# حل الدرس الثالث الاتصال والسلوك الطرفي والنهايات Continuity, End Behavior, and Limits من الوحدة الأولى

| المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الأول | |
|---|---|
| 4 | أوراق عمل الدرس الثالث الاتصال والسلوك الطرفي والنهايات من الوحدة الأولى |
| 5 | أوراق عمل الدرس الأول functions الدوال من الوحدة الأولى |

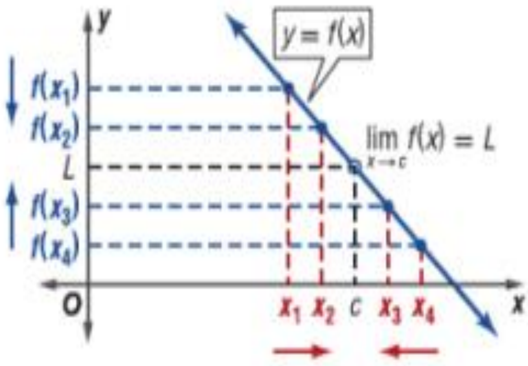
- **Grade 12 general** * **Emirates Schools Establishment**
- **Lesson 1-3:Continuity , End Behavior , and Limits / P(10-14)**

*** day: ……………… , ……/9/2024 * Name :…………………………………………**

## KeyConcept Limits

**Words** If the value of $f(x)$ approaches a unique value $L$ as $x$ approaches $c$ from each side, then the limit of $f(x)$ as $x$ approaches $c$ is $L$.

**Symbols** $\lim\limits_{x \to c} f(x) = L$, which is read *The limit of f(x) as x approaches c is L.*

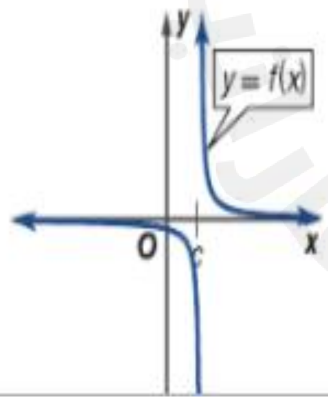



To understand what it means for a function to be continuous from an algebraic perspective, it helps to examine the graphs of **discontinuous functions**, or functions that are not continuous. Functions can have many different types of discontinuity.

## KeyConcept Types of Discontinuity

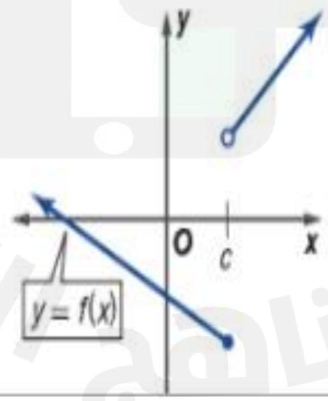
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as $x$ approaches $c$ from the left and right.

Example



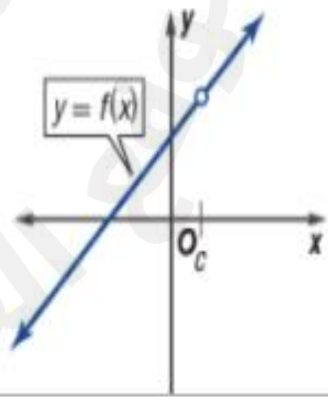

A function has a **jump discontinuity** at $x = c$ if the limits of the function as $x$ approaches $c$ from the left and right exist but have two distinct values.

Example




A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

Example

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**Activity (1):** Determine whether each function is continuous at the given x-value(s), Justify using the continuity test .If discontinuous , identify the type of discontinuity **a infinite** ,**jump**, or **removable** .

**نشاط (1):** حدد ما اذا كانت الدوال التالية متصلة أم لا ، برر اجابتك باستخدام اختبار الاتصال ، اذا كانت الدالة غير متصله ، حدد نوع عدم الاتصال ،**لانهائي** ، **فجوة** ، **قابل للازالة** .

$$a)\quad f(x) = x^2 - 3x + 4 \quad ,at \quad x = 3$$

1) $f(3) =$

2) $\lim_{x\to 3} f(x) =$

3) $\lim_{x\to 3} f(x) \ldots\ldots\ldots\ldots f(3)$ .

**Now since** f(3): ……….., $\lim_{x\to 3} f(x)$: ……………… *and* $\lim_{x\to 3} f(x)$ ……. $f(3)$ ,

then $f(x)$ is …………………………………… At $x = 3$ .

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**b)** $f(x) = \begin{cases} x^2 + 1 & , \quad x \geq 2 \\ 3x - 1 & , \quad x < 2 \end{cases}$ **, at** $x = 2$ **.**

1) $f(2) =$

2) $\lim_{x\to 2} f(x) =$

3) $\lim_{x\to 2} f(x) \ldots\ldots\ldots\ldots f(2)$

**Now since** f(2): ……………., $\lim_{x\to 2} f(x)$ : ……………… *and* $\lim_{x\to 2} f(x)$ ……. $f(2)$ ,

then $f(x)$ is …………………………………… At $x = 2$ .

**c)** $f(x) = \begin{cases} x^2 - 1 & , \quad x \geq -2 \\ x - 5 & , \quad x < -2 \end{cases}$ **, at** $x = -2$ **.**

1) $f(-2) =$

2) $\lim_{x\to -2} f(x) =$

3) $\lim_{x\to -2} f(x) \ldots\ldots\ldots\ldots f(-2)$

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$d) \quad \boldsymbol{f(x) = \frac{x}{x-1}} \qquad , \quad \boldsymbol{when \ \ x = 1}$

1) $f(1) =$

2) $\lim\limits_{x \to 1} f(x) =$

3) $\lim\limits_{x \to 1} f(x) \ldots\ldots\ldots\ldots . f(1)$

**Activity (2):** Determine between which consecutive integers the real zeros of each function are located on the given interval .

**نشاط (2):** حدد الاعداد الصحيحة المتتالية التي تنحصر بينها الاصفار الحقيقية للدالة في الفترة المحددة.

$a) \quad f(x) = x^3 - 4x + 2 \quad ,[-4,4]$

| x | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **F(x)** | | | | | | | | | |

$b) \quad f(x) = x^3 + 2x^2 - 8x + 3 \quad ,[-6,4]$

| x | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **F(x)** | | | | | | | | | | | |

**c) Use the table to determine between which consecutive integers do the real zeros for $f(x) = x^3 - 9x + 4 \ \ , \ lie \ on \ [-4, 4]$**

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | −24 | 4 | 14 | 12 | 4 | -4 | -6 | 4 | 32 |

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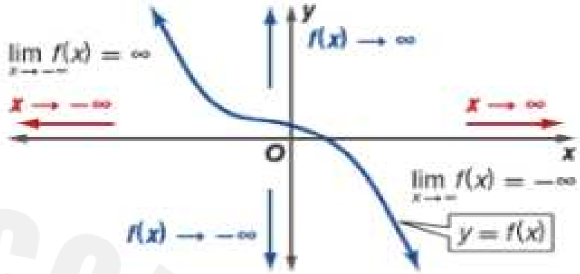
**السلوك الطرفي( End behavior)** : هو ما يحدث لقيمة الدالة f(x) كلما زادت قيمة x أو نقصت بدون اي حدود أي بمعنى الزيادة للغاية او نقصت حتى اصبحت سالبة فسالبة اكثر فاكثر . اي هنا نحدد سلوك الطرف الايمن و سلوك الطرف الايسر.

**2 End Behavior** The **end behavior** of a function describes how a function *behaves* at either *end* of the graph. That is, end behavior is what happens to the value of $f(x)$ as $x$ increases or decreases without bound—becoming greater and greater or more and more negative. To describe the end behavior of a graph, you can use the concept of a limit.

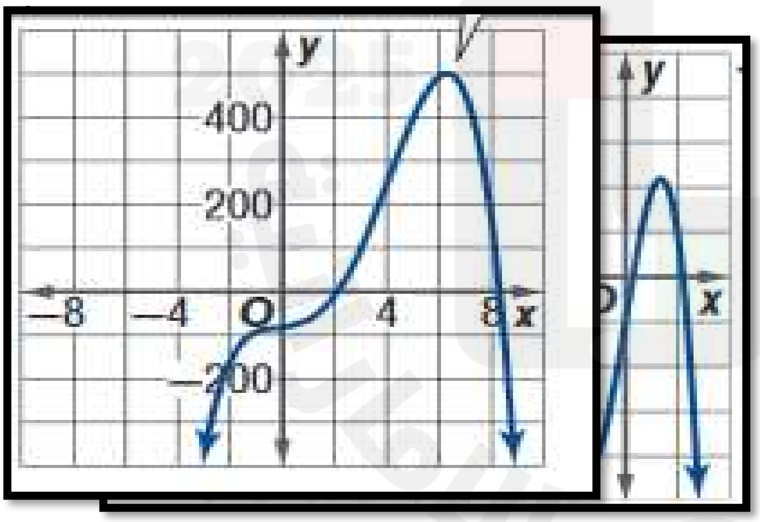
| Left-End Behavior | Right-End Behavior |
|---|---|
| $\lim_{x\to-\infty} f(x)$ | $\lim_{x\to\infty} f(x)$ |

One possibility for the end behavior of the graph of a function is for the value of $f(x)$ to increase or decrease without bound. This end behavior is described by saying that $f(x)$ approaches positive or negative infinity.




**Activity (3):** Use the graph of each function to describe its end behavior, Support the conjecture numerically.

**نشاط (3)** استخدم الرسم البياني للدالة وذلك لوصف السلوك الطرفي الخاص بها،ثم أثبت فرضيتك عدديا .




1) $\lim_{x\to-\infty} f(x) =$

2) $\lim_{x\to\infty} f(x) =$

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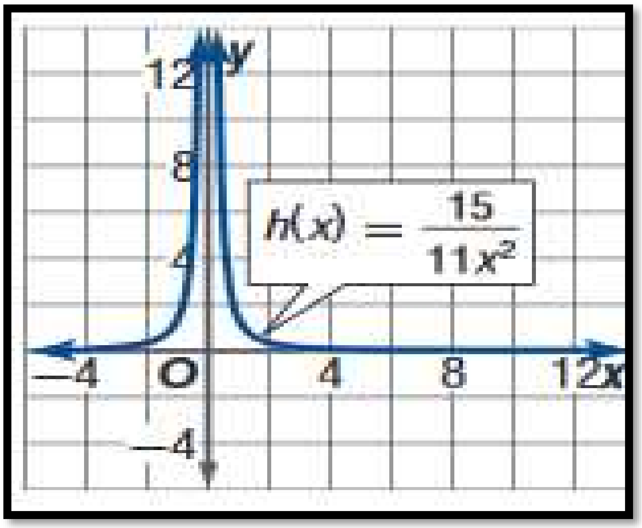
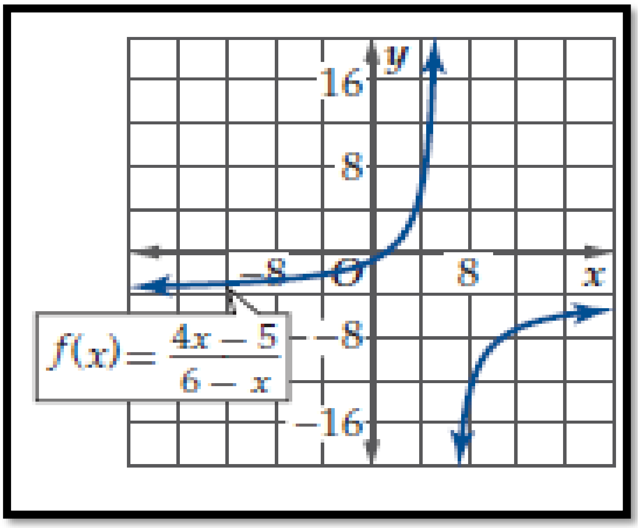
c)




1) $\lim\limits_{x \to -\infty} f(x) =$

2) $\lim\limits_{x \to \infty} f(x) =$

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2) $\lim\limits_{x \to \infty} f(x) =$

**Activity(4) : Describe its endbehavior of each functions:**

a) $f(x) = \frac{4x^3-9x+7}{2x^3+8}$ , 1) $\lim\limits_{x \to -\infty} f(x) =$ 2) $\lim\limits_{x \to \infty} f(x) =$

b) $f(x) = \frac{9x+7}{5-6x}$ , 1) $\lim\limits_{x \to -\infty} f(x) =$ 2) $\lim\limits_{x \to \infty} f(x) =$

c) $f(x) = \frac{4x+1}{2x^2+8x+3}$ , 1) $\lim\limits_{x \to -\infty} f(x) =$ 2) $\lim\limits_{x \to \infty} f(x) =$

d) $f(t) = \frac{3t^2+2t+7}{6t^2+5t-3}$ , 1) $\lim\limits_{x \to -\infty} f(x) =$ 2) $\lim\limits_{x \to \infty} f(x) =$

e) $p(t) = \frac{3200t^2}{4t^2-2t+35}$ , 1) $\lim\limits_{x \to -\infty} f(x) =$ 2) $\lim\limits_{x \to \infty} f(x) =$

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