GRADE 10 ADVANCED

EOT COVERAGE
TERM 2 2022-23
ANSWERS

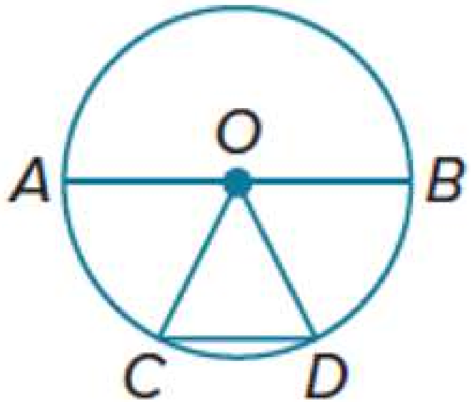
Marks per Main Question الدرجات لكل سؤال أساسي	Part (1) - 3
	Part (2) - 5
	Part (3) - (4 ~ 8)
****Number of Bonus Questions عدد الأسئلة الإضافية	2
Marks per Bonus Question الدرجات لكل سؤال إضافي	5
*** Type of All Questions نوع كافة الأسئلة	Part(1 and 2) MCQ Part (3) FRQ
* Maximum Overall Grade *الدرجة القصوى الممكنة	110
Exam Duration - مدة الامتحان	120 minutes

1	Know the precise definition of circle and find the circumferences of circles.	1 to 14	227

5-1 Circles and Circumference

PART #1

For Exercises 1-3, refer to the circle at the right.

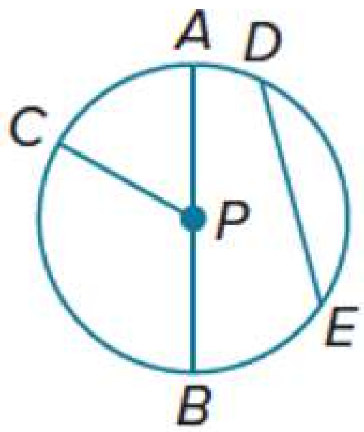


1. Name the circle.

2. Name the radii of the circle.

3. Name the chords of the circle.

For Exercises 4-8, refer to the circle at the right.



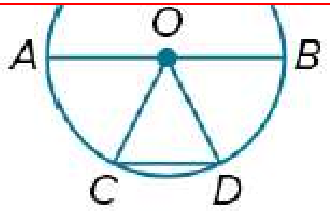
4. Name the circle.

5. Name the radii of the circle.

6. Name the chords of the circle.

7. Name a diameter of the circle.

8. Name a radius not drawn as part of a diameter.



1. Name the circle.

SOLUTION:

The circle has a center at O, so it is named circle O or $\odot O$.

ANSWER:
$\odot O$

2. Name the radii of the circle.

SOLUTION:

The radii, or line segments from the center to a point on circle O are $\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$.

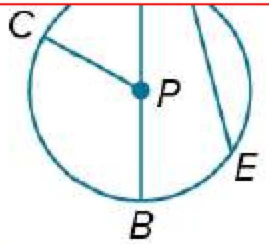
ANSWER:
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$

3. Name the chords of the circle.

SOLUTION:

The chords, or segments with endpoints on circle O are $\overline{AB}$ and $\overline{CD}$.

ANSWER:
$\overline{AB}$ and $\overline{CD}$



4. Name the circle.

SOLUTION:

The circle has a center at P, so it is named circle P or $\odot P$.

ANSWER:
$\odot P$

5. Name the radii of the circle.

SOLUTION:

The radii, or line segments from the center to a point on circle P are $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$.

ANSWER:
$\overline{PA}$, $\overline{PB}$, and $\overline{PC}$

6. Name the chords of the circle.

SOLUTION:

The chords, or segments with endpoints on circle P are $\overline{AB}$ and $\overline{DE}$.

ANSWER:
$\overline{AB}$ and $\overline{DE}$

7. Name a diameter of the circle.

SOLUTION:

A diameter is a chord that passes through the center of a circle. The diameter of circle P is $\overline{AB}$.

ANSWER:
$\overline{AB}$

8. Name a radius not drawn as part of a diameter.

SOLUTION:

Since radii $\overline{PA}$ and $\overline{PB}$ are part of diameter $\overline{AB}$, the radius that is not part of a diameter is $\overline{PC}$.

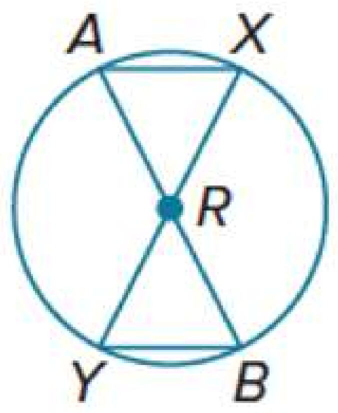
ANSWER:
$\overline{PC}$

1	Know the precise definition of circle and find the circumferences of circles.	1 to 14	227

5-1 Circles and Circumference

PART #1

For Exercises 9-11, refer to ⊙*R*.

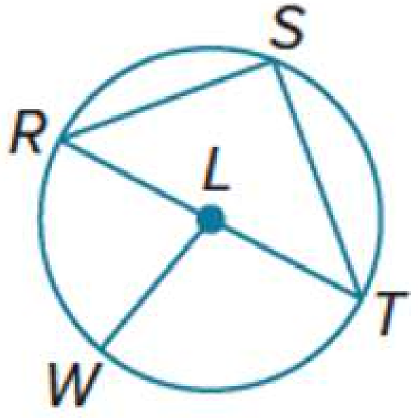


9. If AB = 18 millimeters, find AR.

10. If RY = 10 inches, find AR and AB.

11. Is $\overline{AB} \cong \overline{XY}$? Explain.

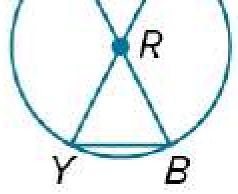
For Exercises 12-14, refer to ⊙*L*.



12. Suppose the radius of the circle is 3.5 yards. Find the diameter.

13. If RT = 19 meters, find LW.

14. If LT = 4.2 inches, what is the diameter of ⊙L?



9. If AB = 18 millimeters, find AR.

SOLUTION:

$r = \frac{d}{2}$ Radius Formul

$r = \frac{18}{2}$ or 9 Substitute and

The radius of circle R is 9 mm.

ANSWER:
9 mm

10. If RY = 10 inches, find AR and AB.

SOLUTION:

RY and AR both represent the measures of two radii of circle R, therefore they have equal measures. Since RY = 10 in., then AR = 10 in.

AB represents the measure of the diameter of circle R. Use the radius and diameter relationships to find AB.

$r = \frac{d}{2}$ Radius Formula

$10 = \frac{d}{2}$ Substitute.

$20 = d$ Simplify.

The measure of the diameter, AB = 20 in.

ANSWER:
AR = 10 in.; AB = 20 in.

11. Is $\overline{AB} \cong \overline{XY}$? Explain.

SOLUTION:

$\overline{AB} \cong \overline{XY}$ because all diameters of the same circle are congruent.

ANSWER:
Yes; all diameters of the same circle are congruent.

12. Suppose the radius of the circle is 3.5 yards. Find the diameter.

SOLUTION:

$r = \frac{d}{2}$ Radius Formula

$3.5 = \frac{d}{2}$ Substitute.

$7 = d$ Simplify.

The measure of the diameter is 7 yards.

ANSWER:
7 yd

13. If RT = 19 meters, find LW.

SOLUTION:

RT is the measure of the diameter of circle L, and LW is the measure of its radius.

$r = \frac{d}{2}$ Radius Formula

$r = \frac{19}{2}$ or 9.5 Substitute and simplify.

The radius of circle L is 9.5 m.

ANSWER:
9.5 m

14. If LT = 4.2 inches, what is the diameter of $\odot L$?

SOLUTION:

LT is the measure of the radius circle L.

$r = \frac{d}{2}$ Radius Formula

$4.2 = \frac{d}{2}$ Substitute.

$8.4 = d$ Simplify.

The measure of the diameter of circle L is 8.4 inches.

ANSWER:
8.4 inches

2	Identify relationships in inscribed polygons, and use those relationships to solve problems.	13 to 24	252

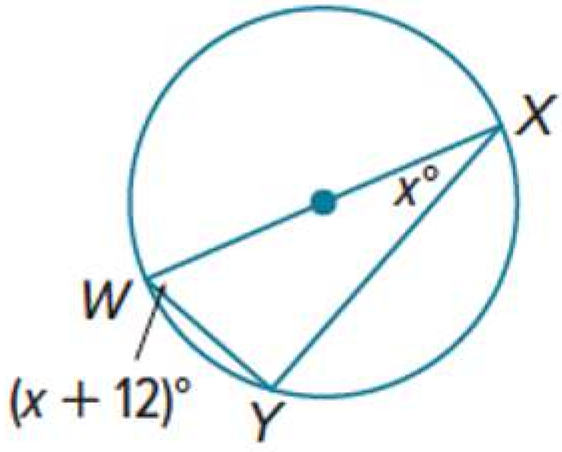
5-4 Inscribed Angles

PART #1

Find each value.

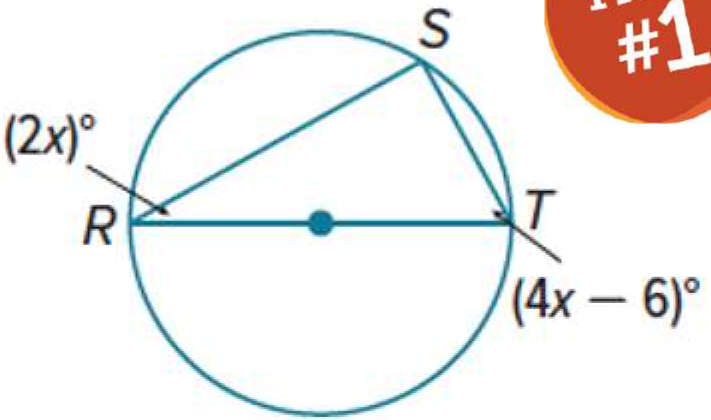
13. x

14. $m\angle W$



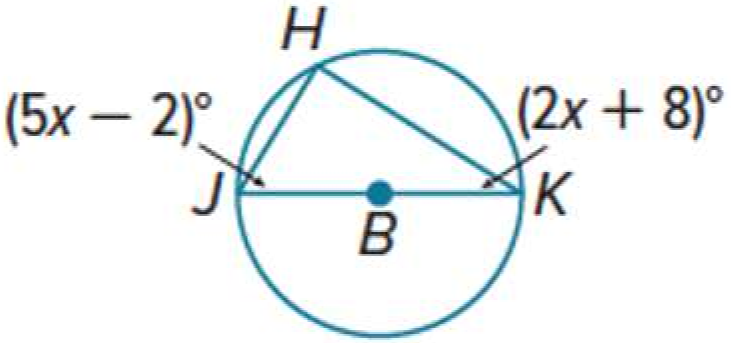
15. x

16. $m\angle T$



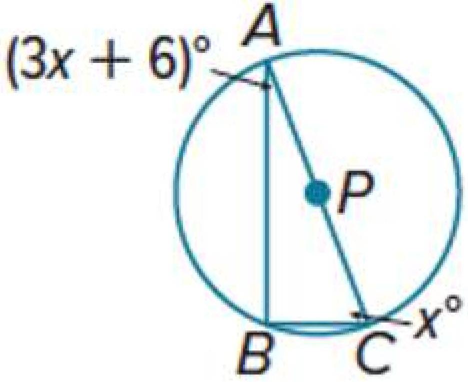
17. $m\angle J$

18. $m\angle K$

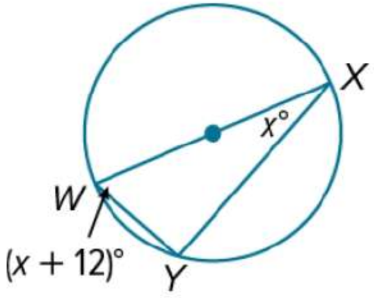


19. $m\angle A$

20. $m\angle C$



13. x

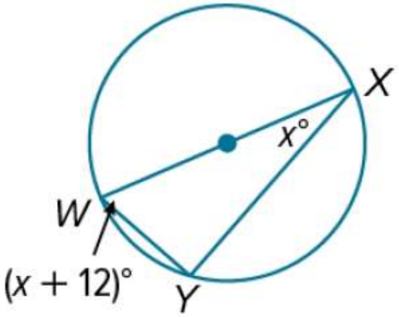


SOLUTION:

$\triangle WXY$ is a right triangle because $\angle Y$ inscribes a semicircle.

$m\angle W + m\angle X = 90$ Acute∠s of a right △ are complen
$x + 12 + x = 90$ Substitution
$2x + 12 = 90$ Simplify.
$2x = 78$ Subtract 12 from each side.
$x = 39$ Divide each side by 2.

14. $m\angle W$



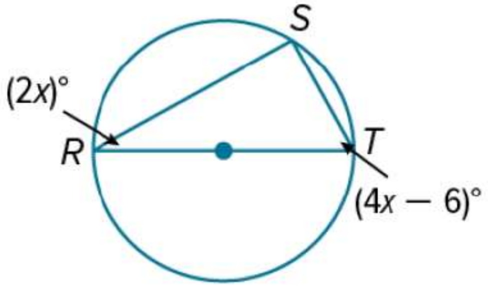
SOLUTION:

$\triangle WXY$ is a right triangle because $\angle Y$ inscribes a semicircle.

$m\angle W + m\angle X = 90$ Acute∠s of a right △ are complem
$x + 12 + x = 90$ Substitution
$2x + 12 = 90$ Simplify.
$2x = 78$ Subtract 12 from each side.
$x = 39$ Divide each side by 2.

So, $m\angle W = 39 + 12$ or $51°$.

15. x

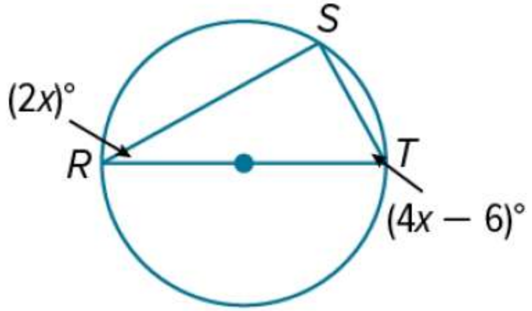


SOLUTION:

$\triangle RST$ is a right triangle because $\angle S$ inscribes a semicircle.

$m\angle R + m\angle T = 90$ Acute∠s of a right △ are comple
$2x + 4x - 6 = 90$ Substitution
$6x + 6 = 90$ Simplify.
$6x = 96$ Subtract 6 from each side.
$x = 16$ Divide each side by 6.

16. $m\angle T$



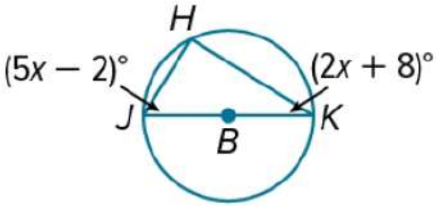
SOLUTION:

$\triangle RST$ is a right triangle because $\angle S$ inscribes a semicircle.

$m\angle R + m\angle T = 90$ Acute∠s of a right △ are comple
$2x + 4x - 6 = 90$ Substitution
$6x - 6 = 90$ Simplify.
$6x = 96$ Add 6 to each side.
$x = 16$ Divide each side by 6.

So, $m\angle T = 4(16) - 6$ or $58°$.

17. $m\angle J$



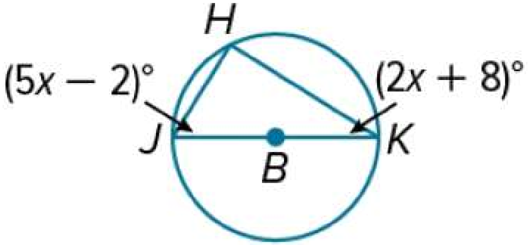
SOLUTION:

$\triangle JHK$ is a right triangle because $\angle H$ inscribes a semicircle.

$m\angle J + m\angle K = 90$ Acute∠s of a right △ are complen
$5x - 2 + 2x + 8 = 90$ Substitution
$7x + 6 = 90$ Simplify.
$7x = 84$ Subtract 6 from each side.
$x = 12$ Divide each side by 7.

So, $m\angle J = 5(12) - 2$ or $58°$.

18. $m\angle K$



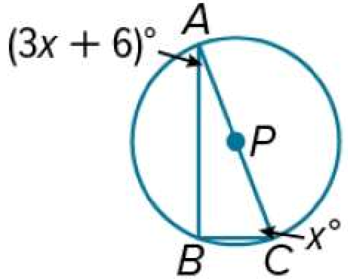
SOLUTION:

$\triangle JHK$ is a right triangle because $\angle H$ inscribes a semicircle.

$m\angle J + m\angle K = 90$ Acute∠s of a right △ are complem
$5x - 2 + 2x + 8 = 90$ Substitution
$7x + 6 = 90$ Simplify.
$7x = 84$ Subtract 6 from each side.
$x = 12$ Divide each side by 7.

So, $m\angle K = 2(12) + 8$ or $32°$.

19. $m\angle A$



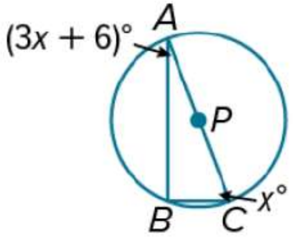
SOLUTION:

$\triangle ABC$ is a right triangle because $\angle B$ inscribes a semicircle.

$m\angle A + m\angle C = 90$ Acute∠s of a right △ are comple
$3x + 6 + x = 90$ Substitution
$4x + 6 = 90$ Simplify.
$4x = 84$ Subtract 6 from each side.
$x = 21$ Divide each side by 4.

So, $m\angle A = 3(21) + 6$ or $69°$.

20. $m\angle C$



SOLUTION:

$\triangle ABC$ is a right triangle because $\angle B$ inscribes a semicircle.

$m\angle A + m\angle C = 90$ Acute∠s of a right △ are complem
$3x + 6 + x = 90$ Substitution
$4x + 6 = 90$ Simplify.
$4x = 84$ Subtract 6 from each side.
$x = 21$ Divide each side by 4.

So, $m\angle C = 21°$.

2	Identify relationships in inscribed polygons and use those relationships to solve problems.	13 to 24	252

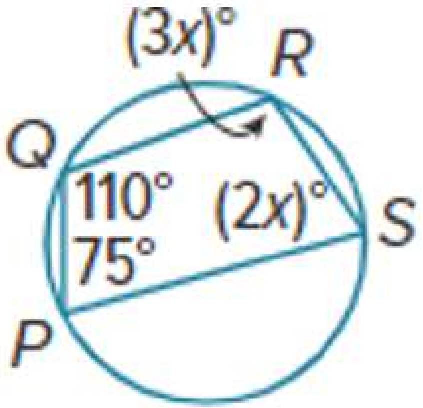
5-4 Inscribed Angles

PART #1

Find each measure.

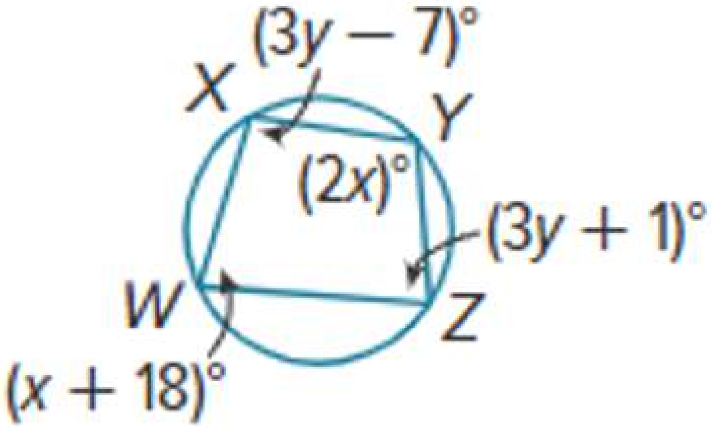
21. $m\angle R$

22. $m\angle S$

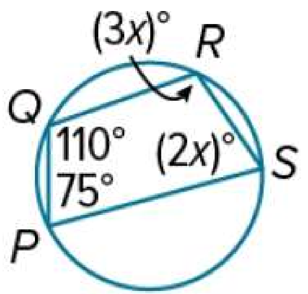


23. $m\angle W$

24. $m\angle X$



21. $m\angle R$



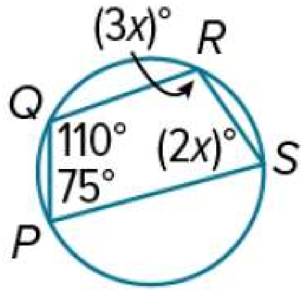
SOLUTION:

Because *PQRS* is inscribed in a circle, its opposite angles are supplementary.

$m\angle P + m\angle R = 180$	Definition of supplementary.
$3x + 75 = 180$	Substitution
$3x = 105$	Su
$x = 35$	Di

So, $m\angle R = 3(35)$ or 105°.

22. $m\angle S$



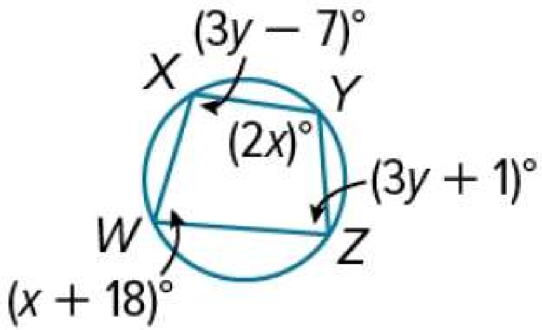
SOLUTION:

Because *PQRS* is inscribed in a circle, its opposite angles are supplementary.

$m\angle Q + m\angle S = 180$	Definition of supplementary.
$110 + 2x = 180$	Substitution
$2x = 70$	Subtract 110 from each side.
$x = 35$	Divide each side by 2.

So, $m\angle S = 2(35)$ or 70°.

23. $m\angle W$



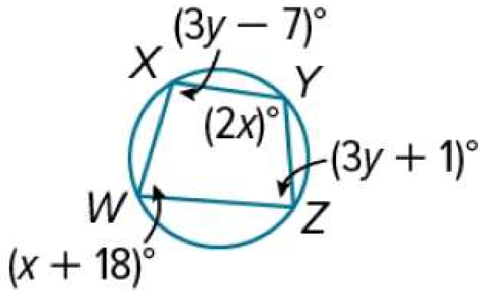
SOLUTION:

Because *WXYZ* is inscribed in a circle, its opposite angles are supplementary.

$m\angle W + m\angle Y = 180$	Definition of supplementary.
$x + 18 + 2x = 180$	Substitution
$3x = 162$	Subtract 18 fro
$x = 54$	Divide each sid

So, $m\angle W = 54 + 18$ or 72°.

24. $m\angle X$



SOLUTION:

Because *WXYZ* is inscribed in a circle, its opposite angles are supplementary.

$m\angle X + m\angle Z = 180$	Definition of supplementary.
$3y - 7 + 3y + 1 = 180$	Substitution
$6y - 6 = 180$	Simplify.
$6y = 186$	Add 6 to each side.
$y = 31$	Divide each side by 6.

So, $m\angle X = 3(31) - 7$ or 86°.

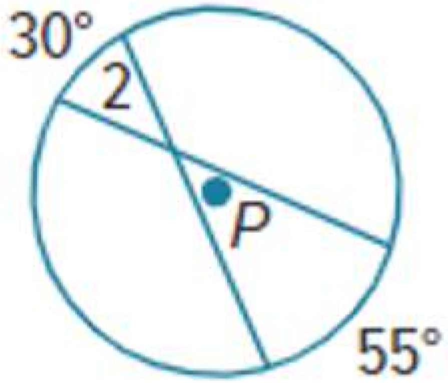
3	Use relationships between tangents and secants to solve problems.	1 to 12	265

5-6 Tangents, Secants, and Angle Measures

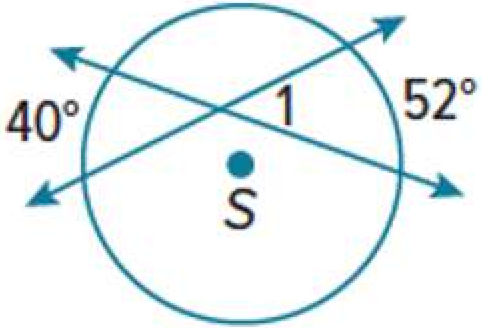
PART #1

Find each measure.

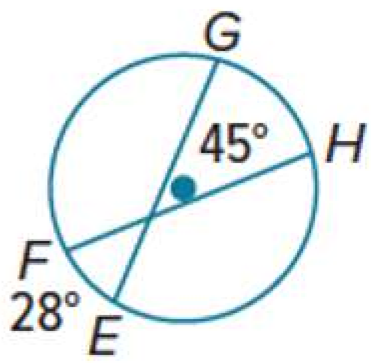
1. $m\angle 2$



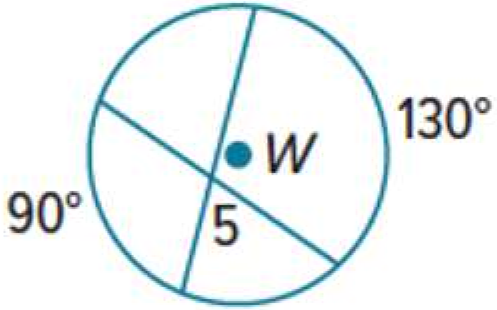
2. $m\angle 1$



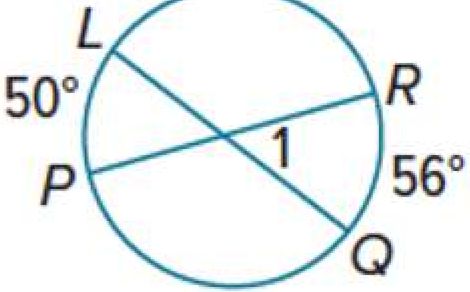
3. $m\widehat{GH}$



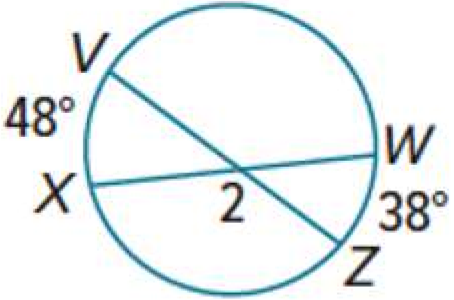
4. $m\angle 5$



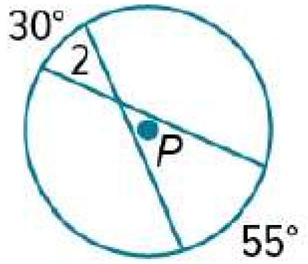
5. $m\angle 1$



6. $m\angle 2$



1. $m\angle 2$

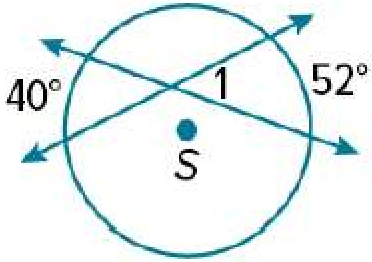


SOLUTION:

$m\angle 2 = \frac{1}{2}(30° + 55°)$ Theorem 10.14

$= \frac{1}{2}(85)$ or 42.5° Simplify.

2. $m\angle 1$

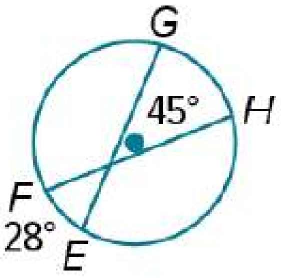


SOLUTION:

$m\angle 1 = \frac{1}{2}(50° + 42°)$ Theorem 10.14

$= \frac{1}{2}(92)$ or 46° Simplify.

3. $m\widehat{GH}$



SOLUTION:

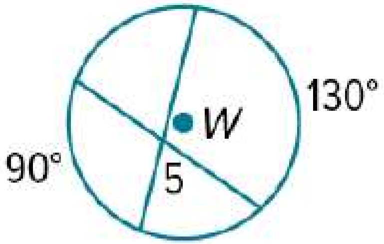
$45° = \frac{1}{2}(28° + m\widehat{GH})$ Theorem 10.14

$45° = 14° + \frac{1}{2}(m\widehat{GH})$ Distributive Property

$31° = \frac{1}{2}(m\widehat{GH})$ Subtract 14 from each side.

$62° = m\widehat{GH}$

4. $m\angle 5$



SOLUTION:

$m\angle W = \frac{1}{2}(90° + 130°)$ Theorem 10.14

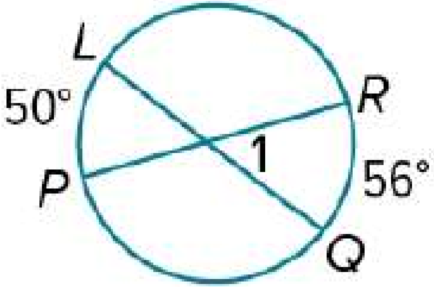
$= \frac{1}{2}(220)$ or 110° Simplify.

$m\angle 5 = 180° - m\angle W$ Definition of linear pair

$= 180° - 110°$ Substitution

$= 70°$ Simplify.

5. $m\angle 1$

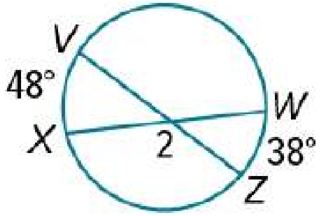


SOLUTION:

$m\angle 1 = \frac{1}{2}(50° + 56°)$ Theorem 10.14

$= \frac{1}{2}(106)$ or 53° Simplify.

6. $m\angle 2$



SOLUTION:

The measure of either angle that forms a linear pair with $\angle 2 = \frac{1}{2}(48 + 38)°$ Theorem 10.14

$= \frac{1}{2}(86)$ or 43° Simplify.

$m\angle 2 = 180° - 43°$ Definition of linear pair

$= 137°$ Simplify.

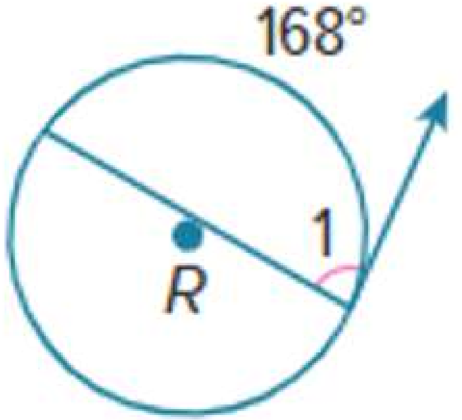
3	Use relationships between tangents and secants to solve problems.	1 to 12	265

5-6 Tangents, Secants, and Angle Measures

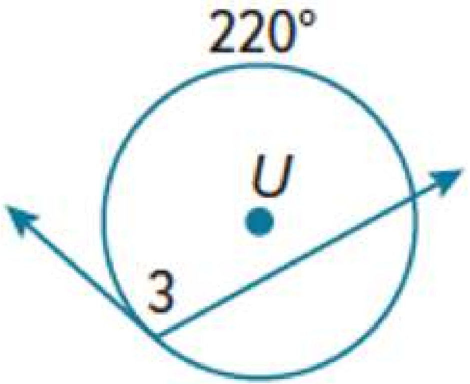
PART #1

Find each measure. Assume that segments that appear to be tangent are tangent.

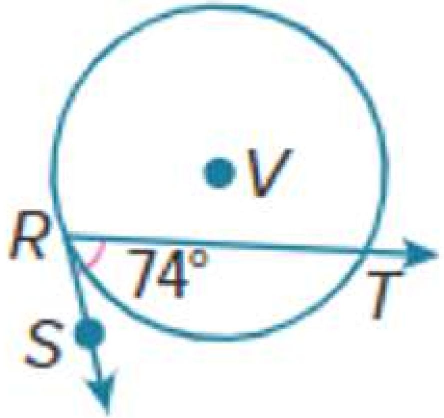
7. $m\angle 1$



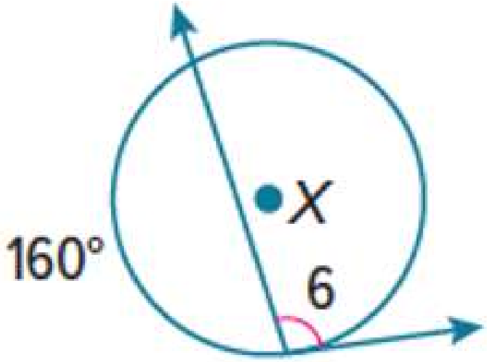
8. $m\angle 3$



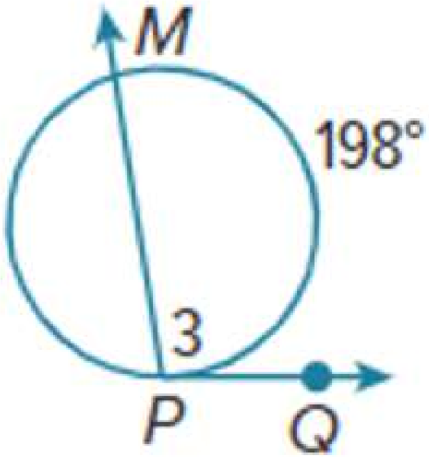
9. $m\widehat{RT}$



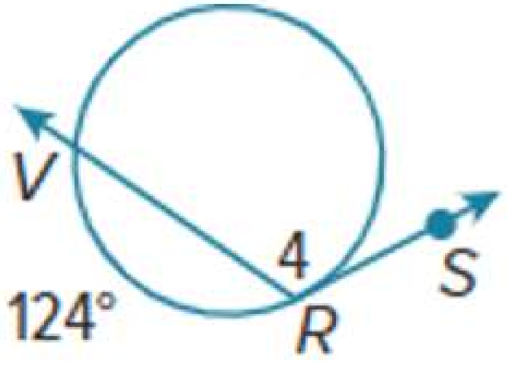
10. $m\angle 6$



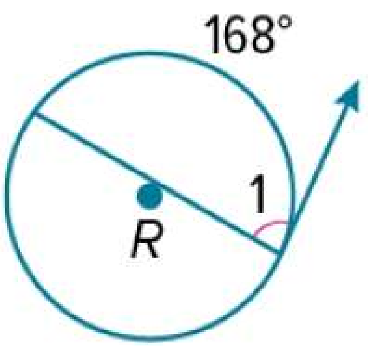
11. $m\angle 3$



12. $m\angle 4$



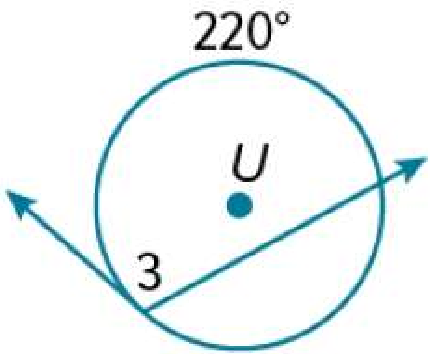
7. $m\angle 1$



SOLUTION:

$m\angle 1 = \frac{1}{2}(168°)$ or $84°$ Theorem 10.15

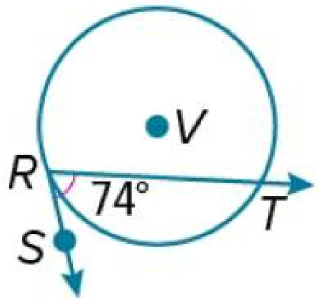
8. $m\angle 3$



SOLUTION:

$m\angle 3 = \frac{1}{2}(220°)$ or $110°$ Theorem 10.15

9. $m\widehat{RT}$



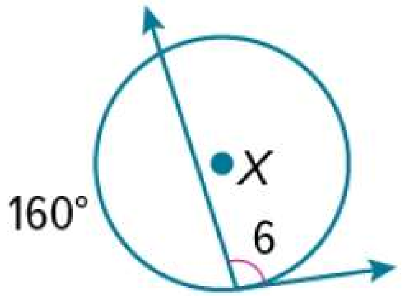
SOLUTION:

$m\angle SRT = \frac{1}{2}m\widehat{RT}$ Theorem 10.15

$74° = \frac{1}{2}m\widehat{RT}$ Substitution

$148° = m\widehat{RT}$ Solve.

10. $m\angle 6$

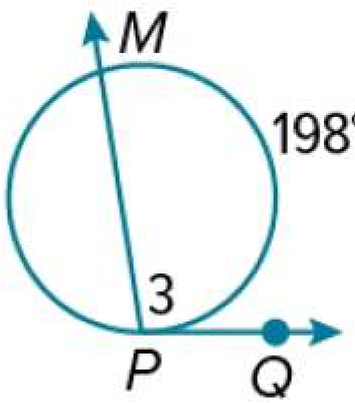


SOLUTION:

$m\angle 6 = \frac{1}{2}(360 - 160)°$ Theorem 10.15

$= \frac{1}{2}(200)$ or $100°$ Solve.

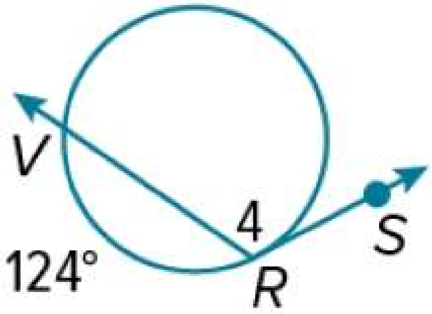
11. $m\angle 3$



SOLUTION:

$m\angle 3 = \frac{1}{2}(198°)$ or $99°$ Theorem 10.15

12. $m\angle 4$



SOLUTION:

$m\angle 4 = \frac{1}{2}(360 - 124)°$ Theorem 10.15

$= \frac{1}{2}(236)$ or $118°$ Solve.

4	Apply the Fundamental Counting Principle to define sample spaces	11 to 15	370

7-1 Sample Spaces

PART #1

Find the number of possible outcomes for each situation.

11. A video game lets you decorate a bedroom using one choice from each category.

Bedroom Décor	Number of Choices
Paint color	8
Comforter set	6
Sheet set	8
Throw rug	5
Lamp	3
Wall hanging	5

12. A cafeteria meal at Angela's work includes one choice from each category.

Cafeteria Meal	Number of Choices
Main dish	3
Side dish	4
Vegetable	2
Salad	2
Salad Dressing	3
Dessert	2
Drink	3

13. **SHOPPING** On a website showcasing outdoor patio plans, there are 4 types of stone, 3 types of edging, 5 dining sets, and 6 grills. Kamar plans to order one item from each category. How many different patio sets can Kamar order?

11. A video game lets you decorate a bedroom using one choice from each category

Bedroom Décor	Number of Choices
Paint color	8
Comforter set	6
Sheet set	8
Throw rug	5
Lamp	3
Wall hanging	5

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$8 \times 6 \times 8 \times 5 \times 3 \times 5 = 28,800$

There are 28,800 ways to decorate the bedroom.

12. A cafeteria meal at Angela's work includes one choice from each category

Cafeteria Meal	Number of Choices
Main dish	3
Side dish	4
Vegetable	2
Salad	2
Salad Dressing	3
Dessert	2
Drink	3

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$3 \times 4 \times 2 \times 2 \times 3 \times 2 \times 3 = 864$

There are 864 meal combinations.

13. **SHOPPING** On a website showcasing outdoor patio plans, there are 4 types of stone, 3 types of edging, 5 dining sets, and 6 grills. Kamar plans to order one item from each category. How many different patio sets can Kamar order?

SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$4 \times 3 \times 5 \times 6 = 360$

There are 360 different patio sets Kamar can order.

4	Apply the Fundamental Counting Principle to define sample spaces	11 to 15	370

7-1 Sample Spaces

PART #1

14. **AUDITIONS** The drama club held tryouts for 6 roles in a one-act play. Five people auditioned for lead female, 3 for lead male, 8 for the best friend, 4 for the mother, 2 for the father, and 3 for the humorous aunt. How many different casts can be created from those who auditioned?

Mixed Exercises

15. **BOARD GAMES** The spinner shown is used in a board game. If the spinner is spun 4 times, how many different possible outcomes are there?

14. **AUDITIONS** The drama club held tryouts for 6 roles in a one-act play. Five people auditioned for lead female, 3 for lead male, 8 for the best friend, 4 for the mother, 2 for the father, and 3 for the humorous aunt. How many different casts can be created from those who auditioned?

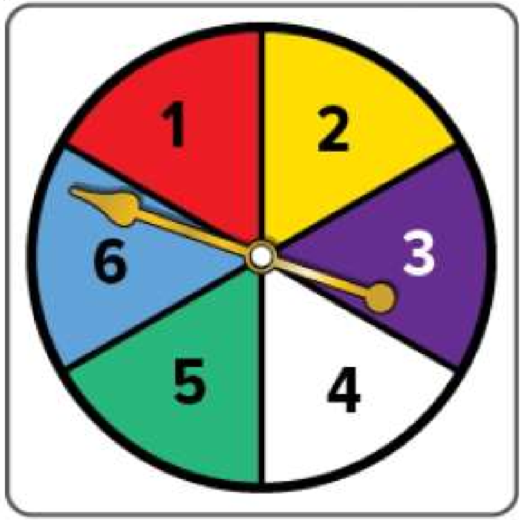
SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$5 \times 3 \times 8 \times 4 \times 2 \times 3 = 2880$

There are 2880 different casts that can be created.

15. **BOARD GAMES** The spinner shown is used in a board game. If the spinner is spun 4 times, how many different possible outcomes are there?



SOLUTION:

Find the number of possible outcomes by using the Fundamental Counting Principle.

$6 \times 6 \times 6 \times 6 = 1296$

There are 1296 different possible outcomes.

5	Describe events as subsets of sample spaces by using complements	11 to 18	378, 379

7-2 Probability and Counting

PART #1

Determine the probability of each event. Round to the nearest hundredth, if necessary.

11. What is the probability of drawing a card from a standard deck and not getting a spade?

12. What is the probability of flipping a coin and not landing on tails?

13. Carmela purchased 10 raffle tickets. If 250 were sold, what is the probability that one of Carmela's tickets will not be drawn?

14. What is the probability of spinning a spinner numbered 1 to 6 and not landing on 5?

11. What is the probability of drawing a card from a standard deck and not getting a spade?

SOLUTION:

Let A be the event of choosing a card that is a spade. Then find the probability of the complement of A.

There are 13 cards that are spades.

There are 52 cards in a deck.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{13}{52} && \text{Substitution} \\ &= \frac{39}{52} \text{ or } \frac{3}{4} && \text{Subtract and simplify.} \end{aligned}$$

The probability of not getting a spade is $\frac{39}{52}$ or $\frac{3}{4}$ or 0.75.

12. What is the probability of flipping a coin and not landing on tails?

SOLUTION:

Let A be the event of a coin landing on tails. Then find the probability of the complement of A.

There is 1 side of a coin that is a tail.

There are 2 sides of a coin.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{1}{2} && \text{Substitution} \\ &= \frac{1}{2} && \text{Subtract and simplify.} \end{aligned}$$

The probability of flipping a coin and not landing on tails is $\frac{1}{2}$ or 0.5.

13. Carmela purchased 10 raffle tickets. If 250 were sold, what is the probability that one of Carmela's tickets will not be drawn?

SOLUTION:

Let A be the event of one of Carmela's tickets being drawn. Then find the probability of the complement of A.

There are 10 raffle tickets that are Carmela's.

There were 250 raffle tickets sold.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{10}{250} && \text{Substitution} \\ &= \frac{240}{250} && \text{Subtract and simplify.} \end{aligned}$$

The probability that one of Carmela's tickets will not be drawn is $\frac{240}{250}$ or 0.96.

14. What is the probability of spinning a spinner numbered 1 to 6 and not landing on 5?

SOLUTION:

Let A be the event of spinning a spinner numbered 1 to 6 and landing on 5. Then find the probability of the complement of A.

There is 1 section on the spinner labeled 5.

There are 6 sections on the spinner.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{1}{6} && \text{Substitution} \\ &= \frac{5}{6} && \text{Subtract and simplify.} \end{aligned}$$

The probability spinning a spinner numbered 1 to 6 and not landing on 5 is $\frac{5}{6}$ or about 0.83.

5	Describe events as subsets of sample spaces by using complements	11 to 18	378, 379

7-2 Probability and Counting

PART #1

15. **STATISTICS** A survey found that about 90% of the junior class is right-handed. If 1 junior is chosen at random out of 100 juniors, what is the probability that he or she is left-handed?

16. **RAFFLE** Raul bought 24 raffle tickets out of 1545 tickets sold. What is the probability that Raul will not win the grand prize of the raffle?

17. **MASCOT** At Riverview High School, 120 students were asked whether they prefer a lion or a timber wolf as the new school mascot. What is the probability that a randomly-selected student will have voted for a lion as the new school mascot?

	Votes
Lion	78
Timber Wolf	42
Total	120

18. **COLLEGE** In Evan's senior class of 240 students, 85% are planning to attend college after graduation. What is the probability that a senior chosen at random is not planning to attend college after graduation?

15. **STATISTICS** A survey found that about 90% of the junior class is right-handed. If 1 junior is chosen at random out of 100 juniors, what is the probability that he or she is left-handed?

SOLUTION:

Let A be the event of choosing a right-handed junior. Then find the probability of the complement of A.

There are 90 juniors that are right-handed because 90% of 100 is $0.9 \times 100 = 90$.

There are 100 juniors.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{90}{100} && \text{Substitution} \\ &= \frac{10}{100} \text{ or } \frac{1}{10} && \text{Subtract and simplify.} \end{aligned}$$

The probability of choosing a junior that is not right-handed, or that is left-handed, is $\frac{10}{100}$ or $\frac{1}{10}$ or 0.10.

16. **RAFFLE** Raul bought 24 raffle tickets out of 1545 tickets sold. What is the probability that Raul will not win the grand prize of the raffle?

SOLUTION:

Let A be the event of one of Raul will win the grand prize of the raffle. Then find the probability of the complement of A.

Raul bought 24 raffle tickets.

There were 1545 raffle tickets sold.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{24}{1545} && \text{Substitution} \\ &= \frac{1521}{1545} \text{ or } \frac{507}{515} && \text{Subtract and simplify.} \end{aligned}$$

The probability that Raul will not win the grand prize of the raffle is $\frac{507}{515}$ or 0.98.

17. **MASCOT** At Riverview High School, 120 students were asked whether they prefer a lion or a timber wolf as the new school mascot. What is the probability that a randomly selected student will have voted for a lion as the new school mascot?

	Votes
Lion	78
Timber Wolf	42
Total	120

SOLUTION:

Let A be the event of a student preferring a lion. The total number of outcomes is the total number of votes, or 120.

There were 78 students that prefer a lion.

$$\begin{aligned} P(A \cap B) &= \frac{\text{number of outcomes in } A}{\text{total number of possible outcomes}} && \text{Probability Rule} \\ &= \frac{78}{120} && \text{Substitution} \\ &= \frac{13}{20} && \text{Simplify.} \end{aligned}$$

18. **COLLEGE** In Evan's senior class of 240 students, 85% are planning to attend college after graduation. What is the probability that a senior chosen at random is not planning to attend college after graduation?

SOLUTION:

Let A be the event of a senior planning to attend college after graduation. Then find the probability of the complement of A.

There are 204 seniors planning to attend college after graduation because 85% of 240 is $0.85 \times 240 = 204$.

There are 240 seniors.

The probability of the complement of A is $P(A') = 1 - P(A)$.

$$\begin{aligned} P(A') &= 1 - P(A) && \text{Probability of a complement} \\ &= 1 - \frac{204}{240} && \text{Substitution} \\ &= \frac{36}{240} \text{ or } \frac{3}{20} && \text{Subtract and simplify.} \end{aligned}$$

The probability that a senior chosen at random is not planning to attend college after graduation is $\frac{36}{240} = \frac{3}{20}$ or 0.15.

6	Solve systems of linear equations by graphing	1 to 12	533

9-4 Solving Systems of Equations Graphically

PART #1

Example 1

Determine the number of solutions for each system. Then state whether the system of equations is *consistent* or *inconsistent* and whether it is *independent* or *dependent*.

1. $y = 3x$
$y = -3x + 2$

2. $y = x - 5$
$-2x + 2y = -10$

3. $2x - 5y = 10$
$3x + y = 15$

4. $3x + y = -2$
$6x + 2y = 10$

5. $x + 2y = 5$
$3x - 15 = -6y$

6. $3x - y = 2$
$x + y = 6$

Examples 2 and 3

Solve the system of equations by graphing.

7. $x - 2y = 0$
$y = 2x - 3$

8. $-4x + 6y = -2$
$2x - 3y = 1$

9. $2x + y = 3$
$y = \frac{1}{2}x - \frac{9}{2}$

10. $y - x = 3$
$y = 1$

11. $2x - 3y = 0$
$4x - 6y = 3$

12. $5x - y = 4$
$-2x + 6y = 4$

1. $y = 3x$
$y = -3x + 2$

SOLUTION:

Both equations are already solved for y. The lines have opposite slopes so there is exactly one solution where the lines intersect making the lines consistent and independent.

2. $y = x - 5$
$-2x + 2y = -10$

SOLUTION:

Solve the equations for y.

$y = x - 5$

$-2x + 2y = -10$	Original equation
$2y = 2x - 10$	Add $2x$ to each side.
$y = x - 5$	Divide each side by 2.

The lines have the same slope and y-intercept. Thus, both equations represent the same line and the system has infinitely many solutions. The system is consistent and dependent.

3. $2x - 5y = 10$
$3x + y = 15$

SOLUTION:

Solve the equations for y.

$2x - 5y = 10$	Original equation
$-5y = -2x + 10$	Subtract $2x$ from each side.
$y = \frac{2}{5}x - 2$	Divide each side by -5.

$3x + y = 15$	Original equation
$y = -3x + 15$	Subtract $3x$ from each side.

The lines have different slopes. Thus, the equations intersect in exactly one point and the system has one solution. The system is consistent and independent.

4. $3x + y = -2$
$6x + 2y = 10$

SOLUTION:

Solve the equations for y.

$3x + y = -2$	Original equation
$y = -3x - 2$	Subtract $3x$ from each side.

$6x + 2y = 10$	Original equation
$2y = -6x + 10$	Subtract $6x$ from each side.
$y = -3x + 5$	Divide each side by 2.

The lines have the same slope. Thus, the lines are parallel and the system has no solution. The system is inconsistent.

5. $x + 2y = 5$
$3x - 15 = -6y$

SOLUTION:

Solve the equations for y.

$x + 2y = 5$	Original equation
$2y = -x + 5$	Subtract x from each side.
$y = -\frac{1}{2}x + \frac{5}{2}$	Divide each side by 2.

$3x - 15 = -6y$	Original equation
$-\frac{1}{2}x + \frac{5}{2} = y$	Divide each side by -6.

The lines have the same slope and y-intercept. Thus, both equations represent the same line and the system has infinitely many solutions. The system is consistent and dependent.

6. $3x - y = 2$
$x + y = 6$

SOLUTION:

Solve the equations for y.

$3x - y = 2$	Original equation
$-y = -3x + 2$	Subtract $3x$ from each side.
$y = 3x - 2$	Divide each side by -1.

$x + y = 6$	Original equation
$y = -x + 6$	Subtract x from each side.

The lines have different slopes. Thus, the equations intersect in exactly one point and the system has one solution. The system is consistent and independent.

7. $x - 2y = 0$
$y = 2x - 3$

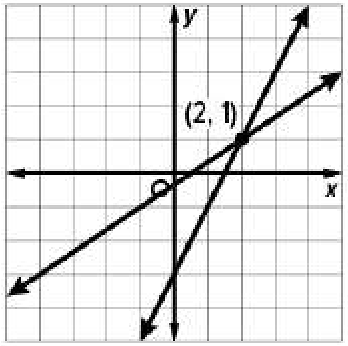
SOLUTION:

Solve each equation for y.

$x - 2y = 0$	Original equation
$-2y = -x$	Subtract x from each side.
$y = \frac{1}{2}x$	Divide each side by -2.

$y = 2x - 3$ Original equation

The lines have different slopes, so there is one solution. Graph the system.



The lines appear to intersect at one point, (2, 1).

Check the solution by substituting the coordinates into each original equation.

$x - 2y = 0$	Original equation	$y = 2x - 3$
$2 - 2(1) = 0$	$x = 2, y = 1$	$1 = 2(2) - 3$
$0 = 0$	True	$1 = 1$

The solution is (2, 1).

8. $-4x + 6y = -2$
$2x - 3y = 1$

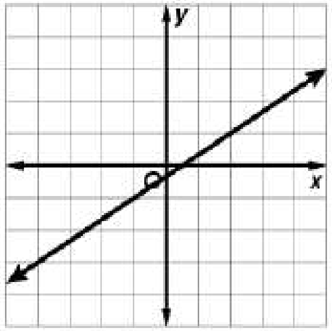
SOLUTION:

Solve each equation for y.

$-4x + 6y = -2$	Original equation
$6y = 4x - 2$	Add $4x$ to each side.
$y = \frac{2}{3}x - \frac{1}{3}$	Divide each side by 6.

$2x - 3y = 1$	Original equation
$-3y = -2x + 1$	Subtract $2x$ from each side.
$y = \frac{2}{3}x - \frac{1}{3}$	Divide each side by -3.

The equations have the same slopes and y-intercepts. So, these equations represent the same line and there are infinitely many solutions. Graph the system.



9. $y = \frac{1}{2}x - \frac{9}{2}$

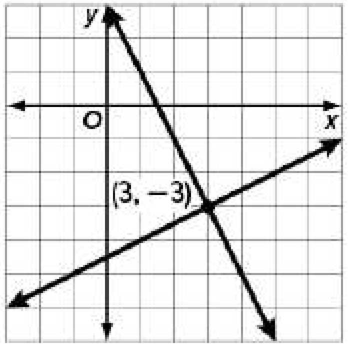
SOLUTION:

Solve each equation for y.

$2x + y = 3$	Original equation
$y = -2x + 3$	Subtract $2x$ from each side.

$y = \frac{1}{2}x - \frac{9}{2}$ Original equation

The lines have different slopes, so there is one solution. Graph the system.



The lines appear to intersect at one point, (3, –3).

Check the solution by substituting the coordinates into each original equation.

$2x + y = 3$	Original equation	$y = \frac{1}{2}x - \frac{9}{2}$
$2(3) - 3 = 3$	$x = 3, y = -3$	$-3 = \frac{1}{2}(3) - \frac{9}{2}$
$3 = 3$	True	$-3 = -3$

The solution is (3, –3).

10. $y - x = 3$
$y = 1$

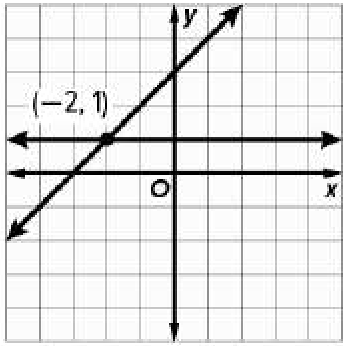
SOLUTION:

Solve each equation for y.

$y - x = 3$ Original equation
$y = x + 3$ Add x to each side.

$y = 1$ Original equation

The lines have different slopes, so there is one solution. Graph the system.



The lines appear to instersect at one point, (–2, 1).

Check the solution by substituting the coordinates into each original equation.

$y - x = 3$	Original equation	$y = 1$
$1 - (-2) = 3$	$x = -2, y = 1$	
$3 = 3$	True	$1 = 1$

The solution is (–2, 1).

11. $2x - 3y = 0$
$4x - 6y = 3$

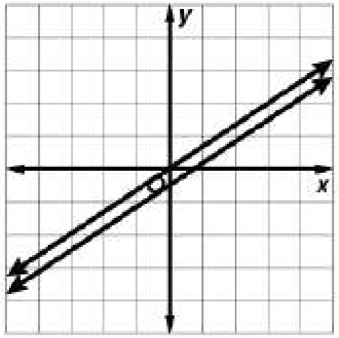
SOLUTION:

Solve each equation for y.

$2x - 3y = 0$ Original equation
$-3y = -2x$ Subtract $2x$ from each side.
$y = \frac{2}{3}x$ Divide each side by -3.

$4x - 6y = 3$ Original equation
$-6y = -4x + 3$ Subtract $4x$ from each side.
$y = \frac{2}{3}x - \frac{1}{2}$ Divide each side by -6.

The equations have the same slopes but different y-intercepts. So, these equations represent parallel line
is no solution. Graph the system.



12. $5x - y = 4$
$-2x + 6y = 4$

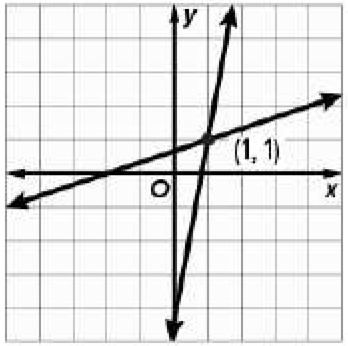
SOLUTION:

Solve each equation for y.

$5x - y = 4$ Original equation
$-y = -5x + 4$ Subtract $5x$ from each side.
$y = 5x - 4$ Divide each side by -1.

$-2x + 6y = 4$ Original equation
$6y = 2x + 4$ Subtract $5x$ from each side.
$y = \frac{1}{3}x + \frac{2}{3}$ Divide each side by 6.

The lines have different slopes, so there is one solution. Graph the system.



The lines appear to instersect at one point, (1, 1).

Check the solution by substituting the coordinates into each original equation.

$5x - y = 4$	Original equation	$-2x + 6y = 4$
$5(1) - 1 = 4$	$x = 1, y = 1$	$-2(1) + 6(1) = 4$
$4 = 4$	True	$4 = 4$

The solution is (1, 1).

7	Solve absolute value inequalities	19 to 30	563

9-9 Solving Absolute Value Equations and Inequalities by Graphing

PART #1

Example 4

Solve each inequality by graphing.

19. $|2x - 6| - 4 \le 0$

20. $|x - 1| - 3 \le 0$

21. $|2x - 1| \ge 4$

22. $|3x + 2| \ge 6$

23. $2|x + 2| < 8$

24. $3|x - 1| < 12$

Example 5

USE TOOLS Use a graphing calculator to solve each inequality.

25. $\left|\frac{1}{4}x + 4\right| - 1 > 0$

26. $\frac{2}{5}|x - 5| + 1 > 0$

27. $|3x - 1| < 2$

28. $|4x + 1| \le 1$

29. $\frac{1}{6}|x - 1| + 1 \le 0$

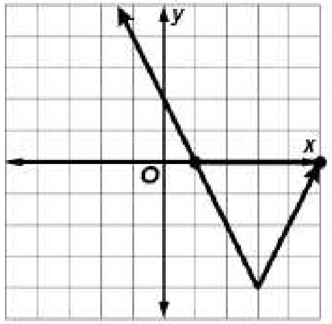
30. $\frac{1}{4}|x + 5| - 1 \le 1$

19. $|2x - 6| - 4 \le 0$

SOLUTION:

The solution set consists of x-values for which the graph of the related function lies below the x-axis, including the x-intercepts. The related function is $f(x) = |2x - 6| - 4$. Graph $f(x)$ by making a table.

x	$f(x)$
–1	4
0	2
1	0
2	–2
3	–4
4	–2
5	0
6	2



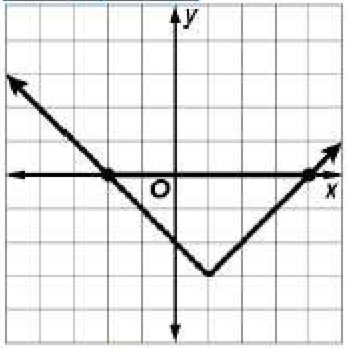
The graph lies below the x-axis between $x = 1$ and $x = 5$. Thus, the solution set is $\{x \mid 1 \le x \le 5\}$ or [1,5]. All values of x between 1 and 5 satisfy the constraints of the original inequality.

20. $|x - 1| - 3 \le 0$

SOLUTION:

The solution set consists of x-values for which the graph of the related function lies below the x-axis, including the x-intercepts. The related function is $f(x) = |x - 1| - 3$. Graph $f(x)$ by making a table.

x	$f(x)$
–4	2
–3	1
–2	0
–1	–1
0	–2
1	–3
2	–2
3	–1
4	0



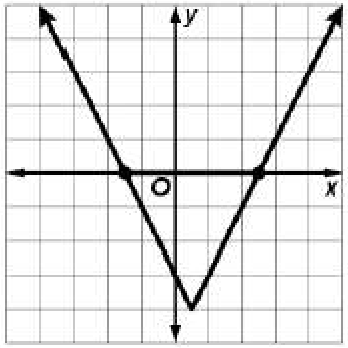
The graph lies below the x-axis between $x = -2$ and $x = 4$. Thus, the solution set is $\{x \mid -2 \le x \le 4\}$ or [–2, 4]. All values of x between –2 and 4 satisfy the constraints of the original inequality.

21. $|2x - 1| \geq 4$

SOLUTION:

The solution set consists of *x*-values for which the graph of the related function lies above the *x*-axis, including the *x*-intercepts. The related function is $f(x) = |2x - 1| - 4$. Graph $f(x)$ by making a table.

x	f(x)
−2	1
−1.5	0
−1	−1
−0.5	−2
0	−3
0.5	−4
1	−3
1.5	−2
2	−1
2.5	0
3	1



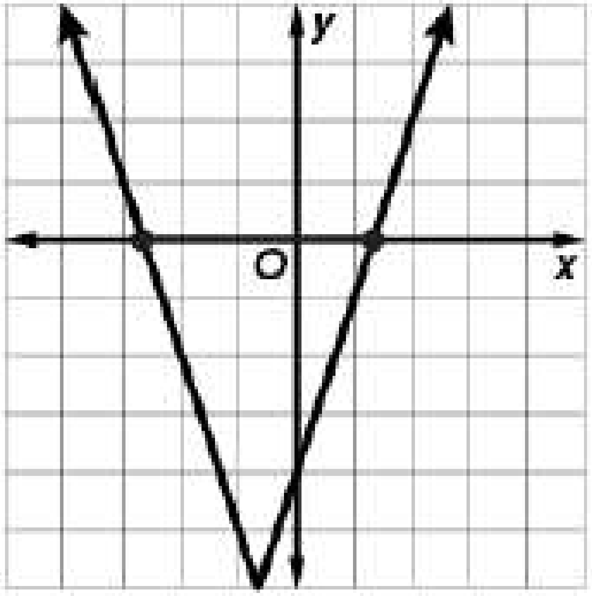
The solution set is $\left\{x \mid x \leq -\frac{3}{2} \text{ or } x \geq \frac{5}{2}\right\}$.

22. $|3x + 2| \geq 6$

SOLUTION:

The solution set consists of *x*-values for which the graph of the related function lies above the *x*-axis, including the *x*-intercepts. The related function is $f(x) = |3x + 2| - 6$. Graph $f(x)$ by making a table.

x	f(x)
−3	1
$-2\frac{2}{3}$	0
$-2\frac{1}{3}$	−1
−2	−2
$-1\frac{2}{3}$	−3
$-1\frac{1}{3}$	−4
−1	−5
$-\frac{2}{3}$	−6
$-\frac{1}{3}$	−5
0	−4
$\frac{1}{3}$	−3
$\frac{2}{3}$	−2
1	−1
$1\frac{1}{3}$	0



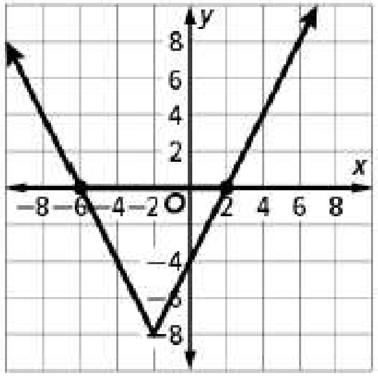
The solution set is $\left\{x \mid x \leq -\frac{8}{3} \text{ or } x \geq \frac{4}{3}\right\}$.

23. $2|x + 2| < 8$

SOLUTION:

The solution set consists of x-values for which the graph of the related function lies below the x-axis, not including the x-intercepts. The related function is $f(x) = 2|x + 2| - 8$. Graph $f(x)$ by making a table.

x	f(x)
–8	4
–6	0
–4	–4
–2	–8
0	–4
2	0
4	4



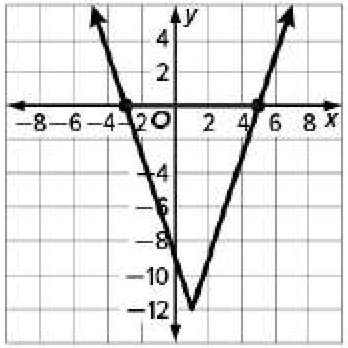
The graph lies below the x-axis between $x = -6$ and $x = 2$. Thus, the solution set is $\{x \mid -6 < x < 2\}$ or $(-6, 2)$.

24. $3|x - 1| < 12$

SOLUTION:

The solution set consists of x-values for which the graph of the related function lies below the x-axis, not including the x-intercepts. The related function is $f(x) = 3|x - 1| - 12$. Graph $f(x)$ by making a table.

x	f(x)
–4	3
–3	0
–2	–3
–1	–6
0	–9
1	–12
2	–9
3	–6
4	–3
5	0
6	3



The solution set is $\{x \mid -3 < x < 5\}$ or $(-3, 5)$.

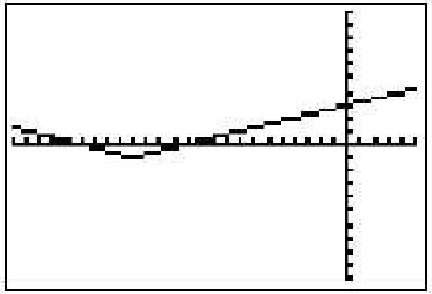
USE TOOLS **Use a graphing calculator to solve each inequality.**

25. $\left|\frac{1}{4}x+4\right|-1>0$

SOLUTION:

Rewriting the inequality results in the function $f(x)=\left|\frac{1}{4}x+4\right|-1>0$.

The > symbol indicates that the solution set consists of x-values for which the graph of the related function lies *above* the x-axis, not including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zero or x-intercepts



[−25, 5] scl: 1 by [−10, 10] scl: 1

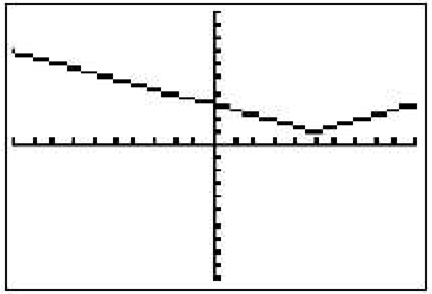
The zeros are located at $x=-20$ and $x=-12$. The graph lies above the x-axis when $x<-20$ and $x>-12$. So the solution set is $\{x \mid x<-20 \text{ or } x>-12\}$.

26. $\frac{2}{5}|x-5|+1>0$

SOLUTION:

Rewriting the inequality results in the function $f(x)=\frac{2}{5}|x-5|+1>0$.

The > symbol indicates that the solution set consists of x-values for which the graph of the related function lies *above* the x-axis, not including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zeros, or x-intercepts



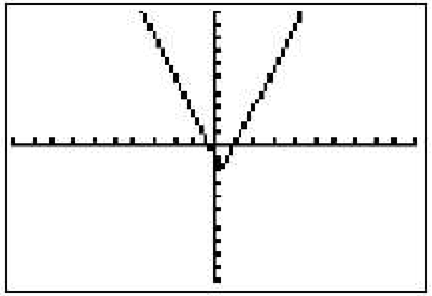
[−10, 10] scl: 1 by [−10, 10] scl: 1

There are no zeros. The graph lies above the x-axis for the entire domain of all real numbers, so the solution set is all real numbers.

27. $|3x - 1| < 2$

SOLUTION:

Rewriting the inequality results in the function $f(x) = |3x - 1| - 2$.
The $<$ symbol indicates that the solution set consists of x-values for which the graph of the related function lies *below* the x-axis, not including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zeros, or x-intercepts



[−10, 10] scl: 1 by [−10, 10] scl: 1

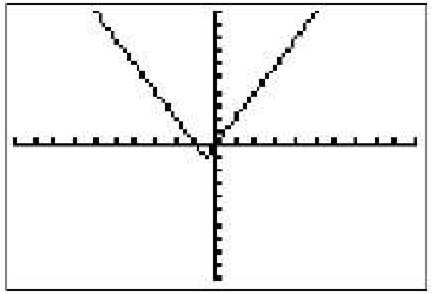
The zeros are located at $x = -\frac{1}{3}$ and $x = 1$. The graph lies below the x-axis when $x > -\frac{1}{3}$ and $x < 1$.

So the solution set is $\left\{x \mid -\frac{1}{3} < x < 1\right\}$.

28. $|4x + 1| \le 1$

SOLUTION:

Rewriting the inequality results in the function $f(x) = |4x + 1| - 1$.
The $\le$ symbol indicates that the solution set consists of x-values for which the graph of the related function lies *below* the x-axis, including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zeros, or x-intercepts



[−5, 5] scl: 0.5 by [−10, 10] scl: 1

The zeros are located at $x = -\frac{1}{2}$ and $x = 0$. The graph lies below the x-axis when $x \ge -\frac{1}{2}$ and $x \le 0$.

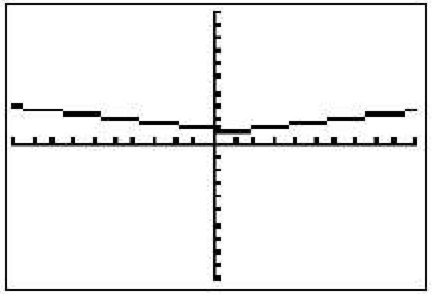
So the solution set is $\left\{x \mid -\frac{1}{2} \le x \le 0\right\}$.

29. $\frac{1}{6}|x-1|+1 \le 0$

SOLUTION:

Rewriting the inequality results in the function $f(x)=\frac{1}{6}|x-1|+1$.

The $\le$ symbol indicates that the solution set consists of x-values for which the graph of the related function lies *below* the x-axis, including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zeros, or x-intercepts



[—10, 10] scl: 1 by [—10, 10] scl: 1

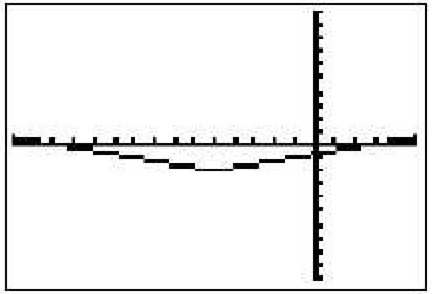
There are no zeros, and the graph never lies below the x-axis, therefore the solution is the empty, or null set.

30. $\frac{1}{4}|x+5|-1 \le 1$

SOLUTION:

Rewriting the inequality results in the function $f(x)=\frac{1}{4}|x+5|-2$.

The $\le$ symbol indicates that the solution set consists of x-values for which the graph of the related function lies *below* the x-axis, including the x-intercepts. Use the **zero** feature from the **CALC** menu to find the zeros, or x-intercepts



[—15, 5] scl: 1 by [—10, 10] scl: 1

The zeros are located at $x=-13$ and $x=3$. The graph lies below the x-axis when $x \ge -13$ and $x \le 3$. So the solution set is $\{x \mid -13 \le x \le 3\}$.

8	Solve quadratic equations by graphing	1 to 12	17

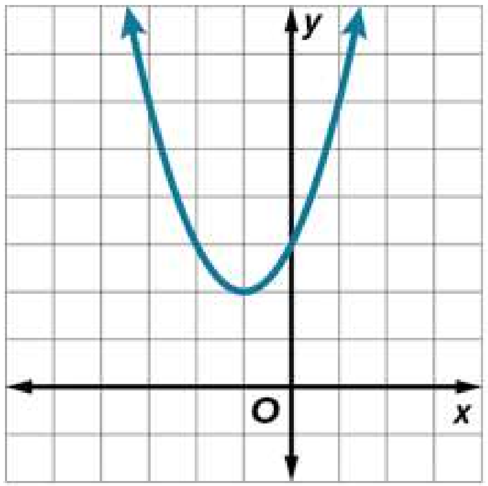
1-2 Solving Quadratic Equations by Graphing

PART #1

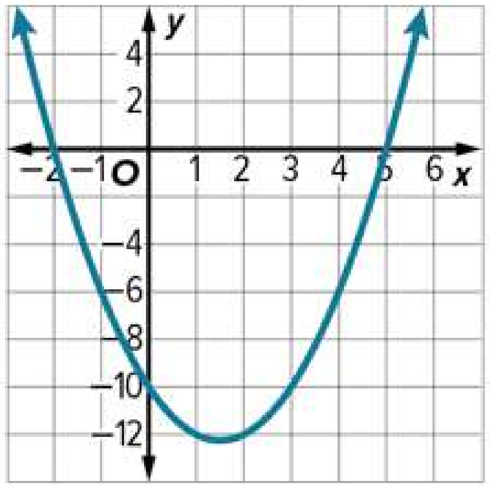
Example 1

Use the related graph of each equation to determine its solutions.

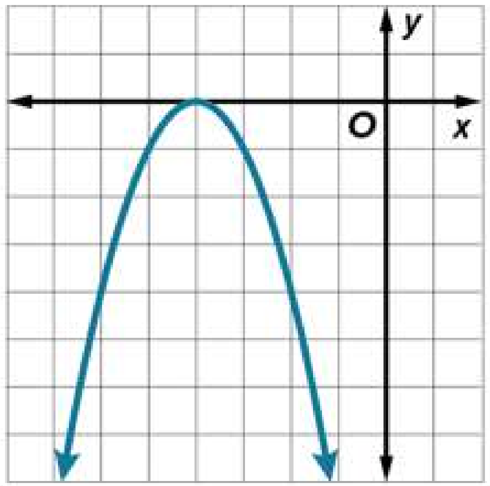
1. $x^2 + 2x + 3 = 0$



2. $x^2 - 3x - 10 = 0$



3. $-x^2 - 8x - 16 = 0$



Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

5. $4x^2 + 4x + 1 = 0$

6. $x^2 + x - 6 = 0$

7. $x^2 + 2x - 3 = 0$

8. $-x^2 - 6x - 9 = 0$

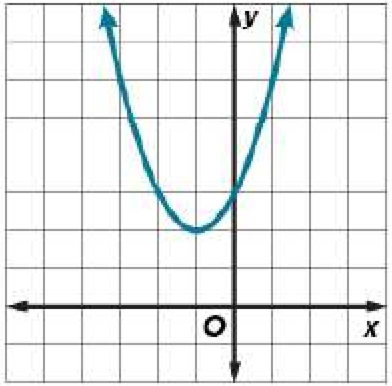
9. $x^2 - 6x + 5 = 0$

10. $x^2 + 2x + 3 = 0$

11. $x^2 - 3x - 10 = 0$

12. $-x^2 - 8x - 16 = 0$

1. $x^2 + 2x + 3 = 0$

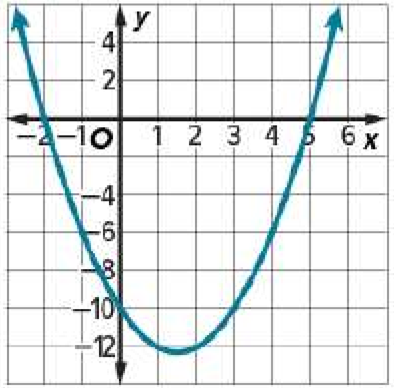


SOLUTION:

There are no zeros of the function.

Therefore, there is no real solution of the equation.

2. $x^2 - 3x - 10 = 0$

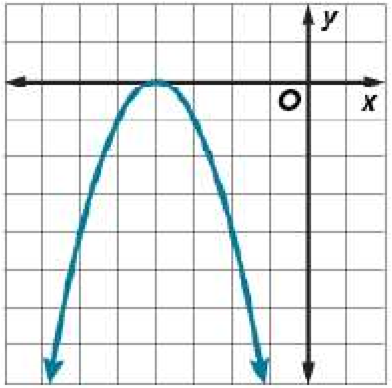


SOLUTION:

The zeros of the function are –2 and 5.

Therefore, the solutions of the equation are –2 and 5.

3. $-x^2 - 8x - 16 = 0$



SOLUTION:

The zero of the function is –4.

Therefore, the solution of the equation is –4.

4. $x^2 - 10x + 21 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetr

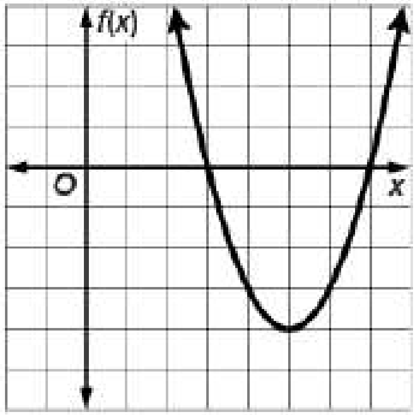
$x = -\frac{-10}{2(1)}$ $a = 1, b = -10$

$x = 5$ Simplify.

Make a table of values.

x	y
3	0
4	–3
5	–4
6	–3
7	0

Plot the points and connect them with a curve.



The zeros of the function are 3 and 7.

5. $4x^2 + 4x + 1 = 0$

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

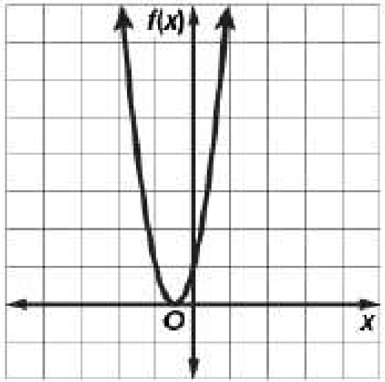
$x = -\frac{4}{2(4)}$ $a = 4, b = 4$

$x = -\frac{1}{2}$ Simplify.

Make a table of values.

x	y
$-1\frac{1}{2}$	4
–1	1
$-\frac{1}{2}$	0
0	1
$\frac{1}{2}$	4

Plot the points and connect them with a curve.



The zero of the function is $-\frac{1}{2}$.

6. $x^2 + x - 6 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

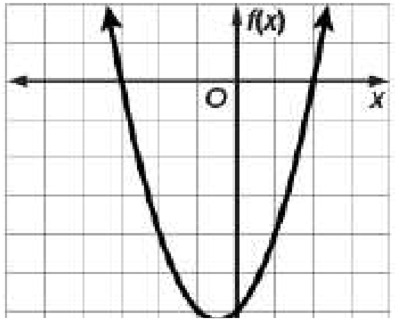
$x = -\frac{1}{2(1)}$ $a = 1, b = 1$

$x = -\frac{1}{2}$ Simplify.

Make a table of values.

x	y
–3	0
–2	–4
–1	–6
$-\frac{1}{2}$	–6.25
0	–6
1	–4
2	0

Plot the points and connect them with a curve.



The zeros of the function are –3 and 2.

Therefore, the solutions of the equation are $x = -3$ or $x = 2$.

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

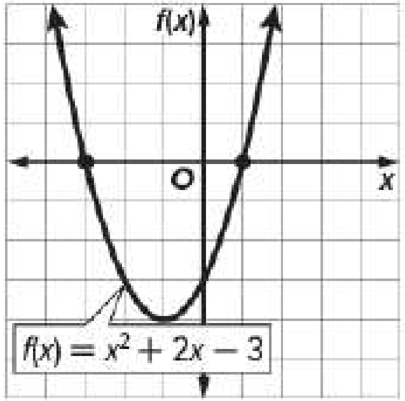
$x = -\frac{2}{2(1)}$ $a = 1, b = 2$

$x = -1$ Simplify.

Make a table of values.

x	y
–3	0
–2	–3
–1	–4
0	–3
1	0

Plot the points and connect them with a curve.



The zeros of the function are –3 and 1.

Therefore, the solutions of the equation are $x = -3$ or $x = 1$.

LUTION:

d the axis of symmetry.

$-\frac{b}{2a}$ Equation of the axis of symmetr

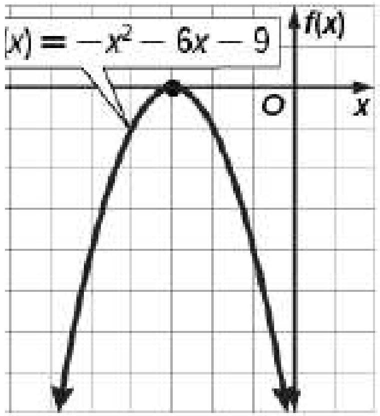
$-\frac{-6}{2(-1)}$ $a = -1, b = -6$

-3 Simplify.

ke a table of values.

x	y
–5	–4
–4	–1
–3	0
–2	–1
–1	–4

t the points and connect them with a curve.



zero of the function is –3.

Therefore, the solution of the equation is $x = -3$.

9. $x^2 - 6x + 5 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

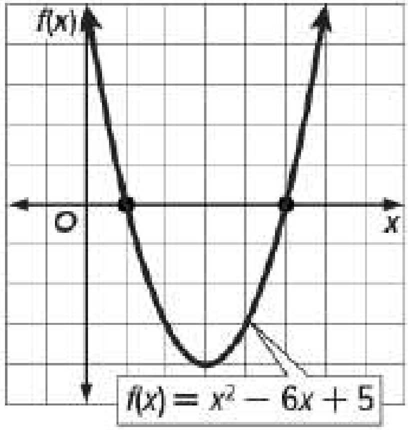
$x = -\frac{-6}{2(1)}$ $a = 1, b = -6$

$x = 3$ Simplify.

Make a table of values.

x	y
1	0
2	–3
3	–4
4	–3
5	0

Plot the points and connect them with a curve.



The zeros of the function are 1 and 5.

10. $x^2 + 2x + 3 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

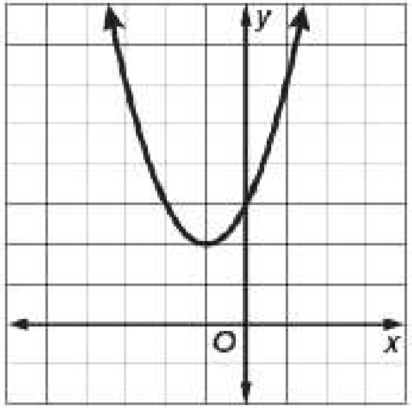
$x = -\frac{2}{2(1)}$ $a = 1, b = 2$

$x = -1$ Simplify.

Make a table of values.

x	y
–3	6
–2	3
–1	2
0	3
1	6

Plot the points and connect them with a curve.



There are no zeros of the function.

11. $x^2 - 3x - 10 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

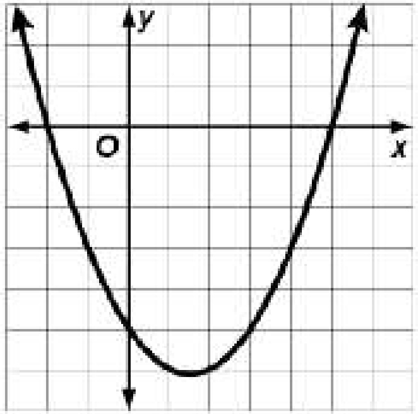
$x = -\frac{-3}{2(1)}$ $a = 1, b = -3$

$x = 1.5$ Simplify.

Make a table of values.

x	y
–2	0
0	–10
1.5	–12.25
3	–10
5	0

Plot the points and connect them with a curve.



The zeros of the function are –2 and 5.

12. $-x^2 - 8x - 16 = 0$

SOLUTION:

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

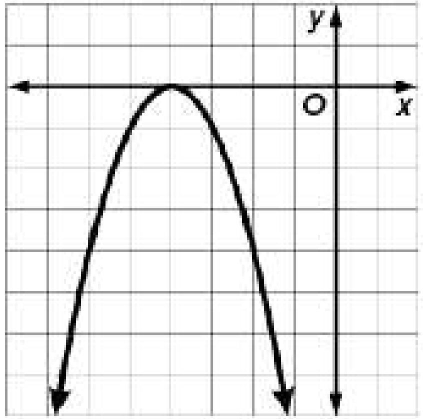
$x = -\frac{-8}{2(-1)}$ $a = -1, b = -8$

$x = -4$ Simplify.

Make a table of values.

x	y
–6	–4
–5	–1
–4	0
–3	–1
–2	–4

Plot the points and connect them with a curve.



The zero of the function is –4.

9	Perform operations with pure imaginary numbers	1 to 18	25

1-3 Complex Numbers

PART #1

Example 1

Simplify.

1. $\sqrt{-48}$

2. $\sqrt{-63}$

3. $\sqrt{-72}$

4. $\sqrt{-24}$

5. $\sqrt{-84}$

6. $\sqrt{-99}$

7. $\sqrt{-23} \cdot \sqrt{-46}$

8. $\sqrt{-6} \cdot \sqrt{-3}$

9. $\sqrt{-5} \cdot \sqrt{-10}$

10. $(3i)(-2i)(5i)$

11. i^{11}

12. $4i(-6i)^2$

Example 3

Solve each equation.

13. $5x^2 + 45 = 0$

14. $4x^2 + 24 = 0$

15. $-9x^2 = 9$

16. $7x^2 + 84 = 0$

17. $5x^2 + 125 = 0$

18. $8x^2 + 96 = 0$

1. $\sqrt{-48}$

SOLUTION:

$$\begin{aligned} \sqrt{-48} &= \sqrt{-1 \cdot 4^2 \cdot 3} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{4^2} \cdot \sqrt{3} && \text{Factor out the imaginary unit.} \\ &= i \cdot 4 \cdot \sqrt{3} \text{ or } 4i\sqrt{3} && \text{Simplify.} \end{aligned}$$

2. $\sqrt{-63}$

SOLUTION:

$$\begin{aligned} \sqrt{-63} &= \sqrt{-1 \cdot 3^2 \cdot 7} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{7} && \text{Factor out the imaginary unit.} \\ &= i \cdot 3 \cdot \sqrt{7} \text{ or } 3i\sqrt{7} && \text{Simplify.} \end{aligned}$$

3. $\sqrt{-72}$

SOLUTION:

$$\begin{aligned} \sqrt{-72} &= \sqrt{-1 \cdot 6^2 \cdot 2} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{2} && \text{Factor out the imaginary unit.} \\ &= i \cdot 6 \cdot \sqrt{2} \text{ or } 6i\sqrt{2} && \text{Simplify.} \end{aligned}$$

4. $\sqrt{-24}$

SOLUTION:

$$\begin{aligned} \sqrt{-24} &= \sqrt{-1 \cdot 2^2 \cdot 6} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{2^2} \cdot \sqrt{6} && \text{Factor out the imaginary unit.} \\ &= i \cdot 2 \cdot \sqrt{6} \text{ or } 2i\sqrt{6} && \text{Simplify.} \end{aligned}$$

SOLUTION:

$$\begin{aligned} \sqrt{-84} &= \sqrt{-1 \cdot 2^2 \cdot 21} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{2^2} \cdot \sqrt{21} && \text{Factor out the imaginary unit.} \\ &= i \cdot 2 \cdot \sqrt{21} \text{ or } 2i\sqrt{21} && \text{Simplify.} \end{aligned}$$

6. $\sqrt{-99}$

SOLUTION:

$$\begin{aligned} \sqrt{-72} &= \sqrt{-1 \cdot 6^2 \cdot 2} && \text{Factor the radicand.} \\ &= \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{2} && \text{Factor out the imaginary unit.} \\ &= i \cdot 6 \cdot \sqrt{2} \text{ or } 6i\sqrt{2} && \text{Simplify.} \end{aligned}$$

7. $\sqrt{-23} \cdot \sqrt{-46}$

SOLUTION:

$$\begin{aligned} \sqrt{-23} \cdot \sqrt{-46} &= i\sqrt{23} \cdot i\sqrt{46} && i = \sqrt{-1} \\ &= i^2 \cdot \sqrt{1058} && \text{Multiply.} \\ &= -1 \cdot \sqrt{23^2} \cdot \sqrt{2} && \text{Simplify.} \\ &= -23\sqrt{2} && \text{Multiply.} \end{aligned}$$

8. $\sqrt{-6} \cdot \sqrt{-3}$

SOLUTION:

$$\begin{aligned} \sqrt{-6} \cdot \sqrt{-3} &= i\sqrt{6} \cdot i\sqrt{3} && i = \sqrt{-1} \\ &= i^2 \cdot \sqrt{18} && \text{Multiply.} \\ &= -1 \cdot \sqrt{3^2} \cdot \sqrt{2} && \text{Simplify.} \\ &= -3\sqrt{2} && \text{Multiply.} \end{aligned}$$

9. $\sqrt{-5}\cdot\sqrt{-10}$

SOLUTION:

$\sqrt{-5}\cdot\sqrt{-10} = i\sqrt{5}\cdot i\sqrt{10}$	$i=\sqrt{-1}$
$= i^2\cdot\sqrt{50}$	Multiply.
$= -1\cdot\sqrt{5^2}\cdot\sqrt{2}$	Simplify.
$= -5\sqrt{2}$	Multiply.

10. $(3i)(-2i)(5i)$

SOLUTION:

$(3i)(-2i)(5i) = (3\cdot -2\cdot 5)\cdot i^3$	Associative and Commutative Prop
$= -30i^3$	Multiply.
$= -30\cdot i^2\cdot i$	Simplify.
$= -30\cdot -1\cdot i$	$i^2=-1$
$= 30i$	Simplify.

11. i^{11}

SOLUTION:

$i^{11} = i^{10}\cdot i$	Product of Powers Property
$= (i^2)^5\cdot i$	Power of a Power Property
$= (-1)^5\cdot i$	$i^2=-1$
$= -i$	Simplify.

12. $(4i)(-6i)^2$

SOLUTION:

$(4i)(-6i)^2 = 4i\cdot 36i^2$	Power of a Product Property
$= 144i^3$	Multiply.
$= 144\cdot i^2\cdot i$	Simplify.
$= 144\cdot -1\cdot i$	$i^2=-1$
$= -144i$	Simplify.

Solve each equation.

13. $5x^2+45=0$

SOLUTION:

$5x^2+45=0$	Original equation
$5(x^2+9)=0$	Factor GCF.
$x^2+9=0$	Divide each side by 5.
$x^2=-9$	Subtract 9 from each side.
$x=\pm\sqrt{-9}$	Square Root Property
$x=\pm 3i$	Simplify.

14. $4x^2+24=0$

SOLUTION:

$4x^2+24=0$	Original equation
$4(x^2+6)=0$	Factor GCF.
$x^2+6=0$	Divide each side by 4.
$x^2=-6$	Subtract 6 from each side.
$x=\pm\sqrt{-6}$	Square Root Property
$x=\pm i\sqrt{6}$	Simplify.

15. $-9x^2=9$

SOLUTION:

$-9x^2=9$	Original equation
$-9x^2-9=0$	Subtract 9 from each side.
$-9(x^2+1)=0$	Factor GCF.
$x^2+1=0$	Divide each side by -9.
$x^2=-1$	Subtract 6 from each side.
$x=\pm\sqrt{-1}$	Square Root Property
$x=\pm i$	Simplify.

16. $7x^2+84=0$

SOLUTION:

$7x^2+84=0$	Original equation
$7(x^2+12)=0$	Factor GCF.
$x^2+12=0$	Divide each side by 7.
$x^2=-12$	Subtract 12 from each side.
$x=\pm\sqrt{-12}$	Square Root Property
$x=\pm 2i\sqrt{3}$	Simplify.

17. $5x^2+125=0$

SOLUTION:

$5x^2+125=0$	Original equation
$5(x^2+25)=0$	Factor GCF.
$x^2+25=0$	Divide each side by 5.
$x^2=-25$	Subtract 25 from each side.
$x=\pm\sqrt{-25}$	Square Root Property
$x=\pm 5i$	Simplify.

18. $8x^2+96=0$

SOLUTION:

$8x^2+96=0$	Original equation
$8(x^2+12)=0$	Factor GCF.
$x^2+12=0$	Divide each side by 8.
$x^2=-12$	Subtract 12 from each side.
$x=\pm\sqrt{-12}$	Square Root Property
$x=\pm 2i\sqrt{3}$	Simplify.

10	Complete the square in a quadratic function to interpret key features of its graph	44 to 51	40

1-5 Solving Quadratic Equations by Completing the Square

PART #1

Examples 9 and 10

Write each function in vertex form. Find the axis of symmetry. Then find the vertex, and determine if it is a *maximum* or *minimum*.

44. $y = x^2 + 2x - 5$

45. $y = x^2 + 6x + 1$

46. $y = -x^2 + 4x + 2$

47. $y = -x^2 - 8x - 5$

48. $y = 2x^2 + 4x + 3$

49. $y = 3x^2 + 6x - 1$

44. $y = x^2 + 2x - 5$

SOLUTION:

$y = x^2 + 2x - 5$	Original equation
$y = (x^2 + 2x) - 5$	Group $ax^2 + bx$.
$y = (x^2 + 2x + 1) - 5 - 1$	Complete the square.
$y = (x + 1)^2 - 6$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = (x + 1)^2 - 6$ is $(-1, -6)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 1$, which is greater than 0, the value of k is a minimum value.

45. $y = x^2 + 6x + 1$

SOLUTION:

$y = x^2 + 6x + 1$	Original equation
$y = (x^2 + 6x) + 1$	Group $ax^2 + bx$.
$y = (x^2 + 6x + 9) + 1 - 9$	Complete the square.
$y = (x + 3)^2 - 8$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = (x + 3)^2 - 8$ is $(-3, -8)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -3$.
Since the value of $a = 1$, which is greater than 0, the value of k is a minimum value.

46. $y = -x^2 + 4x + 2$

SOLUTION:

$y = -x^2 + 4x + 2$	Original equation
$y = (-x^2 + 4x) + 2$	Group $ax^2 + bx$.
$y = -(x^2 - 4x) + 2$	Factor out -1.
$y = -(x^2 - 4x + 4) + 2 - (-1)(4)$	Complete the square.
$y = -(x - 2)^2 + 6$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = -(x - 2)^2 + 6$ is $(2, 6)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = 2$.
Since the value of $a = -1$, which is less than 0, the value of k is a maximum value.

47. $y = -x^2 - 8x - 5$

SOLUTION:

$y = -x^2 - 8x - 5$	Original equation
$y = (-x^2 - 8x) - 5$	Group $ax^2 + bx$.
$y = -(x^2 + 8x) - 5$	Factor out -1.
$y = -(x^2 + 8x + 16) - 5 - (-1)16$	Complete the square.
$y = -(x + 4)^2 + 11$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = -(x + 4)^2 + 11$ is $(-4, 11)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -4$.
Since the value of $a = -1$, which is less than 0, the value of k is a maximum value.

48. $y = 2x^2 + 4x + 3$

SOLUTION:

$y = 2x^2 + 4x + 3$	Original equation
$y = (2x^2 + 4x) + 3$	Group $ax^2 + bx$.
$y = 2(x^2 + 2x) + 3$	Factor.
$y = 2(x^2 + 2x + 1) + 3 - 2(1)$	Complete the square.
$y = 2(x + 1)^2 + 1$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = 2(x + 1)^2 + 1$ is $(-1, 1)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 2$, which is greater than 0, the value of k is a minimum value.

49. $y = 3x^2 + 6x - 1$

SOLUTION:

$y = 3x^2 + 6x - 1$	Original equation
$y = (3x^2 + 6x) - 1$	Group $ax^2 + bx$.
$y = 3(x^2 + 2x) - 1$	Factor.
$y = 3(x^2 + 2x + 1) - 1 - 3(1)$	Complete the square.
$y = 3(x + 1)^2 - 4$	Simplify.

The vertex is (h, k) in the vertex form $y = a(x - h)^2 + k$. The vertex of $y = 3(x + 1)^2 - 4$ is $(-1, -4)$.
The equation of the axis of symmetry is $x = h$. The axis of symmetry is $x = -1$.
Since the value of $a = 3$, which is greater than 0, the value of k is a minimum value.

10	Complete the square in a quadratic function to interpret key features of its graph	44 to 51	40

1-5 Solving Quadratic Equations by Completing the Square

PART #1

50. **FIREWORKS** The height of a firework at an amusement park celebration can be modeled by a quadratic function. Suppose the firework is launched from a platform 2 feet off the ground at a velocity of 96 feet per second. Hint: Use $h(t) = -\frac{1}{2}gt^2 + vt + h_0$, where $g = 32\ \frac{\text{ft}}{\text{s}^2}$.

a. Write a function to represent this situation.

b. Rewrite the function in vertex form.

c. Find the axis of symmetry and the vertex and interpret their meaning in the context of the situation.

51. **DIVING** Malik is participating in a diving championship. For each of his dives, his height above the water can be modeled by a quadratic function. The diving board is 7.5 meters above the water and Malik jumps with a velocity of 4.18 meters per second.
Use $h(t) = -\frac{1}{2}gt^2 + vt + h_0$, where $g = 9.8\ \frac{\text{m}}{\text{s}^2}$.

a. Write a function in vertex form to represent this situation.

b. Find the axis of symmetry and the vertex and interpret their meaning in the context of the situation.

50. **FIREWORKS** The height of a firework at an amusement park celebration can be modeled by a quadratic functio the firework is launched from a platform 2 feet off the ground at a velocity of 96 feet per second.

Use $h(t) = -\frac{1}{2}gt^2 + vt + h_0$, where $g = 32\frac{\text{ft}}{\text{s}^2}$.

a. Write a function to represent this situation.

b. Rewrite the function in vertex form.

c. Find the axis of symmetry and the vertex and interpret their meaning in the context of the situation.

SOLUTION:

a.

$h(t) = -\frac{1}{2}gt^2 + vt + h_0$ Function for projectile motion

$h(t) = -\frac{1}{2}(32)t^2 + 96t + 2$ $g = 32\frac{\text{ft}}{\text{s}^2}$, $v = 96\frac{\text{ft}}{\text{s}}$, $h_0 = 2$ ft

$h(t) = -16t^2 + 96t + 2$ Simplify.

b.

$h(t) = (-16t^2 + 96t) + 2$ Group $ax^2 + bx$.

$h(t) = -(16t^2 - 96t) + 2$ Factor out -1.

$h(t) = -16(t^2 - 6t + 9) + 2 - 16(-9)$ Complete the square.

$h(t) = -16(t - 3)^2 + 146$ Simplify.

c. The axis of symmetry is $t = 3$.

Because the axis of symmetry divides the function into two equal halves, the firework will be at the same height af

it is after 5 seconds.

The vertex is (3, 146).

The vertex is the maximum of the function because $a < 1$. So the firework reached a maximum height of 146 feet at

51. **DIVING** Malik is participating in a diving championship. For each of his dives, his height above the water can be modeled by a quadratic function. The diving board is 7.5 meters above the water and Malik jumps with a velocity of 4.18 meters per second.

Use $h(t) = -\frac{1}{2}gt^2 + vt + h_0$, where $g = 9.8\frac{\text{m}}{\text{s}^2}$.

a. Write the function in vertex form to represent this situation.

b. Find the axis of symmetry and the vertex and interpret their meaning in the context of the situation.

SOLUTION:

a. Write the function.

$h(t) = -\frac{1}{2}gt^2 + vt + h_0$ Function for projectile motion

$h(t) = -\frac{1}{2}(9.8)t^2 + 4.18t + 7.5$ $g = 9.8\frac{\text{m}}{\text{s}^2}$, $v = 4.18\frac{\text{m}}{\text{s}}$, $h_0 = 7.5$ m

$h(t) = -4.9t^2 + 4.18t + 7.5$ Simplify.

Rewrite the function in vertex form.

$h(t) = (-4.9t^2 + 4.18t) + 7.5$ Group $ax^2 + bx$.

$h(t) = -(4.9t^2 - 4.18t) + 7.5$ Factor out -1.

$h(t) = -4.9(t^2 - 0.853t + 0.182) + 7.5 - 4.9(-0.182)$ Complete the square.

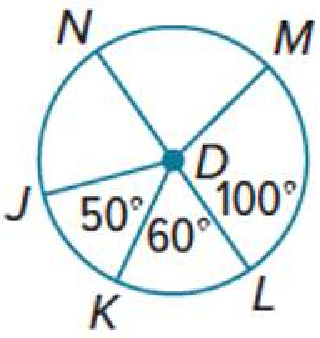
$h(t) = -4.9(t - 0.427)^2 + 8.391$ Simplify.

b. The axis of symmetry is $t = 0.427$; points equidistant from the axis of symmetry represent the times when the diver will be at the same height during his dive. Vertex = (0.427, 8.391); Malik reaches a maximum height of about 8.391 meters approximately 0.427 second after he begins his dive.

11	Find arc lengths and convert between degrees and radians.	20 to 37	238

5-2 Measuring Angles and Arcs

Use ⊙*D* to find the length of each arc to the nearest hundredth. $\overline{NL}$ is a diameter.



20. $\widehat{LM}$ if the radius is 5 inches

21. $\widehat{MN}$ if the diameter is 3 yards

22. $\widehat{KL}$ if *JD* = 7 centimeters

23. $\widehat{NK}$ if *NL* = 12 feet

24. $\widehat{KM}$ if *DM* = 9 millimeters

25. $\widehat{JK}$ if *KD* = 15 inches

Example 6

Write each degree measure in radians as a multiple of π.

26. 120°

27. 45°

28. 30°

29. 90°

30. 180°

31. 225°

Write each radian measure in degrees.

32. $\frac{3\pi}{4}$ radians

33. $\frac{3\pi}{2}$ radians

34. $\frac{\pi}{3}$ radians

35. $\frac{5\pi}{6}$ radians

36. 2π radians

37. $\frac{\pi}{12}$ radians

20. $\widehat{LM}$ if the radius is 5 inches

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{100}{360} \cdot 2\pi(5) \quad \text{Substitution}$$
$$\approx 8.73 \quad \text{Use a calculator.}$$

The length of $\widehat{LM}$ is about 8.73 inches.

ANSWER:
8.73 in.

21. $\widehat{MN}$ if the diameter is 3 yards

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{80}{360} \cdot 2\pi(1.5) \quad \text{Substitution}$$
$$\approx 2.09 \quad \text{Use a calculator.}$$

The length of $\widehat{MN}$ is about 2.09 yards.

ANSWER:
2.09 yd

22. $\widehat{KL}$ if $JD = 7$ centimeters

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{60}{360} \cdot 2\pi(7) \quad \text{Substitution}$$
$$\approx 7.33 \quad \text{Use a calculator.}$$

The length of $\widehat{KL}$ is about 7.33 centimeters.

ANSWER:
7.33 cm

23. $\widehat{NJK}$ if $NL = 12$ feet

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{120}{360} \cdot 2\pi(6) \quad \text{Substitution}$$
$$\approx 12.57 \quad \text{Use a calculator.}$$

The length of $\widehat{NJK}$ is about 12.57 feet.

ANSWER:
12.57 ft

24. $\widehat{KLM}$ if $DM = 9$ millimeters

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{160}{360} \cdot 2\pi(9) \quad \text{Substitution}$$
$$\approx 25.13 \quad \text{Use a calculator.}$$

The length of $\widehat{KLM}$ is about 25.13 millimeters.

ANSWER:
25.13 mm

25. $\widehat{JK}$ if $KD = 15$ inches

SOLUTION:

$$\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}$$
$$= \frac{50}{360} \cdot 2\pi(15) \quad \text{Substitution}$$
$$\approx 13.09 \quad \text{Use a calculator.}$$

The length of $\widehat{JK}$ is about 13.09 inches.

ANSWER:
13.09 in.

26. 120°

SOLUTION:

$120° = 120° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{120\pi}{180}$ or $\frac{2\pi}{3}$ radians Simplify.

ANSWER:
$\frac{2\pi}{3}$ radians

27. 45°

SOLUTION:

$45° = 45° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{45\pi}{180}$ or $\frac{\pi}{4}$ radians Simplify.

ANSWER:
$\frac{\pi}{4}$ radians

28. 30°

SOLUTION:

$30° = 30° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{30\pi}{180}$ or $\frac{\pi}{6}$ radians Simplify.

29. 90°

SOLUTION:

$90° = 90° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{90\pi}{180}$ or $\frac{\pi}{2}$ radians Simplify.

ANSWER:
$\frac{\pi}{2}$ radians

30. 180°

SOLUTION:

$180° = 180° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{180\pi}{180}$ or π radians Simplify.

ANSWER:
π radians

31. 225°

SOLUTION:

$225° = 225° \times \frac{\pi \text{ radians}}{180°}$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{225\pi}{180}$ or $\frac{5\pi}{4}$ radians Simplify.

32. $\frac{3\pi}{4}$ radians

SOLUTION:

$\frac{3\pi}{4}$ radians $= \frac{3\pi}{4}$ radians $\times \frac{180°}{\pi \text{ radians}}$ Multiply by $\frac{180°}{\pi \text{ radians}}$

$= \frac{540}{4}$ or 135° Simplify.

ANSWER:
135°

33. $\frac{3\pi}{2}$ radians

SOLUTION:

$\frac{3\pi}{2}$ radians $= \frac{3\pi}{2}$ radians $\times \frac{180°}{\pi \text{ radians}}$ Multiply by $\frac{180°}{\pi \text{ radians}}$

$= \frac{540}{2}$ or 270° Simplify.

ANSWER:
270°

34. $\frac{\pi}{3}$ radians

SOLUTION:

$\frac{\pi}{3}$ radians $= \frac{\pi}{3}$ radians $\times \frac{180°}{\pi \text{ radians}}$ Multiply by $\frac{180°}{\pi \text{ radians}}$

$= \frac{180}{3}$ or 60° Simplify.

35. $\frac{5\pi}{6}$ radians

SOLUTION:

2π radians $= 2\pi$ radians $\times \frac{180°}{\pi \text{ radians}}$ Mu

$= 360°$ Si

ANSWER:
150°

36. 2π radians

SOLUTION:

2π radians $= 2\pi$ radians $\times \frac{180°}{\pi \text{ radians}}$ Mu

$= 360°$ Si

ANSWER:
360°

37. $\frac{\pi}{12}$ radians

SOLUTION:

$\frac{\pi}{12}$ radians $= \frac{\pi}{12}$ radians $\times \frac{180°}{\pi \text{ radians}}$

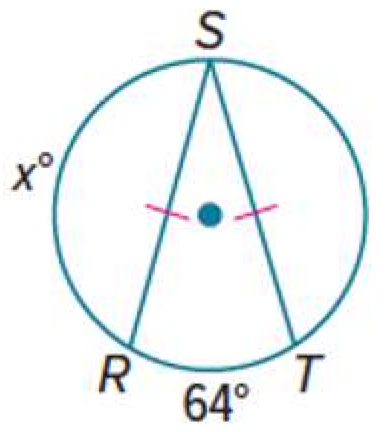
$= \frac{180}{12}$ or 15°

12	Solve problems using the relationships between arcs, chords, and diameters.	1 to 9	245

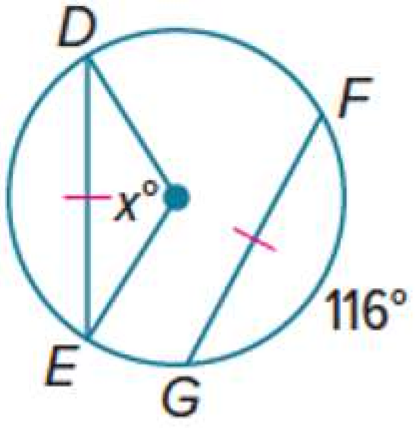
5-3 Arcs and Chords

REGULARITY Find the value of *x*.

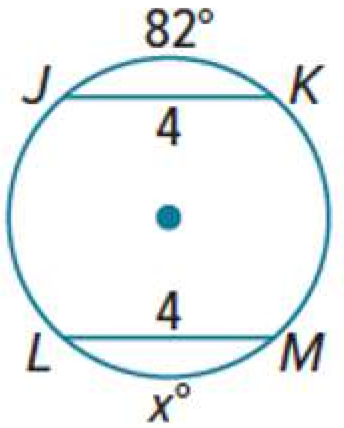
1.



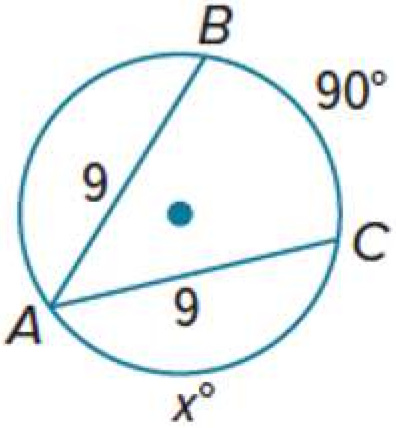
2.



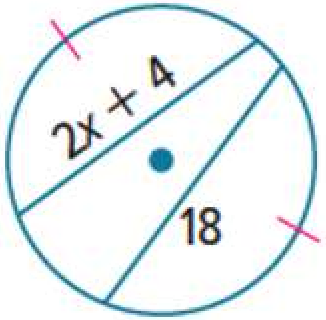
3.



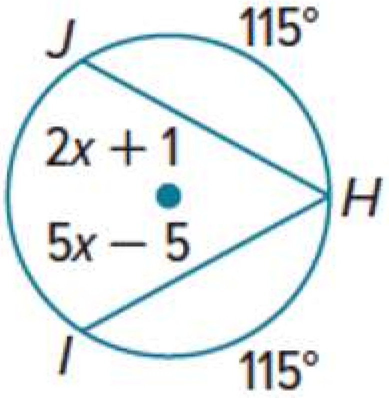
4.



5.

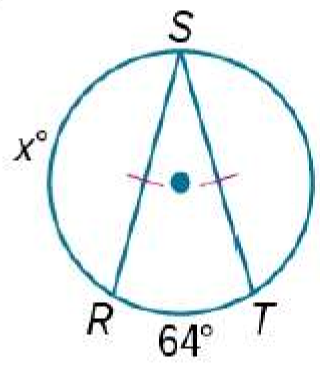


6.



REGULARITY Find the value of x.

1.

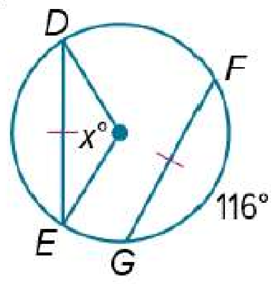


SOLUTION:

$\overline{SR}$ and $\overline{ST}$ are congruent chords, so the corresponding arcs $\widehat{SR}$ and $\widehat{ST}$ are congruent.

$$2x° + 64° = 360°$$
$$2x° = 296°$$
$$x = 148$$

2.

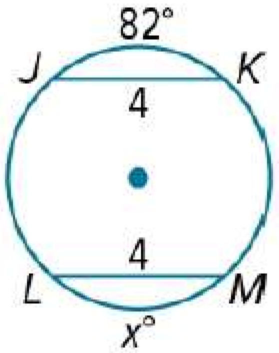


SOLUTION:

$\overline{DE}$ and $\overline{FG}$ are congruent chords, so the corresponding arcs $\widehat{DE}$ and $\widehat{FG}$ are congruent and therefore have equal measures.

Since we are given $m\widehat{FG} = 116°$, then $m\widehat{DE} = 116°$. By definition, a minor arc is equal to the measure of its related central angle. So, the central angle measures $x°$, or 116°; therefore, $x = 116$.

3.

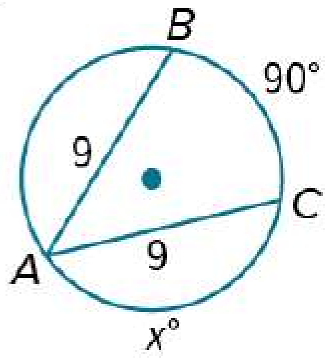


SOLUTION:

$\overline{JK}$ and $\overline{LM}$ are congruent chords, so the corresponding arcs $\widehat{JK}$ and $\widehat{LM}$ are congruent and therefore have equal measures.

Since we are given $m\widehat{JK} = 82°$, then $m\widehat{LM} = 82°$. Therefore $x = 82$.

4.

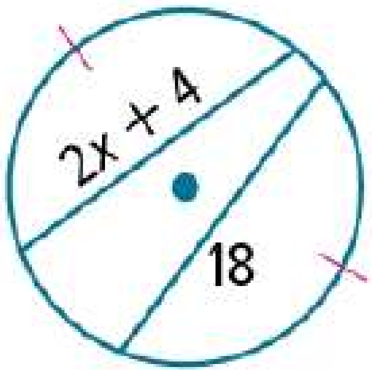


SOLUTION:

$\overline{AB}$ and $\overline{AC}$ are congruent chords, so the corresponding arcs $\widehat{AB}$ and $\widehat{AC}$ are congruent.

$$2x° + 90° = 360°$$
$$2x° = 270°$$
$$x = 135$$

5.

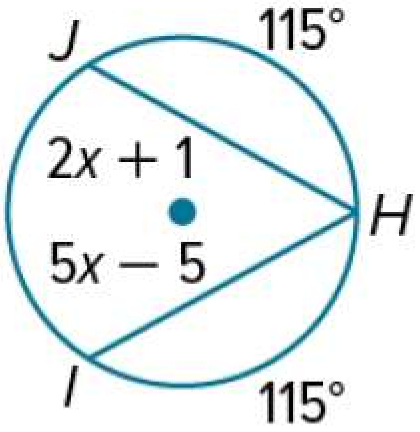


SOLUTION:

The chords shown are congruent, so the corresponding arcs are congruent.

$2x + 4 = 18$	Definition of congruent segments
$2x = 14$	Subtract 4 from each side.
$x = 7$	Divide each side by 2.

6.



SOLUTION:

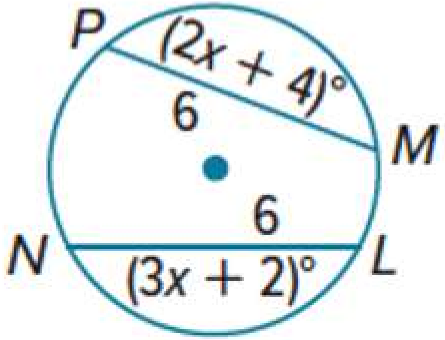
Since the arcs are congruent, the chords are congruent.

$2x + 1 = 5x - 5$	Definition of congruent segments
$1 = 3x - 5$	Subtract $2x$ from each side.
$6 = 3x$	Add 5 to each side.
$2 = x$	Divide each side by 3.

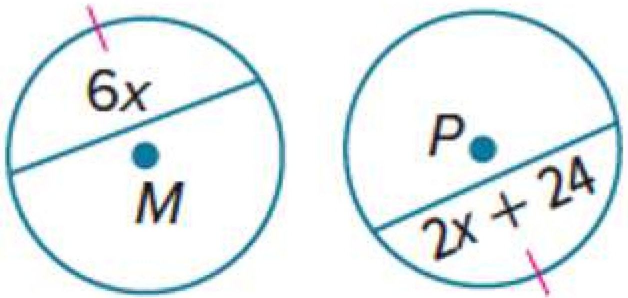
12	Solve problems using the relationships between arcs, chords, and diameters.	1 to 9	245

5-3 Arcs and Chords

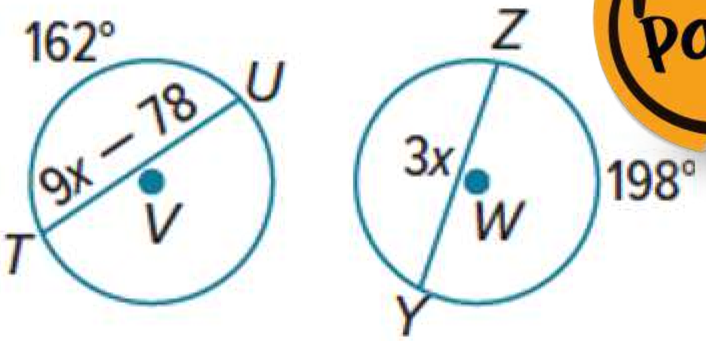
7.



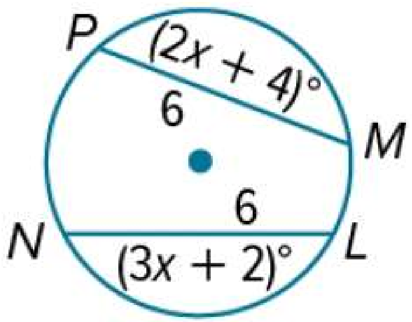
8. $\odot M \cong \odot P$



9. $\odot V \cong \odot W$



7.

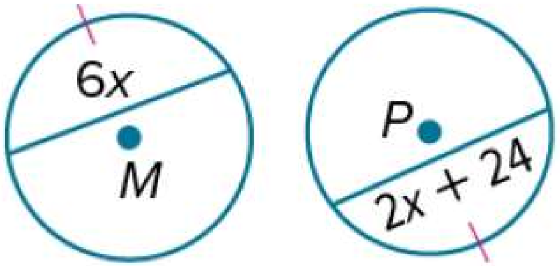


SOLUTION:

The chords shown are congruent, so the corresponding arcs are congruent.

$2x + 4 = 3x + 2$	Definition of congruent segments
$4 = x + 2$	Subtract $2x$ from each side.
$2 = x$	Subtract 2 from each side.

8. $\odot M \cong \odot P$

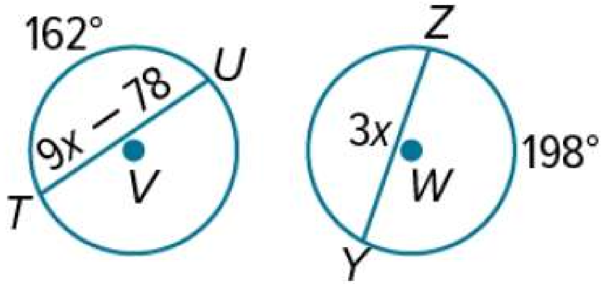


SOLUTION:

Since the arcs are congruent, their corresponding chords are congruent.

$6x = 2x + 24$	Definition of congruent segments
$4x = 24$	Subtract $2x$ from each side.
$x = 6$	Divide each side by 4.

9. $\odot V \cong \odot W$



SOLUTION:

The measure of the arc that corresponds to the chord labeled $3x$ has a measure of $360° - 198° = 162°$. Since the arcs are congruent, their corresponding chords are congruent.

$9x - 78 = 3x$	Definition of congruent segments
$-78 = -6x$	Subtract $9x$ from each side.
$13 = x$	Divide each side by -6.

13	Describe events as subsets of sample spaces by using intersections and unions	1 to 10	377, 378

7-2 Probability and Counting

Part 2

1. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling a number greater than 4. Find $A \cap B$.

2. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling an odd number. Find $A \cap B$.

Use the spinner.

3. Let A be the event of the spinner landing on 4 or 10, and let B be the event of the spinner landing on a section with a number divisible by 4. What are the possible outcomes of each event?
   a. $A = \{\_\_\_\}$

   b. $B = \{\_\_\_\}$

   c. $A \cap B = \{\_\_\_\}$

4. Let P be the event of the spinner landing on a section with a prime number, and let Q be the event of the spinner landing on a section with a number that is a multiple of 3. What are the possible outcomes of each event?
   a. $P = \{\_\_\_\}$

   b. $Q = \{\_\_\_\}$

   c. $P \cap Q = \{\_\_\_\}$

1. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling a number greater than 4. Find $A \cap B$.

SOLUTION:

The possible outcomes for event A are all of the numbers on a die that are even, or $\{2, 4, 6\}$.

The possible outcomes for event B are all of the numbers on a die that are greater than 4, or $\{5, 6\}$.

$A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \{6\}$

2. A fair die is rolled once. Let A be the event of rolling an even number, and let B be the event of rolling an odd number. Find $A \cap B$.

SOLUTION:

The possible outcomes for event A are all of the numbers on a die that are even, or $\{2, 4, 6\}$.

The possible outcomes for event B are all of the numbers on a die that are odd, or $\{1, 3, 5\}$.

$A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \emptyset$

3. Let A be the event of the spinner landing on 4 or 10, and let B be the event of the spinner landing on a section with a number divisible by 4. What are the possible outcomes of each event?

a. $A = \{\_\_\}$

b. $B = \{\_\_\}$

c. $A \cap B = \{\_\_\}$

SOLUTION:

a. The possible outcomes for event A are all of the numbers on the spinner that are 4 or 10, or $\{4, 10\}$.

b. The possible outcomes for event B are all of the numbers on the spinner that are divisible by 4, or $\{4, 8, 12\}$.

c. $A \cap B$ contains all of the outcomes that are in both sample space A and B.

$A \cap B = \{4\}$

4. Let P be the event of the spinner landing on a section with a prime number, and let Q be the event of the spinner landing on a section with a number that is a multiple of 3. What are the possible outcomes of each event?

a. $P = \{\_\_\}$

b. $Q = \{\_\_\}$

c. $P \cap Q = \{\_\_\}$

SOLUTION:

a. The possible outcomes for event P are all of the numbers on the spinner that are prime, or $\{2, 3, 5, 7, 11\}$.

b. The possible outcomes for event Q are all of the numbers on the spinner that are a multiple of 3, or $\{3, 6, 9, 12\}$.

c. $P \cap Q$ contains all of the outcomes that are in both sample space P and Q.

$P \cap Q = \{3\}$

13	Describe events as subsets of sample spaces by using intersections and unions	1 to 10	377, 378

7-2 Probability and Counting

Part 2

5. A card is selected from a standard deck of cards. What is the probability that the card is a diamond and is a seven?

6. A card is selected from a standard deck of cards. What is the probability that the card has a number on it that is divisible by 2 and is black?

Use the spinner.

7. Let *A* be the event that the spinner lands on a vowel. Let *B* be the event that it lands on the letter J. What are the possible outcomes of each event?

a. $A = \{\_\_\_\}$

b. $B = \{\_\_\_\}$

c. $A \cup B = \{\_\_\_\}$

8. Let *X* be the event that the spinner lands on a consonant. Let *Y* be the event that it lands on the letter K. What are the possible outcomes of each event?

a. $X = \{\_\_\_\}$

b. $Y = \{\_\_\_\}$

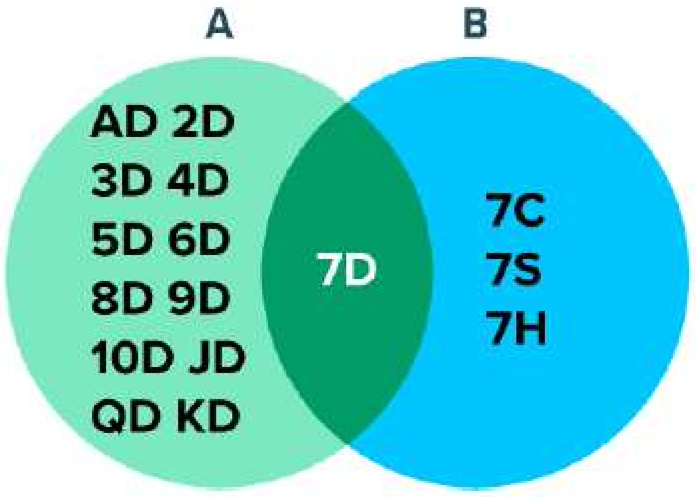
c. $X \cup Y = \{\_\_\_\}$

5. A card is selected from a standard deck of cards. What is the probability that the card is a diamond and is a seven?

SOLUTION:

Let A be the event of choosing a diamond, and let B be the event of choosing a 7. The total number of outcomes is the total number of cards in a deck, or 52.

Write the corresponding number of each card in its correct place in the Venn diagram. In the diagram, D stands for diamonds, H stands for hearts, C stands for clubs, and S stands for spades.



There are 13 cards that are diamonds in a deck of cards and there is only 1 diamond that is also a 7.

$$P(A \cap B) = \frac{\text{number of outcomes in } A \text{ and } B}{\text{total number of possible outcomes}}$$ Probability Rule for Intersections

$$= \frac{1}{52}$$ Substitution

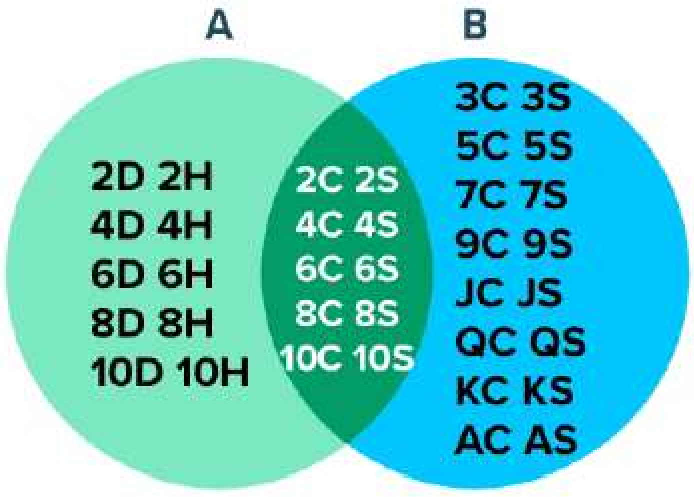
The probability that the card is both a diamond and a 7 is $\frac{1}{52}$, or about 1.92%.

6. A card is selected from a standard deck of cards. What is the probability that the card has a number on it that is divisible by 2
and is black?

SOLUTION:

Let A be the event of choosing a card that has a number on it that is divisible by 2, and let B be the event of choosing a card that is black. The total number of outcomes is the total number of cards in a deck, or 52.

Write the corresponding number of each card in its correct place in the Venn diagram. In the diagram, D stands for diamonds, H stands for hearts, C stands for clubs, and S stands for spades.



There are 20 cards that have a number on it that is divisible by 2 and there are only 10 that is also black cards.

$$P(A \cap B) = \frac{\text{number of outcomes in } A \text{ and } B}{\text{total number of possible outcomes}}$$ Probability Rule for Intersections

$$= \frac{10}{52}$$ Substitution

$$= \frac{5}{26}$$ Simplify.

7. Let A be the event that the spinner lands on a vowel. Let B be the event that it lands on the letter J. What are the possible outcomes
of each event?

a. A = {___}

b. B = {___}

c. $A \cup B$ = {___}

SOLUTION:

a. The possible outcomes for event A are all of the letters on the spinner that are vowels, or {A, E, O, U}.

b. The possible outcomes for event B are all of the letters on the spinner that are J, or {J}.

c. $A \cup B$ contains all of the outcomes that are in either sample space(s) A or B.

$A \cup B$ = {A, E, O, U, J}

8. Let X be the event that the spinner lands on a consonant. Let Y be the event that it lands on the letter K. What are the possible outcomes of each event?

a. X = {___}

b. Y = {___}

c. $X \cup Y$ = {___}

SOLUTION:

a. The possible outcomes for event X are all of the letters on the spinner that are a consonant, or {K, H, S, J}.

b. The possible outcomes for event Y are all of the letters on the spinner that are K, or {K}.

c. $X \cup Y$ contains all of the outcomes that are in either sample space(s) X or Y.

$X \cup Y$ = {K, H, S, J}

13	Describe events as subsets of sample spaces by using intersections and unions	1 to 10	377, 378

7-2 Probability and Counting

Part 2

9. A random number generator is used to generate one integer between 1 and 20. Let C be the event of generating a multiple of 5, and let D be the event of generating a number less than 12. What are the possible outcomes of each event?

a. C = {___}

b. D = {___}

c. $C \cup D$ = {___}

10. A random number generator is used to generate one integer between 1 and 100. Let A be the event of generating a multiple of 10, and let B be the event of generating a factor of 30. What are the possible outcomes of each event?

a. A = {___}

b. B = {___}

c. $A \cup B$ = {___}

9. A random number generator is used to generate one integer between 1 and 20. Let *C* be the event of generating a multiple of 5, and let *D* be the event of generating a number less than 12. What are the possible outcomes of each event?

a. *C* = {__}

b. *D* = {__}

c. $C \cup D$ = {__}

SOLUTION:

a. The possible outcomes for event *C* are all of the numbers that are a multiple of 5, or {5, 10, 15, 20}.

b. The possible outcomes for event *D* are all of the numbers that are less than 12, or {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.

c. $C \cup D$ contains all of the outcomes that are in either sample space(s) *C* or *D*.

$C \cup D$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 20}

10. A random number generator is used to generate one integer between 1 and 100. Let *A* be the event of generating a multiple of 10, and let *B* be the event of generating a factor of 30. What are the possible outcomes of each event?

a. *A* = {__}

b. *B* = {__}

c. $A \cup B$ = {__}

SOLUTION:

a. The possible outcomes for event *A* are all of the numbers that are a multiple of 10, or {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}.

b. The possible outcomes for event *B* are all of the numbers that are a factor of 30, or {1, 2, 3, 5, 6, 10, 15, 30}.

c. $A \cup B$ contains all of the outcomes that are in either sample space(s) *A* or *B*.

$A \cup B$ = {1, 2, 3, 5, 6, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 100}

14	Apply the multiplication rule to situations involving dependent events	6 to 12	401, 402

7-5 Probability and the Multiplication Rule

Part 2

Determine whether the events are *independent* or *dependent*. Explain your reasoning.

6. You roll an even number on a fair die, and then spin a spinner numbered 1 through 5 and it lands on an odd number.

7. An ace is drawn from a standard deck of 52 cards, and is not replaced. Then, a second ace is drawn.

8. In a bag of 3 green and 4 blue marbles, a blue marble is drawn and not replaced. Then, a second blue marble is drawn.

9. You roll two fair dice and roll a 5 on each.

Determine whether the events are *independent* or *dependent*. Explain your reasoning.

6. You roll an even number on a fair die, and then spin a spinner numbered 1 through 5 and it lands on an odd number.

SOLUTION:

The outcome of rolling the fair die in no way changes the probability of the outcome of spinning the spinner.

These two events have no bearing on each other. So, the events are independent.

7. An ace is drawn from a standard deck of 52 cards, and is not replaced. Then, a second ace is drawn.

SOLUTION:

After the first ace is drawn, the card is removed and cannot be chosen again. This affects the probability of the second ace being drawn because the sample space is reduced by one card.

Because the first ace drawn was not replaced, the probability of drawing the second card is affected. events are dependent.

8. In a bag of 3 green and 4 blue marbles, a blue marble is drawn and not replaced. Then, a second blue marble is drawn.

SOLUTION:

After the first blue marble is drawn, the marble is removed and cannot be chosen again. This affects the probability of a second blue marble being drawn because the sample space is reduced by one marble.

Because the first blue marble drawn was not replaced, the probability of the drawing the second marble is affected. So, the events are dependent.

9. You roll two fair dice and roll a 5 on each.

SOLUTION:

The outcome of rolling the first fair die in no way changes the probability of the outcome of rolling the second fair die.

Independent; sample answer: These two rolls have no bearing on each other. So, the events are independent.

14	Apply the multiplication rule to situations involving dependent events	6 to 12	401, 402

7-5 Probability and the Multiplication Rule

10. **LOTTERY** Mr. Hanes places the names of four of his students, Joe, Sofia, Hayden, and Bonita, on slips of paper. From these, he intends to randomly select two students to represent his class at the robotics convention. He draws the name of the first student, sets it aside, then draws the name of the second student. What is the probability he draws Sofia, then Joe?

11. **CARDS** A card is drawn from a standard deck of playing cards and is not replaced. Then a second card is drawn. Find the probability the first card is a jack of spades and the second card is black.

12. **INTRAMURAL SPORTS** The table shows the color and number of jerseys available for the intramural volleyball tournament. If each jersey is given away randomly, what is the probability that the first and second jerseys given away are both red?

Jersey Color	Amount
blue	20
white	15
red	25
black	10

10. **LOTTERY** Mr. Hanes places the names of four of his students, Joe, Sofia, Hayden, and Bonita, on slips of paper intends to randomly select two students to represent his class at the robotics convention. He draws the name of the aside, then draws the name of the second student. What is the probability he draws Sofia, then Joe?

SOLUTION:

These events are dependent because Mr Hanes does not replace the slip of paper he selected. Let A represent sele represent selecting Joe.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ Probability of dependent events

$= \frac{1}{4} \cdot \frac{1}{3}$ or $\frac{1}{12}$ After the first slip labeled Sofia is selected, 3 slips remain, and 1 of those slips is labeled Joe.

So, the probability that he draws Sofia, then Joe is $\frac{1}{12}$ or about 8%.

11. **CARDS** A card is drawn from a standard deck of playing cards and is not replaced. Then a second card is drawn probability the first card is a jack of spades and the second card is black.

SOLUTION:

These events are dependent because the first card is not replace after it is drawn. Let A represent selecting a jack c and B represent selecting a black card.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ Probability of dependent events

$= \frac{1}{52} \cdot \frac{25}{51}$ or $\frac{25}{2652}$ After the first card is selected, 51 cards remain, and 25 of those cards are black.

So, the probability that the first card is a jack of spades and the second card is black is $\frac{25}{2652}$ or about 1%.

12. **INTRAMURAL SPORTS** The table shows the color and number of jerseys available for the intramural volleybal each jersey is given away randomly, what is the probability that the first and second jerseys given away are both rec

Jersey Color	Amount
blue	20
white	15
red	25
black	10

SOLUTION:

These events are dependent because the after the first jersey is given away it is not available when the second jersey Let A represent giving away a red jersey and B represent giving away a jersey that is not red.

$$P(A \text{ and } A) = P(A) \cdot P(A|B)$$ Probability of dependent events

$$= \frac{25}{70} \cdot \frac{24}{69} \text{ or } \frac{20}{161}$$ After the first jersey is given away, 69 jerseys remain, and 24 of those jerseys are red.

So, the probability that the first and second jerseys given away are both red is $\frac{20}{161}$ or about 12%.

15	Apply the addition rule to situations involving events that are not mutually exclusive	7 to 24	410 , 411

7-6 Probability and the Addition Rule

Part 2

CARDS Suppose you pull a card from a standard 52-card deck. Find the probability of each event.

7. The card is a 4.

8. The card is red.

9. The card is a face card.

10. The card is not a face card.

11. *P*(queen or heart)

12. *P*(jack or spade)

13. *P*(five or prime number)

14. *P*(ace or black)

15. A drawing will take place where one ticket is to be drawn from a set of 80 tickets numbered 1 to 80. If a ticket is drawn at random, what is the probability that the number drawn is a multiple of 4 or a factor of 12?

7. The card is a 4.

SOLUTION:

There are 4 cards that are a 4.

So the probability that the card is a 4 is $\frac{4}{52} = \frac{1}{13}$ or about 7.7%.

10. The card is not a face card.

SOLUTION:

A face card is a jack, queen, or king. There are 12 face cards. So, there are 52 – 12 = 40 cards that are not face cards.

So the probability that the card is not face card is $\frac{40}{52} = \frac{10}{13}$ or about 76.9%.

8. The card is red.

SOLUTION:

There are 26 cards that are red.

So the probability that the card is red is $\frac{26}{52} = \frac{1}{2}$ or 50%.

9. The card is a face card.

SOLUTION:

A face card is a jack, queen, or king. There are 12 face cards.

So the probability that the card is face card is $\frac{12}{52} = \frac{3}{13}$ or about 23.1%.

11. P(queen or heart)

SOLUTION:

These are not mutually exclusive events because a card can be a queen and also a heart.

There are 4 queens. There are 13 hearts. There is 1 queen that is also a heart.

$$\begin{aligned} P(\text{queen or heart}) &= P(\text{queen}) + P(\text{heart}) - P(\text{queen and heart}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \end{aligned}$$

So, the probability that a card is a queen or a heart is $\frac{4}{13}$ or about 31%.

12. P(jack or spade)

SOLUTION:

These are not mutually exclusive events because a card can be a jack and also a spade.

There are 4 jacks. There are 13 spades. There is 1 jack that is also a spade.

$$P(\text{jack or spade}) = P(\text{jack}) + P(\text{spade}) - P(\text{jack and spade})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52} \text{ or } \frac{4}{13}$$

So, the probability that a card is a jack or a spade is $\frac{4}{13}$ or about 31%.

13. P(five or prime number)

SOLUTION:

These are not mutually exclusive events because a card can be a five and also a prime number.

There are 4 fives. The numbers 2, 3, 5, and 7 are prime numbers. There are 4 sets of 4 prime numbers, so there are 16 prime numbers. The 4 fives are also prime numbers.

$$P(\text{five or prime number}) = P(\text{five}) + P(\text{prime number}) - P(\text{five and prime number})$$
$$= \frac{4}{52} + \frac{16}{52} - \frac{4}{52}$$
$$= \frac{16}{52} \text{ or } \frac{4}{13}$$

So, the probability that a card is a five or a prime number is $\frac{4}{13}$ or about 31%.

14. P(ace or black)

SOLUTION:

These are not mutually exclusive events because a card can be an ace and also black.

There are 4 aces. There are 26 black cards. There are 2 aces that are also black.

$$P(\text{ace or black}) = P(\text{ace}) + P(\text{black}) - P(\text{ace and black})$$
$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$$
$$= \frac{28}{52} \text{ or } \frac{7}{13}$$

So, the probability that a card is an ace or black is $\frac{7}{13}$ or about 54%.

15. A drawing will take place where one ticket is to be drawn from a set of 80 tickets numbered 1 to 80. If a ticket is drawn at random, what is the probability that the number drawn is a multiple of 4 or a factor of 12?

SOLUTION:

The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, and 80. So, there are 20 multiples of 4.

The factors of 12 are 1, 2, 3, 4, 6, and 12. So, there are 6 factors of 12.

The multiples of 4 that are also factors of 12 are 4 and 12. So, there are 2 multiples of 4 that are also factors of 12.

There are 80 total numbers.

These are not mutually exclusive events because a number can be a multiple of 4 and also a factor of 12.

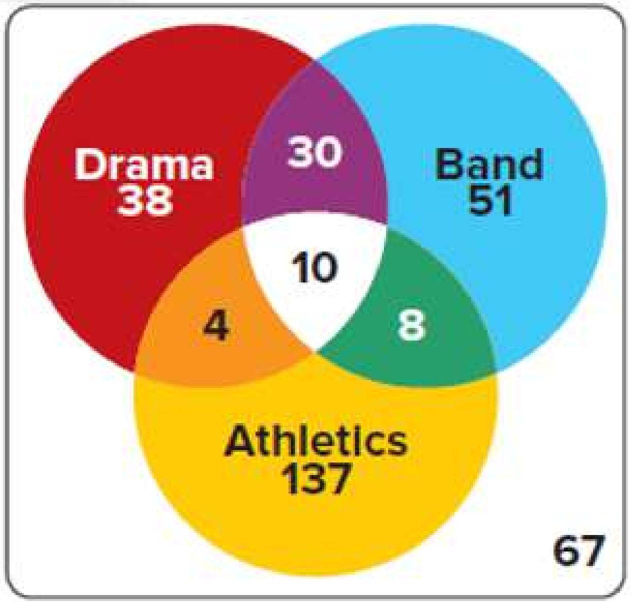
$$P(\text{multiple of 4 or factor of 12}) = P(\text{multiple of 4}) + P(\text{factor of 12}) - P(\text{multiple of 4 and factor of 12})$$
$$= \frac{20}{80} + \frac{6}{80} - \frac{2}{80}$$
$$= \frac{24}{80} \text{ or } \frac{3}{10}$$

15	Apply the addition rule to situations involving events that are not mutually exclusive	7 to 24	410 , 411

7-6 Probability and the Addition Rule

Part 2

16. SCHOOL The Venn diagram shows the extracurricular activities enjoyed by the senior class at Valley View High School.



a. How many students are in the senior class?

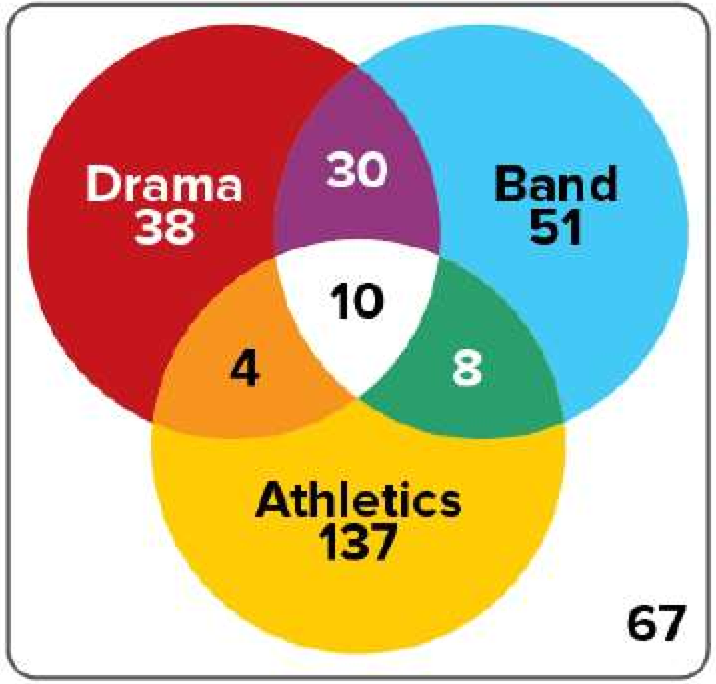
b. How many students participate in athletics?

c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?

d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?

17. BOWLING Cindy's bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that she will bowl either a spare or a strike for any given frame?

18. SPORTS CARDS Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that he randomly selects a baseball or a football card?



a. How many students are in the senior class?

b. How many students participate in athletics?

c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?

d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?

SOLUTION:

a. There are 38 + 30 + 51 + 4 + 10 + 8 + 137 + 67 = 345 students in the senior class.

b. There are 4 + 10 + 8 + 137 = 159 students that participate in athletics.

c. There are 38 + 30 + 10 + 4 = 82 students that participate in drama.

There are 4 + 10 = 14 students that participate in athletics and also participate in drama.

These are not mutually exclusive events because a student can participate in athletics and also participate in drama.

$$P(\text{athletics or drama}) = P(\text{athletics}) + P(\text{drama}) - P(\text{athletics and drama})$$
$$= \frac{159}{345} + \frac{82}{345} - \frac{14}{345}$$
$$= \frac{227}{345}$$

So, the probability that a student participates is athletics or drama is $\frac{227}{345}$ or about 66%.

d. There are 30 students that participate in only drama and band.

$$P(\text{only drama or band}) = \frac{30}{345}$$
$$= \frac{2}{23}$$

So, the probability that a student participates is only drama or band is $\frac{2}{23}$ or about 9%.

17. **BOWLING** Cindy's bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that she will bowl either a spare or a strike for any given frame?

SOLUTION:

These are mutually exclusive events, because the Cindy cannot bowl a spare and a strike.

$$\begin{aligned} P(\text{spare or strike}) &= P(\text{spare}) + P(\text{strike}) \\ &= 45\% + 30\% \\ &= 75\% \end{aligned}$$

So, the probability that Cindy will bowl a spare or a strike is 75% or $\frac{3}{4}$.

18. **SPORTS CARDS** Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that he randomly selects a baseball or a football card?

SOLUTION:

These are mutually exclusive events, because the Dario cannot select a baseball card and a football card.

There are a total of 145 + 102 + 48 = 295 cards.

$$\begin{aligned} P(\text{baseball card or football card}) &= P(\text{baseball card}) + P(\text{football card}) \\ &= \frac{145}{295} + \frac{102}{295} \\ &= \frac{247}{295} \end{aligned}$$

So, the probability that he randomly selects a baseball or a football card is $\frac{247}{295}$ or about 84%.

15	Apply the addition rule to situations involving events that are not mutually exclusive	7 to 24	410 , 411

7-6 Probability and the Addition Rule

19. **SCHOLARSHIPS** A review committee read 3000 application essays for one $5000 college scholarship. Of the applications reviewed, 2865 essays were the required length, 2577 of the applicants had the minimum required grade-point average, and 2486 had the required length and minimum grade-point average. What is the probability that an application essay selected at random will have the required length or the required gradepoint average?

20. **PETS** Ruby's cat had 8 kittens. The litter included 2 orange females, 3 mixed-color females, 1 orange male, and 2 mixed-color males. Ruby wants to keep one kitten. What is the probability that she randomly chooses a kitten that is female or orange?

21. **SPORTS** The table shows the age and number of participants in each sport at a sporting complex. What is the probability that a player is 14 or plays basketball?

Mason Sports Complex			
Age	Soccer	Volleyball	Basketball
14	28	36	42
15	30	26	33
16	35	41	29

19. **SCHOLARSHIPS** A review committee read 3000 application essays for one \$5000 college scholarship. Of the applications reviewed, 2865 essays were the required length, 2577 of the applicants had the minimum required grade-point average, and 2486 had the required length and minimum grade-point average. What is the probability that an application essay selected at random will have the required length or the required gradepoint average?

SOLUTION:

These are not mutually exclusive events because an application essay can have the required length and the required grade point average.

$$P(\text{length or grade point average}) = P(\text{length}) + P(\text{grade point average}) - P(\text{length and grade point average})$$
$$= \frac{2865}{3000} + \frac{2577}{3000} - \frac{2486}{3000}$$
$$= \frac{2956}{3000} \text{ or } \frac{739}{750}$$

So, the probability that an application essay selected at random will ha

point average is $\frac{739}{750}$ or about 98.5%.

20. **PETS** Ruby's cat had 8 kittens. The litter included 2 orange females, 3 mixedcolor females, 1 orange male, and 2 mixed-color males. Ruby wants to keep one kitten. What is the probability that she randomly chooses a kitten that is female or orange?

SOLUTION:

These are not mutually exclusive events because a kitten can be female and orange.

There are 2 + 3 = 5 female kittens. There are 2 + 1 = 3 orange kittens. There are 2 female kittens that are also orange.

$$P(\text{female or orange}) = P(\text{female}) + P(\text{orange}) - P(\text{female and orange})$$
$$= \frac{5}{8} + \frac{3}{8} - \frac{2}{8}$$
$$= \frac{6}{8} \text{ or } \frac{3}{4}$$

So, the probability that she randomly chooses a kitten that is female or orange is $\frac{3}{4}$ or 75%.

21. **SPORTS** The table shows the age and number of participants in each sport at a sporting complex. What is the probability that a player is 14 or plays basketball?

Mason Sports Complex			
Age	**Soccer**	**Volleyball**	**Basketball**
14	28	36	42
15	30	26	33
16	35	41	29

SOLUTION:

These are not mutually exclusive events because a player can be 14 and play basketball.

There are 28 + 36 + 42 = 106 players that are 14. There are 42 + 33 + 29 = 104 players that play basketball. There are 42 players that are 14 and also play basketball.

There are a total of 28 + 36 + 42 + 30 + 26 + 33 + 35 + 41 + 29 = 300 players.

$$P(14 \text{ or basketball}) = P(14) + P(\text{basketball}) - P(14 \text{ and basketball})$$
$$= \frac{106}{300} + \frac{104}{300} - \frac{42}{300}$$
$$= \frac{168}{300} \text{ or } \frac{14}{25}$$

So, the probability that a player is 14 or plays basketball is 56%.

15	Apply the addition rule to situations involving events that are not mutually exclusive	7 to 24	410 , 411

7-6 Probability and the Addition Rule

Part 2

22. **USE A MODEL** Vicente and Kelly are designing a board game. They decide that the game will use a pair of dice and the players will have to find the sum of the numbers rolled. VIcente and Kelly created the table shown to help determine probabilities. Each player will roll the pair of dice twice during that player's turn.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2,6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a. What is the probability of rolling a pair or two numbers that have a sum of seven?

b. What is the probability of rolling two numbers whose sum is an even number or not rolling a 2? Round to the nearest thousandth.

23. **PARKS** The table shows Parks and Recreation Department classes and the number of participants ages 7-9. What is the probability that a participant chosen at random is in drama or is an 8-year-old?

Age	Swimming	Drama	Art
7	40	35	25
8	30	21	14
9	20	44	11

24. **FLOWER GARDEN** Erin is planning her summer garden. The table shows the number of bulbs she has according to type and color of flower. If Erin randomly selects one of the bulbs, what is the probability that she selects a bulb for a yellow flower or a dahlia?

Flower	Orange	Yellow	White
Dahlia	5	4	3
Lily	3	1	2
Gladiolus	2	5	6
Iris	0	1	4

22. **USE A MODEL** Vicente and Kelly are designing a board game. They decide that the game will use a pair of dice and the players will have to find the sum of the numbers rolled. Vicente and Kelly created the table shown to help determine probabilities. Each player will roll the pair of dice twice during that player's turn.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

SOLUTION:

a. These events are mutually exclusive because two numbers cannot be a pair and a sum of seven.

There are 6 pairs. There are 6 numbers that have a sum of seven. There are 36 possible outcomes.

$$P(\text{pair or sum of seven}) = P(\text{pair}) + P(\text{sum of seven})$$
$$= \frac{6}{36} + \frac{6}{36}$$
$$= \frac{12}{36} \text{ or } \frac{1}{3}$$

So, the probability of rolling a pair or two numbers that have a sum of seven is $\frac{1}{3}$ or about 0.333.

b. These events are not mutually exclusive because two numbers can have a sum that is an even number and also not a 2.
There are 18 numbers that have a sum that is an even number. There are 25 numbers where a 2 is not rolled. There are 13 numbers that have a sum that is an even number and a 2 is not rolled.

$$P(\text{sum is even or not a 2}) = P(\text{sum is even}) + P(\text{not a 2}) - P(\text{sum is even and not a 2})$$
$$= \frac{18}{36} + \frac{25}{36} - \frac{13}{36}$$
$$= \frac{30}{36} \text{ or } \frac{5}{6}$$

So, probability of rolling two numbers whose sum is an even number or not rolling a 2 is about 0.833.

23. **PARKS** The table shows Parks and Recreation Department classes and the number of participants ages 7–9. What is the probability that a participant chosen at random is in drama or is an 8-year-old?

Age	Swimming	Drama	Art
7	40	35	25
8	30	21	14
9	20	44	11

SOLUTION:

These are not mutually exclusive events because a participant can be in drama and an 8-year-old.

There are $35 + 21 + 44 = 100$ participants in drama. There are $30 + 21 + 14 = 65$ participants that are 8 years old. There are 21 participants that are in drama and also are 8 years old.

There are a total of $40 + 35 + 25 + 30 + 21 + 14 + 20 + 44 + 11 = 240$ participants.

$$P(\text{drama or 8 years old}) = P(\text{drama}) + P(\text{8 years old}) - P(\text{drama and 8 years old})$$
$$= \frac{100}{240} + \frac{65}{240} - \frac{21}{240}$$
$$= \frac{144}{240} \text{ or } \frac{11}{60}$$

So, the probability that a participant chosen at random is in drama or is an 8-year-old is 0.6.

24. **FLOWER GARDEN** Erin is planning her summer garden. The table shows the number of bulbs she has according to type and color of flower. If Erin randomly selects one of the bulbs, what is the probability that she selects a bulb for a yellow flower or a dahlia?

Flower	Orange	Yellow	White
Dahlia	5	4	3
Lily	3	1	2
Gladiolus	2	5	6
Iris	0	1	4

SOLUTION:

These are not mutually exclusive events because a bulb can be for a yellow flower and a dahlia.

There are 4 + 1 + 5 + 1 = 11 bulbs for yellow flowers. There are 5 + 4 + 3 = 12 bulbs for dahlias. There are 4 bulbs that are for yellow flowers and also for dahlias.

There are a total of 5 + 4 + 3 + 3 + 1 + 2 + 2 + 5 + 6 + 0 + 1 + 4 = 36 bulbs.

$$P(\text{yellow or dahlia}) = P(\text{yellow}) + P(\text{dahlia}) - P(\text{yellow and dahlia})$$
$$= \frac{11}{36} + \frac{12}{36} - \frac{4}{36}$$
$$= \frac{19}{36}$$

So, the the probability that she selects a bulb for a yellow flower or a dahlia is about 52.8%.

16	Recognize and explain the concepts of conditional probability and independence in everyday situations	1 to 10	415

7-7 Conditional Probability

Part 2

1. **CLUBS** The Spanish Club is having a potluck lunch where each student brings in a cultural dish. The 10 students randomly draw cards numbered with consecutive integers from 1 to 10. Students who draw odd numbers will bring main dishes. Students who draw even numbers will bring desserts. If Cynthia is bringing a dessert, what is the probability that she drew the number 10?

2. A card is randomly drawn from a standard deck of 52 cards. What is the probability that the card is a king of diamonds, given that the card drawn is a king?

3. **GAME** In a game, a spinner with the 7 colors of the rainbow is spun. Find the probability that the color spun is blue, given the color is one of the three primary colors: red, yellow, or blue.

4. Fifteen cards numbered 1–15 are placed in a hat. What is the probability that the card has a multiple of 3 on it, given that the card picked is an odd number?

1. **CLUBS** The Spanish Club is having a potluck lunch where each student brings in a cultural dish. The 10 students randomly draw cards numbered with consecutive integers from 1 to 10. Students who draw odd numbers will bring main dishes. Students who draw even numbers will bring desserts. If Cynthia is bringing a dessert, what is the probability that she drew the number 10?

SOLUTION:

Let $P(A)$ = the probability that the number is even.
Let $P(A \text{ and } B)$ = the probability that the number is both 10 and even.
There are 10 available integers.
The sample space for event A contains 5 outcomes: {2, 4, 6, 8, 10}.
So, $P(A) = \frac{5}{10}$ or $\frac{1}{2}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {10}.
So, $P(A \text{ and } B) = \frac{1}{10}$.

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Formula for conditional probability

$= \frac{\frac{1}{10}}{\frac{1}{2}}$ Substitute.

$= \frac{1}{5}$ or 20%

2. A card is randomly drawn from a standard deck of 52 cards. What is the probability that the card is a king of diamonds, given that the card drawn is a king?

SOLUTION:

Let $P(A)$ = the probability that the card is a king.
Let $P(A \text{ and } B)$ = the probability that the card is a king and a diamond.
There are 52 possible cards.
The sample space for event A contains 4 outcomes: {king of hearts, king of diamonds, king of spades, king of clubs}
So, $P(A) = \frac{4}{52}$ or $\frac{1}{13}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {king of diamonds}.
So, $P(A \text{ and } B) = \frac{1}{52}$.

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Formula for conditional probability

$= \frac{\frac{1}{52}}{\frac{1}{13}}$ Substitute.

$= \frac{1}{4}$ or 25%

3. **GAME** In a game, a spinner with the 7 colors of the rainbow is spun. Find the probability that the color spun is blue, given the color is one of the three primary colors: red, yellow, or blue.

SOLUTION:

Let $P(A)$ = the probability that the color spun is a primary color.
Let $P(A \text{ and } B)$ = the probability that the color spun is blue.
There are 7 possible colors.
The sample space for event A contains 4 outcomes: {red, orange, yellow, green, blue, indigo, violet}
So, $P(A) = \frac{3}{7}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {blue}.
So, $P(A \text{ and } B) = \frac{1}{7}$.

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Formula for conditional probability

$= \frac{\frac{1}{7}}{\frac{3}{7}}$ Substitute.

$= \frac{1}{3}$ Simplify.

4. Fifteen cards numbered 1–15 are placed in a hat. What is the probability that the card has a multiple of 3 on it, given that the card picked is an odd number?

SOLUTION:

Let $P(A)$ = the probability that the number is odd.
Let $P(A \text{ and } B)$ = the probability that the number is odd and a multiple of three.
There are 15 possible numbers.
The sample space for event A contains 8 outcomes: {1, 3, 5, 7, 9, 11, 13, 15}
So, $P(A) = \frac{8}{15}$.

The sample space for $P(A \text{ and } B)$ contains 3 outcomes: {3, 9, 15}.
So, $P(A \text{ and } B) = \frac{3}{15}$.

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Formula for conditional probability

$= \frac{\frac{3}{15}}{\frac{8}{15}}$ Substitute.

$= \frac{3}{8}$ Simplify.

16	Recognize and explain the concepts of conditional probability and independence in everyday situations	1 to 10	415

7-7 Conditional Probability

Part 2

5. A blue marble is selected at random from a bag of 3 red and 9 blue marbles and not replaced. What is the probability that a second marble selected will be blue?

6. A die is rolled. If the number rolled is less than 5, what is the probability that it is the number 2?

7. If two dice are rolled, what is the probability that the sum of the faces is 4, given that the first die rolled is odd?

8. A spinner numbered 1 through 12 is spun. Find the probability that the number spun is an 11 given that the number spun was an odd number.

9. If two dice are rolled, what is the probability that the sum of the faces is 8, given that the first die rolled is even?

10. **PICNIC** A school picnic offers students hamburgers, hot dogs, chips, and a drink.

    a. At the picnic, 60% of the students order a hamburger and 48% of the students order a hamburger and chips. What is the conditional probability that a student who orders a hamburger also orders chips?

    b. If 50% of the students ordered chips, are the events of ordering a hamburger and ordering chips independent? Explain.

    c. If 80% of the students who ordered a hot dog also ordered a drink and 35% of all the students ordered a hot dog, find the probability that a student at the picnic orders a hot dog and drink. Explain.

5. A blue marble is selected at random from a bag of 3 red and 9 blue marbles and not replaced. What is the probability that a second marble selected will be blue?

SOLUTION:

Let $P(A)$ = the probability that the first marble selected is blue.
Let $P(B)$ = the probability that the second marble selected is blue.
Let $P(A \text{ and } B)$ = the probability that the first and second marbles are both blue.

Since there are a total of 12 marbles to begin with, 9 of which are blue, $P(A) = \frac{9}{12}$ or $\frac{3}{4}$

Since the first marble was blue, now there are 3 red and 8 blue marbles in the sample space, $P(B) = \frac{8}{11}$, or about 73%.

Another way to look at this:

$P(A \text{ and } B) = \frac{3}{4} \cdot \frac{8}{11} = \frac{6}{11}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$

$$= \frac{\frac{6}{11}}{\frac{3}{4}} \quad \text{Substitute.}$$

$$= \frac{8}{11} \quad \text{Simplify.}$$

6. A die is rolled. If the number rolled is less than 5, what is the probability that it is the number 2?

SOLUTION:

Let $P(A)$ = the probability that the number is less than 5.
Let $P(A \text{ and } B)$ = the probability that the number is less than 5 and 2.
There are 6 possible numbers.
The sample space for event A contains 4 outcomes: {1, 2, 3, 4}

So, $P(A) = \frac{4}{6}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {2}.

So, $P(A \text{ and } B) = \frac{1}{6}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$

$$= \frac{\frac{1}{6}}{\frac{4}{6}}$$

$$= \frac{1}{4}$$

7. If two dice are rolled, what is the probability that the sum of the faces is 4, given that the first die rolled is odd?

SOLUTION:

Let $P(A)$ = the probability that the first die rolled is odd.
Let $P(A \text{ and } B)$ = the probability that the first die rolled is odd and the sum of the faces is 4.
There are 36 possible combinations of dice rolls.
The sample space for event A contains 18 outcomes: {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}.

So, $P(A) = \frac{18}{36} = \frac{1}{2}$.

The sample space for $P(A \text{ and } B)$ contains 2 outcomes: {(1, 3), (3, 1)}.

So, $P(A \text{ and } B) = \frac{2}{36} = \frac{1}{18}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$

$$= \frac{\frac{1}{18}}{\frac{1}{2}} \quad \text{Substitute.}$$

$$= \frac{1}{9} \quad \text{Simplify.}$$

8. A spinner numbered 1 through 12 is spun. Find the probability that the number spun is an 11 given that the number spun was an odd number.

SOLUTION:

Let $P(A)$ = the probability that the number spun is odd.
Let $P(A \text{ and } B)$ = the probability that the number spun is odd and 11.
There are 12 possible numbers.
The sample space for event A contains 6 outcomes: {1, 3, 5, 7, 9, 11}

So, $P(A) = \frac{6}{12}$.

The sample space for $P(A \text{ and } B)$ contains 1 outcome: {11}.

So, $P(A \text{ and } B) = \frac{1}{12}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Formula for conditional probability}$$

$$= \frac{\frac{1}{12}}{\frac{6}{12}} \quad \text{Substitute.}$$

$$= \frac{1}{6} \quad \text{Simplify.}$$

So, the probability of spinning a number that is 11 given that it is an odd numbers is about 17%.

9. If two dice are rolled, what is the probability that the sum of the faces is 8, given that the first die rolled is even?

SOLUTION:

Let $P(A)$ = the probability that the first die rolled is even.
Let $P(A \text{ and } B)$ = the probability that the first die rolled is even and the sum of the faces is 8.
There are 36 possible combinations of dice rolls.
The sample space for event A contains 18 outcomes: {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.
So, $P(A) = \frac{18}{36} = \frac{1}{2}$.
The sample space for $P(A \text{ and } B)$ contains 3 outcomes: {(2, 6), (4, 4), (6, 2)}.
So, $P(A \text{ and } B) = \frac{1}{12}$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability

$$= \frac{\frac{1}{12}}{\frac{1}{2}}$$ Substitute.

$$= \frac{1}{6}$$ Simplify.

10. **PICNIC** A school picnic offers students hamburgers, hot dogs, chips, and a drink.

a. At the picnic, 60% of the students order a hamburger and 48% of the students order a hamburger and chips. What is the conditional probability that a student who orders a hamburger also orders chips?

b. If 50% of the students ordered chips, are the events of ordering a hamburger and ordering chips independent? Explain.

c. If 80% of the students who ordered a hot dog also ordered a drink and 35% of all the students ordered a hot dog, find the probability that a student at the picnic orders a hot dog and drink. Explain.

SOLUTION:

Let $P(A)$ = the probability that student orders a hamburger.
Let $P(A \text{ and } B)$ = the probability that student orders a hamburger and chips.
So, $P(A) = 60\%$ or $\frac{3}{5}$.

The sample space for $P(A \text{ and } B) = 48\%$ or $\frac{12}{25}$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$ Formula for conditional probability

$$= \frac{\frac{12}{25}}{\frac{3}{5}}$$ Substitute.

17	Solve systems of equations by using the substitution method	1 to 6	539

9-5 Solving Systems of Equations Algebraically

Part 2

Use substitution to solve each system of equations.

1. $2x - y = 9$
$x + 3y = -6$

2. $2x - y = 7$
$6x - 3y = 14$

3. $2x + y = 5$
$3x - 3y = 3$

4. $3x + y = 7$
$4x + 2y = 16$

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

6. $2x + y = 8$
$3x + \frac{3}{2}y = 12$

1. $2x - y = 9$
$x + 3y = -6$

SOLUTION:

Because the coefficient of x in Equation 2 is 1, solve for x in that equation.

$x + 3y = -6$ Equation 2
$x = -3y - 6$ Subtract $3y$ from each side.

Substitute the expression. Substitute for x. Then solve for y.

$2x - y = 9$ Equation 1
$2(-3y - 6) - y = 9$ $x = -3y - 6$
$-6y - 12 - y = 9$ Distributive Property
$-7y - 12 = 9$ Simplify.
$-7y = 21$ Add 12 to each side.
$y = -3$ Divide each side by -7.

Substitute the value of y into one of the original equations to solve for x.

$x + 3y = -6$ Equation 2
$x + 3(-3) = -6$ $y = -3$
$x - 9 = -6$ Multiply.
$x = 3$ Add.

The solution is (3, –3).

2. $2x - y = 7$
$6x - 3y = 14$

SOLUTION:

Because the coefficient of y in Equation 1 is –1, solve for y in that equation.

$2x - y = 7$ Equation 1
$-y = -2x + 7$ Subtract $2x$ from each side.
$y = 2x - 7$ Divide each side by -1.

Substitute the expression. Substitute for y. Then solve for x.

$6x - 3y = 14$ Equation 2
$6x - 3(2x - 7) = 14$ $y = 2x - 7$
$6x - 6x + 21 = 14$ Distributive Property
$21 = 14$ False

This system has no solution because $21 = 14$ is not true.

3. $2x + y = 5$
$3x - 3y = 3$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$2x + y = 5$ Equation 1
$y = -2x + 5$ Subtract $2x$ from each side.

Substitute the expression. Substitute for y. Then solve for x.

$3x - 3y = 3$ Equation 2
$3x - 3(-2x + 5) = 3$ $y = -2x + 5$
$3x + 6x - 15 = 3$ Distributive Property
$9x - 15 = 3$ Simplify.
$9x = 18$ Add 15 to each side.
$x = 2$ Divide each side by 9.

Substitute the value of x into one of the original equations to solve for y.

$2x + y = 5$ Equation 1
$2(2) + y = 5$ $x = 2$
$4 + y = 5$ Multiply.
$y = 1$ Subtract.

The solution is (2, 1).

4. $3x + y = 7$
$4x + 2y = 16$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$3x + y = 7$	Equation 1
$y = -3x + 7$	Subtract $3x$ from each side.

Substitute the expression. Substitute for y. Then solve for x.

$4x + 2y = 16$	Equation 2
$4x + 2(-3x + 7) = 16$	$y = -3x + 7$
$4x - 6x + 14 = 16$	Distributive Property
$-2x + 14 = 16$	Simplify.
$-2x = 2$	Subtract 14 from each side.
$x = -1$	Divide each side by -2.

Substitute the value of x into one of the original equations to solve for y.

$3x + y = 7$	Equation 1
$3(-1) + y = 7$	$x = -1$
$-3 + y = 7$	Multiply.
$y = 10$	Add.

The solution is (–1, 10).

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

SOLUTION:

Because the coefficient of y in Equation 1 is –1, solve for y in that equation.

$4x - y = 6$	Equation 1
$-y = -4x + 6$	Subtract $4x$ from each side.
$y = 4x - 6$	Divide each side by -1.

Substitute the expression. Substitute for y. Then solve for x.

$2x - \frac{y}{2} = 4$	Equation 2
$2x - \frac{4x - 6}{2} = 4$	$y = 4x - 6$
$2x - (2x - 3) = 4$	Simplify fraction.
$2x - 2x + 3 = 4$	Distributive Property
$3 = 4$	False

This system has no solution because 3= 4 is not true.

6. $2x + y = 8$
$3x + \frac{3}{2}y = 12$

SOLUTION:

Because the coefficient of y in Equation 1 is 1, solve for y in that equation.

$2x + y = 8$	Equation 1
$y = -2x + 8$	Subtract $2x$ from each side.

Substitute the expression. Substitute for y. Then solve for x.

$3x + \frac{3}{2}y = 12$	Equation 2
$3x + \frac{3}{2}(-2x + 8) = 12$	$y = -2x + 8$
$3x - 3x + 12 = 12$	Distributive Property
$12 = 12$	True

This system has infinitely many solutions because 12= 12 is true.

18	Solve systems of linear equations in three variables	1 to 12	557

9-8 Systems of Equations in Three Variables

Solve each system of equations.

1. $2x + 3y - z = 0$
$x - 2y - 4z = 14$
$3x + y - 8z = 17$

2. $2p - q + 4r = 11$
$p + 2q - 6r = -11$
$3p - 2q - 10r = 11$

3. $a - 2b + c = 8$
$2a + b - c = 0$
$3a - 6b + 3c = 24$

4. $3s - t - u = 5$
$3s + 2t - u = 11$
$6s - 3t + 2u = -12$

5. $2x - 4y - z = 10$
$4x - 8y - 2z = 16$
$3x + y + z = 12$

6. $p - 6q + 4r = 2$
$2p + 4q - 8r = 16$
$p - 2q = 5$

7. $2a + c = -10$
$b - c = 15$
$a - 2b + c = -5$

8. $x + y + z = 3$
$13x + 2z = 2$
$-x - 5z = -5$

9. $2m + 5n + 2p = 6$
$5m - 7n = -29$
$p = 1$

10. $f + 4g - h = 1$
$3f - g + 8h = 0$
$f + 4g - h = 10$

11. $-2c = -6$
$2a + 3b - c = -2$
$a + 2b + 3c = 9$

12. $3x - 2y + 2z = -2$
$x + 6y - 2z = -2$
$x + 2y = 0$

1. $x - 2y - 4z = 14$
$3x + y - 8z = 17$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 2 by –2 to eliminate x.

$x - 2y - 4z = 14$	Equation 2
$-2(x - 2y - 4z) = (-2)14$	Multiply each side by -2.
$-2x + 4y + 8z = -28$	Multiply.

Add Equations 1 and 2 to eliminate x.

$2x + 3y - z = 0$	Equation 1
$-2x + 4y + 8z = -28$	Equation 2
$7y + 7z = -28$	Add the equations.

Multiply Equation 2 by –3 to eliminate x.

$x - 2y - 4z = 14$	Equation 2
$-3(x - 2y - 4z) = (-3)14$	Multiply each side by -3.
$-3x + 6y + 12z = -42$	Multiply.

Add Equations 2 and 3 to eliminate x.

$-3x + 6y + 12z = -42$	Equation 2
$3x + y - 8z = 17$	Equation 3
$7y + 4z = -25$	Add the equations.

Add reduced equations to eliminate y.

$7y + 7z = -28$	Reduced Equation 1
$7y + 4z = -25$	Reduced Equation 2
$3z = -3$	Subtract the second equatio
$z = -1$	Divide each side by 3.

Use substitution to solve for y.

$7y + 7z = -28$	Reduced Equation 1
$7y + 7(-1) = -28$	Substitution
$7y - 7 = -28$	Multiply.
$7y = -21$	Add 7 to each side.
$y = -3$	Divide each side by 7.

Use substitution to solve for x.

$2x + 3y - z = 0$	Equation 1
$2x + 3(-3) - (-1) = 0$	Substitution
$2x - 9 + 1 = 0$	Multiply.
$2x - 8 = 0$	Simplify.
$2x = 8$	Add 8 to each side.
$x = 4$	Divide each side by 2.

The ordered triple is (4, –3, –1).

$2p - q + 4r = 11$
2. $p + 2q - 6r = -11$
$3p - 2q - 10r = 11$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 2 by –2 to eliminate p.

$p + 2q - 6r = -11$	Equation 2
$-2(p + 2q - 6r) = -2(-11)$	Multiply each side by negative 2.
$-2p - 4q + 12r = 22$	Multiply.

Add Equations 1 and 2 to eliminate p.

$2p - q + 4r = 11$	Equation 1
$-2p - 4q + 12r = 22$	Equation 2
$-5q + 16r = 33$	Add the equations.

Multiply Equation 2 by –3 to eliminate p.

$p + 2q - 6r = -11$	Equation 2
$-3(p + 2q - 6r) = (-3) - 11$	Multiply each side by -3.
$-3p - 6q + 18r = 33$	Multiply.

Add Equations 2 and 3 to eliminate x.

$-3p - 6q + 18r = 33$	Equation 2
$3p - 2q - 10r = 11$	Equation 3
$-8q + 8r = 44$	Add the equations.

Multiply Reduced Equation 2 by –2.

$-8q + 8r = 44$	Reduced Equation 2
$-2(-8q + 8r) = (-2)44$	Multiply each side by -2.
$16q - 16r = -88$	Multiply.

Add reduced equations to eliminate r.

$-5q + 16r = 33$	Reduced Equation 1
$16q - 16r = -88$	Reduced Equation 2
$11q = -55$	Add the equations.
$q = -5$	Divide each side by 11.

Use substitution to solve for r.

$-5q + 16r = 33$	Reduced Equation 1
$-5(-5) + 16r = 33$	Substitution
$25 + 16r = 33$	Multiply.
$16r = 8$	Subtract 25 from each side.
$r = \frac{1}{2}$	Divide each side by 16.

Use substitution to solve for p.

$p + 2q - 6r = -11$	Equation 2
$p + 2(-5) - 6\left(\frac{1}{2}\right) = -11$	Substitution
$p - 10 - 3 = -11$	Multiply.
$p - 13 = -11$	Simplify.
$p = 2$	Add 13 to each side.

The ordered triple is $\left(2, -5, \frac{1}{2}\right)$.

3. $a - 2b + c = 8$
$2a + b - c = 0$
$3a - 6b + 3c = 24$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 1 by –3 and add the equations to eliminate *y*.

$a - 2b + c = 8$	Equation 1
$-3(a - 2b + c) = -3(8)$	Multiply each side by -3.
$-3a + 6b - 3c = -24$	Multiply.

Add Equations 1 and 3 to eliminate *a*.

$-3a + 6b - 3c = -24$	Equation 1
$3a + 6b + 3c = 24$	Equation 3
$0 = 0$	Add the equations.

The equation 0 = 0 is always true. This indicates that the first and third equations represent the same plane.

Check the third plane. Add Equation 1 and 2 to eliminate *c*.

$a - 2b + c = 8$	Equation 1
$2a + b - c = 0$	Equation 2
$3a - b = 8$	Add the equations.

The planes intersect in a line, because the resultant equation is in two variables. So, there are an infinite number of solutions.

4. $3s - t - u = 5$
$3s + 2t - u = 11$
$6s - 3t + 2u = -12$

SOLUTION:

Select two of the equations and eliminate one of the variables.

Subtract Equation 2 from Equation 1 to eliminate *s*.

$3s - t - u = 5$	Equation 1
$3s + 2t - u = 11$	Equation 2
$-3t = -6$	Subtract the equations.
$t = 2$	Divide each side by -3.

Multiply Equation 2 by 2 to eliminate *u*.

$3s + 2t - u = 11$	Equation 2
$2(3s + 2t - u) = (2)11$	Multiply each side by 2.
$6s + 4t - 2u = 22$	Multiply.

Add Equations 2 and 3 to eliminate *u*.

$6s + 4t - 2u = 22$	Equation 2
$6s - 3t + 2u = -12$	Equation 3
$12s + t = 10$	Add the equations.

Use substitution to solve for *t*.

$12s + t = 10$	Reduced Equation 2
$12s + 2 = 10$	Substitution
$12s = 8$	Subtract 2 from each side.
$s = \frac{8}{12} = \frac{2}{3}$	Divide each side by 12.

Use substitution to solve for *u*.

$3s - t - u = 5$	Equation 1
$3\left(\frac{2}{3}\right) - 2 - u = 5$	Substitution
$2 - 2 - u = 5$	Multiply.
$-u = 5$	Subract.
$u = -5$	Divide each side by -1.

The ordered triple is $\left(\frac{2}{3}, 2, -5\right)$.

5. $\begin{aligned} 2x - 4y - z &= 10 \\ 4x - 8y - 2z &= 16 \\ 3x + y + z &= 12 \end{aligned}$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 1 by –2 and add the equations to eliminate x.

$2x - 4y - z = 10$	Equation 1
$-2(2x - 4y - z) = -2(10)$	Multiply each side by -2.
$-4x + 8y + 2z = -20$	Multiply.

Add Equations 1 and 2 to eliminate x.

$-4x + 8y + 2z = -20$	Equation 1
$4x - 8y - 2z = 16$	Equation 2
$0 = -4$	Add the equations.

The equation 0 = –4 is false. This indicates that the first and second equations have no solution. Since the first two equations have no solution, that implies there can be no solution for all three planes.

6. $\begin{aligned} p - 6q + 4r &= 2 \\ 2p + 4q - 8r &= 16 \\ p - 2q &= 5 \end{aligned}$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 1 by 2 and add to Equation 2 to eliminate r.

$p - 6q + 4r = 2$	Equation 1
$2(p - 6q + 4r) = 2(2)$	Multiply each side by 2.
$2p - 12q + 8r = 4$	Multiply.

$2p - 12q + 8r = 4$	Equation 1
$2p + 4q - 8r = 16$	Equation 2
$4p - 8q = 20$	Add the equations.

Multiply Equation 3 by –4.

$p - 2q = 5$	Equation 3
$-4(p - 2q) = -4(5)$	Multiply each side by -4.
$-4p + 8q = -20$	Multiply.

Add Equation 3 and Reduced Equation 1 to eliminate p.

$-4p + 8q = -20$	Equation 3
$4p - 8q = 20$	Reduced Equation 1
$0 = 0$	Add the equations.

The equation 0 = 0 is always . This indicates that Equation 3 and Reduced Equation 1 represent the same plane.

The planes intersect in a line, because the resultant equation from Equations 1 and 2 is in two variables. So, there are an infinite number of solutions.

7. $2a + c = -10$
$b - c = 15$
$a - 2b + c = -5$

SOLUTION:

Select two of the equations and eliminate one of the variables. Add Equation 1 and 2 to eliminate c.

$2a + 0b + c = -10$	Equation 1
$0a + b - c = 15$	Equation 2
$2a + b = 5$	Add the equations.

Add Equation 2 and 3 to eliminate c.

$0a + b - c = 15$	Equation 2
$a - 2b + c = -5$	Equation 3
$a - b = 10$	Add the equations.

Add reduced equations to eliminate b.

$2a + b = 5$	Reduced Equation 1
$a - b = 10$	Reduced Equation 2
$3a = 15$	Add the equations.
$a = 5$	Divide each side by 3.

Use substitution to solve for b.

$a - b = 10$	Reduced Equation 2
$5 - b = 10$	Substitution
$-b = 5$	Subtract 5 from each side.
$b = -5$	Divide each side by -1.

Use substitution to solve for c.

$b - c = 15$	Equation 2
$-5 - c = 15$	Substitution
$-c = 20$	Add 5 to each side.

The ordered triple is (5, –5, –20).

$x + y + z = 3$
8. $13x + 2z = 2$
$-x - 5z = -5$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 3 by 13 to eliminate x.

$-x - 5z = -5$	Equation 3
$13(-x - 5z) = 13(-5)$	Multiply each side by 13.
$-13x - 65z = -65$	Multiply.

Add Equations 2 and 3 to eliminate x.

$13x + 2z = 2$	Equation 2
$-13x - 65z = -65$	Equation 3
$-63z = -63$	Add the equations.
$z = 1$	Divide each side by -63.

Use substitution to solve for x.

$-x - 5z = -5$	Equation 3
$-x - 5(1) = -5$	Substitution
$-x - 5 = -5$	Multiply.
$-x = 0$	Add 5 to each side.
$x = 0$	Divide each side by -1.

Use substitution to solve for y.

$x + y + z = 3$	Equation 1
$0 + y + 1 = 3$	Substitution
$y + 1 = 3$	Simplify.
$y = 2$	Subtract 1 from each side.

The ordered triple is (0, 2, 1).

9. $$\begin{aligned} 2m + 5n + 2p &= 6 \\ 5m - 7n &= -29 \\ p &= 1 \end{aligned}$$

SOLUTION:

Select two of the equations and eliminate one of the variables. Substitute the value of p into Equation 1 to eliminate p.

$2m + 5n + 2p = 6$	Equation 1
$2m + 5n + 2(1) = 6$	Substitute 1 for p.
$2m + 5n + 2 = 6$	Multiply.
$2m + 5n = 4$	Subtract 2 from each side.

Multiply Equation 2 by 2.

$5m - 7n = -29$	Equation 2
$2(5m - 7n) = 2(-29)$	Multiply by 2.
$10m - 14n = -58$	Multiply.

Multiply Reduced Equation 1 by –5.

$2m + 5n = 4$	Reduced Equation 1
$-5(2m + 5n) = -5(4)$	Multiply by -5.
$-10m - 25n = -20$	Multiply.

Add Equation 2 and Reduced Equation 1 to eliminate m.

$10m - 14n = -58$	Equation 2
$-10m - 25n = -20$	Reduced Equation 1
$-39n = -78$	Add the equations.
$n = 2$	Divide each side by -39.

Use substitution to solve for m.

$5m - 7n = -29$	Equation 2
$5m - 7(2) = -29$	Substitution
$5m - 14 = -29$	Multiply.
$5m = -15$	Add 14 to each side.
$m = -3$	Divide each side by 5.

The ordered triple is (–3, 2, 1).

10. $$\begin{aligned} f + 4g - h &= 1 \\ 3f - g + 8h &= 0 \\ f + 4g - h &= 10 \end{aligned}$$

SOLUTION:

Select two of the equations and eliminate one of the variables. Multiply Equation 1 by –1 and add the equations to eliminate f.

$f + 4g - h = 1$	Equation 1
$-1(f + 4g - h) = -1(1)$	Multiply each side by -1.
$-f - 4g + h = -1$	Multiply.

Add Equations 1 and 3 to eliminate f.

$-f - 4g + h = -1$	Equation 1
$f + 4g - h = 10$	Equation 3
$0 = 9$	Add the equations.

The equation 0 = 9 is false. This indicates that the first and third equations have no solution. Since these two equations have no solution, that implies there can be no solution for all three planes.

$-2c = -6$

11. $2a + 3b - c = -2$

$a + 2b + 3c = 9$

SOLUTION:

Solve Equation 1 for c.

$-2c = -6$ Equation 1

$c = 3$ Divide each side by -2.

Multiply Equation 3 by –2.

$a + 2b + 3c = 9$ Equation 3

$-2(a + 2b + 3c) = -2(9)$ Multiply by -2.

$-2a - 4b - 6c = -18$ Multiply.

Add Equation 2 and 3 to eliminate a.

$2a + 3b - c = -2$ Equation 2

$-2a - 4b - 6c = -18$ Equation 3

$-b - 7c = -20$ Add the equations.

Use substitution to solve for b.

$-b - 7c = -20$ Reduced Equation 1

$-b - 7(3) = -20$ Substitution

$-b - 21 = -20$ Multiply.

$-b = 1$ Add 21 to each side.

$b = -1$ Divide each side by -1.

Use substitution to solve for a.

$a + 2b + 3c = 9$ Equation 3

$a + 2(-1) + 3(3) = 9$ Substitution

$a - 2 + 9 = 9$ Multiply.

$a + 7 = 9$ Simplify.

$a = 2$ Subtract 7 from each side.

The ordered triple is (2, –1, 3).

$3x - 2y + 2z = -2$

12. $x + 6y - 2z = -2$

$x + 2y = 0$

SOLUTION:

Select two of the equations and eliminate one of the variables. Add Equations 1 and 2 to eliminate z.

$3x - 2y + 2z = -2$ Equation 1

$x + 6y - 2z = -2$ Equation 2

$4x + 4y = -4$ Add the equations.

Multiply equation 3 by –4.

$x + 2y = 0$ Equation 3

$-4(x + 2y) = -4(0)$ Multiply each side by -4.

$-4x - 8y = 0$ Multiply.

Add reduced Equation 1 and Equation 3 to find y.

$4x + 4y = -4$ Reduced Equation 1

$-4x - 8y = 0$ Equation 3.

$-4y = -4$ Add the equations.

$y = 1$ Divide each side by -4.

Use substitution to solve for x.

$x + 2y = 0$ Equation 3

$x + 2(1) = 0$ Substitution

$x + 2 = 0$ Multiply.

$x = -2$ Subtract 2 from each side.

Use substitution to solve for z.

$x + 6y - 2z = -2$ Equation 2

$-2 + 6(1) - 2z = -2$ Substitution

$-2 + 6 - 2z = -2$ Multiply.

$4 - 2z = -2$ Add -2 and 6.

$-2z = -6$ Subtract 4 from each side.

$z = 3$ Divide each side by -2.

The ordered triple is (–2, 1, 3).

19	Graph quadratic functions	1 to 6	9

1-1 Graphing Quadratic Functions

Part 2

Example 1

Graph each function. Then state the domain and range.

1. $f(x) = x^2 + 6x + 8$

2. $f(x) = -x^2 - 2x + 2$

3. $f(x) = 2x^2 - 4x + 3$

4. $f(x) = -2x^2$

5. $f(x) = x^2 - 4x + 4$

6. $f(x) = x^2 - 6x + 8$

For $f(x) = x^2 + 6x + 8$, $a = 1$, $b = 6$, and $c = 8$. c is the y-intercept, so the y-intercept is 8.

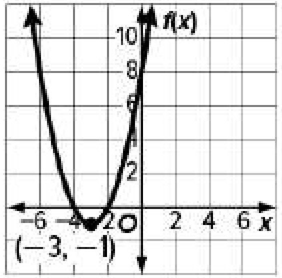
Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry.}$$
$$= -\frac{6}{2(1)} \quad a = 1, b = 6$$
$$= -3 \quad \text{Simplify.}$$

The equation of the axis of symmetry is $x = -3$, so the x-coordinate of the vertex is -3. Because $a > 0$, the vertex is a minimum.

Graph the function.

x	$x^2 + 6x + 8$	$(x, f(x))$
−5	$(-5)^2 + 6(-5) + 8$	3
−4	$(-4)^2 + 6(-4) + 8$	0
−3	$(-3)^2 + 6(-3) + 8$	−1
−2	$(-2)^2 + 6(-2) + 8$	0
−1	$(-1)^2 + 6(-1) + 8$	3



Analyze the graph.

The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq -1\}$.

For $f(x) = -x^2 - 2x + 2$, $a = -1$, $b = -2$, and $c = 2$. c is the y-intercept, so the y-intercept is 2.

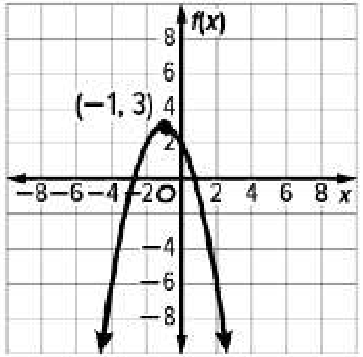
Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry.}$$
$$= -\frac{-2}{2(-1)} \quad a = -1, b = -2$$
$$= -1 \quad \text{Simplify.}$$

The equation of the axis of symmetry is $x = -1$, so the x-coordinate of the vertex is -1. Because $a < 0$, the vertex is a maximum.

Graph the function.

x	$-x^2 - 2x + 2$	$(x, f(x))$
−3	$-(-3)^2 - 2(-3) + 2$	−1
−2	$-(-2)^2 - 2(-2) + 2$	2
−1	$-(-1)^2 - 2(-1) + 2$	3
0	$-(0)^2 - 2(0) + 2$	2
1	$-(1)^2 - 2(1) + 2$	−1



The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \leq 3\}$.

For $f(x) = 2x^2 - 4x + 3$, $a = 2$, $b = -4$, and $c = 3$. c is the y-intercept, so the y-intercept is 3.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

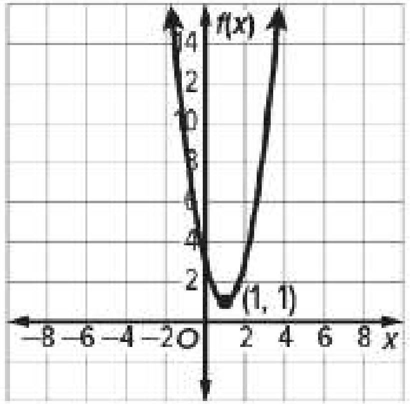
$= -\frac{-4}{2(2)}$ $a = 2, b = -4$

$= 1$ Simplify.

The equation of the axis of symmetry is $x = 1$, so the x-coordinate of the vertex is 1. Because $a > 0$, the vertex is a minimum.

Graph the function.

x	$2x^2 - 4x + 3$	(x, f(x))
−1	$2(-1)^2 - 4(-1) + 3$	9
0	$2(0)^2 - 4(0) + 3$	3
1	$2(1)^2 - 4(1) + 3$	1
2	$2(2)^2 - 4(2) + 3$	3
3	$2(3)^2 - 4(3) + 3$	9



The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq 1\}$.

For $f(x) = -2x^2$, $a = -2$, $b = 0$, and $c = 0$. c is the y-intercept, so the y-intercept is 0.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

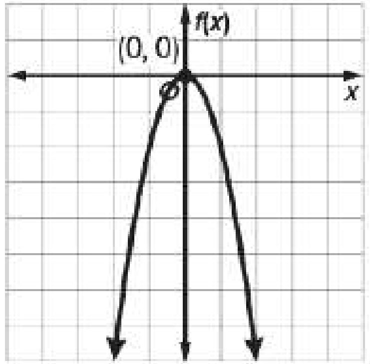
$= -\frac{0}{2(-2)}$ $a = -2, b = 0$

$= 0$ Simplify.

The equation of the axis of symmetry is $x = 0$, so the x-coordinate of the vertex is 0. Because $a < 0$, the vertex is a maximum.

Graph the function.

x	$-2x^2$	(x, f(x))
−2	$-2(-2)^2$	−8
−1	$-2(-1)^2$	−2
0	$-2(0)^2$	0
1	$-2(1)^2$	−2
2	$-2(2)^2$	−8



The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \leq 0\}$.

5. $f(x) = x^2 - 4x + 4$

SOLUTION:

For $f(x) = x^2 - 4x + 4$, $a = 1$, $b = -4$, and $c = 4$. c is the y-intercept, so the y-intercept is 4.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

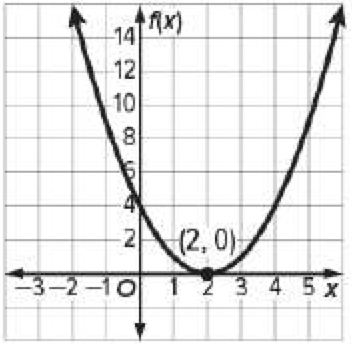
$= -\frac{-4}{2(1)}$ $a = 1, b = -4$

$= 2$ Simplify.

The equation of the axis of symmetry is $x = 2$, so the x-coordinate of the vertex is 2. Because $a > 0$, the vertex is a minimum.

Graph the function.

x	$x^2 - 4x + 4$	$(x, f(x))$
0	$(0)^2 - 4(0) + 4$	4
1	$(1)^2 - 4(1) + 4$	1
2	$(2)^2 - 4(2) + 4$	0
3	$(3)^2 - 4(3) + 4$	1
4	$(4)^2 - 4(4) + 4$	4



Analyze the graph.
The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq 0\}$

6. $f(x) = x^2 - 6x + 8$

SOLUTION:

For $f(x) = x^2 - 6x + 8$, $a = 1$, $b = -6$, and $c = 8$. c is the y-intercept, so the y-intercept is 8.

Find the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry.

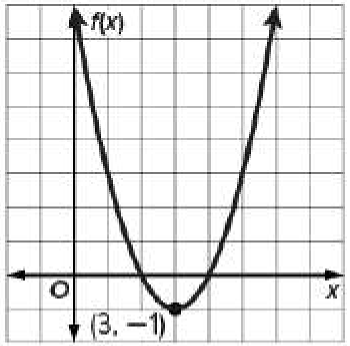
$= -\frac{-6}{2(1)}$ $a = 1, b = -6$

$= 3$ Simplify.

The equation of the axis of symmetry is $x = 3$, so the x-coordinate of the vertex is 3. Because $a > 0$, the vertex is a minimum.

Graph the function.

x	$x^2 - 6x + 8$	$(x, f(x))$
1	$(1)^2 - 6(1) + 8$	3
2	$(2)^2 - 6(2) + 8$	0
3	$(3)^2 - 6(3) + 8$	−1
4	$(4)^2 - 6(4) + 8$	0
5	$(5)^2 - 6(5) + 8$	3



Analyze the graph.
The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq -1\}$.

20	Solve quadratic equations by factoring	1 to 6	31

1-4 Solving Quadratic Equations by Factoring

Part 2

Solve each equation by factoring. Check your solution.

1. $6x^2 - 2x = 0$

2. $x^2 = 7x$

3. $20x^2 = -25x$

4. $x^2 + x - 30 = 0$

5. $x^2 + 14x + 33 = 0$

6. $x^2 - 3x = 10$

1. $6x^2 - 2x = 0$

SOLUTION:

$6x^2 - 2x = 0$	Original equation
$2x(3x) - 2x(1) = 0$	Factor the GCF.
$2x(3x - 1) = 0$	Distributive Property
$2x = 0$ or $3x - 1 = 0$	Zero Product Property
$x = 0 \quad x = \frac{1}{3}$	Solve.

2. $x^2 = 7x$

SOLUTION:

$x^2 = 7x$	Original equation
$x^2 - 7x = 0$	Subtract $7x$ from each side.
$x(x) - x(7) = 0$	Factor the GCF.
$x(x - 7) = 0$	Distributive Property
$x = 0$ or $x - 7 = 0$	Zero Product Property
$x = 0 \quad x = 7$	Solve.

3. $20x^2 = -25x$

SOLUTION:

$20x^2 = -25x$	Original equation
$20x^2 + 25x = 0$	Add $25x$ to each side.
$5x(4x) + 5x(5) = 0$	Factor the GCF.
$5x(4x + 5) = 0$	Distributive Property
$5x = 0$ or $4x + 5 = 0$	Zero Product Property
$x = 0 \quad x = -\frac{5}{4}$	Solve.

4. $x^2 + x - 30 = 0$

SOLUTION:

$x^2 + x - 30 = 0$	Original equation
$(x + 6)(x - 5) = 0$	Factor the trinomial.
$x + 6 = 0$ or $x - 5 = 0$	Zero Product Property
$x = -6 \quad x = 5$	Solve.

5. $x^2 + 14x + 33 = 0$

SOLUTION:

$x^2 + 14x + 33 = 0$	Original equation
$(x + 11)(x + 3) = 0$	Factor the trinomial.
$x + 11 = 0$ or $x + 3 = 0$	Zero Product Property
$x = -11 \quad x = -3$	Solve.

6. $x^2 - 3x = 10$

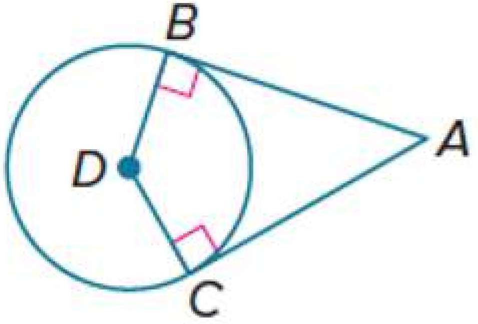
SOLUTION:

$x^2 - 3x = 10$	Original equation
$x^2 - 3x - 10 = 0$	Subtract 10 from each side.
$(x - 5)(x + 2) = 0$	Factor the trinomial.
$x - 5 = 0$ or $x + 2 = 0$	Zero Product Property
$x = 5 \quad x = -2$	Solve.

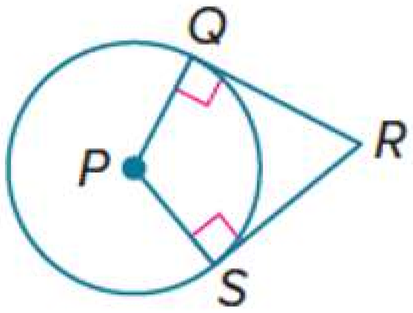
21	Describe relationships between central and circumscribed angles, and use those relationships to solve problems	19 to 26	260

PART #3

19. If $m\angle BDC = 12x°$ and $m\angle A = (4x + 4)°$, find $m\angle A$.



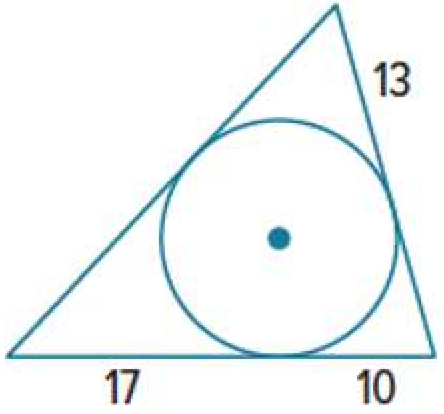
20. If $m\angle QPS = (15x + 8)°$ and $m\angle R = (10x - 3)°$, find $m\angle R$.



Example 6

Each polygon is circumscribed about a circle. Find the perimeter of each polygon.

21.



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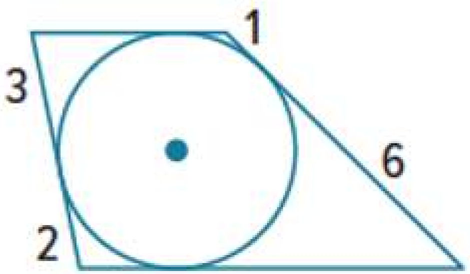

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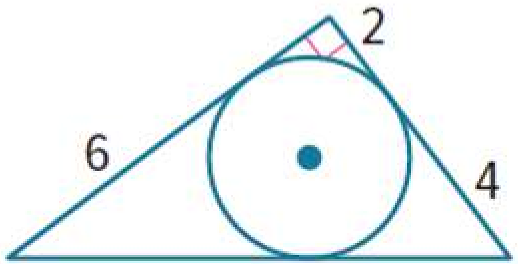
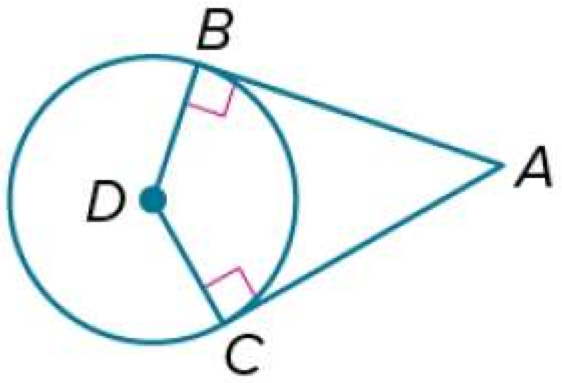


19. If $m\angle BDC = 12x°$ and $m\angle A = (4x + 4)°$, find $m\angle A$.



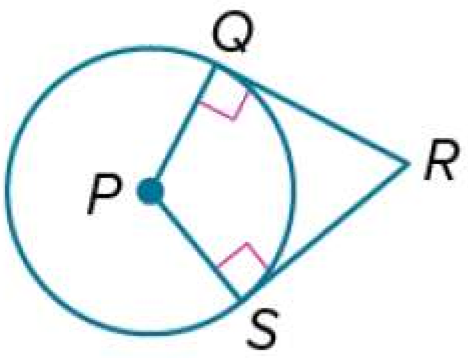
SOLUTION:

Because $\overline{BA}$ and $\overline{CA}$ are tangent to circle D, $m\angle BDC$ and $\angle A$ are supplementary.

$m\angle BDC + m\angle A = 180$ Definition of supplementary angles
$12x + 4x + 4 = 180$ Substitution
$16x + 4 = 180$ Simplify.
$16x = 176$ Subtract 4 from each side.
$x = 11$ Divide each side by 16.

So, $m\angle A = 4(11) + 4$ or 48°.

20. If $m\angle QPS = (15x + 8)°$ and $m\angle R = (10x - 3)°$, find $m\angle R$.



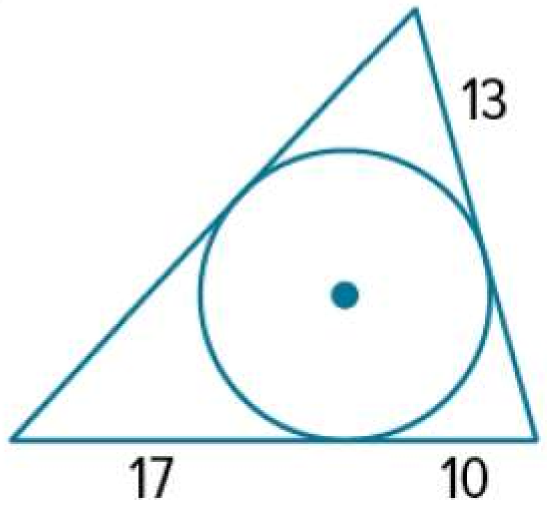
SOLUTION:

Because $\overline{QR}$ and $\overline{SR}$ are tangent to circle P, $m\angle QPS$ and $\angle R$ are supplementary.

$m\angle QPS + m\angle R = 180$ Definition of supplementary angles
$15x + 8 + 10x - 3 = 180$ Substitution
$25x + 5 = 180$ Simplify.
$25x = 175$ Subtract 5 from each side.
$x = 7$ Divide each side by 25.

So, $m\angle R = 10(7) - 3$ or 67°.

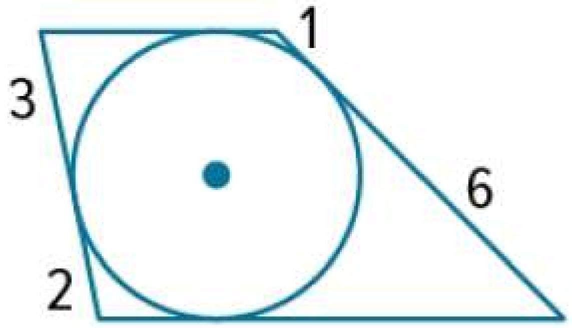
21.



SOLUTION:

Perimeter = 2(17) + 2(10) + 2(13) = 34 + 20 + 26 = 80.

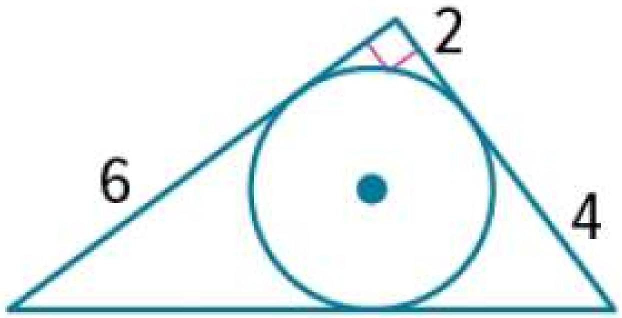
22.



SOLUTION:

Perimeter = 2(3) + 2(1) + 2(2) + 2(6) = 6 + 2 + 4 + 12 = 24

23.



SOLUTION:

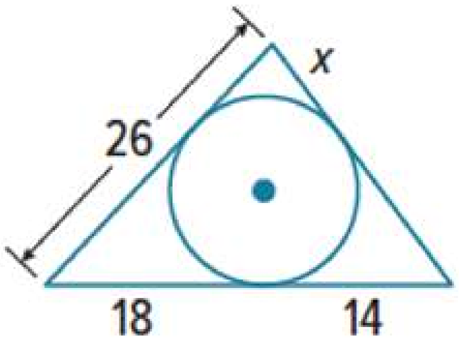
Perimeter = 2(2) + 2(4) + 2(6) = 4 + 8 + 12 = 24

21	Describe relationships between central and circumscribed angles, and use those relationships to solve problems	19 to 26	260

PART #3

Each polygon is circumscribed about a circle. Find the value of *x*. Then find the perimeter of each polygon.

24.



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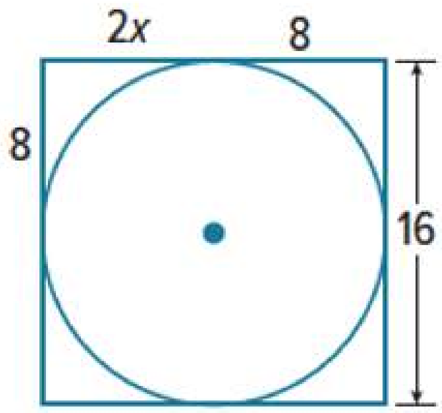

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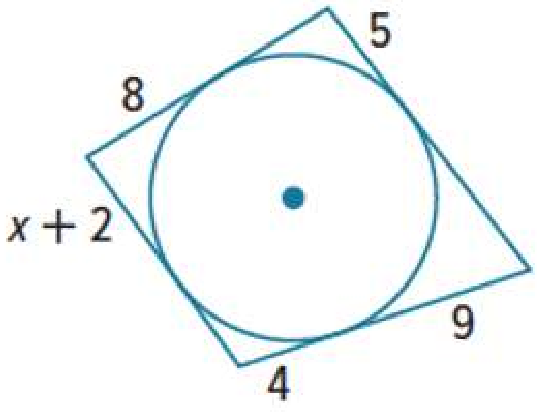

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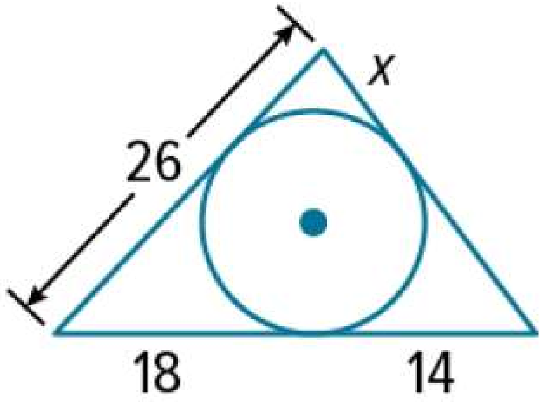
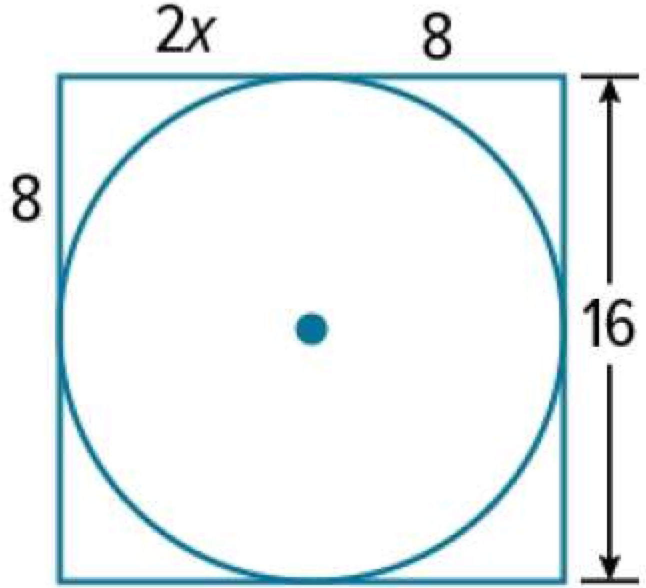


SOLUTION:

$x = 26 - 18$, or 8

Perimeter = 2(8) + 2(14) + 2(18) = 16 + 28 + 36 = 80

25.

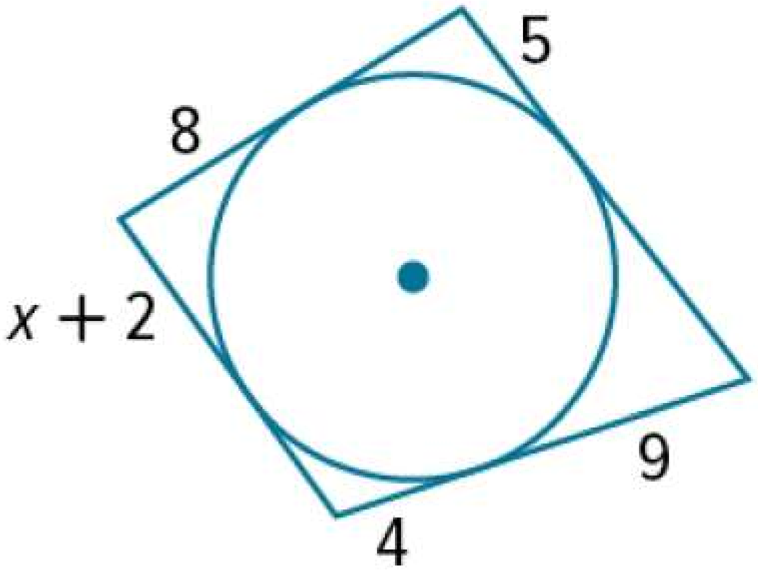


SOLUTION:

$2x = 8$, so $x = 4$

Perimeter = 8(8) = 64

26.



SOLUTION:

$x + 2 = 8$, so $x = 6$

Perimeter = 2(8) + 2(5) + 2(4) + 2(9) = 52

22	Find and interpret the average rate of change of quadratic functions given symbolically, in tables, and in graphs	13 to 24	10

1-1 Graphing Quadratic Functions

PART #3

Determine the average rate of change of $f(x)$ over the specified interval.

13. $f(x) = x^2 - 10x + 5$; interval $[-4, 4]$

14. $f(x) = 2x^2 + 4x - 6$; interval $[-3, 3]$

15. $f(x) = 3x^2 - 3x + 1$; interval $[-5, 5]$

16. $f(x) = 4x^2 + x + 3$; interval $[-2, 2]$

17. $f(x) = 2x^2 - 11$; interval $[-3, 3]$

18. $f(x) = -2x^2 + 8x + 7$; interval $[-4, 4]$

13. $f(x) = x^2 - 10x + 5$; interval [–4, 4]

SOLUTION:

The average rate of change is equal to $\frac{f(4) - f(-4)}{4 - (-4)}$.

First find $f(4)$ and $f(-4)$.

$f(4) = (4)^2 - 10(4) + 5$ or -19

$f(-4) = (-4)^2 - 10(-4) + 5$ or 61

Then substitute to find the average rate of change.

$$\frac{f(4) - f(-4)}{4 - (-4)} = \frac{-19 - 61}{4 - (-4)} = -10$$

The average rate of change of the function over the interval [–4, 4] is –10.

14. $f(x) = 2x^2 + 4x - 6$; interval [–3, 3]

SOLUTION:

The average rate of change is equal to $\frac{f(3) - f(-3)}{3 - (-3)}$.

First find $f(3)$ and $f(-3)$.

$f(3) = 2(3)^2 + 4(3) - 6$ or 24

$f(-3) = 2(-3)^2 + 4(-3) - 6$ or 0

Then substitute to find the average rate of change.

$$\frac{f(3) - f(-3)}{3 - (-3)} = \frac{24 - 0}{3 - (-3)} = 4$$

The average rate of change of the function over the interval [–3, 3] is 4.

15. $f(x) = 3x^2 - 3x + 1$; interval [–5, 5]

SOLUTION:

The average rate of change is equal to $\frac{f(5) - f(-5)}{5 - (-5)}$.

First find $f(5)$ and $f(-5)$.

$f(5) = 3(5)^2 - 3(5) + 1$ or 61

$f(-5) = 3(-5)^2 - 3(-5) + 1$ or 91

Then substitute to find the average rate of change.

$$\frac{f(5) - f(-5)}{5 - (-5)} = \frac{61 - 91}{5 - (-5)} = -3$$

The average rate of change of the function over the interval [–5, 5] is –3.

16. $f(x) = 4x^2 + x + 3$; interval [–2, 2]

SOLUTION:

The average rate of change is equal to $\frac{f(2)-f(-2)}{2-(-2)}$.

First find $f(2)$ and $f(-2)$.

$f(2) = 4(2)^2 + (2) + 3$ or 21

$f(-2) = 4(-2)^2 + (-2) + 3$ or 17

Then substitute to find the average rate of change.

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{21-17}{2-(-2)} = 1$$

The average rate of change of the function over the interval [–2, 2] is 1.

17. $f(x) = 2x^2 - 11$; interval [–3, 3]

SOLUTION:

The average rate of change is equal to $\frac{f(3)-f(-3)}{3-(-3)}$.

First find $f(3)$ and $f(-3)$.

$f(3) = 2(3)^2 - 11$ or 7

$f(-3) = 2(-3)^2 - 11$ or 7

Then substitute to find the average rate of change.

$$\frac{f(3)-f(-3)}{3-(-3)} = \frac{7-7}{3-(-3)} = 0$$

The average rate of change of the function over the interval [–3, 3] is 0.

18. $f(x) = -2x^2 + 8x + 7$; interval [–4, 4]

SOLUTION:

The average rate of change is equal to $\frac{f(4)-f(-4)}{4-(-4)}$.

First find $f(4)$ and $f(-4)$.

$f(4) = -2(4)^2 + 8(4) + 7$ or 7

$f(-4) = -2(-4)^2 + 8(-4) + 7$ or -57

Then substitute to find the average rate of change.

$$\frac{f(4)-f(-4)}{4-(-4)} = \frac{7-(-57)}{4-(-4)} = 8$$

The average rate of change of the function over the interval [–4, 4] is 8.

23	Use permutations to compute probabilities	5 to 7	393

1-1 Graphing Quadratic Functions

PART #3

Determine the average rate of change of $f(x)$ over the specified interval.

19. interval [–3, 3]

x	$f(x)$
–3	0
–2	3
–1	–4
0	–3
1	0
2	5
3	12

20. interval [–4, 4]

x	$f(x)$
–4	–27
–2	–3
0	5
2	–3
4	–27

21. interval [–2, 2]

x	$f(x)$
–2	–3
–1	–3
0	–1
1	3
2	9

22. interval [–5, 5]

x	$f(x)$
–5	–39
–3	–15
–1	1
0	6
1	9
3	9
5	1

23. interval [–3, 3]

x	$f(x)$
–3	27
–2	12
–1	3
0	0
1	3
2	12
3	27

24. interval [–2, 2]

x	$f(x)$
–2	12
–1	5
0	0
1	–3
2	–4

19. interval [–3, 3]

x	$f(x)$
–3	0
–2	3
–1	–4
0	–3
1	0
2	5
3	12

SOLUTION:

The average rate of change is equal to $\frac{f(3)-f(-3)}{3-(-3)}$.

First find $f(3)$ and $f(-3)$ from the table.

$f(3) = 12$
$f(-3) = 0$

Then substitute to find the average rate of change.

$$\frac{f(3)-f(-3)}{3-(-3)} = \frac{12-(0)}{3-(-3)} = 2$$

The average rate of change of the function over the interval [–3, 3] is 2.

20. interval [–4, 4]

x	$f(x)$
–4	–27
–2	–3
0	5
2	–3
4	–27

SOLUTION:

The average rate of change is equal to $\frac{f(4)-f(-4)}{4-(-4)}$.

First find $f(4)$ and $f(-4)$ from the table.

$f(4) = -27$
$f(-4) = -27$

Then substitute to find the average rate of change.

$$\frac{f(4)-f(-4)}{4-(-4)} = \frac{-27-(-27)}{4-(-4)} = 0$$

The average rate of change of the function over the interval [–4, 4] is 0.

21. interval [–2, 2]

x	$f(x)$
–2	–3
–1	–3
0	–1
1	3
2	9

SOLUTION:

The average rate of change is equal to $\frac{f(2)-f(-2)}{2-(-2)}$.

First find $f(2)$ and $f(-2)$ from the table.

$f(2) = 9$
$f(-2) = -3$

Then substitute to find the average rate of change.

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{9-(-3)}{2-(-2)} = 3$$

The average rate of change of the function over the interval [–2, 2] is 3.

22. interval [–5, 5]

x	$f(x)$
–5	–39
–3	–15
–1	1
0	6
1	9
3	9
5	1

SOLUTION:

The average rate of change is equal to $\frac{f(5)-f(-5)}{5-(-5)}$.

First find $f(5)$ and $f(-5)$ from the table.

$f(5) = 1$
$f(-5) = -39$

Then substitute to find the average rate of change.

$$\frac{f(5)-f(-5)}{5-(-5)} = \frac{1-(-39)}{5-(-5)} = 4$$

The average rate of change of the function over the interval [–5, 5] is 4.

23. interval [–3, 3]

x	$f(x)$
–3	27
–2	12
–1	3
0	0
1	3
2	12
3	27

SOLUTION:

The average rate of change is equal to $\frac{f(3)-f(-3)}{3-(-3)}$.

First find $f(3)$ and $f(-3)$ from the table.

$f(3) = 27$
$f(-3) = 27$

Then substitute to find the average rate of change.

$$\frac{f(3)-f(-3)}{3-(-3)} = \frac{27-27}{3-(-3)} = 0$$

The average rate of change of the function over the interval [–3, 3] is 0.

24. interval [–2, 2]

x	$f(x)$
–2	12
–1	5
0	0
1	–3
2	–4

SOLUTION:

The average rate of change is equal to $\frac{f(2)-f(-2)}{2-(-2)}$.

First find $f(2)$ and $f(-2)$ from the table.

$f(2) = -4$
$f(-2) = 12$

Then substitute to find the average rate of change.

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{-4-(12)}{2-(-2)} = -4$$

The average rate of change of the function over the interval [–2, 2] is –4.

23	Use permutations to compute probabilities	5 to 7	393

7-4 Probability with Permutations and Combinations

PART #3

5. **PHONE NUMBERS** What is the probability that a 7-digit telephone number generated using the digits 2, 3, 2, 5, 2, 7, and 3 is the number 222-3357?

6. **IDENTIFICATION** A store randomly assigns their employees work identification numbers to track productivity. Each number consists of 5 digits ranging from 1-9. If the digits cannot repeat, find the probability that a randomly generated number is 25938.

7. **STUDENT COUNCIL** The table shows the finalists for class president. The order in which they will give their speeches will be chosen randomly.

   a. What is the probability that Denny, Kelli, and Chaminade are the first 3 speakers, in any order?

   b. What is the probability that Denny is first, Kelli is second, and Chaminade is third?

Class President Finalists
Alan Shepherd
Chaminade Hudson
Denny Murano
Kelli Baker
Tanika Johnson
Jerome Murdock
Marlene Lindeman

5. **PHONE NUMBERS** What is the probability that a 7-digit telephone number generated using the digits 2, 3, 2, 5, 2, 7, and 3 is the number 222-3357?

SOLUTION:

There is a total of seven numbers. Of these numbers, 2 occurs 3 times, 3 occurs 2 times, and 5 and 7 occur 1 time each. So, the number of distinguishable permutations of these numbers is $\frac{7!}{3! \cdot 2!} = 420$.

There is only one favorable arrangement, 222–3357.

The probability that a permutation of these numbers selected at random results in the correct phone number, 222–3357, is $\frac{1}{420}$.

6. **IDENTIFICATION** A store randomly assigns their employees work identification numbers to track productivity. Each number consists of 5 digits ranging from 1–9. If the digits cannot repeat, find the probability that a randomly generated number is 25938.

SOLUTION:

Because the digits cannot repeat, order in this situation is important. The number of possible outcomes in the sample space is the number of permutations of 9 digits taken 5 at a time, ${}_9P_5$.

$${}_9P_5 = \frac{9!}{(9-5)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 15{,}120$$

There is only one favorable arrangement, 25938.

The probability that a permutation of these numbers selected at random results in the correct identification number, 25938, is $\frac{1}{15{,}120}$.

7. **STUDENT COUNCIL** The table shows the finalists for class president. The order in which they will give their speeches will be chosen randomly.

Class President Finalists
Alan Shepherd
Chaminade Hudson
Denny Murano
Kelli Baker
Tanika Johnson
Jerome Murdock
Marlene Lindeman

a. What is the probability that Denny, Kelli, and Chaminade are the first 3 speakers, in any order?
b. What is the probability that Denny is first, Kelli is second, and Chaminade is third?

SOLUTION:

a. The number of possible outcomes in the sample space is the number of permutations of 7 finalists taken 3 at a time, ${}_7P_3$.

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 210$$

The number of favorable outcomes is the number of permutations of Denni, Kelly, and Chaminade, or 3!.

The probability that Denni, Kelly, and Chaminade are selected at random in any order, is $\frac{3!}{210}$ or $\frac{1}{35}$.

b. The probability that Denny is first, Kelli is second, and Chaminade is third has the same number of possible outcomes as part a, 210. However, now the favorable outcome is 1 because the order is restricted. The probability is $\frac{1}{210}$.

BONUS

PART #3

24	A learning outcome from the TERM 2 Topics	Undisclosed	Undisclosed
25	A learning outcome from the TERM 2 Topics	Undisclosed	Undisclosed