Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

1. For each at bat, a player can either get on base or make an out. Suppose a player bats twice.

SOLUTION:

Organized List:

Pair each possible outcome for the first at bat with the possible outcomes for the second at bat.

S, S   O, O
S, O   O, S

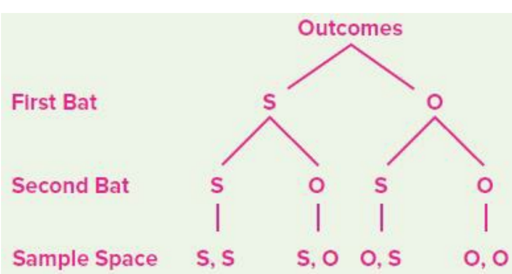
Table:

List the outcomes of the first at bat in the left column and those of the second at bat in the top row.

Outcomes	Safe	Out
Safe	S, S	S, O
Out	O, S	O, O

Tree Diagram:

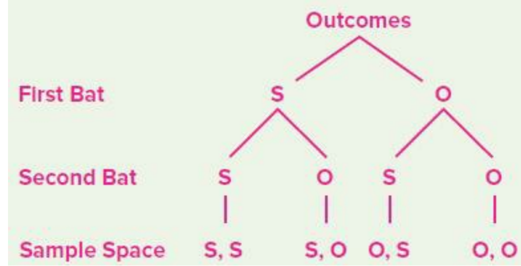
The top group is all of the outcomes for the first at bat. The second group includes all of the outcomes for the second at bat. The last group shows the sample space.



ANSWER:

S, S   O, O
S, O   O, S

Outcomes	Safe	Out
Safe	S, S	S, O
Out	O, S	O, O



2. Quinton sold the most tickets in his school for the annual Autumn Festival. As a reward, he gets to choose twice from a grab bag with tickets that say “free juice” or “free notebook.”

SOLUTION:

Organized List:

Pair each possible outcome for the first choice with the possible outcomes for the second choice.

J, J   N, N
J, N   N, J

Table:

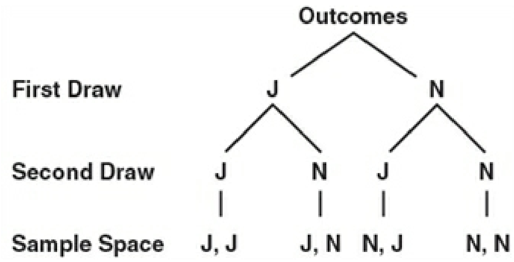
List the outcomes of the first choice in the left column and those of the second choice in the top row.

Outcomes	Juice	Notebook
Juice	J, J	J, N
Notebook	N, J	N, N

Tree Diagram:

The top group is all of the outcomes for the first choice. The second group includes all of the outcomes for the second choice. The last group shows the sample space.

12-1 Representing Sample Spaces

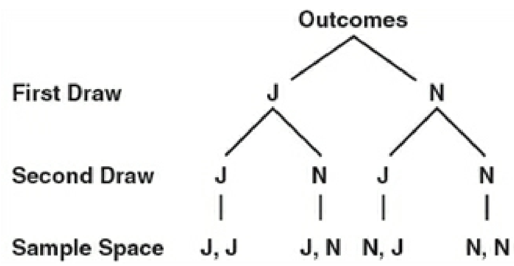


ANSWER:

J, J N, N

J, N N, J

Outcomes	Juice	Notebook
Juice	J, J	J, N
Notebook	N, J	N, N



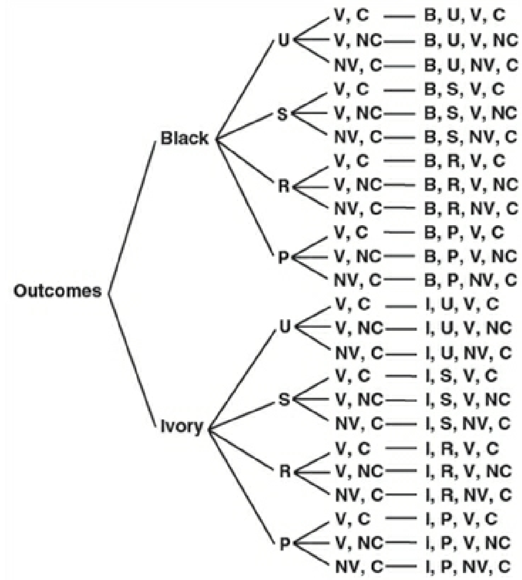
3. **TUXEDOS** Patrick is renting a prom tuxedo from the catalog shown. Draw a tree diagram to represent the sample space for this situation.

SOLUTION:

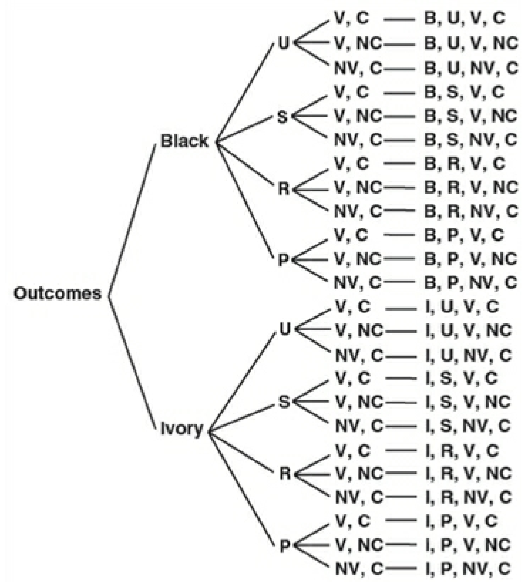
The sample space is the result of four stages. The suit color only has two options, so do it first.

- Suit Color (Black or Ivory)
- Tie Color (U, S, R, or P)
- Vest (V or NV)
- Cummerbund (C or NC)

Draw a tree diagram with four stages.



ANSWER:



Find the number of possible outcomes for each situation.

4. Desirée is preparing for homecoming and must decide what to wear. Assume one of each is chosen.

Options	Number of Choices
dress	15
shoes	5
purse	3
earrings	4
necklace	2

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

The dress can be chosen in 15 different ways, shoes in 5 different ways, purse in 3 different ways, earrings in 4 different ways, and the necklace in 2 different ways. Therefore, choosing an outfit for homecoming can be done in $15 \times 5 \times 3 \times 4 \times 2 = 1800$ ways.

ANSWER:

1800

5. Marcos is creating a new menu for his restaurant. Assume one of each item is ordered.

Menu Titles	Number of Choices
appetizer	8
soup	4
salad	6
entree	12
dessert	9

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

An appetizer can be ordered in 8 different ways, soup in 4 different ways, salad in 6 different ways, entree in 12 different ways, and dessert in 9 different ways. Therefore, an order with one item from each set can be done in $8 \times 4 \times 6 \times 12 \times 9 = 20{,}736$ ways.

ANSWER:

20,736

STRUCTURE Represent the sample space for each experiment by making an organized list, a table, and a tree diagram.

12-1 Representing Sample Spaces

6. Gina is a junior and has a choice for the next two years of either playing volleyball or basketball during the winter quarter.

SOLUTION:

Organized List:

Pair each possible outcome for the first year with the possible outcomes for the second year.

V, V   B, B   V, B   B, V

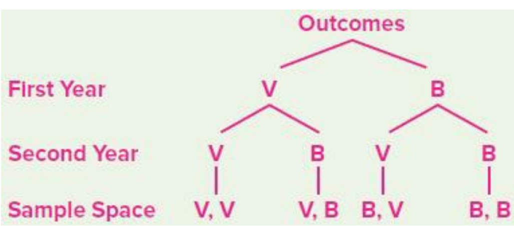
Table:

List the outcomes of the first year in the left column and those of the second year in the top row.

Outcomes	Volleyball	Basketball
Volleyball	V, V	V, B
Basketball	B, V	B, B

Tree Diagram:

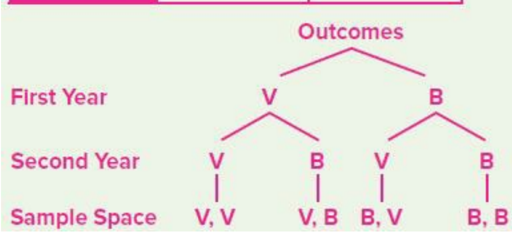
The top group is all of the outcomes for the first year. The second group includes all of the outcomes for the second year. The last group shows the sample space.



ANSWER:

V, V   B, B   V, B   B, V

Outcomes	Volleyball	Basketball
Volleyball	V, V	V, B
Basketball	B, V	B, B



7. Two different history classes in New York City are taking a trip to either the Smithsonian or the Museum of Natural History.

SOLUTION:

Organized List:

Pair each possible outcome for the first class with the possible outcomes for the second class.

S, S   N, N   S, N   N, S

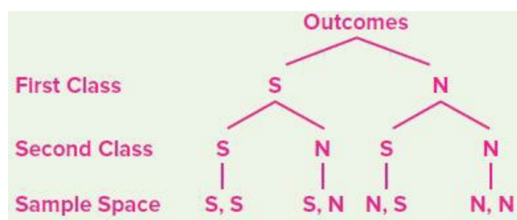
Table:

List the outcomes of the first class in the left column and those of the second class in the top row.

Outcomes	Smithsonian	Natural
Smithsonian	S, S	S, N
Natural	N, S	N, N

Tree Diagram:

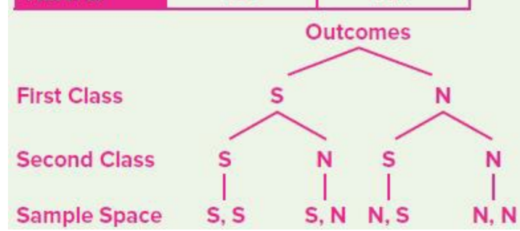
The top group is all of the outcomes for the first class. The second group includes all of the outcomes for the second class. The last group shows the sample space.



ANSWER:

S, S   N, N   S, N   N, S

Outcomes	Smithsonian	Natural
Smithsonian	S, S	S, N
Natural	N, S	N, N



8. Simeon has an opportunity to travel abroad as a foreign exchange student during each of his last two years of college. He can choose between Ecuador or Italy.

SOLUTION:

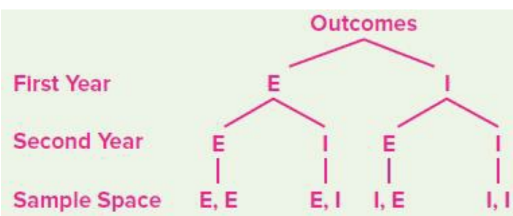
Organized List:
Pair each possible outcome for the first year with the possible outcomes for the second year.

E, E I, I E, I I, E

Table:
List the outcomes of the first year in the left column and those of the second year in the top row.

Outcomes	Italy	Ecuador
Italy	I, I	I, E
Ecuador	E, I	E, E

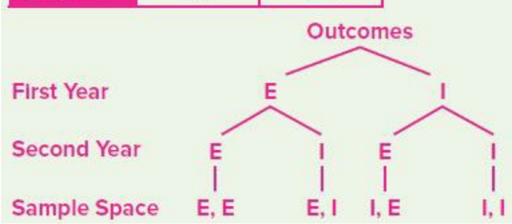
Tree Diagram:
The top group is all of the outcomes for the first year. The second group includes all of the outcomes for the second year. The last group shows the sample space.



ANSWER:

E, E I, I E, I I, E

Outcomes	Italy	Ecuador
Italy	I, I	I, E
Ecuador	E, I	E, E



9. A new club is formed, and a meeting time must be chosen. The possible meeting times are Monday or Thursday at 5:00 or 6:00 p.m.

SOLUTION:

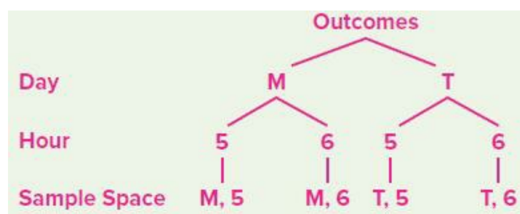
Organized List:
Pair each possible outcome for the days with the possible outcomes for the times.

M, 5 T, 5 M, 6 T, 6

Table:
List the outcomes of the days in the left column and those of the times in the top row.

Outcomes	5	6
Monday	M, 5	M, 6
Thursday	T, 5	T, 6

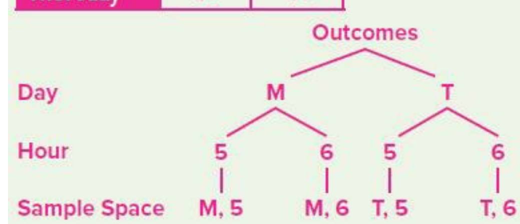
Tree Diagram:
The top group is all of the outcomes for the days. The second group includes all of the outcomes for the times. The last group shows the sample space.



ANSWER:

M, 5 T, 5 M, 6 T, 6

Outcomes	5	6
Monday	M, 5	M, 6
Thursday	T, 5	T, 6



10. An exam with multiple versions has exercises with triangles. In the first exercise, there is an obtuse triangle or an acute triangle. In the second exercise, there is an isosceles triangle or a scalene triangle.

SOLUTION:

Organized List:

Pair each possible outcome for the angles with the possible outcomes for the sides.

A, I A, S O, I O, S

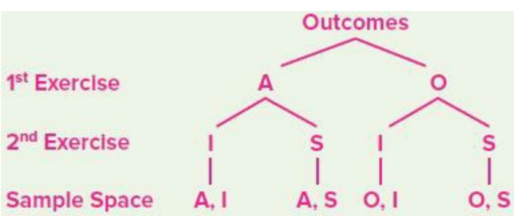
Table:

List the outcomes of the angles in the left column and those of the sides in the top row.

Outcomes	Isosceles	Scalene
Acute	A, I	A, S
Obtuse	O, I	O, S

Tree Diagram:

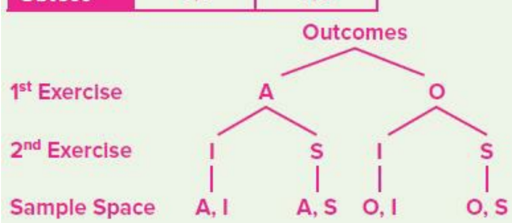
The top group is all of the outcomes for the 1st exercise. The second group includes all of the outcomes for the 2nd exercise. The last group shows the sample space.



ANSWER:

A, I A, S O, I O, S

Outcomes	Isosceles	Scalene
Acute	A, I	A, S
Obtuse	O, I	O, S



11. **PAINTING** In an art class, students are working on two projects where they can use one of two different types of paints for each project. Represent the sample space for this experiment by making an organized list, a table, and a tree diagram.

SOLUTION:

Organized List:

Pair each possible outcome for the first project with the possible outcomes for the second project.

O, O A, A O, A A, O

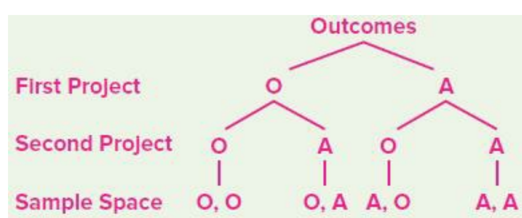
Table:

List the outcomes of the first project in the left column and those of the second project in the top row.

Outcomes	Oil	Acrylic
Oil	O, O	O, A
Acrylic	A, O	A, A

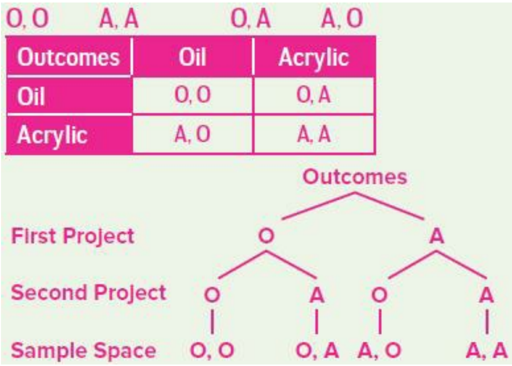
Tree Diagram:

The top group is all of the outcomes for the first project. The second group includes all of the outcomes for the second project. The last group shows the sample space.



ANSWER:

12-1 Representing Sample Spaces



Draw a tree diagram to represent the sample space for each situation.

12. **BURRITOS** At a burrito stand, customers have the choice of beans, pork, or chicken with rice or no rice, and cheese and/or salsa.

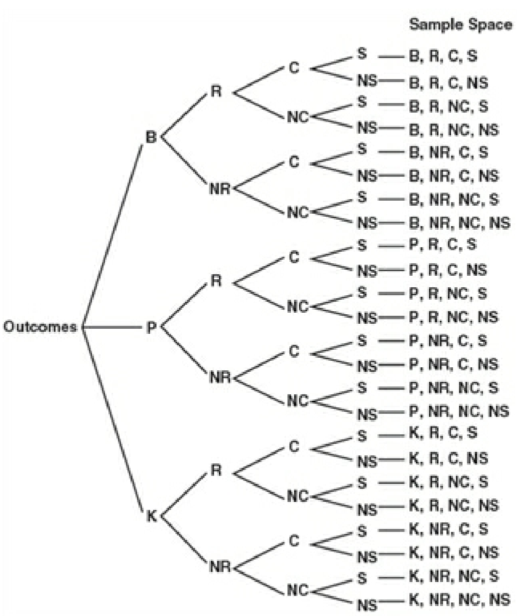
SOLUTION:

The sample space is the result of four stages.

- Choices (B, P, K)
- Rice (R or NR)
- Cheese (C or NC)
- Salsa (S or NS)

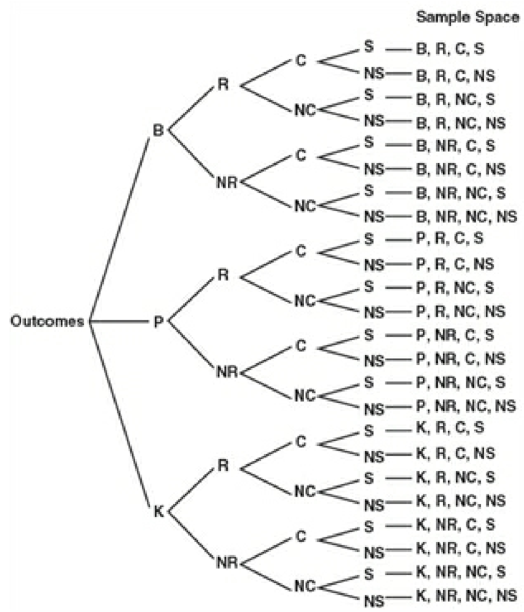
Draw a tree diagram with four stages.

B = beans, P = pork, K = chicken, R = rice, NR = no rice, C = cheese, NC = no cheese, S = salsa, and NS = no salsa



ANSWER:

B = beans, P = pork, K = chicken, R = rice, NR = no rice, C = cheese, NC = no cheese, S = salsa, and NS = no salsa



13. **TRANSPORTATION** Blake is buying a vehicle and has a choice of sedan, truck, or van with leather or fabric interior, and a GPS and/or sunroof.

SOLUTION:

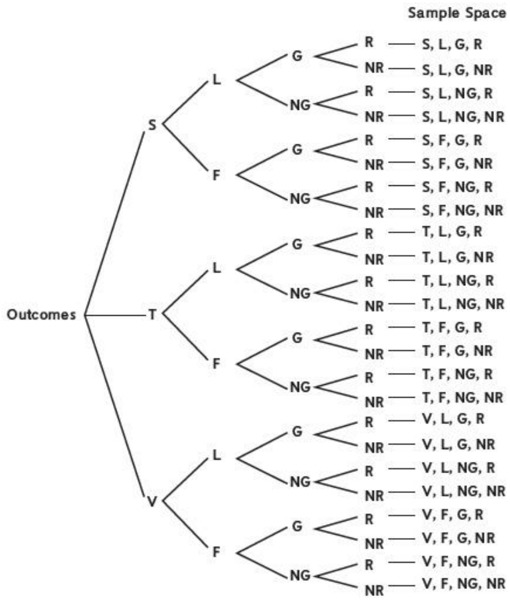
The sample space is the result of four stages.

- Vehicle (S, T, or V)
- Interior (L or F)
- GPS (G or NG)
- Sunroof (S or NS)

Draw a tree diagram with four stages.

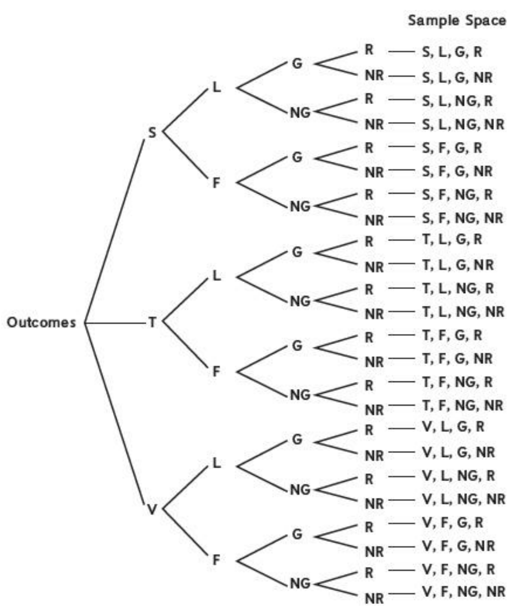
S = sedan, T = truck, V = van, L = leather, F = fabric, G = GPS, NG = no GPS, R = sunroof, NR = no sunroof

12-1 Representing Sample Spaces



ANSWER:

S = sedan, T = truck, V = van, L = leather, F = fabric, G = GPS, NG = no GPS, R = sunroof, NR = no sunroof



14. **TREATS** Ping and her friends go to a frozen yogurt parlor which has a sign like the one shown. Draw a tree diagram for all possible combinations of cones with peanuts and/or sprinkles.

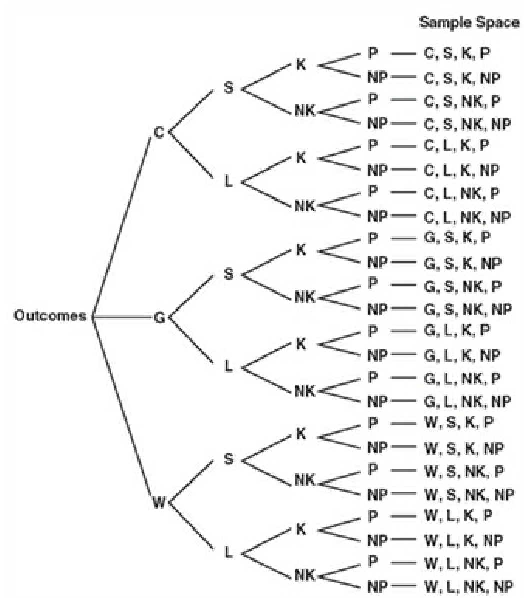
SOLUTION:

The sample space is the result of four stages.

- Cone (C, G, or W)
- Flavor (S or L)
- Sprinkles (K or NK)
- Peanuts (P or NP)

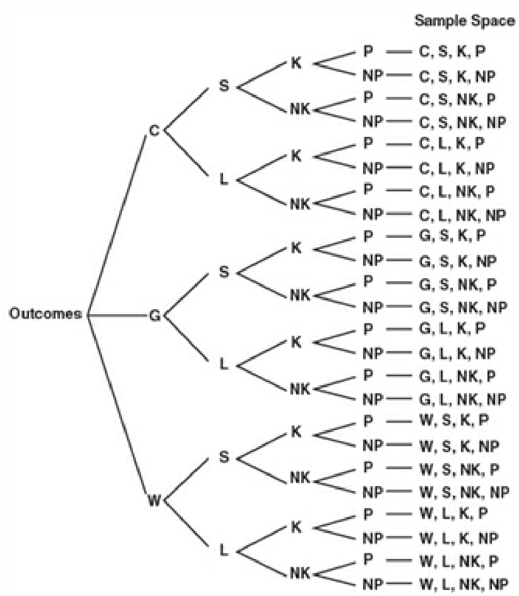
Draw a tree diagram with four stages.

C = cake cone, G = sugar cone, W = waffle cone, S = strawberry, L = lime, P = peanuts, NP = no peanuts, K = sprinkles, NK = no sprinkles



ANSWER:

C = cake cone, G = sugar cone, W = waffle cone, S = strawberry, L = lime, P = peanuts, NP = no peanuts, K = sprinkles, NK = no sprinkles



MODELING In Exercises 15–18, find the number of possible outcomes for each situation.

15. In the Junior Student Council elections, there are 3 people running for secretary, 4 people running for treasurer, 5 people running for vice president, and 2 people running for class president.

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

A secretary can be chosen in 3 different ways, treasurer in 4 different ways, vice president in 5 different ways, and class president in 2 different ways. Therefore, the Junior Student Council can be formed in $3 \times 4 \times 5 \times 2 = 120$ ways.

ANSWER:

120

16. When signing up for classes during his first semester of college at Texas A&M, Frederico has 4 class spots to fill with a choice of 4 literature classes, 2 math classes, 6 history classes, and 3 film classes.

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Frederico can choose his literature classes in 4 different ways, math classes in 2 different ways, history classes in 6 different ways, and film classes in 3 different ways. Therefore, he can choose his classes in $4 \times 2 \times 6 \times 3 = 144$ ways.

ANSWER:

144

17. Niecy is choosing one each of 6 colleges, 5 majors, 2 minors, and 4 clubs.

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Niecy can choose any of the 6 colleges, any of the 5 majors, any of the 2 minors, and any of the 4 clubs. Therefore, she has $6 \times 5 \times 2 \times 4 = 240$ ways to make her choice.

ANSWER:

240

18. Evita works at a restaurant where she has to wear a white blouse, black pants or skirt, and black shoes. She has 5 blouses, 4 pants, 3 skirts, and 6 pairs of black shoes.

SOLUTION:

By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Evita can wear any of her 5 blouses and any of the 6 pairs of shoes. She can combine them with either a pant of her choice from 4 pants or a skirt from 3 skirts. Therefore, she can choose her outfit in 5 × 6 × (3 + 4) = 210 ways.

ANSWER:

210

19. **ART** For an art class assignment, Mr. Green gives students their choice of two quadrilaterals to use as a base. One must have sides of equal length, and the other must have at least one set of parallel sides. Represent the sample space by making an organized list, a table, and a tree diagram.

SOLUTION:

Organized List:

Pair each possible outcome for the first quadrilateral with the possible outcomes for the second quadrilateral.

H = rhombus, P = parallelogram, R = rectangle, S = square, T = trapezoid;
H, P; H, R; H, S; H, T; H, H; S, P; S, R; S, S; S, T; S, H

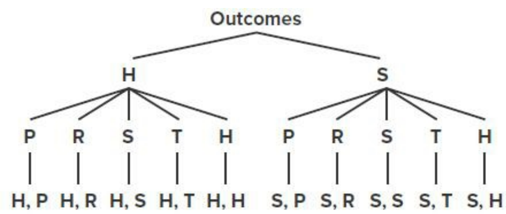
Table:

List the outcomes of the first quadrilateral in the left column and those of the second quadrilateral in the top row.

Outcomes	Rhombus	Square
parallelogram	H, P	S, P
rectangle	H, R	S, R
square	H, S	S, S
trapezoid	H, T	S, T
rhombus	H, H	S, H

Tree Diagram:

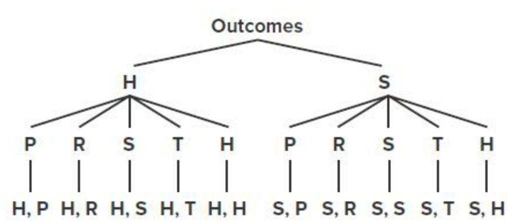
The top row is all of the outcomes for the first quadrilateral. The second row includes all of the outcomes for the second quadrilateral. The last row shows the sample space.



ANSWER:

H = rhombus, P = parallelogram, R = rectangle, S = square, T = trapezoid;
H, P; H, R; H, S; H, T; H, H; S, P; S, R; S, S; S, T; S, H

Outcomes	Rhombus	Square
parallelogram	H, P	S, P
rectangle	H, R	S, R
square	H, S	S, S
trapezoid	H, T	S, T
rhombus	H, H	S, H



20. **BREAKFAST** A hotel restaurant serves omelets with a choice of vegetables, ham, or sausage that come with a side of hash browns, grits, or toast.

a. How many different outcomes of omelet and one side are possible if a vegetable omelet comes with just one vegetable?
b. Find the number of possible outcomes for a vegetable omelet if you can get any or all vegetables on any omelet.

SOLUTION:
a. There are 4 different vegetables, which means 4 different types of vegetable omelets. Each has a choice of 3 different sides. So, there are 4(3) = 12 different vegetable omelet orders.

There are another 3 outcomes for a ham omelet (with one of three different sides) and another 3 for a sausage omelette (with one of three different sides).

So, in all, there are 12 + 3 + 3 = 18 different outcomes.

b. There is $\binom{4}{4}=1$ way to have all 4 vegetables.

There are $\binom{4}{3}=4$ ways to have 3 vegetables.

There are $\binom{4}{2}=6$ ways to have 2 vegetables.

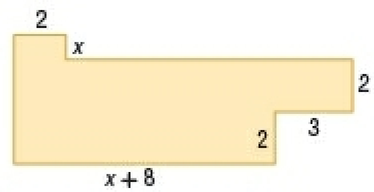
There are $\binom{4}{1}=4$ ways to have 1 vegetable.

You must have 1 vegetable or it is not a vegetable omelet!

Therefore, there are 15 different possible vegetable omelets. Multiply by 3 for the different choices of sides. So, there are 45 different possible outcomes for a vegetable omelet.

ANSWER:
a. 18
b. 45

21. **COMPOSITE FIGURES** Carlito is calculating the area of the composite figure shown. List six different ways he can do this.



SOLUTION:
The figure can be divided into different sets of squares and rectangles. The area of the composite figure is the sum of the areas of the smaller squares and rectangles. Some of the ways the figure can be divided and the resulting sum of the areas are shown below.

$4(x + 6) + 2(3) + 2(x + 4)$;
$2(x + 11) + 2(x + 8) + 2(x)$;
$2(x + 4) + 2(x + 9) + 2(x + 6)$;
$2(x) + 2(3) + 4(x + 8)$;
$2(x) + 2(x + 8) + 2(3) + 2(x + 8)$;
$2(x) + 2(3) + 2(4) + 2(x + 6) + 2(x + 6)$

ANSWER:
Sample answer: 6 different ways: $4(x + 6) + 2(3) + 2(x + 4)$; $2(x + 11) + 2(x + 8) + 2(x)$; $2(x + 4) + 2(x + 9) + 2(x + 6)$;
$2(x) + 2(3) + 4(x + 8)$; $2(x) + 2(x + 8) + 2(3) + 2(x + 8)$; $2(x) + 2(3) + 2(4) + 2(x + 6) + 2(x + 6)$

22. **TRANSPORTATION** Miranda got a new bicycle lock that has a four-number combination. Each number in the combination is from 0 to 9.

a. How many combinations are possible if there are no restrictions on the number of times Miranda can use each number?
b. How many combinations are possible if Miranda can use each number only once? Explain.

SOLUTION:
a. By the Fundamental Counting Principle, the number of possible outcomes in a sample space can be found by multiplying the number of possible outcomes from each stage or event.

Miranda can use any of the 10 digits in the first, second, third, and fourth places. Therefore, total number of combinations are 10 × 10 × 10 × 10 = 10,000.

b. Since Miranda can use each number only once, the combination cannot have any duplicates, like 9935 or 8888. Therefore, the number of combinations is not 10 × 10 × 10 × 10. We need to see how many possibilities there are for each number in the combination.

There are 10 possibilities for the first number in the combination. If the first number is 4, then the second number can be anything except 4. Therefore, there are only 9 possibilities for the second number in the combination. If the second number is 5, then the third number cannot be 4 or 5, so there are 8 possibilities for the third number in the combination. If the third number is 9, then the fourth number can be anything except 4, 5, or 9. Therefore, there are 7 possibilities for the fourth number in the combination.

The number of possible combinations is 10 × 9 × 8 × 7 or 5040.

ANSWER:
a. 10,000
b. 5040; Sample answer: There are 10 possibilities for the first number in the combination. Because Miranda can use each number only once, there are only 9 possibilities for the second number in the combination, 8 possibilities for the third number in the combination, and 7 possibilities for the fourth number in the combination. The number of possible combinations is 10 × 9 × 8 × 7 or 5040.

23. **GAMES** Cody and Monette are playing a board game in which you roll two dot cubes per turn.

a. In one turn, how many outcomes result in a sum of 8?
b. How many outcomes in one turn result in an odd sum?

SOLUTION:
a. A sum of 8 can occur in 5 different ways, (2, 6), (6, 2), (3, 5), (5, 3), and (4, 4) where the first number in the ordered pair represents the outcome of the first roll and the second number represents that of the second roll.

b. A sum of an odd number can occur in 18 different ways, as shown.

Outcome	Sum	Outcome	Sum
(1,2)	3	(2,1)	3
(1,4)	5	(4,1)	5
(1,6)	7	(6,1)	7
(2,3)	5	(3,2)	5
(2,5)	7	(5,2)	7
(3,4)	7	(4,3)	7
(3,6)	9	(6,3)	9
(4,5)	9	(5,4)	9
(5,6)	11	(6,5)	11

ANSWER:
a. 5
b. 18

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a sequence of events. In the first stage of a two-stage experiment, you spin Spinner 1 below. If the result is red, you flip a coin. If the result is yellow, you roll a dot cube. If the result is green, you roll a number cube. If the result is blue, you spin Spinner 2.

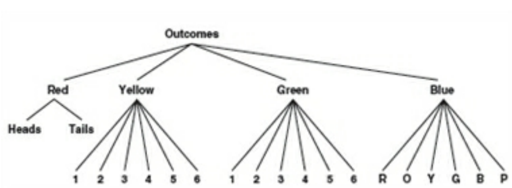
12-1 Representing Sample Spaces

a. Geometric Draw a tree diagram to represent the sample space for the experiment.
b. Logical Draw a Venn diagram to represent the possible outcomes of the experiment.
c. Analytical How many possible outcomes are there?
d. Verbal Could you use the Fundamental Counting Principle to determine the number of outcomes? Explain.

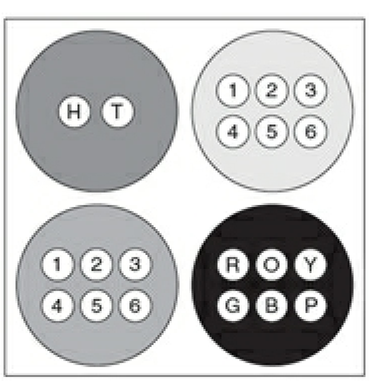
SOLUTION:

a.

Tree Diagram:
The top group is all of the outcomes for the first task (spin). The second group includes all of the outcomes for the second task.



b. Each outcome of the first spin is independent, so the four circles representing these outcomes will not intersect. All of the outcomes for the second task shall be included in each respective circle because they cannot occur in any other circle (they are independent).

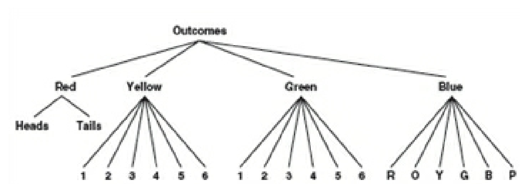


c. The total number of outcomes is 6 + 6 + 6 + 2 = 20. The complete list is provided in part **a** and each outcome is also shown in the Venn diagram in part **b**.

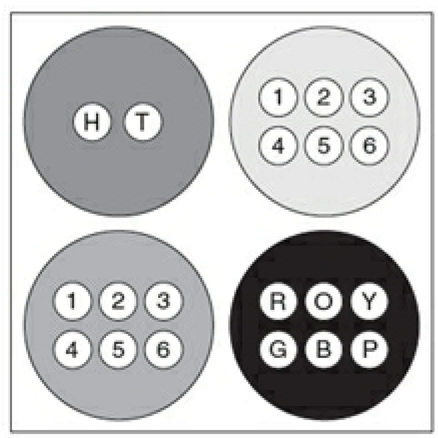
d. No; The Fundamental Counting Principle relies on independent outcomes. Since the second stage of the experiment depends on what happens in the first stage of the experiment, you cannot multiply the number of outcomes for each stage. You have to find the number of possible outcomes for each stage and add them.

ANSWER:

a.



b.



c. 20

d. Sample answer: No; since the second stage of the experiment depends on what happens in the first stage of the experiment, you cannot multiply the number of outcomes for each stage. You have to find the number of possible outcomes for each stage and add them.

25. **CONSTRUCT ARGUMENTS** A box contains *n* different objects. If you remove three objects from the box, one at a time, without putting the previous object back, how many possible outcomes exist? Explain your reasoning.

SOLUTION:

We have 3 stages: taking out each object three times.

Since we are not putting the objects back in the box, and there will be fewer objects in the box for each stage, these stages are not independent.

Each stage will have a different amount of possible outcomes. There are *n* objects in the box when you remove the first object, so after you remove one object, there are $n - 1$ possible outcomes. After you remove the second object, there are $n - 2$ possible outcomes.

The number of possible outcomes is the product of the number of outcomes of each experiment or $n(n - 1)(n - 2) = n^3 - 3n^2 + 2n$.

ANSWER:

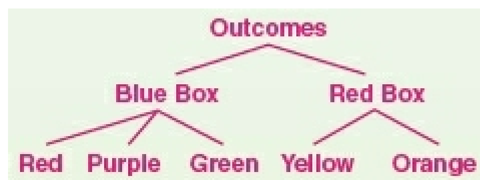
$n^3 - 3n^2 + 2n$; Sample answer: There are *n* objects in the box when you remove the first object, so after you remove one object, there are $n - 1$ possible outcomes. After you remove the second object, there are $n - 2$ possible outcomes. The number of possible outcomes is the product of the number of outcomes of each experiment or $n(n - 1)(n - 2)$.

26. **CHALLENGE** Sometimes a tree diagram for an experiment is not symmetrical. Describe a two-stage experiment where the tree diagram is asymmetrical. Include a sketch of the tree diagram. Explain.

SOLUTION:

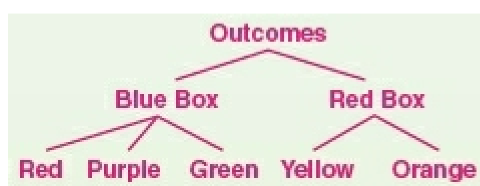
In order to make a tree diagram asymmetrical, set up the events so that there are a different number of outcomes for each event. For example, rolling a number cube has 6 outcomes and flipping a coin has only 2 outcomes.

Sample answer: In an experiment, you choose between a blue box and a red box. You then remove a ball from the box that you chose without looking into the box. The blue box contains a red ball, a purple ball, and a green ball. The red box contains a yellow ball and an orange ball.



ANSWER:

Sample answer: In an experiment, you choose between a blue box and a red box. You then remove a ball from the box that you chose without looking into the box. The blue box contains a red ball, a purple ball, and a green ball. The red box contains a yellow ball and an orange ball.



27. **WRITING IN MATH** Explain why it is not possible to represent the sample space for a multistage experiment by using a table.

SOLUTION:

You can list the possible outcomes for one stage of an experiment in the columns and the possible outcomes for the other stage of the experiment in the rows. Because a table is two dimensional, it would be impossible to list the possible outcomes for three or more stages of an experiment. Therefore, tables can only be used to represent the sample space for a two-stage experiment.

For example, consider two coin flips. All four outcomes can be shown in the table, with one stage in the column, and the other stage in the row.

	Heads	**Tails**
Heads	H, H	H, T
Tails	T, H	T, T

Now, consider three coin flips. Where do we place the third stage? It cannot be placed in the bottom row, the far right column, or anywhere inside the table. It would only work if the table were three-dimensional.

ANSWER:

Sample answer: You can list the possible outcomes for one stage of an experiment in the columns and the possible outcomes for the other stage of the experiment in the rows. Since a table is two dimensional, it would be impossible to list the possible outcomes for three or more stages of an experiment. Therefore, tables can only be used to represent the sample space for a two-stage experiment.

28. **CONSTRUCT ARGUMENTS** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

When an outcome falls outside the sample space, it is a failure.

SOLUTION:

Never; the sample space is the set of all possible outcomes. An outcome cannot fall outside the sample space. A failure occurs when the outcome is in the sample space, but is not a favorable outcome.

For example, when you are flipping a coin, it can be heads or tails, and nothing else. If heads is the favorable outcome, then tails is the unfavorable outcome. However, tails is still in the sample space because it is a *possible* outcome.

ANSWER:

Sample answer: Never; the sample space is the set of all possible outcomes. An outcome cannot fall outside the sample space. A failure occurs when the outcome is in the sample space, but is not a favorable outcome.

29. **CHALLENGE** A multistage experiment has n possible outcomes at each stage. If the experiment is performed with k stages, write an equation for the total number of possible outcomes P. Explain.

SOLUTION:

If there are n outcomes for each stage, then there are:

$n \times n$ or n^2 outcomes for 2 stages,

$n \times n \times n$ or n^3 outcomes for 3 stages, and

$n \times n \times n ... \times n$ or n^k outcomes for k stages.

$P = n^k$; Sample answer: The total number of possible outcomes is the product of the number of outcomes for each of the stages 1 through k. Because there are k stages, you are multiplying n by itself k times which is n^k.

ANSWER:

$P = n^k$; Sample answer: The total number of possible outcomes is the product of the number of outcomes for each of the stages 1 through k. Because there are k stages, you are multiplying n by itself k times which is n^k.

30. **WRITING IN MATH** Explain when it is necessary to show all of the possible outcomes of an experiment by using a tree diagram and when using the Fundamental Counting Principle is sufficient.

SOLUTION:

Sample answer: Drawing a tree diagram is necessary if you want to show the sample space for an experiment or if you want to know the number of times a certain outcome occurs. One example would be listing the different options of pairs to be chosen from a group of people.

Using the Fundamental Counting Principle only tells you how many possible outcomes there are, so it is only useful when you want to know how many outcomes there are. One example would be to find out the probability of rolling a 1, 1, 1, 1, 1 when rolling 5 six-sided number cubes. Listing all of the different outcomes would be time-consuming and irrelevant to finding the probability.

ANSWER:

Sample answer: Drawing a tree diagram is necessary if you want to show the sample space for an experiment or if you want to know the number of times a certain outcome occurs. Using the Fundamental Counting Principle only tells you how many possible outcomes there are, so it is only useful when you want to know how many outcomes there are.

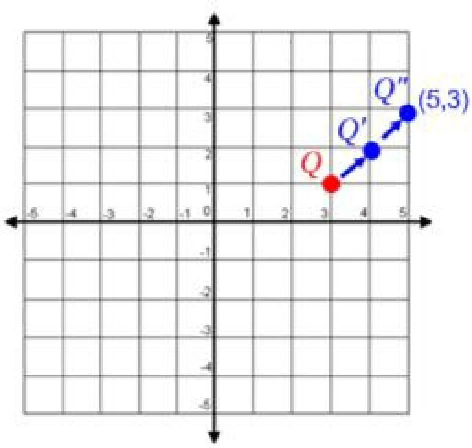
31. Nathaniel performs an experiment that involves tossing a coin two times. First he plots the point $Q(3, 1)$. Then he tosses a coin. If the coin lands on heads, he translates point Q along $\langle 1, 1\rangle$. If the coin lands on tails, he translates point Q along $\langle -1, -1\rangle$. Then he tosses the coin again and repeats the process on the image of point Q. Nathaniel notes the final image of the point. Which of the following is not in the sample space for this experiment?

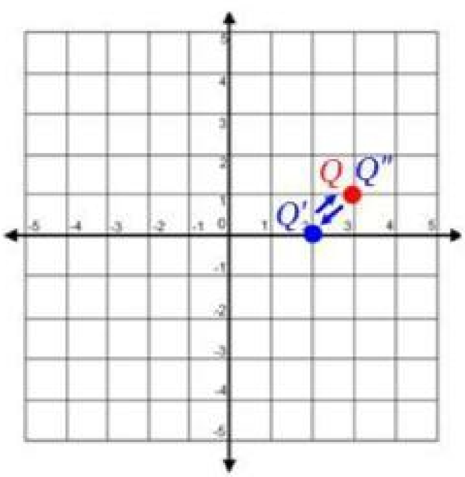
A (5, 3)
B (4, 2)
C (3, 1)
D (1, –1)

SOLUTION:
One way to solve this problem is to graph point Q and determine the possible outcomes of translating it and then its image in some combination of the vectors $\langle 1, 1\rangle$ or $\langle -1, -1\rangle$.

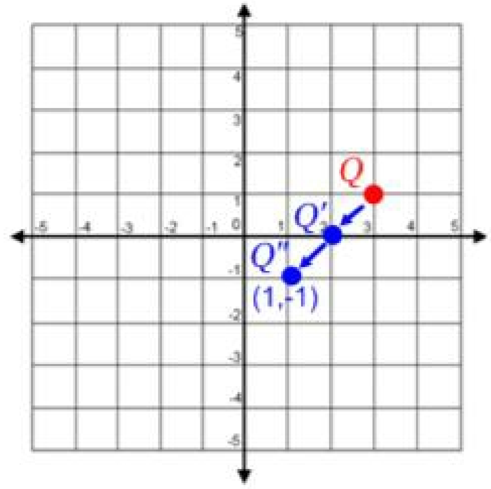
Option A: Start at (3, 1), then roll two heads.
Option C: Start at (3, 1), then roll a head and a tail

(Rolling a tail and a head would have the same final result)





Option D: Start at (3, 1), then roll two tails.



Therefore, (4, 2) is not in this sample space. So, the correct answer is choice B.

ANSWER:
B

32. Brad's password must be five digits long; he must use the numbers 0–9, and the digits must not repeat. What is the maximum number of different passwords that Brad can have?

SOLUTION:
Think about how many choices Brad has for each digit of his password:

The first digit has 10 choices, 0–9.
The second digit has one less because the digits cannot repeat. So, it has 9 choices.
The third digit has two less. So, it has 8 choices.
The fourth digit has three less. So, it has 7 choices.
And, the fifth digit has four less. So, it has 6 choices.

Therefore, the maximum number of different passwords is $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$ or 30,240.

ANSWER:
30,240

33. Becky performs an experiment that involves tossing a coin three times. First she plots the point $R(-4, 2)$. She tosses a coin and transforms point R according to the rules in the table. Then she tosses the coin again and transforms the image of point R. Then she tosses the coin a third time and transforms the most recent image. Becky notes the final image of the point. Which of the following is not a possible outcome for this experiment?

Result of Toss	Transformation
Heads	$(x, y) \rightarrow (-y, x)$
Tails	$(x, y) \rightarrow (x + 2, y)$

A (2, 2)
B (–2, 0)
C (4, 0)
D (–2, 2)
E (0, –2)

SOLUTION:
One way to think about this problem is to create a table and consider all the possibilities that can happen with three tosses of a coin. The point (–2, 2) is not one of the possibilities. So, the correct answer is choice D.

ANSWER:
D

34. Amani plots the point $P(2, -1)$. Then she tosses a coin. If the coin lands on heads, she reflects point P in the x-axis. If the coin lands on tails, she reflects point P in the y-axis. Then she tosses the coin again and repeats the process on the image of point P. How many different final images are possible?

A 1
B 2
C 3
D 4

SOLUTION:
One way to think about this problem is to sketch a graph and plot point P, and then perform the experiment to determine how many different final images are possible. In doing so, it can be determined that there are only 2 different final images possible. So, the correct answer is choice B.

ANSWER:
B

35. There are 3 trails leading to camp A from your starting position. There are 3 trails from camp A to camp B. How many different routes are there from the starting position to camp B? Draw a tree diagram to illustrate your answer.

SOLUTION:
Using the Fundamental Counting Principal, $3 \times 3 = 9$. So, there are 9 different routes.

This can also be shown using a tree diagram, where the first level shows the paths to Camp A and the second level shows the paths to Camp B.

ANSWER:
9

36. Bag A contains 10 marbles of which 2 are red and 8 are black. Bag B contains 12 marbles of which 4 are red and 8 are black. A coin is tossed, and if it comes up heads, a marble is drawn from bag A and its color is noted. If it comes up tails, a marble is drawn from bag B and its color is noted. Find the sample space for this experiment.

SOLUTION:

From the given information, we know that when the coin comes up heads then a marble is drawn from bag A, which contains red and black marbles. So, we can have (heads, red) and (heads, black). We also know that when the coin comes up tails then a marble is drawn from bag B, which contains red and black marbles. So, we can have (tails, red) and (tails, black).

Therefore, the sample space is {(heads, red), (heads, black), (tails, red), (tails, black)}.

ANSWER:

{(heads, red), (heads, black), (tails, red), (tails, black)}

37. A bag contains four apples and six bananas. A fruit is taken from the bag and eaten. Then another fruit is taken and eaten. This process continues.

a. Find the sample space for the following experiments.

i. one fruit is eaten.
ii. two fruits are eaten.
iii. three fruits are eaten.

b. For $n \leq 4$, write an expression for the number of elements in the sample space when n fruits are eaten.

SOLUTION:

a.

i. Since the bag contains apples and bananas, and only one fruit is eaten the sample space is (apple, banana).

ii. When two fruits are eaten, the possible outcomes are (apple, banana), (banana, apple), (apple, apple), (banana, banana), so the sample space is {(apple, banana), (banana, apple), (apple, apple), (banana, banana)}.

iii. When tree fruits are eaten, the possible outcomes are (apple, banana, apple), (apple, banana, banana), (banana, apple, apple), (banana, apple, banana), (apple, apple, apple), (apple, apple, banana), (banana, banana, banana), (banana, banana, apple), so the sample space is {(apple, banana, apple), (apple, banana, banana), (banana, apple, apple), (banana, apple, banana), (apple, apple, apple), (apple, apple, banana), (banana, banana, banana), (banana, banana, apple)}.

b. The expression would be 2^n.

ANSWER:

a.

i. (apple, banana)
ii. {(apple, banana), (banana, apple), (apple, apple), (banana, banana)}
iii. {(apple, banana, apple), (apple, banana, banana), (banana, apple, apple), (banana, apple, banana), (apple, apple, apple), (apple, apple, banana), (banana, banana, banana), (banana, banana, apple)}

b. 2^n