8} f(x) \approx 3.606$, and $\lim_{x\to 8} f(x) = f(8)$.

3. $h(x) = \frac{x^2 - 36}{x + 6}$; at $x = -6$ and $x = 6$

ANSWER:

Discontinuous at $x = -6$; $h(-6)$ is undefined and $\lim_{x \to -6} h(x) = -12$, so $h(x)$ has a removable discontinuity at $x = -6$. Continuous at $x = 6$. $h(6) = 0$, $\lim_{x \to 6} h(x) = 0$, and $\lim_{x \to 6} h(x) = h(6)$.

4. $h(x) = \frac{x^2 - 25}{x + 5}$; at $x = -5$ and $x = 5$

ANSWER:

Discontinuous at $x = -5$; $h(-5)$ is undefined and $\lim_{x \to -5} h(x) = -10$, so $h(x)$ has a removable discontinuity at $x = -5$. Continuous at $x = 5$. $h(5) = 0$, $\lim_{x \to 5} h(x) = 0$, and $\lim_{x \to 5} h(x) = h(5)$.

5. $g(x) = \frac{x}{x - 1}$; at $x = 1$

ANSWER:

Discontinuous; $g(1)$ is undefined and $g(x)$ approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right, so $g(x)$ has an infinite discontinuity at $x = 1$.

6. $g(x) = \frac{2 - x}{2 + x}$; at $x = 2$ and $x = -2$

ANSWER:

Discontinuous at $x = -2$; $g(-2)$ is undefined and $g(x)$ approaches $-\infty$ as x approaches -2 from the left and ∞ as x approaches -2 from the right, so $g(x)$ has an infinite discontinuity at $x = -2$. Continuous at $x = 2$; $g(2) = 0$, $\lim_{x \to 2} g(x) = 0$, and $\lim_{x \to 2} g(x) = g(2)$.

7. $h(x) = \dfrac{x-4}{x^2-5x+4}$; at $x = 1$ and $x = 4$

ANSWER:

Discontinuous at $x = 1$; $h(1)$ is undefined and $h(x)$ approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right, so $h(x)$ has an infinite discontinuity at $x = 1$. Discontinuous at $x = 4$; $h(4)$ is undefined and $\lim_{x \to 4} h(x) = \dfrac{1}{3}$, so $h(x)$ has a removable discontinuity at $x = 4$.

8. $h(x) = \dfrac{x(x-6)}{x^3}$; at $x = 0$ and $x = 6$

ANSWER:

Discontinuous at $x = 0$; $h(0)$ is undefined and $h(x)$ approaches $-\infty$ as x approaches 0 from both sides, so $h(x)$ has an infinite discontinuity at $x = 0$. Continuous at $x = 6$; $h(6) = 0$, $\lim_{x \to 6} h(x) = 0$, and $\lim_{x \to 6} h(x) = h(6)$.

9. $f(x) = \begin{cases} 4x-1 & \text{if } x \le -6 \\ -x+2 & \text{if } x > -6 \end{cases}$; at $x = -6$

ANSWER:

Discontinuous at $x = -6$; $f(x)$ approaches -25 as x approaches -6 from the left and 8 as x approaches -6 from the right, so $f(x)$ has a jump discontinuity at $x = -6$.

10. $f(x) = \begin{cases} x^2-1 & \text{if } x > -2 \\ x-5 & \text{if } x \le -2 \end{cases}$; at $x = -2$

ANSWER:

Discontinuous at $x = -2$; $f(x)$ approaches -7 as x approaches -2 from the left and 3 as x approaches -2 from the right, so $f(x)$ has a jump discontinuity at $x = -2$.

34. $g(x) = -4x^2 + 3x - 4; [-1, 3]$

SOLUTION:

$g(3) = -4(3)^2 + 3(3) - 4$
$g(3) = -36 + 9 - 4$
$g(3) = = -31$

$g(-1) = -4(-1)^2 + 3(-1) - 4$
$g(-1) = -4 + (-3) - 4$
$g(-1) = -11$

$$\frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{-31 - (-11)}{4} = \frac{-20}{4} = -5$$

35. $g(x) = 3x^2 - 8x + 2; [4, 8]$

SOLUTION:

$g(8) = 3(8)^2 - 8(8) + 2$
$g(8) = 192 - 64 + 2$
$g(8) = 130$

$g(4) = 3(4)^2 - 8(4) + 2$
$g(4) = 48 - 32 + 2$
$g(4) = 18$

$$\frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{g(8) - g(4)}{8 - 4} = \frac{130 - 18}{4} = \frac{112}{4} = 28$$

36. $f(x) = 3x^3 - 2x^2 + 6; [2, 6]$

SOLUTION:

$f(x) = 3x^3 - 2x^2 + 6; [2, 6]$
$f(6) = 3(6)^3 - 2(6)^2 + 6$
$f(6) = 648 - 72 + 6$
$f(6) = 582$
$f(3) = 3(3)^3 - 2(3)^2 + 6$
$f(3) = 81 - 18 + 6$
$f(3) = 22$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(6) - f(3)}{6 - 3} = \frac{582 - 22}{3} = \frac{560}{4} = 140$$

37. $f(x) = -2x^3 - 4x^2 + 2x - 8;\ [-2, 3]$

SOLUTION:

$f(3) = -2(3)^3 - 4(3)^2 + 2(3) - 8$
$f(3) = -54 - 36 + 6 - 8$
$f(3) = -92$

$f(-2) = -2(-2)^3 - 4(-2)^2 + 2(-2) - 8$
$f(-2) = 16 - 16 + (-4) - 8$
$f(-2) = -12$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(3) - f(-2)}{3 - (-2)} = \frac{-92 - (-12)}{5} = \frac{-80}{5} = -16$$

38. $f(x) = 3x^4 - 2x^2 + 6x - 1;\ [5, 9]$

SOLUTION:

$f(9) = 3(9)^4 - 2(9)^2 + 6(9) - 1$
$f(9) = 19683 - 162 + 54 - 1$
$f(9) = 19{,}574$

$f(5) = 3(5)^4 - 2(5)^2 + 6(5) - 1$
$f(5) = 1875 - 50 + 30 - 1$
$f(5) = 1854$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(9) - f(5)}{9 - 5} = \frac{19{,}574 - 1854}{4} = \frac{17{,}720}{4} = 4430$$

39. $f(x) = -2x^4 - 5x^3 + 4x - 6;\ [-1, 5]$

SOLUTION:

$f(5) = -2(5)^4 - 5(5)^3 + 4(5) - 6$
$f(5) = -1250 - 625 + 20 - 6$
$f(5) = -1861$

$f(-1) = -2(-1)^4 - 5(-1)^3 + 4(-1) - 6$
$f(-1) = -2 + 5 - 4 - 6$
$f(-1) = -7$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{-1861 - (-7)}{6} = \frac{-1854}{6} = -309$$

40. $h(x) = -x^5 - 5x^2 + 6x - 9$; [3, 6]

SOLUTION:

$h(6) = -(6)^5 - 5(6)^2 + 6(6) - 9$
$h(6) = -7776 - 180 + 36 - 9$
$h(6) = -7929$

$h(3) = -(3)^5 - 5(3)^2 + 6(3) - 9$
$h(3) = -243 - 45 + 18 - 9$
$h(3) = -279$

$$\frac{h(x_2) - h(x_1)}{x_2 - x_1} = \frac{h(6) - h(3)}{6 - 3}$$
$$= \frac{-7929 + 279}{3}$$
$$= \frac{-7650}{3}$$

25 **FINANCIAL LITERACY** Ahmed acquired an inheritance of AED 20,000 at age 8, but he will not have access to it until he turns 18. (Examples 4 and 5)

a. If his inheritance is placed in a savings account earning 4.6% interest compounded monthly, how much will Ahmed's inheritance be worth on his 18th birthday?

b. How much will Ahmed's inheritance be worth if it is placed in an account earning 4.2% interest compounded continuously?

ANSWER:

a. \$31,653.63

b. \$30,439.23

26. **FINANCIAL LITERACY** Eman invests AED 1200 in a certificate of deposit (CD). The table shows the interest rates offered by the bank on 3- and 5-year CDs. (Examples 4 and 5)

CD Offers		
Years	3	5
Interest	3.45%	4.75%
Compounded	continuously	monthly

a. How much would her investment be worth with each option?

b. How much would her investment be worth if the 5-year CD was compounded continuously?

ANSWER:

a. 3-year CD: \$1330.85, 5-year CD: \$1520.98

b. \$1521.69

27. $N_0 = 15{,}831,\ r = -4.2\%$

ANSWER:

t	5	10	15	20	50
N	12,774	10,308	8317	6711	1853

28. $N_0 = 23{,}112,\ r = 0.8\%$

ANSWER:

t	5	10	15	20	50
N	24,051	25,029	26,046	27,105	34,424

29. $N_0 = 17{,}692,\ k = 2.02\%$

ANSWER:

t	5	10	15	20	50
N	19,572	21,652	23,953	26,499	48,575

30. $N_0 = 9689,\ k = -3.7\%$

ANSWER:

t	5	10	15	20	50
N	8053	6693	5562	4623	1523

18	حل مسائل تتضمن نموًا وتضاءلًا أسيًا.	Exercises (25-33)	P166
	Solve problems involving exponential growth and decay.		

31. **WATER** Worldwide water usage in 1950 was about 294.2 million gallons. If water usage has grown at the described rate, estimate the amount of water used in 2000 and predict the amount in 2050. (Example 6)

a. 3% annually **b.** 3.05% continuously

ANSWER:

a. about 1289.75 million or 1.29 billion gallons; about 5654.12 million or 5.65 billion gallons

b. about 1351.89 million or 1.35 billion gallons; about 6212.13 million or 6.21 billion gallons

18	حل مسائل تتضمن نموًا وتضاءلًا أسيًا. Solve problems involving exponential growth and decay.	Exercises (25-33)	P166

32. **WAGES** Yasmin receives a 3.5% raise at the end of each year from her employer to account for inflation. When she started working for the company in 1994, she was earning a salary of AED 31,000. (Example 6)

a. What was Yasmin's salary in 2000 and 2004?

b. If Yasmin continues to receive a raise at the end of each year, how much money will she earn during her final year if she plans on retiring in 2024?

ANSWER:

a. about \$38,107; about \$43,729

b. about \$87,011

19	إيجاد قيم الدوال المثلثية للزوايا الحادة للمثلثات القائمة الزاوية.	Exercises (1-18)	P229
	Find values of trigonometric functions for acute angles of right triangles.		

1. 

ANSWER:

$\sin\theta = \frac{4\sqrt{2}}{9}, \cos\theta = \frac{7}{9}, \tan\theta = \frac{4\sqrt{2}}{7}, \csc\theta = \frac{9\sqrt{2}}{8}, \sec\theta = \frac{9}{7}, \cot\theta = \frac{7\sqrt{2}}{8}$

2. 

ANSWER:

$\sin\theta = \frac{2\sqrt{14}}{15}, \cos\theta = \frac{13}{15}, \tan\theta = \frac{2\sqrt{14}}{13}, \csc\theta = \frac{15\sqrt{14}}{28}, \sec\theta = \frac{15}{13}, \cot\theta = \frac{13\sqrt{14}}{28}$

3. 

ANSWER:

$\sin\theta = \frac{9\sqrt{97}}{97}, \cos\theta = \frac{4\sqrt{97}}{97}, \tan\theta = \frac{9}{4}, \csc\theta = \frac{\sqrt{97}}{9}, \sec\theta = \frac{\sqrt{97}}{4}, \cot\theta = \frac{4}{9}$

4. 35

ANSWER:

$\sin\theta = \frac{12}{37}, \cos\theta = \frac{35}{37}, \tan\theta = \frac{12}{35}, \csc\theta = \frac{37}{12}, \sec\theta = \frac{37}{35}, \cot\theta = \frac{35}{12}$

5.

ANSWER:

$\sin\theta = \frac{\sqrt{165}}{29}, \cos\theta = \frac{26}{29}, \tan\theta = \frac{\sqrt{165}}{26}, \csc\theta = \frac{29\sqrt{165}}{165}, \sec\theta = \frac{29}{26}, \cot\theta = \frac{26\sqrt{165}}{165}$

6. 30

ANSWER:

$\sin\theta = \frac{6}{7}, \cos\theta = \frac{\sqrt{13}}{7}, \tan\theta = \frac{6\sqrt{13}}{13}, \csc\theta = \frac{7}{6}, \sec\theta = \frac{7\sqrt{13}}{13}, \cot\theta = \frac{\sqrt{13}}{6}$

19	إيجاد قيم الدوال المثلثية للزوايا الحادة للمثلثات القائمة الزاوية.	Exercises (1-18)	P229
	Find values of trigonometric functions for acute angles of right triangles.		

7.

ANSWER:

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$

8.

ANSWER:

$\sin\theta = \frac{\sqrt{17}}{17}$, $\cos\theta = \frac{4\sqrt{17}}{17}$, $\tan\theta = \frac{1}{4}$, $\csc\theta = \sqrt{17}$, $\sec\theta = \frac{\sqrt{17}}{4}$, $\cot\theta = 4$

9. $\sin\theta = \frac{4}{5}$

ANSWER:

$\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

19	إيجاد قيم الدوال المثلثية للزوايا الحادة للمثلثات القائمة الزاوية. **Find values of trigonometric functions for acute angles of right triangles.**	**Exercises (1-18)**	**P229**

10. $\cos\theta = \frac{6}{7}$

ANSWER:

$\sin\theta = \frac{\sqrt{13}}{7}$, $\tan\theta = \frac{\sqrt{13}}{6}$, $\csc\theta = \frac{7\sqrt{13}}{13}$, $\sec\theta = \frac{7}{6}$, $\cot\theta = \frac{6\sqrt{13}}{13}$

11. $\tan\theta = 3$

ANSWER:

$\sin\theta = \frac{3\sqrt{10}}{10}$, $\cos\theta = \frac{\sqrt{10}}{10}$, $\csc\theta = \frac{\sqrt{10}}{3}$, $\sec\theta = \sqrt{10}$, $\cot\theta = \frac{1}{3}$

12. $\sec\theta = 8$

ANSWER:

$\sin\theta = \frac{3\sqrt{7}}{8}$, $\cos\theta = \frac{1}{8}$, $\tan\theta = 3\sqrt{7}$, $\csc\theta = \frac{8\sqrt{7}}{21}$, $\cot\theta = \frac{\sqrt{7}}{21}$

13. $\cos\theta = \frac{5}{9}$

ANSWER:

$\sin\theta = \frac{2\sqrt{14}}{9}$, $\tan\theta = \frac{2\sqrt{14}}{5}$, $\csc\theta = \frac{9\sqrt{14}}{28}$, $\sec\theta = \frac{9}{5}$, $\cot\theta = \frac{5\sqrt{14}}{28}$

14. $\tan\theta = \frac{1}{4}$

ANSWER:

$\sin\theta = \frac{\sqrt{17}}{17}$, $\cos\theta = \frac{4\sqrt{17}}{17}$, $\csc\theta = \sqrt{17}$, $\sec\theta = \frac{\sqrt{17}}{4}$, $\cot\theta = 4$

15. $\cot\theta = 5$

ANSWER:

$\sin\theta = \frac{\sqrt{26}}{26}$, $\cos\theta = \frac{5\sqrt{26}}{26}$, $\tan\theta = \frac{1}{5}$, $\csc\theta = \sqrt{26}$, $\sec\theta = \frac{\sqrt{26}}{5}$

16. $\csc\theta = 6$

ANSWER:

$\sin\theta = \frac{1}{6}$, $\cos\theta = \frac{\sqrt{35}}{6}$, $\tan\theta = \frac{\sqrt{35}}{35}$, $\sec\theta = \frac{6\sqrt{35}}{35}$, $\cot\theta = \sqrt{35}$

17. $\sec\theta = \frac{9}{2}$

ANSWER:

$\sin\theta = \frac{\sqrt{77}}{9}, \cos\theta = \frac{2}{9}, \tan\theta = \frac{\sqrt{77}}{2}, \csc\theta = \frac{9\sqrt{77}}{77}, \cot\theta = \frac{2\sqrt{77}}{77}$

18. $\sin\theta = \frac{8}{13}$

ANSWER:

$\cos\theta = \frac{\sqrt{105}}{13}, \tan\theta = \frac{8\sqrt{105}}{105}, \csc\theta = \frac{13}{8}, \sec\theta = \frac{13\sqrt{105}}{105}, \cot\theta = \frac{\sqrt{105}}{8}$

14. $y = 3\sin\left(x - \frac{\pi}{4}\right)$

SOLUTION:

In this function, $a = 3$, $b = 1$, $c = -\frac{\pi}{4}$, and $d = 0$.
Because $d = 0$, there is no vertical shift.

Amplitude: $|a| = |3|$ or 3

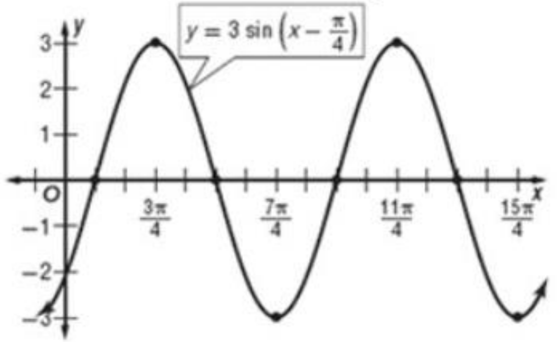
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{\frac{\pi}{4}}{|1|}$ or $-\frac{\pi}{4}$

Midline: $y = d$ or $y = 0$

Graph $y = 3\sin x$ shifted $\frac{\pi}{4}$ units to the right.



15. $y = \cos\left(\frac{x}{3} + \frac{\pi}{2}\right)$

SOLUTION:

In this function, $a = 1$, $b = \frac{1}{3}$, $c = \frac{\pi}{2}$, and $d = 0$.
Because $d = 0$, there is no vertical shift.

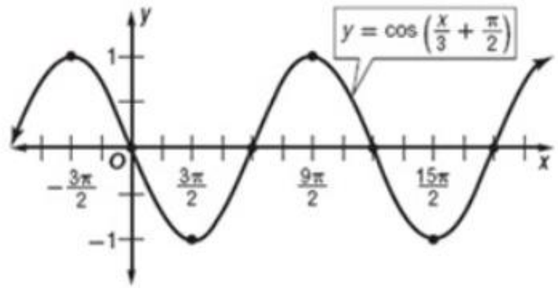
Amplitude: $|a| = |1|$ or 1

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{1}{3}\right|}$ or 6π

Frequency: $\frac{|b|}{2\pi} = \frac{\left|\frac{1}{3}\right|}{2\pi}$ or $\frac{1}{6\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{\frac{\pi}{2}}{\left|\frac{1}{3}\right|}$ or $-\frac{3\pi}{2}$

Midline: $y = d$ or $y = 0$



16. $y = 0.25 \cos x + 3$

SOLUTION:

In this function, $a = \frac{1}{4}$, $b = 1$, $c = 0$, and $d = 3$.

Amplitude: $|a| = \left|\frac{1}{4}\right|$ or $\frac{1}{4}$

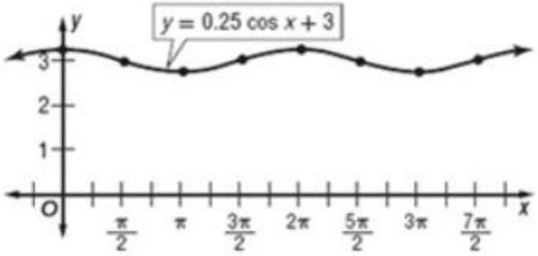
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{0}{|1|}$ or 0

Midline: $y = d$ or $y = 3$

Graph $y = \frac{1}{4} \cos x$ shifted 3 units up.



17. $y = \sin 3x - 2$

SOLUTION:

In this function, $a = 1$, $b = 3$, $c = 0$, and $d = -2$.

Amplitude: $|a| = |1|$ or 1

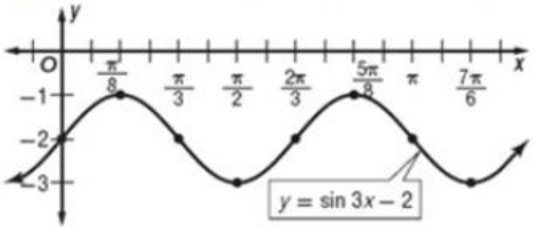
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$

Frequency: $\frac{|b|}{2\pi} = \frac{|3|}{2\pi}$ or $\frac{3}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{0}{|3|}$ or 0

Midline: $y = d$ or $y = -2$

Graph $y = \sin 3x$ shifted 2 units down.



18. $y = \cos\left(x - \frac{3\pi}{2}\right) - 1$

SOLUTION:

In this function, $a = 1$, $b = 1$, $c = -\frac{3\pi}{2}$, and $d = -1$.

.

Amplitude: $|a| = |1|$ or 1

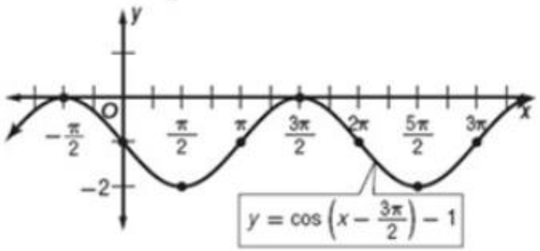
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{-\frac{3\pi}{2}}{|1|}$ or $\frac{3\pi}{2}$

Midline: $y = d$ or $y = -1$

Graph $y = \cos x$ shifted $\frac{3\pi}{2}$ units to the right and 1 unit down.



19. $y = \sin\left(x + \frac{5\pi}{6}\right) + 4$

SOLUTION:

In this function, $a = 1$, $b = 1$, $c = \frac{5\pi}{6}$, and $d = 4$.

.

Amplitude: $|a| = |1|$ or 1

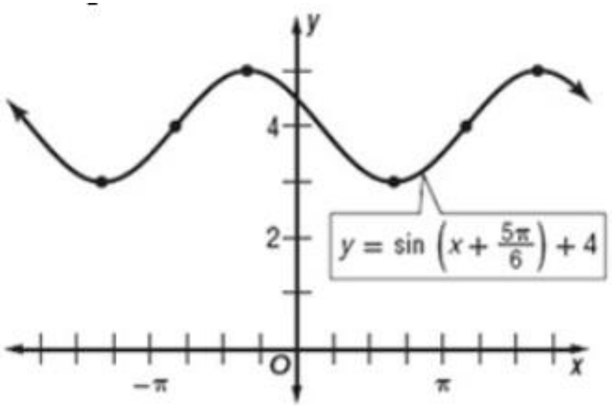
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{\frac{5\pi}{6}}{|1|}$ or $-\frac{5\pi}{6}$

Midline: $y = d$ or $y = 4$

Graph $y = \sin x$ shifted $\frac{5\pi}{6}$ units to the left and 4 units up.



21	اثبات صحة المتطابقات المثلثية. Verify trigonometric identities.	Exercises (1-15)	P316

14	اثبات صحة المتطابقات المثلثية. Verify trigonometric identities.	Exercises (1-10)	P316

11. $\frac{1}{1tan2\theta}+\frac{1}{1cot2\theta}=1$

SOLUTION:

$$\frac{1}{1-\tan^2\theta}+\frac{1}{1-\cot^2\theta}$$

$$=\frac{1}{1-\frac{\sin^2\theta}{\cos^2\theta}}+\frac{1}{1-\frac{\cos^2\theta}{\sin^2\theta}}$$

Quotient Identity

$$=\frac{1}{\frac{\cos^2\theta}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}}+\frac{1}{\frac{\sin^2\theta}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}}$$

Rewrite 1 using the common denominator.

$$=\frac{1}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}+\frac{1}{\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta}}$$

Write denominators as fractions with common denominators.

$$=\frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta-\cos^2\theta}$$

Simplify fractions.

$$=\frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}+\frac{-1}{-1}\cdot\frac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}$$

Factor out -1.

$$=\frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}+\frac{-\sin^2\theta}{\cos^2\theta-\sin^2\theta}$$

Common denominator

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}$$

Write as a fraction with a common denominator.

$$=1$$

Divide out common factor of $\left(\cos^2\theta-\sin^2\theta\right)$.

12. $\frac{1}{csc\theta+1}+\frac{1}{csc\theta 1}=2\sec^2\theta\sin\theta$

SOLUTION:

$$\frac{1}{\csc\theta+1}+\frac{1}{\csc\theta-1}$$

$$=\frac{\csc\theta-1}{\csc\theta-1}\cdot\frac{1}{\csc\theta+1}+\frac{\csc\theta+1}{\csc\theta+1}\cdot\frac{1}{\csc\theta-1}$$ Common denominator

$$=\frac{\csc\theta-1}{\csc^2\theta-1}+\frac{\csc\theta+1}{\csc^2\theta-1}$$ Multiply.

$$=\frac{2\csc\theta}{\csc^2\theta-1}$$ Write as a fraction with a common denominator.

$$=\frac{2\csc\theta}{\cot^2\theta}$$ Pythagorean Identity

$$=\frac{2\left(\frac{1}{\sin\theta}\right)}{\frac{\cos^2\theta}{\sin^2\theta}}$$ Reciprocal and Quotient Identities

$$=\frac{2}{\sin\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta}$$ Multiply by the reciprocal of the denominator.

$$=\frac{2\sin\theta}{\cos^2\theta}$$ Multiply.

$$=\left(\frac{2}{\cos^2\theta}\right)\sin\theta$$ Factor.

$$=2\sec^2\theta\sin\theta$$ Reciprocal Identity

13. $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$

SOLUTION:

$(\csc\theta - \cot\theta)(\csc\theta + \cot\theta)$

$= \csc^2\theta - \cot^2\theta$ Multiply.

$= 1$ Pythagorean Identity

14. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

SOLUTION:

$\cos^4\theta - \sin^4\theta$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$ Factor.

$= 1(\cos^2\theta - \sin^2\theta)$ Pythagorean Identity

$= \cos^2\theta - \sin^2\theta$ Multiply.

15. $\frac{1}{1sin\grave{e}} + \frac{1}{1+sin\grave{e}} = 2\sec^2\theta$

SOLUTION:

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$$

$$= \frac{1+\sin\theta}{1+\sin\theta} \cdot \frac{1}{1-\sin\theta} + \frac{1-\sin\theta}{1-\sin\theta} \cdot \frac{1}{1+\sin\theta}$$

$$= \frac{1+\sin\theta}{1-\sin^2\theta} + \frac{1-\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta$$