حل تجميعة صفحات الكتاب وفق الهيكل الوزاري منهج ريفيل

مؤسسة الإمارات للتعليم المدرسي
EMIRATES SCHOOLS ESTABLISHMENT

Al Shawamekh School
Mathematics
Grade 10 Advanced
Reveal

MCQ:

Final Exam Revision

Name : Answer Sheet ....

Grade 10 (   )

Trimester 3
2023-2024

Q	Learning Outcome	Exercise	Page
1	Graph and analyze power functions	1 to 6	79

M2L1: Polynomial Functions

Describe the end behavior of each function using the leading coefficient and degree and state the domain and range.

1. $f(x) = 3x^4$

leading coefficient is 3 (positive).
degree is 4 (even).
End behavior:
as $x \to -\infty, f(x) \to \infty$
and as $x \to \infty$ and $f(x) \to \infty$.
domain is all real numbers.
range is all real numbers greater than or equal to 0

2. $f(x) = -2x^3$

leading coefficient is -2 (negative).
degree is 3 (odd).
End behavior:
as $x \to -\infty, f(x) \to \infty$
and as $x \to \infty$ and $f(x) \to -\infty$.
domain is all real numbers.
range is all real numbers.

3. $f(x) = -\frac{1}{2}x^5$

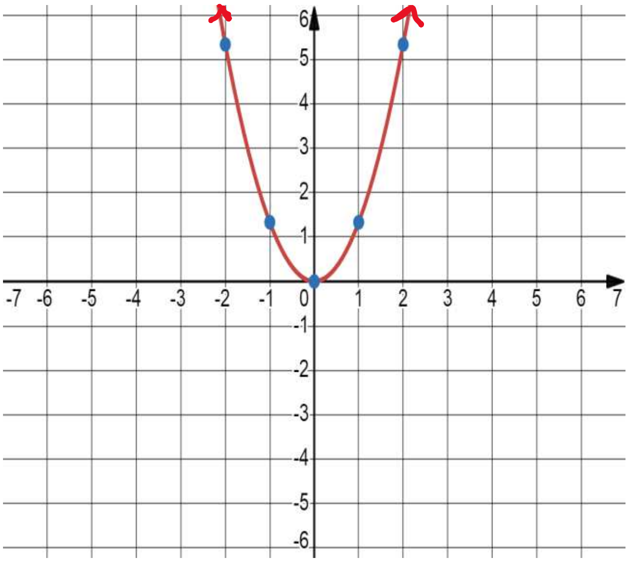
leading coefficient is $\frac{-1}{2}$ (negative).
degree is 5 (odd).
End behavior:
as $x \to -\infty, f(x) \to \infty$
and as $x \to \infty$ and $f(x) \to -\infty$.
domain is all real numbers.
range is all real numbers.

4. $f(x) = \frac{3}{4}x^6$

leading coefficient is $\frac{3}{4}$ (positive).
degree is 6 (even).
End behavior:
as $x \to -\infty, f(x) \to \infty$
and as $x \to \infty$ and $f(x) \to \infty$.
domain is all real numbers.
range is all real numbers greater than or equal to 0

5. USE A MODEL The shape of a parabolic reflector inside a flashlight can be modeled by the function $f(x) = \frac{4}{3}x^2$. Graph the function $f(x)$, and state the domain and range.

x	y
-2	5.3
-1	1.3
0	0
1	1.3
2	5.3

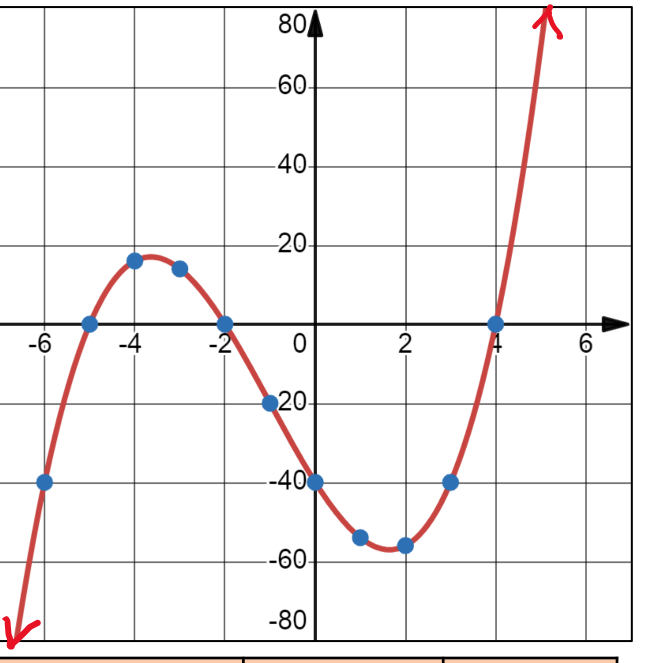


domain is all real numbers or $D = (-\infty, \infty)$
range is all real numbers ≥ 0 or $R = [0, \infty)$

6. **MACHINE EFFICIENCY** A company uses the function $f(x) = x^3 + 3x^2 - 18x - 40$ to model the change in efficiency of a machine based on its position x. Graph the function and state the domain and range.

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	-40	0	16	14	0	-20	-40	-54	-56	-40	0

domain is all real numbers or $D = (-\infty, \infty)$
range is all real numbers or $R = (-\infty, \infty)$



Q	Learning Outcome	Exercise	Page
2	Graph and analyze polynomial functions.	7 to 21	79+80

M2L1: Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable.
If it is not a polynomial in one variable, explain why.

7. n + 8

degree is 1 (odd).
leading coefficient is 1 (positive).

8. $(2x - 1)(4x^2 + 3)$

degree is 3 (odd).
leading coefficient is 8 (positive).

9. $-5x^5 + 3x^3 - 8$

degree is 5 (odd).
leading coefficient is -5 (negative).

10. $18 - 3y + 5y^2 - y^5 + 7y^6$

degree is 6 (even).
leading coefficient is 7 (positive).

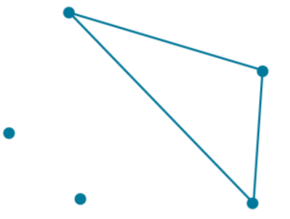
11. $u^3 + 4u^2t^2 + t^4$

This is not a **polynomial** in one variable because there are two variables, u and t.

12. $2r - r^2 + \frac{1}{r^2}$

This is not a **polynomial** because there is a negative exponent.

13. TRIANGLES Dylan drew n dots on a piece of paper making sure that no set of 3 points were collinear. The number of triangles that can be made using the dots as vertices is equal to $f(n) = \frac{1}{6}(n^3 - 3n^2 + 2n)$, when $n \geq 0$.



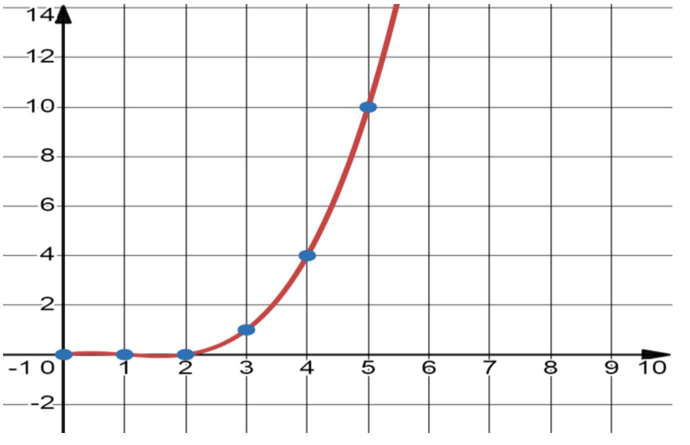
a. If Dylan drew 15 dots, how many triangles can be made?

Substitute 15 for n into the function

$f(n) = \frac{1}{6}((15)^3 - 3(15)^2 + 2(15)) = 455$

b. Sketch a graph of the function.

x	y
0	0
1	0
2	0
3	1
4	4
5	10



14. DRILLING The volume of a drill bit can be estimated by the formula for a cone, $V = \frac{1}{3}\pi hr^2$, where h is the height of the bit and r is its radius. Substituting $\frac{\sqrt{3}}{3}r$ for h, the volume of the drill bit can be estimated by $V = \frac{\sqrt{3}}{9}\pi r^3$.

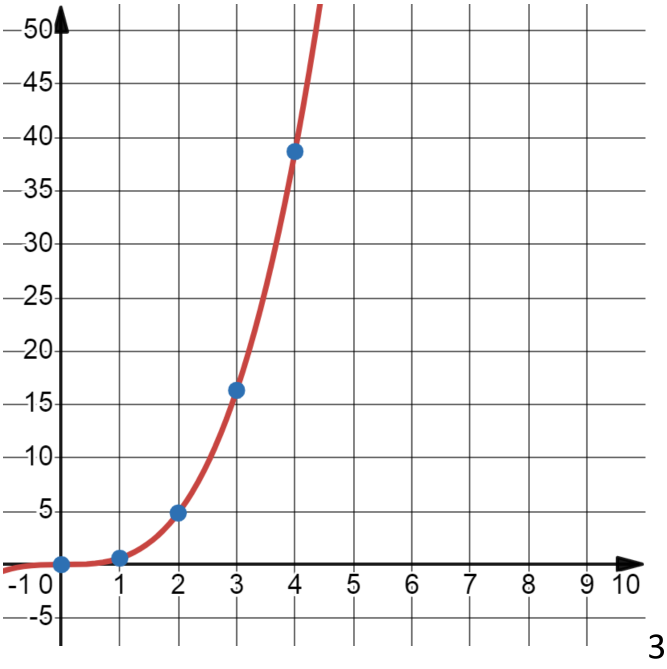
a. What is the volume of a drill bit with a radius of 3 centimeters?

Substitute 3 for r into the function

$$V = \frac{\sqrt{3}}{9}\pi(3)^3 = 3\pi\sqrt{3}\ cm^3$$

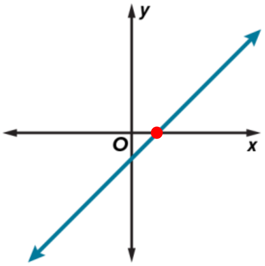
b. Sketch a graph of the function in the context of the situation.

x	y
0	0
1	0.60
2	4.84
3	16.32
4	38.70



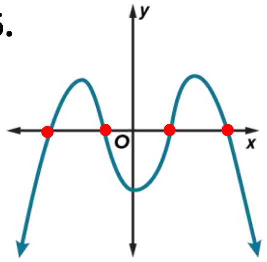
Use the graph to state the number of real zeros of the function.

15.



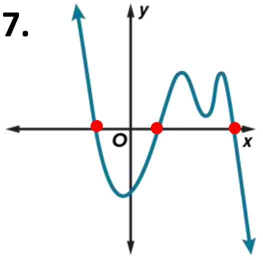
1 real zero

16.



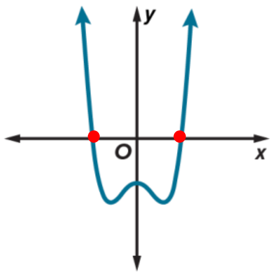
4 real zeros

17.



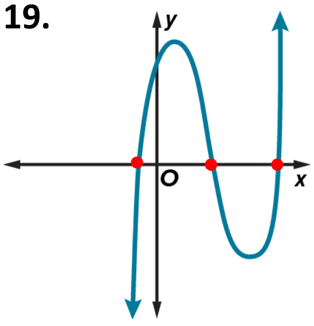
3 real zeros

18.



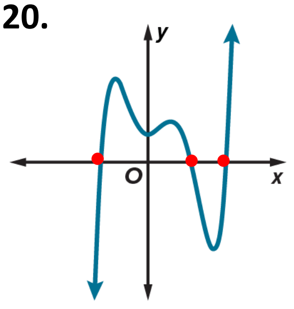
2 real zeros

19.



3 real zeros

20.

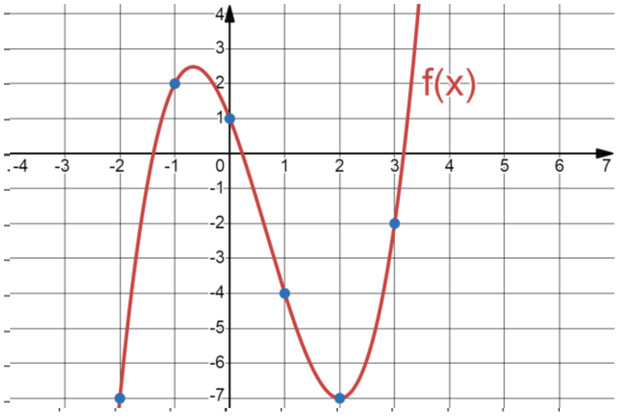


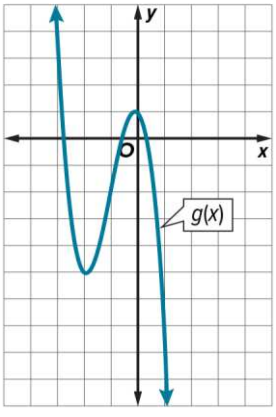
3 real zeros

21. Examine $f(x) = x^3 - 2x^2 - 4x + 1$ and $g(x)$ shown in the graph.

a. Which function has the greater relative maximum?

b. Compare the zeros, x- and y-intercepts, and end behavior of $f(x)$ and $g(x)$.





Compare	$\boldsymbol{f}(x)$	$\boldsymbol{g}(x)$
Relative Maximum	at approximately 2.5	at approximately 1
Zeros	−1.39, 0.23, 3.16	−2.75, −0.5, 0.25
x-intercepts	−1.39, 0.23, 3.16	−2.75, −0.5, 0.25
y-intercept	1	1
End behavior	$as\ x \to -\infty\ , f(x) \to -\infty$ and $as\ x \to \infty\ , f(x) \to \infty$	$as\ x \to -\infty\ , f(x) \to \infty$ and $as\ x \to \infty\ , f(x) \to -\infty$

Q	Learning Outcome	Exercise	Page
3	Find extrema of polynomial functions.	5 to 11	89

M2L2: Analyzing Graphs of Polynomial Functions ·

Use a table to graph each function. Then estimate the *x*-coordinates at which relative maxima and relative minima occur.

5. $f(x) = -2x^3 + 12x^2 - 8x$

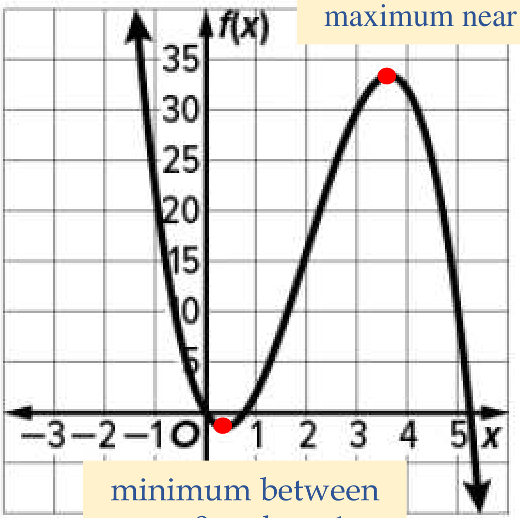
Using table:

x	$f(x)$
−2	80
−1	22
0	0
1	2
2	16
3	30
4	32
5	10

minimum between $x = 0$ and $x = 1$.

maximum near $x = 4$

Using Graph:



maximum near $x = 4$

minimum between $x = 0$ and $x = 1$.

6. $f(x) = 2x^3 - 4x^2 - 3x + 4$

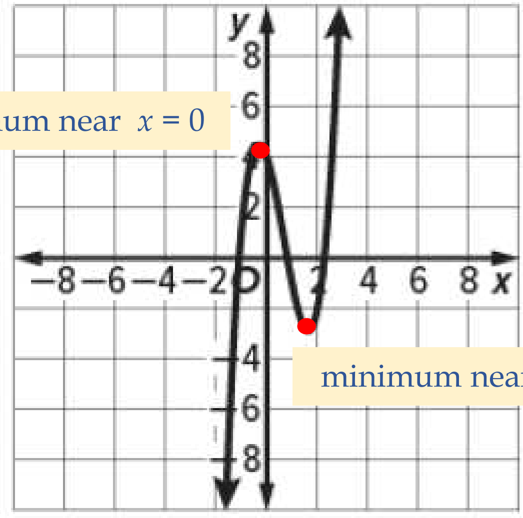
Using table:

x	$f(x)$
−4	−176
−3	−77
−2	−22
−1	1
0	4
1	−1
2	−2
3	13
4	56

maximum near $x = 0$

minimum near $x = 2$

Using Graph:



maximum near $x = 0$

minimum near $x = 2$

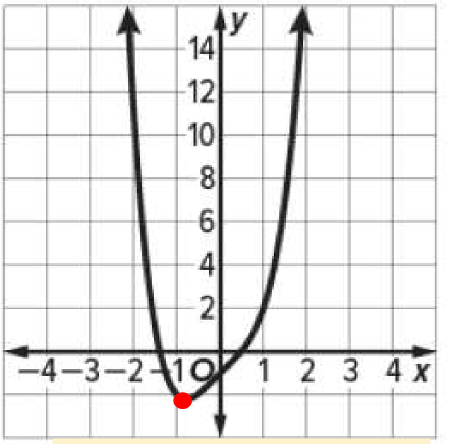
7. $f(x) = x^4 + 2x - 1$

Using table:

x	$f(x)$
–4	247
–3	74
–2	11
–1	–2
0	–1
1	2
2	19
3	86
4	263

minimum near $x = -1$

Using Graph:



minimum near $x = -1$

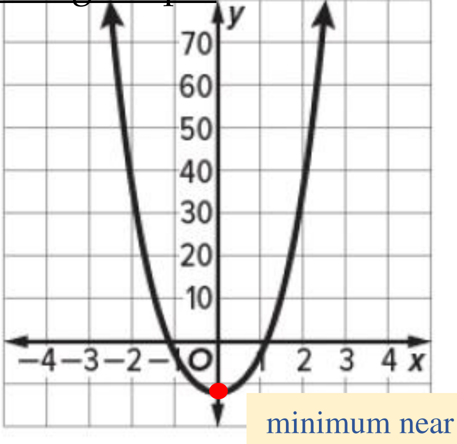
8. $f(x) = x^4 + 8x^2 - 12$

Using table:

x	$f(x)$
–4	372
–3	141
–2	36
–1	–3
0	–12
1	–3
2	36
3	141
4	372

minimum near $x = 0$

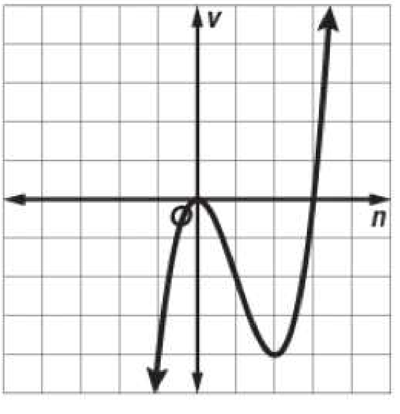
Using Graph:



minimum near $x = 0$

9.BUSINESS A banker models the expected value v of a company in millions of dollars by using the formula $v = n^3 - 3n^2$, where n is the number of years in business. Graph the function and describe its key features over the relevant domain.

x	$f(x)$
0	0
1	–2
2	–4
3	0
4	16
5	50
6	108
7	196
8	320



Because n is the number of years in business

Domain: D={n | $n \geq 0$}

Range: R={v | $v \geq -4$}.

Relative minimum *near* 2 years of business.

y-intercept is at (0, 0),

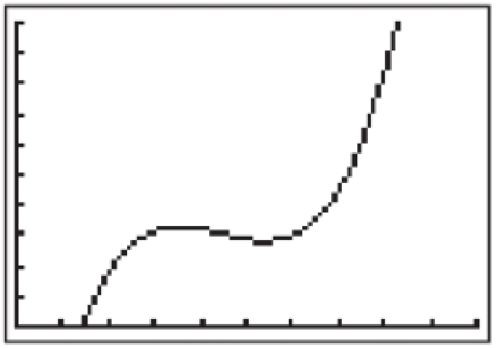
n-intercept is at (0,0) and (3, 0).

End behavior:

as $x \to \infty$ and $f(x) \to \infty$.

graph does not have symmetry

10. **HEIGHT** A plant's height is modeled by the function $f(x) = 1.5x^3 - 20x^2 + 85x - 84$, where x is the number of weeks since the seed was planted and $f(x)$ is the height of the plant. Graph the function and describe its key features over its relevant domain.



Domain: $D=\{x \mid x \geq 1.4\}$
Range: $R=\{f(x) \mid f(x) \geq 0\}$.

Relative minimum *near* 5 weeks of growth
Relative maximum *near* 4 weeks of growth

No y-intercept
x-intercept is at (1.4,0)

End behavior: as $x \to \infty$ and $f(x) \to \infty$.

graph does not have symmetry

Q	**Learning Outcome**	**Exercise**	**Page**
4	Multiply polynomials.	13 to 28	97

M2L3: Operations with Polynomials.

Multiply.

13. $3p(np - z)$

$= 3p(np) + 3p(-z)$ Distributive Property
$= 3np^2 - 3pz$ Multiply.

14. $4x(2x^2 + y)$

$= 4x(2x^2) + 4x(y)$ Distributive Property
$= 8x^3 + 4xy$ Multiply.

15. $-5(2c^2 - d^2)$

$= -5(2c^2) + (-5)(-d^2)$ Distributive Property
$= -10c^2 + 5d^2$ Multiply.

16. $x^2(2x + 9)$

$= x^2(2x) + x^2(9)$ Distributive Property
$= 2x^3 + 9x^2$ Multiply.

17. $(a - 5)^2$

$= (a-5)(a-5)$ Rewrite as two binomials.
$= a(a) + a(-5) + (-5)(a) + (-5)(-5)$ FOIL Method
$= a^2 - 5a - 5a + 25$ Multiply.
$= a^2 - 10a + 25$ Combine like terms.

18. $(2x - 3)(3x - 5)$

$= 2x(3x) + 2x(-5) + (-3)(3x) + (-3)(-5)$ FOIL Method
$= 6x^2 - 10x - 9x + 15$ Multiply.
$= 6x^2 - 19x + 15$ Combine like terms.

19. $(x-y)(x^2+2xy+y^2)$

$=x(x^2)+x(2xy)+x(y^2)+(-y)(x^2)+(-y)(2xy)+(-y)(y^2)$	Distributive Property
$=x^3+2x^2y+xy^2-x^2y-2xy^2-y^3$	Multiply.
$=x^3+(2x^2y-x^2y)+(xy^2-2xy^2)-y^3$	Group like terms.
$=x^3+x^2y-xy^2-y^3$	Combine like terms.

20. $(a+b)(a^3-3ab-b^2)$

$=a(a^3)+a(-3ab)+a(-b^2)+b(a^3)+b(-3ab)+b(-b^2)$	Distributive Property
$=a^4-3a^2b-ab^2+a^3b-3ab^2-b^3$	Multiply.
$=a^4+a^3b-3a^2b+(-ab^2-3ab^2)-b^3$	Group like terms.
$=a^4+a^3b-3a^2b-4ab^2-b^3$	Combine like terms.

21. $(x-y)(x+y)(2x+y)$

$= x(x)+x(y)+(-y)(x)+(-y)(y)\ (2x+y)$

$= (x^2+xy-xy-y^2)\ (2x+y)$

$= (x^2-y^2)\ (2x+y)$

$= (x^2)(2x)+(x^2)(y)+(-y^2)(2x)+(-y^2)(y)$

$= 2x^3+x^2y-2xy^2-y^3$

22. $(a+b)(2a+3b)(2x-y)$

$= a(2a)+a(3b)+(b)(2a)+(b)(3b)\ (2x-y)$

$= (2a^2+3ba+2b+3b^2)\ (2x-y)$

$= (2a^2+5ba+3b^2)\ (2x-y)$

$= (2a^2)(2x)+(2a^2)(-y)+(5ba)(2x)+(5ba)(-y)+(3b^2)(2x)+(3b^2)(-y)$

$= 4a^2x-2a^2y+10abx-5aby+6b^2x-3b^2y$

23. $(r-2t)(r+2t)$

$=r(r)+r(2t)+(-2t)(r)+(-2t)(2t)$	FOIL Method
$=r^2+2rt-2rt-4t^2$	Multiply.
$=r^2-4t^2$	Combine like terms.

24. $(3y+4)(2y-3)$

$=3y(2y)+3y(-3)+4(2y)+4(-3)$	FOIL Method
$=6y^2-9y+8y-12$	Multiply.
$=6y^2-y-12$	Combine like terms.

25. $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

$$= x^3(2x^2) + x^3(-x) + (x^3)(2) + (-3x^2)(2x^2) + (-3x^2)(-x) + (-3x^2)(2) + 1(2x^2) + 1(-x) + 1(2)$$ Distributive Property

$$= 2x^5 - x^4 + 2x^3 - 6x^4 + 3x^3 - 6x^2 + 2x^2 - x + 2$$ Multiply.

$$= 2x^5 + (-x^4 - 6x^4) + (2x^3 + 3x^3) + (-6x^2 + 2x^2) - x + 2$$ Group like terms.

$$= 2x^5 - 7x^4 + 5x^3 - 4x^2 - x + 2$$ Combine like terms.

26. $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

$$= 4x^5(3x) + 4x^5(-1) + x^3(3x) + x^3(-1) + (-7x^2)(3x) + (-7x^2)(-1) + 2(3x) + 2(-1)$$ Distributive Property

$$= 12x^6 - 4x^5 + 3x^4 - x^3 - 21x^3 + 7x^2 + 6x - 2$$ Multiply.

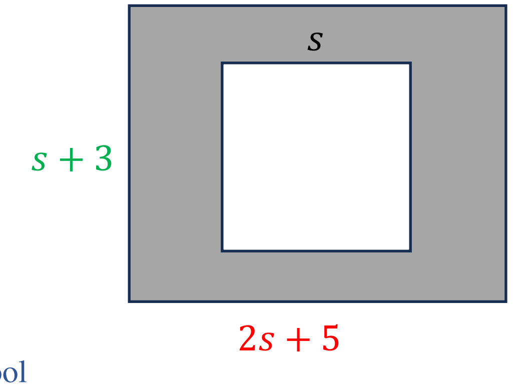
$$= 12x^6 - 4x^5 + 3x^4 + (-x^3 - 21x^3) + 7x^2 + 6x - 2$$ Group like terms.

$$= 12x^6 - 4x^5 + 3x^4 - 22x^3 + 7x^2 + 6x - 2$$ Combine like terms.

27. CONSTRUCTION A rectangular deck is built around a square pool. The pool has side length s. The length of the deck is 5 units longer than twice the side length of the pool. The width of the deck is 3 units longer than the side length of the pool. What is the area of the deck in terms of s?

The area of the deck and the pool

$$A = w \times l$$
$$= (2s + 5)(s + 3)$$
$$= (2s)(s) + (2s)(3) + (5)(s) + (5)(3)$$
$$= 2s^2 + 6s + 5s + 15$$



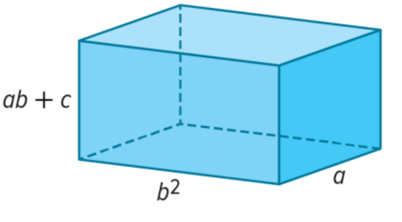
The area of the pool $A = s^2$

The area of the deck = Area of the deck and the pool - of the pool

$$= 2s^2 + 11s + 15 - s^2 = s^2 + 11s + 15$$

28. VOLUME The volume of a rectangular prism is given by the product of its length, width, and height. A rectangular prism has a length of b^2 units, a width of a units, and a height of $ab + c$ units. What is the volume of the rectangular prism? Express your answer in simplified form.

$$V = w \times l \times h$$
$$= (a)(b^2)(ab + c)$$
$$= ab^2(ab + c)$$
$$= ab^2(ab) + ab^2(c)$$
$$= a^2b^3 + ab^2c$$ cubic units.



Q	Learning Outcome	Exercise	Page
5	Divide polynomials by using synthetic division.	11 to 16	105

M2L4: Dividing Polynomials .

Simplify using synthetic division.

11. $(3v^2 - 7v - 10)(v - 4)^{-1}$

$= (3v^2 - 7v - 10) \div (v - 4)$

$v - 4 = 0$

$v = 4$

+4	3	−7	−10
		12	20
	3	5	10

$3v + 5 + \dfrac{10}{v - 4}$.

12. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

$(3t^4 + 4t^3 - 32t^2 - 5t - 20) \div (t + 4)$

$t + 4 = 0$

$t = -4$

−4	3	4	−32	−5	−20
	↓	−12	32	0	20
	3	−8	0	−5	0

$3t^3 - 8t^2 - 5.$

13. $\dfrac{y^3 + 6}{y + 2}$

$= (y^3 + 0y^2 + 0y + 6) \div (y + 2)$

$y + 2 = 0$

$y = -2$

−2	1	0	0	6
	↓	−2	4	−8
	1	−2	4	−2

$y^2 - 2y + 4 - \dfrac{2}{y + 2}$.

14. $\dfrac{2x^3 - x^2 - 18x + 32}{2x - 6}$

$2x - 6 = 0$

$2x = 6$

$x = 3$

	2	-1	-18	32
3		6	15	-9
	2	5	-3	32

$= x^2 + \frac{5}{2}x - \frac{3}{2} + \frac{32}{(2x - 6)}$

15. $(4p^3 - p^2 + 2p) \div (3p - 1)$

$$= \frac{4}{3}x^2 + \frac{1}{3}x + \frac{19}{29} + \frac{19}{27(3p-1)}$$

$3p - 1 = 0$
$3p = 1$
$p = \frac{1}{3}$

	4	-1	2	0
$\frac{1}{3}$		$\frac{4}{3}$	$\frac{1}{9}$	$\frac{19}{27}$
	$\frac{4}{3}$	$\frac{\frac{1}{3}}{3}$	$\frac{\frac{19}{9}}{3}$	$\frac{19}{27}$

16. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

$$= (3c^4 + 6c^3 + 0c^2 - 2c + 4) \div (c + 2)$$

$$3c^3 - 2 + \frac{8}{c+2}.$$

$c + 2 = 0$
$c = -2$

-2	3	6	0	−2	4
	↓	−6	0	0	4
	3	0	0	−2	8

Q	Learning Outcome	Exercise	Page
6	Solve polynomial equations by graphing.	1 to 14	121

M3L1: Solving Polynomial Equations by Graphing.

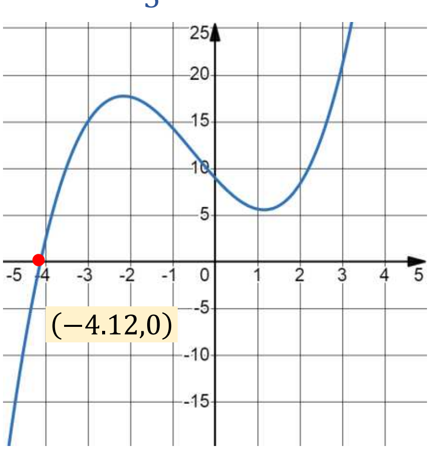
Use a graphing calculator to solve each equation by graphing. If necessary, round to the nearest hundredth.

1. $\frac{2}{3}x^3 + x^2 - 5x = -9$

Method 1: Related function

$\frac{2}{3}x^3 + x^2 - 5x + 9 = 0$

$f(x) = \frac{2}{3}x^3 + x^2 - 5x + 9$

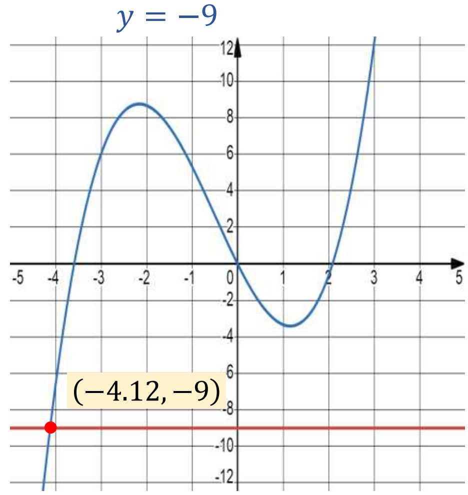


The real zero is -4.12

Method 2: Solve system of equations

$y = \frac{2}{3}x^3 + x^2 - 5x$

$y = -9$



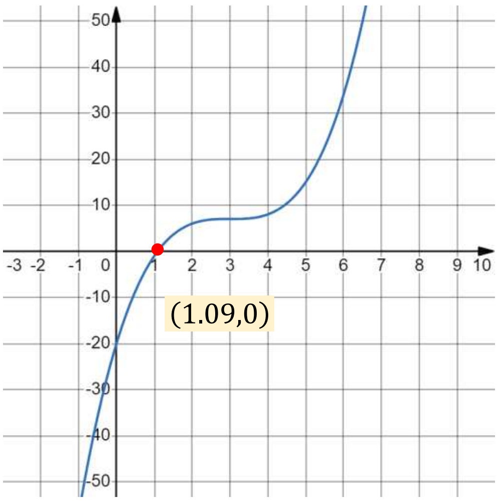
The real zero is -4.12

2. $x^3 - 9x^2 + 27x = 20$

Method 1: Related function

$x^3 + 9x^2 + 27x - 20 = 0$

$f(x) = x^3 + 9x^2 + 27x - 20$

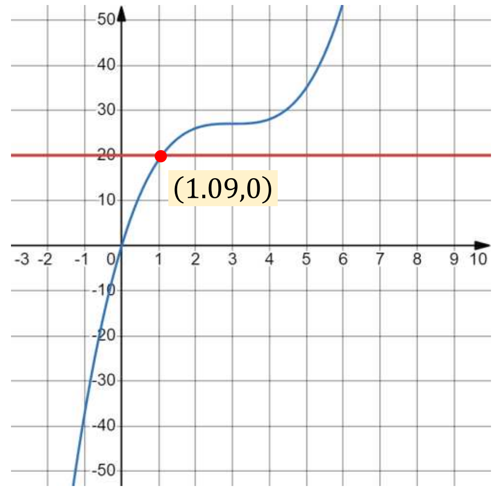


The real zero is about 1.09

Method 2: Solve system of equations

$y = x^3 + 9x^2 + 27x$

$y = 20$



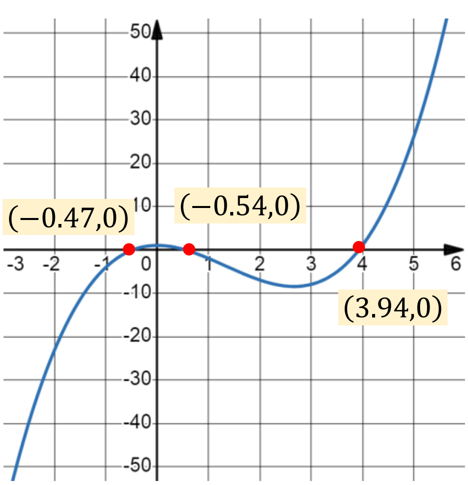
The real zero is about 1.09

3. $x^3 + 1 = 4x^2$

Method 1: Related function

$x^3 + 1 - 4x^2 = 0$

$f(x) = x^3 - 4x^2 + 1$

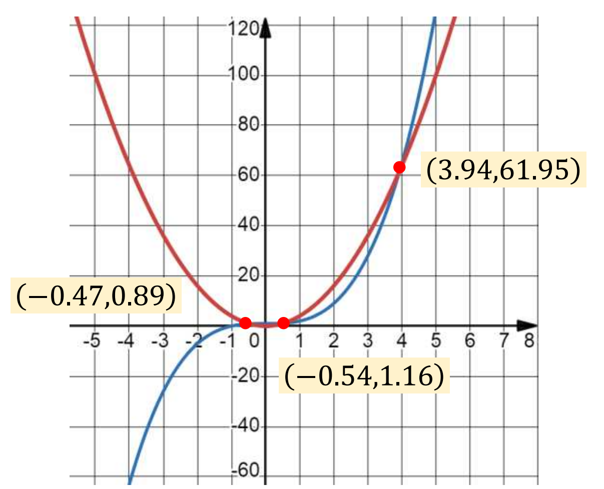


The real zeros are about
−0.47, 0.54, and 3.94.

Method 2: Solve system of equations

$y = x^3 + 1$

$y = 4x^2$



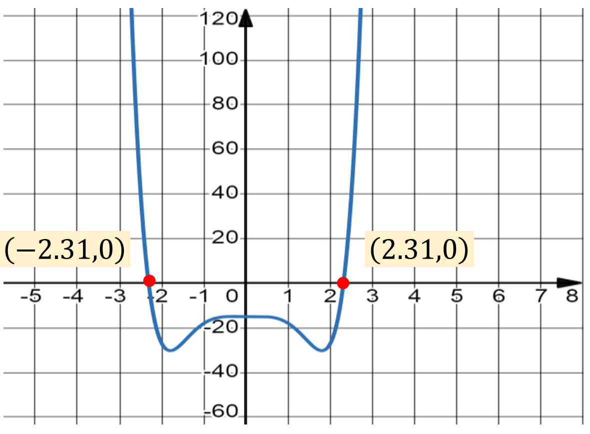
The real zeros are about
−0.47, 0.54, and 3.94.

4. $x^6 - 15 = 5x^4 - x^2$

Method 1: Related function

$x^6 - 15 - 5x^4 + x^2 = 0$

$f(x) = x^6 - 5x^4 + x^2 - 15$

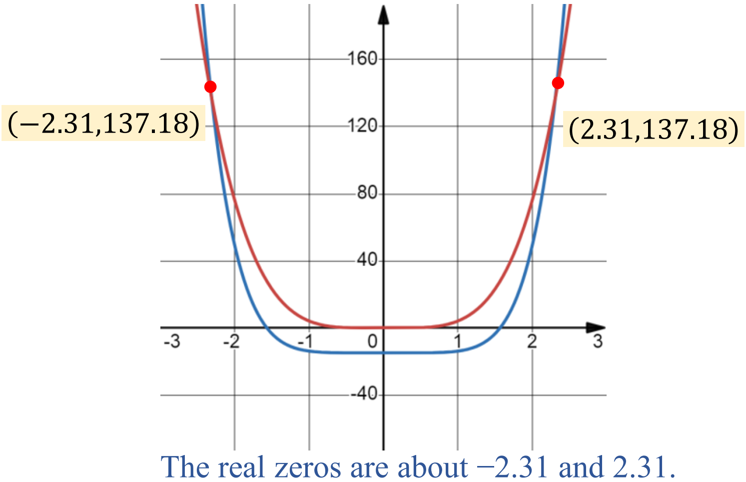


The real zeros are about −2.31 and 2.31.

Method 2: Solve system of equations

$y = x^6 - 15$

$y = 5x^4 - x^2$



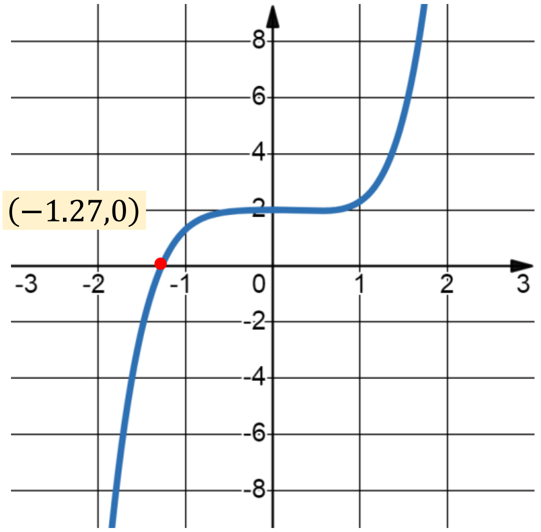
The real zeros are about −2.31 and 2.31.

5. $\frac{1}{2}x^5 = \frac{1}{5}x^2 - 2$

Method 1: Related function

$\frac{1}{2}x^5 - \frac{1}{5}x^2 + 2 = 0$

$f(x) = \frac{1}{2}x^5 - \frac{1}{5}x^2 + 2$

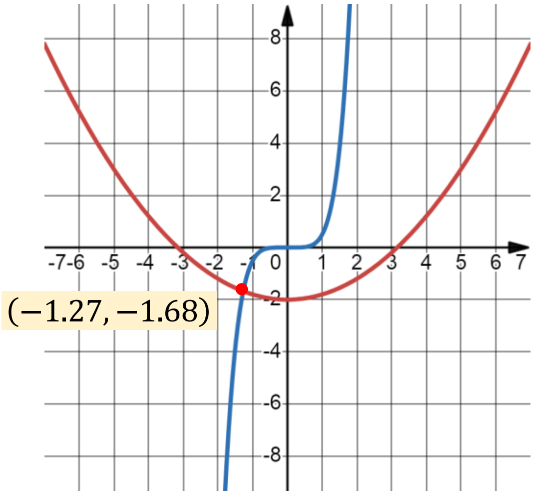


The real zeros is about −1.27.

Method 2: Solve system of equations

$y = \frac{1}{2}x^5$

$y = \frac{1}{5}x^2 - 2$



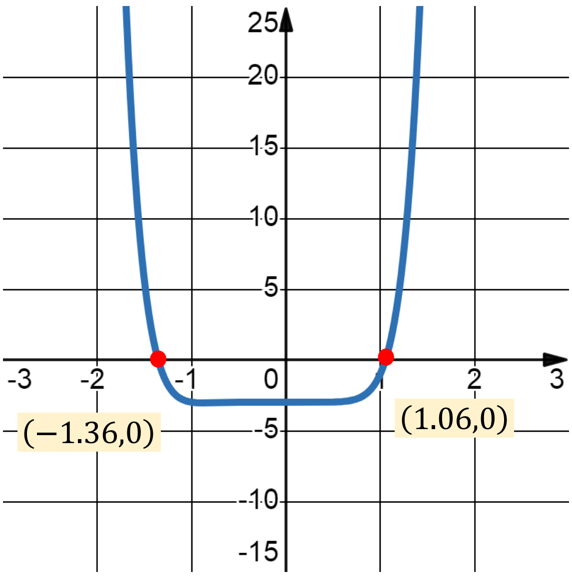
The real zeros is about −1.27.

6. $x^8 = -x^7 + 3$

Method 1: Related function

$x^8 + x^7 - 3 = 0$

$f(x) = x^8 + x^7 - 3$

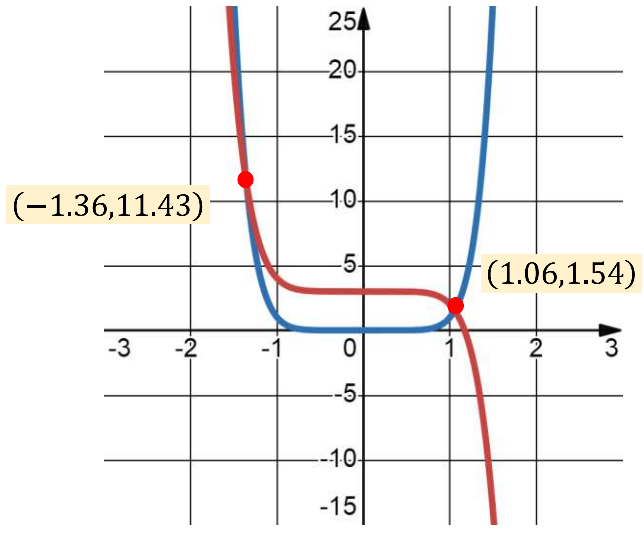


The real zeros are about −1.36 and 1.06.

Method 2: Solve system of equations

$y = x^8$

$y = -x^7 + 3$



The real zeros are about −1.36 and 1.06.

7. SHIPPING A shipping company will ship a package for $7.50 when the volume is no more than 15,000 cm^3. Grace needs to ship a package that is $3x - 5$ cm long, $2x$ cm wide, and $x + 20$ cm tall.

a. Write a polynomial equation to represent the situation if Grace plans to spend a maximum of $7.50.

$w \times l \times h = V$

$(2x)(3x - 5)(x + 20) = 15000$

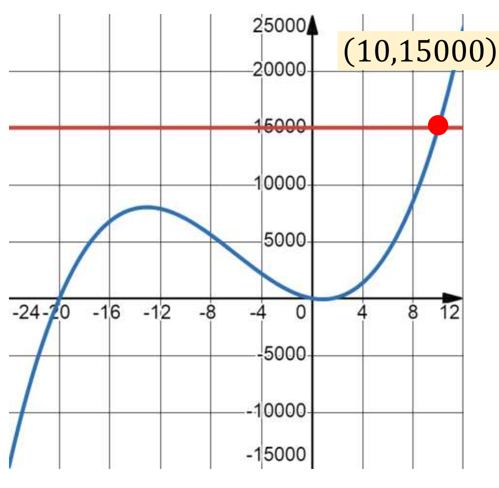
$[(2x)(3x) + (2x)(-5)](x + 20) = 15000$

$[6x^2 - 10x](x + 20) = 15000$

$[6x^2](x) + [6x^2](20) + [-10x](x)\ [-10x](20) = 15000$

$6x^3 + 120x^2 - 10x^2 - 200x = 15000$

$6x^3 + 110x^2 - 200x = 15000$



b. Write and solve a system of equations.

$y = 6x^3 + 110x^2 - 200x$

$y = 15000$

The real solution is 10, so $x = 10$ cm.

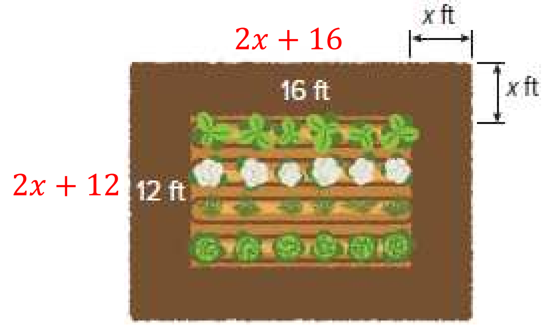
c. What should the dimensions of the package be to have the maximum volume?

length: $3x - 5 = 3(10) - 5 = 25$ cm

width: $2x = 2(20) = 20$ cm

height: $x + 20 = 10 + 20 = 30$ cm

8.GARDEN A rectangular garden is 12 feet across and 16 feet long. It is surrounded by a border of mulch that is a uniform width, *x*. The maximum area for the garden, plus border, is 285 ft^2.



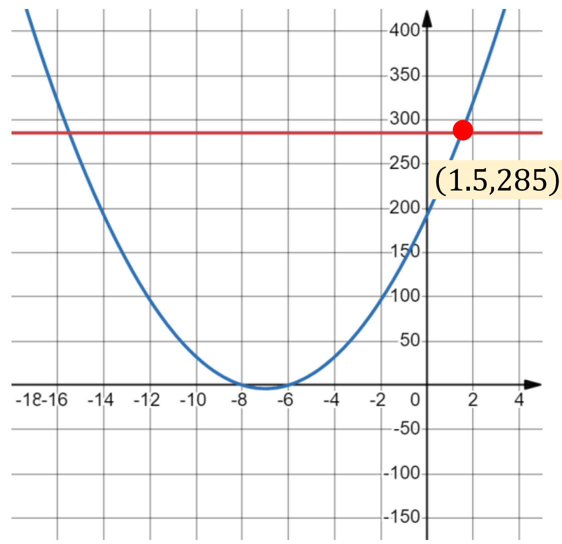
a. Write a polynomial equation to represent the situation.

$$w \times l = A$$
$$(2x + 16)(2x + 12) = 285$$
$$4x^2 + 24x + 32x + 192 = 285$$
$$4x^2 + 56x + 192 = 285$$

b. Write and solve a system of equations.

$$y = 4x^2 + 56x + 192$$
$$y = 285$$

The real solution is 1.5, so $x = 1.5$ ft.



c. What are the dimensions of the garden plus boarder?

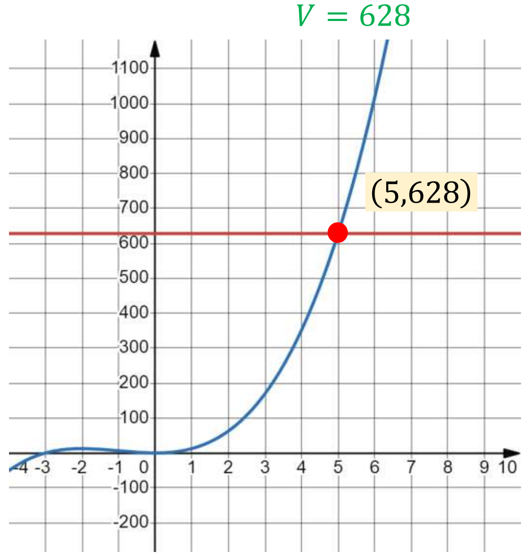
length: $2x + 16 = 2(1.5) + 16 = 19$ ft
width: $2x + 12 = 2(1.5) + 12 = 15$ ft

9.PACKAGING A juice manufacturer is creating new cylindrical packaging. The height of the cylinder is to be 3 units longer than the radius of the can. The cylinder is to have a volume of 628 cubic inches. Use 3.14 for π. $h = r + 3$

$V = 628$

a. Write a polynomial equation to support the model.

$$V = \pi r^2 h$$
$$(3.14)r^2(r + 3) = 628$$
$$3.14r^2(r + 3) = 628$$
$$3.14r^3 + 9.42r^2 = 628$$



b. Write and solve a system of equations.

$$y = 3.14r^3 + 9.42r^2$$
$$y = 628$$

The real solution is 5, so $x = 5$ units.

c. What is the radius and height of the new packaging?

radius: $x = 5$ in.
height: $x + 3 = 5 + 3 = 8$ in.

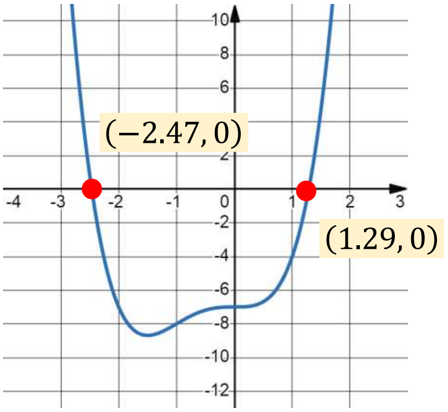
Solve each equation. If necessary, round to the nearest hundredth.

10. $x^4 + 2x^3 = 7$

Method 1: Related function

$x^4 + 2x^3 - 7 = 0$

$f(x) = x^4 + 2x^3 - 7$

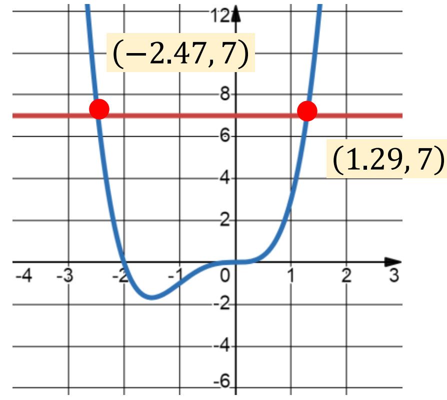


The real zeros are about −2.47 and 1.29.

Method 2: Solve system of equations

$y = x^4 + 2x^3$

$y = 7$



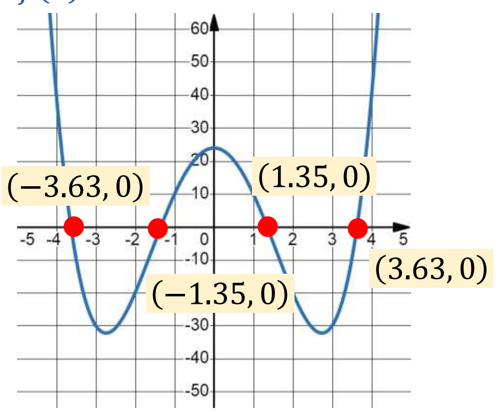
The real zeros are about −2.47 and 1.29.

11. $x^4 - 15x^2 = -24$

Method 1: Related function

$x^4 - 15x^2 + 24 = 0$

$f(x) = x^4 - 15x^2 + 24$

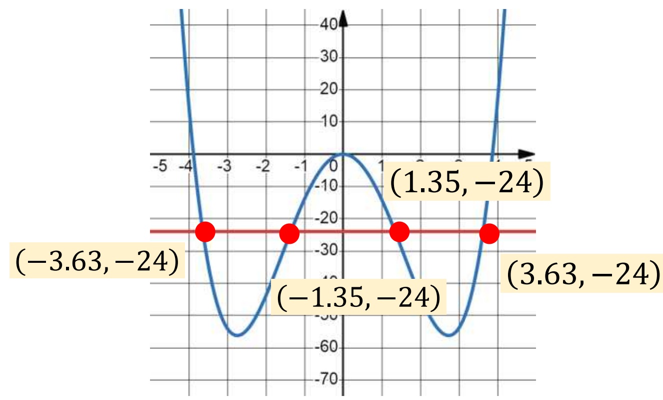


The real zeros are about −3.63, −1.35, 1.35, and 3.63.

Method 2: Solve system of equations

$y = x^4 - 15x^2$

$y = -24$



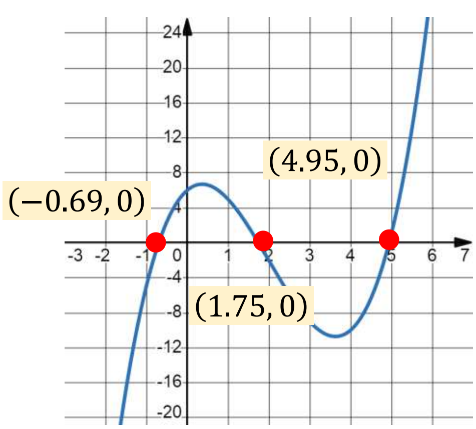
The real zeros are about −3.63, −1.35, 1.35, and 3.63.

12. $x^3 - 6x^2 + 4x = -6$

Method 1: Related function

$x^3 - 6x^2 + 4x + 6 = 0$

$f(x) = x^3 - 6x^2 + 4x + 6$

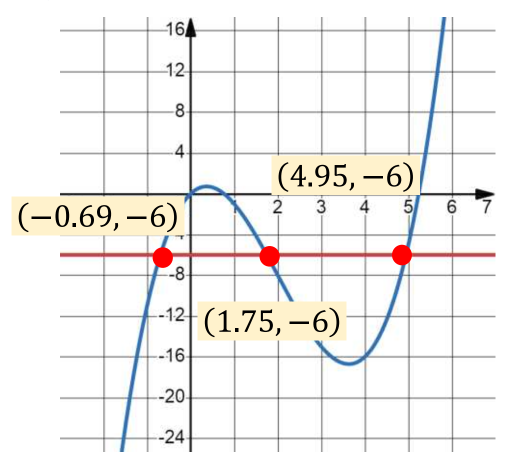


The real zeros are about −0.69, 1.75, and 4.95

Method 2: Solve system of equations

$y = x^3 - 6x^2 + 4x$

$y = -6$

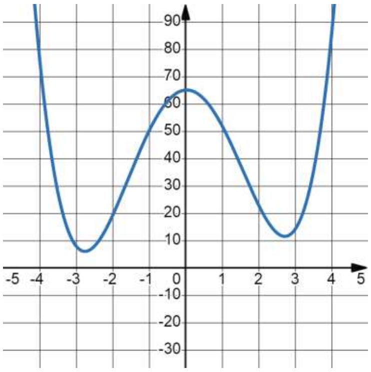


The real zeros are about −0.69, 1.75, and 4.95

13. $x^4 - 15x^2 + x + 65 = 0$

Method 1: Related function

$f(x) = x^4 - 15x^2 + x + 65$

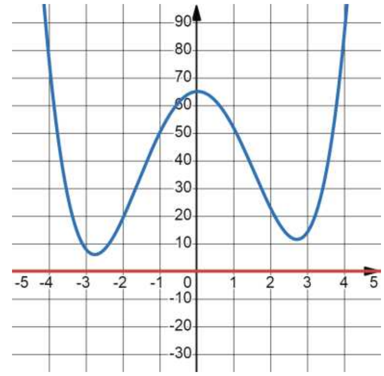


There are no real zeros

Method 2: Solve system of equations

$y = x^4 - 15x^2 + x + 65$

$y = 0$



There are no real zeros

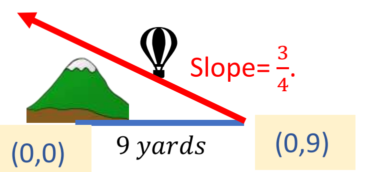
14. **BALLOON** Treyvon is standing 9 yards from the base of a hill that has a slope of $\frac{3}{4}$. He throws a water balloon from a height of 2 yards. Its path is modeled by $h(x) = -0.1x^2 + 0.8x + 2$, where h is the height of the balloon in yards and x is the distance the balloon travels in yards.

a. Write a polynomial equation to represent the situation.

Expression represents the distance the balloon travels in yards.

$$y - y_1 = m\,(x - x_1) \qquad ;\ m = \frac{3}{4}\ and\ (0,9)$$

$$y = \frac{3}{4}(x - 9)$$



The equation that represents how far from Treyvon the balloon will be when it hits the hill

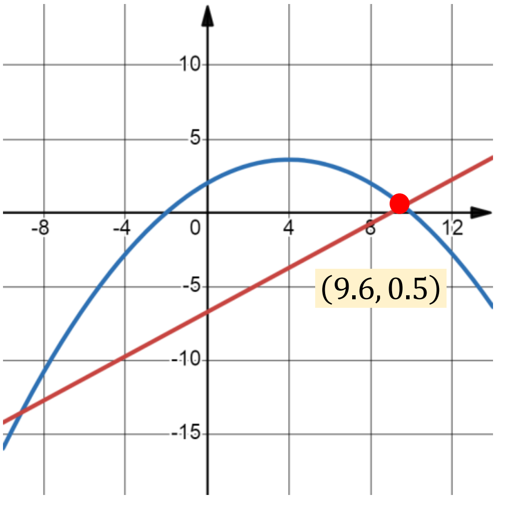
$$h(x) = -0.1x^2 + 0.8x + 2$$

b. How far from Treyvon will the balloon hit the hill?

$$y = \frac{3}{4}(x - 9)$$

$$y = -0.1x^2 + 0.8x + 2$$

The real solution is the x-coordinate of the intersection, which is about $x = 9.6$ yd.



Q	Learning Outcome	Exercise	Page
7	Solve polynomial equations by writing them in quadratic form and factoring.	16 to 27	128

M3L2: Solving Polynomial Equations Algebraically .

Write each expression in quadratic form, if possible.

16. $x^4 + 12x^2 - 8$

$(x^2)^2 + 12(x^2) - 8$

17. $-15x^4 + 18x^2 - 4$

$-15(x^2)^2 + 18(x^2) - 4$

18. $8x^6 + 6x^3 + 7$

$8(x^3)^2 + 6(x^3) + 7$

19. $5x^6 - 2x^2 + 8$

the expression cannot be written in quadratic form

20. $9x^8 - 21x^4 + 12$

$9(x^4)^2 - 21(x^4) + 12$

21. $16x^{10} + 2x^5 + 6$

$16(x^5)^2 + 2(x^5) + 6$

Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

$$\begin{aligned} x^4 + 6x^2 + 5 &= 0 \\ (x^2)^2 + 6(x^2) + 5 &= 0 \\ u^2 + 6u + 5 &= 0 \\ (u+5)(u+1) &= 0 \\ u = -5 &\text{ or } u = -1 \\ x^2 = -5 &\text{ or } x^2 = -1 \\ x = \pm i\sqrt{5} &\text{ or } x = \pm i \end{aligned}$$

23. $x^4 - 3x^2 - 10 = 0$

$$\begin{aligned} x^4 - 3x^2 - 10 &= 0 \\ (x^2)^2 - 3(x^2) - 10 &= 0 \\ u^2 - 3u - 10 &= 0 \\ (u-5)(u+2) &= 0 \\ u = 5 &\text{ or } u = -2 \\ x^2 = 5 &\text{ or } x^2 = -2 \\ x = \pm\sqrt{5} &\text{ or } x = \pm i\sqrt{2} \end{aligned}$$

24. $4x^4 - 14x^2 + 12 = 0$

$$\begin{aligned} 4x^4 - 14x^2 + 12 &= 0 \\ (2x^2)^2 - 7(2x^2) + 12 &= 0 \\ u^2 - 7u + 12 &= 0 \\ (u-4)(u-3) &= 0 \\ u = 4 &\text{ or } u = 3 \\ 2x^2 = 4 &\text{ or } 2x^2 = 3 \\ x^2 = 2 &\text{ or } x^2 = \frac{3}{2} \\ x = \pm\sqrt{2} &\text{ or } x = \pm\frac{\sqrt{3}}{2} \end{aligned}$$

25. $9x^4 - 27x^2 + 20 = 0$

$$\begin{aligned} 9x^4 - 27x^2 + 20 &= 0 \\ (3x^2)^2 - 9(3x^2) + 20 &= 0 \\ u^2 - 9u + 20 &= 0 \\ (u-4)(u-5) &= 0 \\ u = 4 &\text{ or } u = 5 \\ 3x^2 = 4 &\text{ or } 3x^2 = 5 \\ x^2 = \frac{4}{3} &\text{ or } x^2 = \frac{5}{3} \\ x = \pm\frac{2\sqrt{3}}{3} &\text{ or } x = \pm\frac{\sqrt{15}}{3} \end{aligned}$$

Solve each equation.

26. $4x^4 - 5x^2 - 6 = 0$

$$4x^4 - 5x^2 - 6 = 0$$
$$4(x^2)^2 - 5(x^2) - 6 = 0$$
$$4u^2 - 5u - 6 = 0$$
$$(u-2)(4u+3) = 0$$
$$u = 2 \text{ or } u = -\frac{3}{4}$$
$$x^2 = 2 \text{ or } x^2 = -\frac{3}{4}$$
$$x = \pm\sqrt{2} \text{ or } x = \pm i\frac{\sqrt{3}}{2}$$

27. $24x^4 + 14x^2 - 3 = 0$

$$24x^4 + 14x^2 - 3 = 0$$
$$24(x^2)^2 + 14(x^2) - 3 = 0$$
$$24u^2 + 14u - 3 = 0$$
$$(6u-1)(4u+3) = 0$$
$$u = \frac{1}{6} \text{ or } u = -\frac{3}{4}$$
$$x^2 = \frac{1}{6} \text{ or } x^2 = -\frac{3}{4}$$
$$x = \pm\frac{\sqrt{6}}{6} \text{ or } x = \pm i\frac{\sqrt{3}}{2}$$

Q	Learning Outcome	Exercise	Page
8	Evaluate functions by using synthetic substitution.	45 to 58	140 +141

M3L4: The Remainder and Factor Theorems.

45. REASONING Jessica evaluates the polynomial $p(x) = x^3 - 5x^2 + 3x + 5$ for a factor using synthetic substitution. Some of his work is shown below. Find the values of *a* and *b*.

a	**1**	**–5**	**3**	**5**
		11	**66**	**759**
	1	**6**	**69**	**b**

$1 \times a = 11 \;\rightarrow\; a = 11$

$5 + 759 = b \;\rightarrow b = 764$

46. STATE YOUR ASSUMPTION The revenue from streaming music services in the United States from 2005 to 2016 can be modeled by $y = 0.26x^5 - 7.48x^4 + 79.20x^3 - 333.33x^2 + 481.68x + 99.13$, where *x* is the number of years since 2005 and *y* is the revenue in millions of U.S. dollars.

a. Estimate the revenue from streaming music services in 2010.

a. Since 2010 – 2005 = 5, use synthetic substitution to determine $y(5)$.

5	0.26	−7.48	79.20	−333.33	481.68	99.13
		1.3	−30.9	241.5	−459.15	112.65
	0.26	−6.18	48.3	−91.83	22.53	211.78

In 2010, the revenue from streaming music services was approximately $211.78 million.

b. What might the revenue from streaming music services be in 2020? What assumption did you make to make your prediction?

b. Since 2020 – 2005 = 15, use synthetic substitution to determine $y(15)$.

15	0.26	−7.48	79.20	−333.33	481.68	99.13
		3.9	−53.7	382.5	737.55	18,288.45
	0.26	−3.58	25.5	49.17	1219.23	18,387.58

In 2020, the revenue from streaming music services will be approximately $18,387.58 million.

There is an assumption that the model still represents the situation after 15 years.

47. **NATURAL EXPONENTIAL FUNCTION** The natural exponential function $y = e^x$ is a special function that is applied in many fields such as physics, biology, and economics. It is not a polynomial function, however for small values of x, the value of e^x is very closely approximated by the polynomial function $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$. Use synthetic substitution to determine $f(0.1)$.

0.1	$\frac{1}{6}$	$\frac{1}{2}$	1	1
		$\frac{1}{60}$	$\frac{31}{600}$	$\frac{631}{6000}$
	$\frac{1}{6}$	$\frac{31}{60}$	$1\frac{31}{600}$	$1\frac{631}{6000}$

So, $f(0.1) = 1\frac{631}{6000}$.

Find values of k so that each remainder is 3.

48. $(x^2 - x + k) \div (x - 1)$

1	1	-1	k
		1	0
	1	0	3

$$k + 0 = 3$$
$$k = 3$$

49. $(x^2 + kx - 17) \div (x - 2)$

2	1	k	-17
		2	$2(k+2)$
	1	$k+2$	3

$$-17 + 2(k+2) = 3$$
$$-17 + 2k + 4 = 3$$
$$-13 + 2k = 3$$
$$2k = 16$$
$$k = 8$$

50. $(x^2 + 5x + 7) \div (x + k)$

$-k$	1	5	7
		$-k$	$-k(5-k)$
	1	$5-k$	3

$$7 + [-k(5-k)] = 3$$
$$7 - 5k + k^2 = 3$$
$$k^2 - 5k + 4 = 0$$
$$(k-1)(k-4) = 0$$
$$k - 1 = 0 \text{ or } x - 4 = 0$$
$$k = 1 \text{ or } x = 4$$

51. $(x^3 + 4x^2 + x + k) \div (x + 2)$

-2	1	4	1	k
		-2	-4	6
	1	2	-3	3

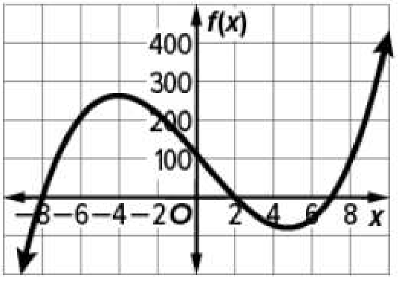
$$k + 6 = 3$$
$$k = -3$$

52. If $f(-8) = 0$ and $f(x) = x^3 - x^2 - 58x + 112$, find all the zeros of $f(x)$ and use them to graph the function.

-8	1	-1	-58	112
		-8	72	-112
	1	-9	14	0

$$x^3 - x^2 - 58x + 122 = (x+8)(x^2 - 9x + 14)$$
$$= (x+8)(x-7)(x-2)$$

The zeros of $f(x)$ are at –8, 7, and 2.



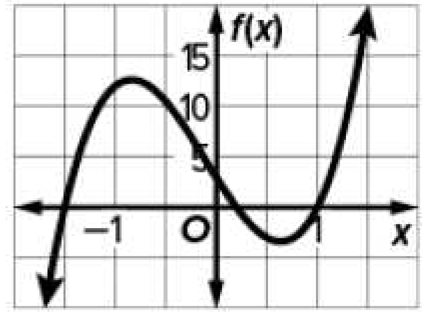
53. REASONING If $P(1) = 0$ and $P(x) = 10x^3 + kx^2 - 16x + 3$, find all the factors of $P(x)$ and use them to graph the function. Explain your reasoning.

$$k - 3 = 0$$
$$k = 3$$

1	10	k	-16	3
		10	$k + 10$	$k - 6$
	10	$k + 10$	$k - 6$	$k - 3$
	10	13	-3	

$$10x^3 + 3x^2 - 16x + 3 = (x - 1)(10x^2 + 13x - 3)$$
$$= (x + 8)(5x - 1)(2x + 3)$$

The zeros of $f(x)$ are at –1.5, 0.2, and 1.



54. GEOMETRY The volume of a box with a square base is $V(x) = 2x^3 + 15x^2 + 36x + 27$. If the height of the box is $(2x + 3)$ units, what are the measures of the sides of the base in terms of x?

$$2x^3 + 15x^2 + 36x + 27 = (2x + 3)(x^2 + 6x + 9)$$
$$= (2x + 3)(x + 3)(x + 3)$$

The sides of the base are $(x + 3)$.

$-\frac{3}{2}$	2	15	36	27
		-3	-18	-27
	2	12	18	0

55. GEOMETRY The average value of a franchise in the National Football League from 2000 to 2018 can be modeled by $y = -0.037x^5 + 1.658x^4 - 24.804x^3 + 145.100x^2 - 207.594x + 482.008$, where x is the number of years since 2000 and y is the value in millions of U.S. dollars.

$x = 0$

a. Complete the table of estimated values. Round to the nearest million.

	$x = 3$	$x = 12$	$x = 21$	$x = 25$
Year	2003	2012	2021	2025
Estimated Average Franchise Value (millions $)	621	1197	1740	−15255

b. What assumption did you make to make your predictions? Do you think the assumption is valid? Explain.

The assumption that is made is that the model still represents the situation after 25 years. This assumption does not seem valid because the average value is unlikely to fall so quickly.

56. CONSTRUCT ARGUMENTS Divide the polynomial function $f(x) = 4x^3 - 10x + 8$ by the factor $(x + 5)$. Then state and confirm the Remainder Theorem for this particular polynomial function and factor.

-5	4	0	-10	8
		-20	100	-450
	4	-20	90	-442

$f(-5) = 4(-5)3 - 10(-5) + 8 = -500 + 50 + 8 = -442.$

This means that $f(-5) = -442$.

57. **REGULARITY** The polynomial function $P(x)$ is symmetric in the y-axis and contains the point (2, –5). What is the remainder when $P(x)$ is divided by $(x + 2)$? Explain your reasoning.

The point (2,-5), *so* $P(-2) = -5$,
The remainder when $P(x)$ is divided by $(x + 2)$ is -5.

58. **STRUCTURE** Verify the Remainder Theorem for the polynomial $x^2 + 3x + 5$ and the factor $(x - \sqrt{3})$ by first using synthetic division and then evaluating for $x = \sqrt{3}$

synthetic division

$$\begin{array}{r|rrr} \sqrt{3} & 1 & 3 & 5 \\ & & \sqrt{3} & 3\sqrt{3}+3 \\ \hline & 1 & 3+\sqrt{3} & 3\sqrt{3}+8 \end{array}$$

evaluating

$$\left(\sqrt{3}\right)^2 + 3\sqrt{3} + 5 = 3 + 3\sqrt{3} + 5 = 3\sqrt{3} + 8$$

Q	Learning Outcome	Exercise	Page
9	Use the Factor Theorem to determine factors of polynomials.	23 to 30	140 +141

M3L4: The Remainder and Factor Theorems.

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

23. $x^3 - 3x + 2$; $x + 2$

$$x^3 - 3x + 2 = (x+2)(x^2 - 2x + 1)$$
$$= (x+2)(x-1)(x-1)$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

24. $x^4 + 2x^3 - 8x - 16$; $x + 2$

$$x^4 + 2x^3 - 8x - 16 = (x+2)(x^3 - 8)$$
$$= (x+2)(x-2)(x^2 + 2x + 4)$$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 0 & -8 & -16 \\ & & -2 & 0 & 0 & 16 \\ \hline & 1 & 0 & 0 & -8 & 0 \end{array}$$

25. $x^3 + 5x^2 + 2x - 8$; $x + 2$

$$x^3 + 5x^2 + 2x - 8 = (x+2)(x^2 + 3x - 4)$$
$$= (x+2)(x+4)(x-1)$$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 2 & -8 \\ & & -2 & -6 & 8 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

26. $x^3 - x^2 - 5x - 3$; $x - 3$

$$x^3 - x^2 - 5x - 3 = (x-3)(x^2 + 2x + 1)$$
$$= (x-3)(x+1)(x+1)$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -5 & -3 \\ & & 3 & 6 & 3 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

27. $2x^3 + 17x^2 + 23x - 42$; $x - 1$

$$2x^3 + 17x^2 + 23x - 42 = (x-1)(2x^2 + 19x + 42)$$
$$= (x-1)(x+6)(2x+7)$$

$$\begin{array}{r|rrrr} 1 & 2 & 17 & 23 & -42 \\ & & 2 & 19 & 42 \\ \hline & 2 & 19 & 42 & 0 \end{array}$$

28. $2x^3 + 7x^2 - 53x - 28$; $x - 4$

$$2x^3 + 7x^2 - 53x - 28 = (x-4)(2x^2 + 15x + 7)$$
$$= (x-4)(x+7)(2x+1)$$

4	2	7	−53	−28
		8	60	28
	2	15	7	0

29. $x^4 + 2x^3 + 2x^2 - 2x - 3$; $x - 1$

$$x^4 + 2x^3 + 2x^2 - 2x - 3 = (x-1)(x^3 + 3x^2 + 5x + 3)$$
$$= (x-1)(x+1)(x^2 + 2x + 3)$$

1	1	2	2	−2	−3
		1	3	5	3
	1	3	5	3	0

−1	1	3	5	3
		−1	−2	−3
	1	2	3	0

30. $3x^3 - 19x^2 - 15x + 7$; $x - 7$

$$3x^3 - 19x^2 - 15x + 7 = (x-7)(3x^2 + 2x - 1)$$
$$= (x-7)(x+1)(3x-1)$$

7	3	−19	−15	7
		21	14	−7
	3	2	−1	0

Q	Learning Outcome	Exercise	Page
10	Use the Fundamental Theorem of Algebra to determine the numbers and types of roots of polynomial equations.	1 to 28	149

M3L5: Roots and Zeros .

Solve each equation. State the number and type of roots.

1. $5x + 12 = 0$

$$5x = -12$$
$$x = -\frac{12}{5}$$

The polynomial has degree 1,
There is one root in the set of complex numbers.
The equation has one real root $\frac{-12}{5}$.

2. $x^2 - 4x + 40 = 0$

$$x = 2 \pm 6i$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two imaginary roots, 2 + 6i and 2 – 6i.,

3. $x^5 + 4x^3 = 0$

$$x^5 + 4x^3 = 0$$
$$x^3(x^2+4) = 0$$
$$x^3 = 0 \text{ or } x^2 + 4 = 0$$
$$x = 0 \text{ or } x^2 = -4$$
$$x = \pm\sqrt{-4}$$
$$x = \pm 2i$$

The polynomial has degree 5,
There is five roots in the set of complex numbers.
The equation has one real repeated root, 0, and two imaginary roots, 2i and −2i.

4. $x^4 - 625 = 0$

$$
\begin{aligned}
x^4 - 625 &= 0 \\
\left[(x^2)^2 - 25^2\right] &= 0 \\
(x^2+25)(x^2-25) &= 0 \\
(x^2+25)(x^2-5^2) &= 0 \\
(x^2+25)(x+5)(x-5) &= 0 \\
x^2+25 = 0 \text{ or } x+5 = 0 \text{ or } x-5 &= 0 \\
x^2 = -25 \text{ or } x = -5 \text{ or } x &= 5 \\
x = \sqrt{-25} \text{ or } x = -5 \text{ or } x &= 5 \\
x = \pm 5i \text{ or } x = -5 \text{ or } x &= 5
\end{aligned}
$$

The polynomial has degree 4,
There is four roots in the set of complex numbers.
The equation has two real roots, −5 and 5, and two imaginary roots, 5i and −5i.

5. $4x^2 - 4x - 1 = 0$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two real roots, $\frac{1+\sqrt{2}}{2}$ and $\frac{1-\sqrt{2}}{2}$.

6. $x^5 - 81x = 0$

$$
\begin{aligned}
x^5 - 81x &= 0 \\
x(x^4 - 81) &= 0 \\
x\left[(x^2)^2 - 9^2\right] &= 0 \\
x(x^2+9)(x^2-9) &= 0 \\
x(x^2+9)(x^2-3^2) &= 0 \\
x(x^2+9)(x+3)(x-3) &= 0 \\
x = 0 \text{ or } x^2+9 = 0 \text{ or } x+3 = 0 \text{ or } x-3 &= 0 \\
x = 0 \text{ or } x^2 = -9 \text{ or } x = -3 \text{ or } x &= 3 \\
x = 0 \text{ or } x = \sqrt{-9} \text{ or } x = -3 \text{ or } x &= 3 \\
x = 0 \text{ or } x = \pm 3i \text{ or } x = -3 \text{ or } x &= 3
\end{aligned}
$$

The polynomial has degree 5,
There is five roots in the set of complex numbers.
The equation has three real roots, 0, −3, and 3, and two imaginary roots, −3i and 3i.

7. $2x^2 + x - 6 = 0$

$$x = -2 \text{ or } x = \frac{3}{2}$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two real roots, -2 and $\frac{3}{2}$.

8. $4x^2 + 1 = 0$

$$x = \frac{\pm i}{2} \text{ or } \pm\frac{1}{2}i$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two imaginary roots, $\frac{1}{2}i$ and $-\frac{1}{2}i$.

9. $x^3 + 1 = 0$

$$
\begin{aligned}
x^3 + 1 &= 0 \\
x^3 + 1^3 &= 0 \\
(x+1)(x^2-x+1) &= 0 \\
x+1 = 0 \text{ or } x^2-x+1 &= 0 \\
x = -1 \text{ or } x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \\
x = -1 \text{ or } x &= \frac{1 \pm \sqrt{-3}}{2} \\
x = -1 \text{ or } x &= \frac{1 \pm i\sqrt{3}}{2}
\end{aligned}
$$

The polynomial has degree 3,
There is three roots in the set of complex numbers.
The equation has one real root, −1, and two imaginary roots $\frac{1+i\sqrt{3}}{2}$ and $\frac{1-i\sqrt{3}}{2}$

10. $2x^2 - 5x + 14 = 0$

$$x = \frac{5 \pm i\sqrt{87}}{4}$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two imaginary roots, $\frac{5+i\sqrt{87}}{4}$ and $\frac{5-i\sqrt{87}}{4}$.

11. $-3x^2 - 5x + 8 = 0$

$$x = -\frac{8}{3} \text{ or } x = 1$$

The polynomial has degree 2,
There is two roots in the set of complex numbers.
The equation has two real roots, $-\frac{8}{3}$ and 1.

12. $8x^3 - 27 = 0$

$$\begin{aligned} 8x^3 - 27 &= 0 \\ x^3 + 1^3 &= 0 \\ (2x-3)(4x^2+6x+9) &= 0 \\ 2x - 3 = 0 \text{ or } 4x^2 + 6x + 9 &= 0 \\ x = \frac{3}{2} \text{ or } x &= \frac{-6 \pm \sqrt{6^2 - 4(4)(9)}}{2(4)} \\ x = \frac{3}{2} \text{ or } x &= \frac{-6 \pm \sqrt{-108}}{8} \\ x = \frac{3}{2} \text{ or } x &= \frac{-6 \pm 6i\sqrt{3}}{8} \\ x = \frac{3}{2} \text{ or } x &= \frac{-3 \pm 3i\sqrt{3}}{4} \end{aligned}$$

The polynomial has degree 3,
There is three roots in the set of complex numbers.
The equation has one real root, $\frac{3}{2}$, and two imaginary roots,
$\frac{-3+3i\sqrt{3}}{4}$ and $\frac{-3-3i\sqrt{3}}{4}$

13. $16x^4 - 625 = 0$

$$\begin{aligned} 16x^4 - 625 &= 0 \\ [(4x^2)^2 - 25^2] &= 0 \\ (4x^2+25)(4x^2-25) &= 0 \\ (4x^2+25)[(2x)^2 - 5^2] &= 0 \\ (4x^2+25)(2x+5)(2x-5) &= 0 \\ 4x^2 + 25 = 0 \text{ or } 2x + 5 = 0 \text{ or } 2x - 5 &= 0 \\ x^2 = -\frac{25}{4} \text{ or } x = -\frac{5}{2} \text{ or } x &= \frac{5}{2} \\ x = \sqrt{-\frac{25}{4}} \text{ or } x = -\frac{5}{2} \text{ or } x &= \frac{5}{2} \\ x = \pm\frac{5}{2}i \text{ or } x = -\frac{5}{2} \text{ or } x &= \frac{5}{2} \end{aligned}$$

The polynomial has degree 4,
There is four roots in the set of complex numbers.
The equation has two real roots, $\frac{-5}{2}$ and $\frac{5}{2}$, and two imaginary roots, $\frac{-5}{2}i$ and $\frac{5}{2}i$

14. $x^3 - 6x^2 + 7x = 0$

$$\begin{aligned} x^3 - 6x^2 + 7x &= 0 \\ x(x^2 - 6x + 7) &= 0 \\ x = 0 \text{ or } x^2 - 6x + 7 &= 0 \\ x = 0 \text{ or } x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} \\ x = 0 \text{ or } x &= \frac{6 \pm \sqrt{8}}{2} \\ x = 0 \text{ or } x &= \frac{6 \pm 2\sqrt{2}}{2} \\ x = 0 \text{ or } x &= 3 \pm \sqrt{2} \end{aligned}$$

The polynomial has degree 3,
There is three roots in the set of complex numbers.
The equation has three real roots, 0 , $3 - \sqrt{2}$ and $3 + \sqrt{2}$

15. $x^5 - 8x^3 + 16x = 0$

$$x^5 - 8x^3 + 16x = 0$$
$$x(x^4 - 8x^2 + 16) = 0$$
$$x[(x^2)^2 - 8(x^2) + 16] = 0$$
$$x(u^2 - 8u + 16) = 0$$
$$x(u-4)^2 = 0$$
$$x(x^2-4)^2 = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 4 = 0 \quad \text{or} \quad x^2 - 4 = 0$$
$$x = 0 \qquad x^2 = 4 \qquad x^2 = 4$$
$$x = \pm 2 \qquad x = \pm 2$$

The polynomial has degree 5,
There is five roots in the set of complex numbers.
The equation has five real roots, −2, −2, 0, 2, and 2

16. $x^5 + 2x^3 + x = 0$

$$x^5 + 2x^3 + x = 0$$
$$x(x^4 + 2x^2 + 1) = 0$$
$$x[(x^2)^2 + 2(x^2) + 1] = 0$$
$$x(u^2 + 2u + 1) = 0$$
$$x(u+1)^2 = 0$$
$$x(x^2+1)^2 = 0$$
$$x = 0 \quad \text{or} \quad x^2 + 1 = 0 \quad \text{or} \quad x^2 + 1 = 0$$
$$x = 0 \qquad x^2 = -1 \qquad x^2 = -1$$
$$x = \pm i \qquad x = \pm i$$

The polynomial has degree 5,
There is five roots in the set of complex numbers.
The equation has one real root, 0 and four imaginary roots,
$-i$, $-i$, i and i.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

17. $g(x) = 3x^3 - 4x^2 - 17x + 6$

So, there are 2 or 0 positive real zeros.

$g(-x) = -3x^3 - 4x^2 + 17x + 6$

So, there are 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	1	0	3
0	1	2	3

18. $h(x) = 4x^3 - 12x^2 - x + 3$

So, there are 2 or 0 positive real zeros.

$h(-x) = -4x^3 - 12x^2 + x + 3$

So, there are 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	1	0	3
0	1	2	3

19. $f(x) = x^3 - 8x^2 + 2x - 4$ So, there are 3 or 1 positive real zeros.

$f(-x) = -x^3 - 8x^2 - 2x - 4$ So, there are 0 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
3	0	0	3
1	0	2	3

20. $p(x) = x^3 - x^2 + 4x - 6$ So, there are 3 or 1 positive real zeros.

$p(-x) = -x^3 - x^2 - 4x - 6$ So, there are 0 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
3	0	0	3
1	0	2	3

21. $q(x) = x^4 + 7x^2 + 3x - 9$ So, there is 1 positive real zero.

$q(-x) = x^4 + 7x^2 - 3x - 9$ So, there is 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
1	1	2	4

22. $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$ So, there are 2 or 0 positive real zeros.

$f(x) = x^4 + x^3 - 5x^2 - 6x + 1$ So, there are 2 or 0 negative real zeros.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	2	0	4
2	0	2	4
0	2	2	4
0	0	4	4

23. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$ So, there are 2 or 0 positive real zeros.

$f(-x) = x^4 + 5x^3 + 2x^2 - 5x + 7$ So, there are 2 or 0 negative real zeros.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	2	0	4
2	0	2	4
0	2	2	4
0	0	4	4

23. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$

So, there are 2 or 0 positive real zeros.

$f(-x) = x^4 + 5x^3 + 2x^2 - 5x + 7$

So, there are 2 or 0 negative real zeros.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	2	0	4
2	0	2	4
0	2	2	4
0	0	4	4

24. $f(x) = 2x^3 - 7x^2 - 2x + 12$

So, there are 2 or 0 positive real zeros.

$f(-x) = -2x^3 - 7x^2 + 2x + 12$

So, there are 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	1	0	3
0	1	2	3

25. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$

So, there are 2 or 0 positive real zeros.

$f(-x) = 3x^5 + 5x^4 + 4x^2 - 8$

So, there are 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	1	3	5
0	1	4	5

26. $f(x) = x^4 - 2x^2 - 5x + 19$

So, there are 2 or 0 positive real zeros.

$f(-x) = x^4 - 2x^2 + 5x + 19$

So, there are 2 or 0 negative real zeros.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	2	0	4
2	0	2	4
0	2	2	4
0	0	4	4

27. $f(x) = 4x^6 - 5x^4 - x^2 + 24$

So, there are 2 or 0 positive real zeros.

$f(-x) = 4x^6 - 5x^4 - x^2 + 24$

So, there are 2 or 0 negative real zeros.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	2	2	6
2	0	4	6
0	2	4	6
0	0	6	6

28. $f(x) = -x^5 + 14x^3 + 18x - 36$

So, there are 2 or 0 positive real zeros.

$f(-x) = +x^5 - 14x^3 - 18x - 36$

So, there is 1 negative real zero.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
2	1	2	5
0	1	4	5

Q	Learning Outcome	Exercise	Page
11	Determine the numbers and types of roots of polynomial equations, find zeros, and use zeros to graph polynomial functions.	29 to 45	150

M3L5: Roots and Zeros .

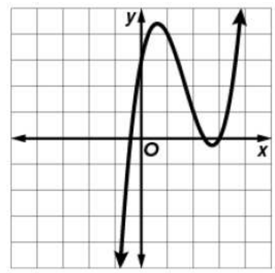
Find all of the zeros of each function and use them to sketch a rough graph.

29. $h(x) = x^3 - 5x^2 + 5x + 3$

$= (x - 3)(x^2 - 2x - 1)$

The zeros are 3 , $1 + \sqrt{2}$ and $1 - \sqrt{2}$

	1	-5	5	3
3		3	-6	-3
	1	-2	-1	0

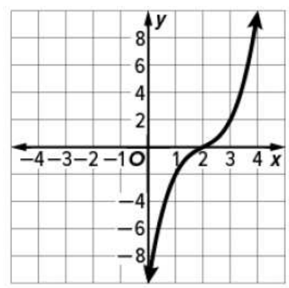


30. $g(x) = x^3 - 6x^2 + 13x - 10$

$= (x - 2)(x^2 - 4x + 5)$

The zeros are 2 , $2 + i$ and $2 - i$

	1	-6	13	-10
2		2	-8	10
	1	-4	5	0



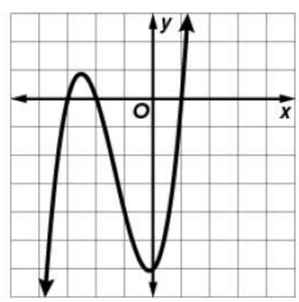
31. $h(x) = x^3 + 4x^2 + x - 6$

$= (x + 2)(x^2 + 2x - 3)$

$= (x + 2)(x + 3)(x - 1)$

The zeros are -2 , -3 and 1

	1	4	1	-6
-2		-2	-4	6
	1	2	-3	0



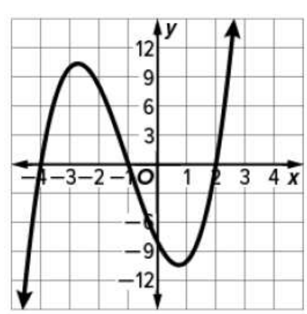
32. $q(x) = x^3 + 3x^2 - 6x - 8$

$= (x - 2)(x^2 + 5x + 4)$

$= (x - 2)(x + 4)(x + 1)$

The zeros are 2 , 4 and -1

	1	3	-6	-8
2		2	10	8
	1	5	4	0



33. $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

$= (x-1)(x^3 - 2x^2 - 7x - 4)$

	1	-3	-5	3	4
1		1	-2	-7	-4
	1	-2	-7	-4	0

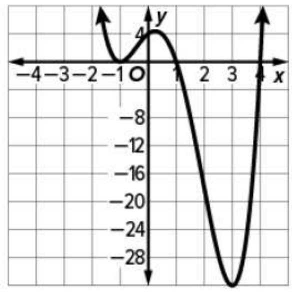
$= (x-1)(x+1)(x^2 - 3x - 4)$

$= (x-1)(x+1)(x-4)(x+1)$

	1	-2	-7	-4
-1		-1	3	4
	1	-3	-4	0

The function has zeros at −1, −1, 1, and 4.

-1 is repeated



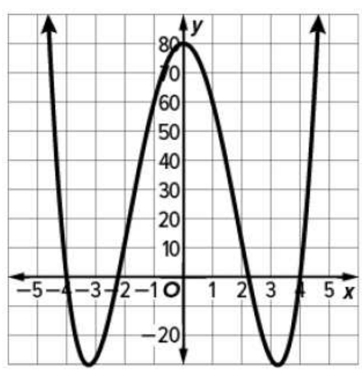
34. $f(x) = x^4 - 21x^2 + 80$

$= (x+4)(x^3 - 4x^2 - 5x + 20)$

	1	0	-21	0	80
-4		-4	16	20	-80
	1	-4	-5	20	0

$= (x+4)(x-4)(x^2 - 5)$

	1	-4	-5	20
4		4	0	-20
	1	0	-5	0



The function has zeros at −4, 4, $\sqrt{5}$, and $-\sqrt{5}$.

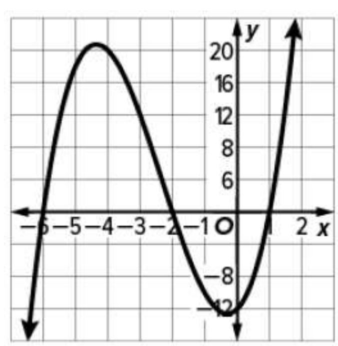
35. $f(x) = x^3 + 7x^2 + 4x - 12$

$= (x-1)(x^2 + 8x + 12)$

$= (x-1)(x+2)(x+6)$

The zeros are 1 , −2 and −6

	1	7	4	-12
1		1	8	12
	1	8	12	0



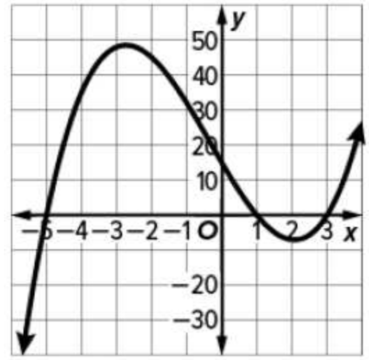
36. $f(x) = x^3 + x^2 - 17x + 15$

$= (x-1)(x^2+2x-15)$

$= (x-1)(x+5)(x-3)$

	1	1	-17	15
1		1	2	-15
	1	2	-15	0

The zeros are 1 , −5 and 3



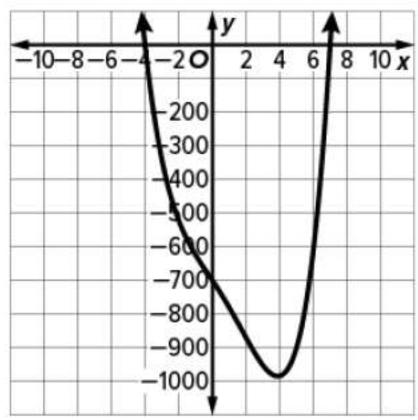
37. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$

$= (x+4)(x^3 - 7x^2 + 25x - 175)$

	1	-3	-3	-75	-700
-4		-4	28	-100	700
	1	-7	25	-175	0

$= (x+4)(x-7)(x^2+25)$

	1	-7	25	-175
7		7	0	175
	1	0	25	0



The zeros are $-4, 7, -5i$ *and* $5i$.

38. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$

$= (x+3)(x^3 + 3x^2 + 64x + 192)$

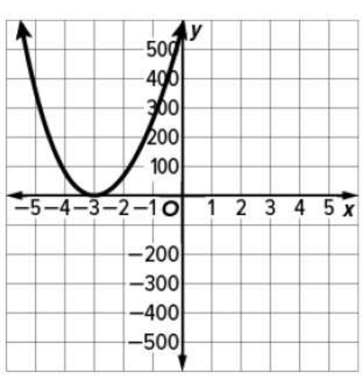
	1	6	73	384	576
-3		-3	-9	-192	-576
	1	3	64	192	0

$= (x+3)(x+3)(x^2+64)$

	1	3	64	192
-3		-3	0	-192
	1	0	64	0

The zeros are
$-3, -3, -8i$ *and* $8i$.

-3 is repeated



39. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$

$= (x-4)(x^3 - 4x^2 + 4x - 16)$

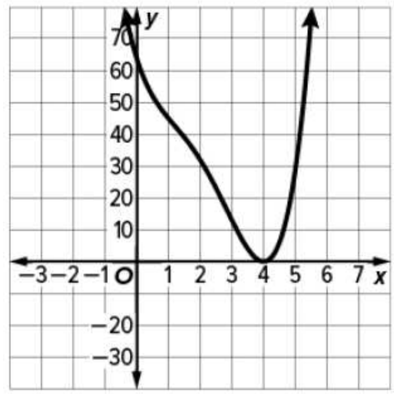
	1	-8	20	-32	64
4		4	-16	16	-64
	1	-4	4	-16	0

$= (x-4)(x-4)(x^2+4)$

	1	-4	4	-16
4		4	0	16
	1	0	4	0

The zeros are $4, 4, -2i$ *and* $2i$.

4 is repeated



40. $f(x) = x^5 - 8x^3 - 9x$

$= (x)(x^4 - 8x^2 - 9)$

	1	0	-8	0	-9	0
0		0	0	0	0	0
	1	0	-8	0	-9	0

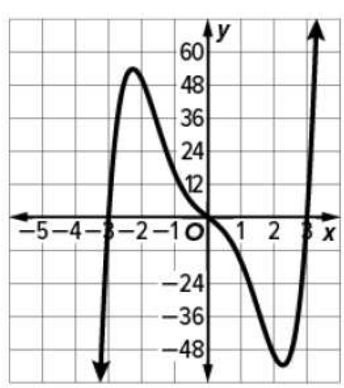
$= (x)(x+3)(x^3 - 3x^2 + x - 3)$

	1	0	-8	0	-9
-3		-3	9	-3	9
	1	-3	1	-3	0

$= (x)(x+3)(x-3)(x^2+1)$

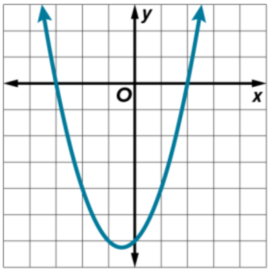
	1	-3	1	-3
3		3	0	3
	1	0	1	0

The zeros are $0, -3, 3, -i$ *and* i.



Write a polynomial that could be represented by each graph

41.

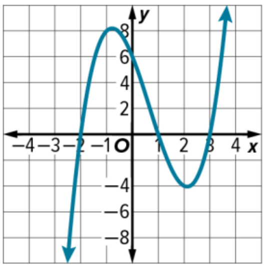


x-intercepts at $x = -3$ and $x = 2$
so, its factors are $x + 3$ and $x - 2$

$$y = (x+3)(x-2)$$
$$= x^2 + x - 6$$

$y = x^2 + x - 6.$

42.

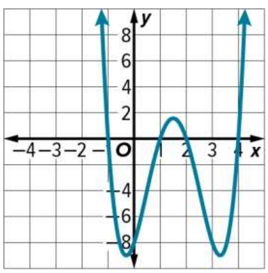


x-intercepts at $x = -2$, $x = 1$ and $x = 3$
so, its factors are $x + 2$, $x - 1$ and $x - 3$

$$y = (x+2)(x-1)(x-3)$$
$$= (x^2 + x - 2)(x - 3)$$
$$= x^3 - 2x^2 - 5x + 6$$

$y = x^3 - 2x^2 - 5x + 6.$

43.



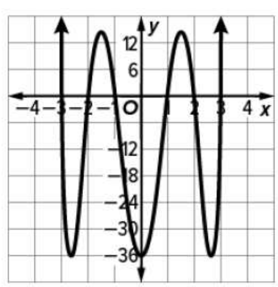
x-intercepts at $x = -1$, $x = 1$, $x = 2$ and $x = 4$
so, its factors are $x + 1$, $x - 1$, $x - 2$ and $x - 4$

$$y = (x+1)(x-1)(x-2)(x-4)$$
$$= (x^2 - 1)(x^2 - 6x + 8)$$
$$= x^4 - 6x^3 + 7x^2 + 6x - 8$$

$y = x^4 - 6x^3 + 7x^2 + 6x - 8.$

44. FISH Some fish jump out of the water. When a fish is out of the water, it's location is above sea level. When a fish dives back into the water, its location is below sea level. A biologist can use polynomial functions to model the location of fish compared to see level. A biologist noticed that a fish is at sea level at –3, –2, –1, 1, 2, and 3 seconds from noon. Graph a polynomial function that could represent the location of the fish compared to sea level *y*, in centimeters, *x* seconds from noon.

$$y = (x+3)(x-3)(x+2)(x-2)(x+1)(x-1)$$
$$= (x^2 - 9)(x^2 - 4)(x^2 - 1)$$
$$= (x^4 - 13x^2 + 36)(x^2 - 1)$$
$$= x^6 - 14x^4 + 49x^2 - 36$$

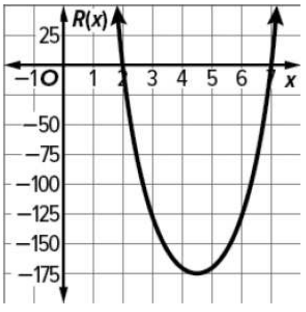


45. **BUSINESS** After introducing a new product, a company's profit is modeled by a polynomial function. In 2012 ($x = 2$) and 2017 ($x = 7$), the company's profit on the product was $0. Graph a polynomial function that could represent the amount of profit, p(*x*), in thousands of dollars, *x* years since 2010. $x = 0$

$$y = (x-2)(x-7) = x^2 - 9x + 14$$

Suppose the company multiplies this function by 31.25.

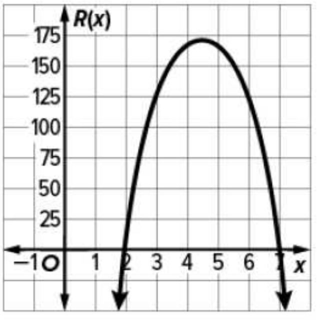
$y = 31.25x^2 - 281.25x + 437.5$



The graph show negative profits between 2012 and 2017.

Suppose the company multiplies this function by −31.25.

$y = -31.25x^2 + 281.25x - 437.5$



The graph show positive profits between 2012 and 2017.

Q	Learning Outcome	Exercise	Page
12	Find compositions of functions.	9 to 18	164

M4L1: Operations on Functions.

For each pair of functions, find *f* ∘ *g* and *g* ∘ *f*, if they exist. State the domain and range for each.

9. *f* = {(–8, –4), (0, 4), (2, 6), (–6, –2)}
g = {(4, –4), (–2, –1), (–4, 0), (6, –5)}

find $f \circ g$

$f(g(4) = f(-4) = undefined$

$f(g(-2) = f(-1) = undefined$

$f(g(-4) = f(0) = 4$

$f(g(6) = f(-5) = undefined$

$g(x)$	$f(x)$
$(4, -4)$	$\rightarrow undefined$
$(-2, -1)$	$\rightarrow undefined$
$(-4, 0)$	$\rightarrow (0, 4)$
$(6, -5)$	$\rightarrow undefined$

$f \circ g = \{(-4, 4)\}.$

$D = \{-4\}. R = \{4\}.$

find $g \circ f$

$g(f(-8) = g(-4) = 0$

$g(f(0) = g(4) = -4$

$g(f(2) = g(6) = -5$

$g(f(-6) = g(-2) = -1$

$f(x)$	$g(x)$
$(-8, -4)$	$\rightarrow (-4, 0)$
$(0, 4)$	$\rightarrow (4, -4)$
$(2, 6)$	$\rightarrow (6, -5)$
$(-6, -2)$	$\rightarrow (-2, -1)$

$g \circ f = \{(-8, 0), (0, -4), (2, -5), (-6, -1)\}$

$D = \{-8, -6, 0, 2\}. R = \{-5, -4, -1, 0\}.$

10. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
$g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$

find $f \circ g$

$f(g(6) = f(8) = 12$

$f(g(-12) = f(-5) = undefined$

$f(g(0) = f(5) = undefined$

$f(g(5) = f(1) = undefined$

$g(x) \quad f(x)$

$(6,8) \to (8,12)$

$(-12,-5) \to undefined$

$(0,5) \to undefined$

$(5,1) \to undefined$

$$f \circ g = \{(6,12)\}.$$

$$D = \{6\}. R = \{12\}.$$

find $g \circ f$

$g(f(-7) = g(0) = 5$

$g(f(4) = g(5) = 1$

$g(f(8) = g(12) = undefined$

$g(f(-3) = g(6) = 8$

$f(x) \quad g(x)$

$(-7,0) \to (0,5)$

$(4,5) \to (5,1)$

$(8,12) \to undefined$

$(-3,6) \to (6,8)$

$$g \circ f = \{(-7,5),(4,1),(-3,8)\}$$

$$D = \{-7,-3,4\}. R = \{1,5,8\}.$$

11. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
$g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$

find $f \circ g$

$f(g(-8) = f(2) = undefined$

$f(g(-4) = f(1) = undefined$

$f(g(3) = f(-3) = undefined$

$f(g(5) = f(7) = undefined$

$g(x) \quad f(x)$

$(-8,2) \to undefined$

$(-4,1) \to undefined$

$(3,-3) \to undefined$

$(5,7) \to undefined$

$$f \circ g = \{ \quad \}.$$

$$D = \{ \quad \}. R = \{ \quad \}.$$

find $g \circ f$

$g(f(5) = g(13) = undefined$

$g(f(-4) = g(-2) = undefined$

$g(f(-8) = g(-11) = undefined$

$g(f(3) = g(1) = undefined$

$f(x) \quad g(x)$

$(5,13) \to undefined$

$(-4,-2) \to undefined$

$(-8,-11) \to undefined$

$(3,1) \to undefined$

$$g \circ f \; undefined$$

$$D = \emptyset \; , R = \emptyset$$

12. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
$g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$

find $f \circ g$

$f(g(-6) = f(1) = undefined$

$f(g(-18) = f(13) = undefined$

$f(g(-14) = f(9) = undefined$

$f(g(-2) = f(-3) = undefined$

$g(x)$	$f(x)$
$(-6, 1)$	$\rightarrow undefined$
$(-18, 13)$	$\rightarrow undefined$
$(-14, 9)$	$\rightarrow undefined$
$(-2, -3)$	$\rightarrow undefined$

$f \circ g \quad undefined$

$D = \{ \quad \}. R = \{ \quad \}.$

find $g \circ f$

$g(f(-4) = g(-14) = 9$

$g(f(0) = g(-6) = 1$

$g(f(-6) = g(-18) = 13$

$g(f(2) = g(-2) = -3$

$f(x)$	$g(x)$
$(-4, -14)$	$\rightarrow (-14, 9)$
$(0, -6)$	$\rightarrow (-6, 1)$
$(-6, -18)$	$\rightarrow (-18, 13)$
$(2, -2)$	$\rightarrow (-2, -3)$

$g \circ f = \{(-4, 9), (0, 1), (-6, 13), (2, -3)\}$

$D = \{-6, -4, 0, 2\} \quad , R = \{-3, 1, 9, 13\}.$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each.

13. $f(x) = 2x$
$g(x) = x + 5$

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x+5) \\ &= 2(x+5) \\ &= 2x+10\end{aligned}$$

D = {all real numbers}
R = {all real numbers}

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x) \\ &= 2x+5 \\ &= 2x+5\end{aligned}$$

D = {all real numbers}
R = {all real numbers}

14. $f(x) = -3x$
$g(x) = -x + 8$

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(-x+8) \\ &= -3(-x+8) \\ &= 3x-24\end{aligned}$$

D = {all real numbers}
R = {all real numbers}

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(-3x) \\ &= -(-3x)+8 \\ &= 3x+8\end{aligned}$$

D = {all real numbers}
R = {all real numbers}

15. $f(x) = x^2 + 6x - 2$

$g(x) = x - 6$

$$[f \circ g](x) = f[g(x)] = f(x-6) = (x-6)^2 + 6(x-6) - 2 = x^2 - 6x - 2$$

D = {all real numbers}
R = $\{y \mid y \geq -11\}$.

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$

$$y = (3)^2 - 6(3) - 2 = -11$$

$$[g \circ f](x) = g[f(x)] = g(x^2 + 6x - 2) = x^2 + 6x - 2 - 6 = x^2 + 6x - 8$$

D = {all real numbers}
R = $\{y \mid y \geq -17\}$.

$$x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$$

$$y = (-3)^2 + 6(-3) - 8 = -17$$

16. $f(x) = 2x^2 - x + 1$

$g(x) = 4x + 3$

$$[f \circ g](x) = f[g(x)] = f(4x+3) = 2(4x+3)^2 - (4x+3) + 1 = 32x^2 + 44x + 16$$

D = {all real numbers}
R = $\{y \mid y \geq -0.875\}$.

$$x = \frac{-b}{2a} = \frac{-(44)}{2(32)} = \frac{-11}{16}$$

$$y = 32\left(\frac{-11}{16}\right)^2 + 44\left(\frac{-11}{16}\right) + 16 = \frac{7}{8} = 0.875$$

$$[g \circ f](x) = g[f(x)] = g(2x^2 - x + 1) = 4(2x^2 - x + 1) + 3 = 8x^2 - 4x + 7$$

D = {all real numbers}
R = $\{y \mid y \geq 6.5\}$.

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(8)} = \frac{1}{4}$$

$$y = 8\left(\frac{1}{4}\right)^2 - 4\left(\frac{1}{4}\right) + 7 = \frac{13}{2} = 6.5$$

17. USE A MODEL Mr. Rivera wants to purchase a riding lawn mower, which is on sale for 15% off the original price. The sales tax in his area is 6.5%. Let x represent the original cost of the lawn mower. Write two functions representing the price of the lawn mower $p(x)$ after the discount and the price of the lawn mower $t(x)$ after sales tax. Write a composition of functions that represents the price of the riding lawn mower. How much will Mr. Rivera pay for a riding lawn mower that originally cost $1350?

original price:

$$P(x) = (100\% - 15\%)x$$

$$P(x) = 0.85x$$

The price after taxes

$$T(x) = (100\% + 6.5\%)x$$

$$T(x) = 1.065x$$

$$T(P(x)) = T(0.85x) = 1.065(0.85x) = 0.90525x$$

How much will Mr. Rivera pay for a riding lawn mower that originally cost $1350?

$$0.90525(1350) = 1222.09\$$$

18. REASONING A sporting goods store is offering a 20% discount on shoes. Mariana also has a $5 off coupon that can be applied to her purchase. She is planning to buy a pair of shoes that originally costs $89. Will the final price be lower if the discount is applied before the coupon or if the coupon is applied before the discount? Justify your response.

discount :

$D(x) = (100\% - 20\%)x$

$D(x) = 0.80x$

coupon:

$C(x) = x - 5$

Coupon then discount

$D(C(x)) = D(x - 5)$

$= 0.80(x - 5)$

$D(C(89) = 0.80(89 - 5)$

$= 67.2\$$

discount then coupon

$C(D(x)) = C(0.80x)$

$= (0.80x) - 5$

$C(D(x)) = (0.80(89)) - 5$

$= 66.2\$$

The final price of the shoes will be less if the discount is applied before the coupon.

Q	Learning Outcome	Exercise	Page
13	Verify that two relations are inverses by using compositions.	17 to 24	172

M4L2: Inverse Relations and Functions.

Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

17. $f(x) = x - 1$

$g(x) = 1 - x$

Find $[f \circ g](x)$.

$$[f \circ g](x) = f[g(x)] = f(1 - x) = (1 - x) - 1 = -x$$

Because $[f \circ g](x)$ is not the identity function, $f(x)$ and $g(x)$ are not inverses.

18. $f(x) = 2x + 3$

$g(x) = \frac{1}{2}(x - 3)$

Find $[f \circ g](x)$.

$$[f \circ g](x) = f[g(x)] = f\left(\frac{1}{2}(x-3)\right) = 2\left(\frac{1}{2}(x-3)\right) + 3 = 2\left(\frac{1}{2}x - \frac{3}{2}\right) + 3 = x - 3 + 3 = x$$

Find $[g \circ f](x)$

$$[g \circ f](x) = g[f(x)] = g[2x + 3] = \frac{1}{2}((2x + 3) - 3) = \frac{1}{2}(2x) = x$$

Because $[f \circ g](x)$ and $[g \circ f](x)$ is the identity function, $f(x)$ and $g(x)$ are inverses.

19. $f(x) = 5x - 5$

$g(x) = \frac{1}{5}x + 1$

Find $[f \circ g](x)$.

$$[f \circ g](x) = f[g(x)] = f\left(\frac{1}{5}x + 1\right) = 5\left(\frac{1}{5}x + 1\right) - 5 = x + 5 - 5 = x$$

Find $[g \circ f](x)$.

$$[g \circ f](x) = g[f(x)] = g(5x - 5) = \frac{1}{5}(5x - 5) + 1 = x - 1 + 1 = x$$

Because $[f \circ g](x)$ and $[g \circ f](x)$ is the identity function, $f(x)$ and $g(x)$ are inverses.

20. $f(x) = 2x$

$g(x) = \frac{1}{2}x$

Find $[f \circ g](x)$

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{2}x\right] \\ &= 2\left(\frac{1}{2}x\right) \\ &= x\end{aligned}$$

Find $[g \circ f](x)$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g[2x] \\ &= \frac{1}{2}(2x) \\ &= x\end{aligned}$$

Because $[f \circ g](x)$ and $[g \circ f](x)$ is the identity function, $f(x)$ and $g(x)$ are inverses.

21. $h(x) = 6x - 2$

$g(x) = \frac{1}{6}x + 3$

Find $[h \circ g](x)$.

$$\begin{aligned}[h \circ g](x) &= h[g(x)] \\ &= h\left(\frac{1}{6}x + 3\right) \\ &= 6\left(\frac{1}{6}x + 3\right) - 2 \\ &= x + 18 - 2 \\ &= 16\end{aligned}$$

Because $[h \circ g](x)$ is not the identity function, $h(x)$ and $g(x)$ are not inverses.

22. $f(x) = 8x - 10$

$g(x) = \frac{1}{8}x + \frac{5}{4}$

Find $[f \circ g](x)$

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{8}x + \frac{5}{4}\right] \\ &= 8\left(\frac{1}{8}x + \frac{5}{4}\right) - 10 \\ &= x + 10 - 10 \\ &= x\end{aligned}$$

Find $[g \circ f](x)$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g[8x - 10] \\ &= \frac{1}{8}(8x - 10) + \frac{5}{4} \\ &= x - \frac{5}{4} + \frac{5}{4} \\ &= x\end{aligned}$$

Because $[f \circ g](x)$ and $[g \circ f](x)$ is the identity function, $f(x)$ and $g(x)$ are inverses.

23. GEOMETRY The formula for the volume of a right circular cone with a height of 2 feet is $V = \frac{2}{3}\pi r^2$.

Determine whether $r = \sqrt{\frac{3V}{2\pi}}$ is the inverse of the original function.

Find $V \circ r$.

$$\begin{aligned}V &= \frac{2}{3}\pi r^2 \\ &= \frac{2}{3}\pi\left(\sqrt{\frac{3V}{2\pi}}\right)^2 \\ &= \frac{2}{3}\pi\left(\frac{3V}{2\pi}\right) \\ &= V\end{aligned}$$

Find $r \circ V$.

$$\begin{aligned}r &= \sqrt{\frac{3V}{2\pi}} \\ &= \sqrt{\frac{3\left(\frac{2}{3}\pi r^2\right)}{2\pi}} \\ &= \sqrt{r^2} \\ &= r\end{aligned}$$

$r = \sqrt{\frac{3V}{2\pi}}$ is the inverse of $V = \frac{2}{3}\pi r^2$.

24. GEOMETRY The formula for the area of a trapezoid is $A = \frac{h}{2}(a + b)$. Determine whether $h = 2A - (a + b)$ is the inverse of the original function.

Find $A \circ h$.	Find $h \circ A$.
$A = \frac{h}{2}(a+b)$	$h = 2A-(a+b)$
$= \frac{2A-(a+b)}{2}(a+b)$	$=2\left[\frac{h}{2}(a+b)\right]-(a+b)$
$= \frac{(a+b)[2A-(a+b)]}{2}$	$=h(a+b)-(a+b)$

$h = 2A-(a+b)$ is not the inverse of $A = \frac{h}{2}(a+b)$.

Q	Learning Outcome	Exercise	Page
14	Simplify expressions in exponential or radical form.	13 to 30	179+ 180

M4L3: nth Roots and Rational Exponents .

Write each expression in radical form, or write each radical in exponential form.

13. $8^{\frac{1}{5}}$ $= \sqrt[5]{8^1}$ $= \sqrt[5]{8}$

14. $4^{\frac{2}{7}}$ $= \sqrt[7]{4^2}$ $= \sqrt[7]{16}$

15. $(x^3)^{\frac{3}{2}}$ $= x^{\frac{9}{2}}$ $= \sqrt{x^9}$

16. $\sqrt{17}$ $= 17^{\frac{1}{2}}$

17. $\sqrt[3]{5xy^2}$ $= 5^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}$

18. $\sqrt[4]{625x^2}$ $= 625^{\frac{1}{4}}x^{\frac{2}{4}}$ $= 5x^{\frac{1}{2}}$

19. ORBITING The distance in millions of miles a planet is from the Sun in terms of t, the number of Earth days it takes for the planet to orbit the Sun, can be modeled by the expression $\sqrt[3]{6t^2}$. Write the expression in exponential form.

$\sqrt[3]{6t^2} = 6^{\frac{1}{3}}t^{\frac{2}{3}}$

20. DEPRECIATION The depreciation rate is calculated by the expression $1 - \left(\frac{T}{P}\right)^{\frac{1}{n}}$, where n is the age of the item in years, T is the resale price in dollars, and P is the original price in dollars. Write the expression in radical form for an 8 year old car that was originally purchased for \$52,425.

$$1-\left(\frac{T}{P}\right)^{\frac{1}{n}} = 1-\left(\frac{T}{52{,}425}\right)^{\frac{1}{8}} \quad n=8,\ P=52{,}425$$

$$= 1-\sqrt[8]{\frac{T}{52{,}425}} \quad b^{\frac{1}{n}} = \sqrt[n]{b}$$

Evaluate each expression.

21. $27^{\frac{1}{3}}$ $= 3$

22. $256^{\frac{1}{4}}$ $= 4$

23. $16^{-\frac{3}{2}}$ $= \frac{1}{64}$

24. $81^{-\frac{1}{4}}$ $= \frac{1}{3}$

25. $1024^{\frac{3}{5}}$ $= 64$

26. $16^{-\frac{5}{4}}$ $= \frac{1}{32}$

Simplify each expression.

27. $x^{\frac{1}{3}} \bullet x^{\frac{2}{5}} = x^{\frac{1}{3}+\frac{2}{5}} = x^{\frac{11}{15}}$

28. $a^{\frac{4}{9}} \bullet a^{\frac{1}{4}} = a^{\frac{4}{9}+\frac{1}{4}} = x^{\frac{25}{36}}$

29. $b^{-\frac{3}{4}} = \frac{1}{b^{\frac{3}{4}}} \cdot \frac{b^{\frac{1}{4}}}{b^{\frac{1}{4}}} = \frac{1.\, b^{\frac{1}{4}}}{b^{\frac{3}{4}}.\, b^{\frac{1}{4}}} = \frac{b^{\frac{1}{4}}}{b^{\frac{3}{4}+\frac{1}{4}}} = \frac{b^{\frac{1}{4}}}{b}$

30. $y^{-\frac{4}{5}} = \frac{1}{y^{\frac{4}{5}}} \cdot \frac{y^{\frac{1}{5}}}{y^{\frac{1}{5}}} = \frac{y^{\frac{1}{5}}}{y^{\frac{4}{5}+\frac{1}{5}}} = \frac{y^{\frac{1}{5}}}{y}$

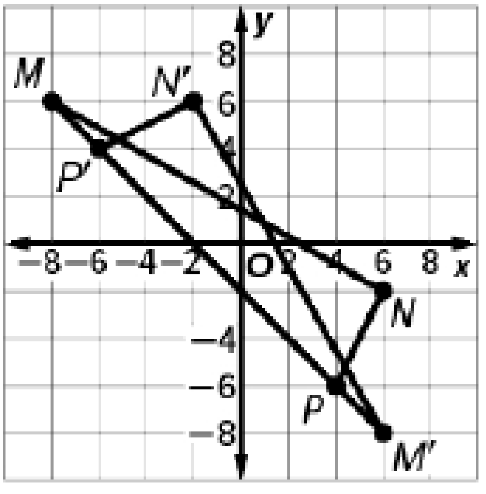
Q	Learning Outcome	Exercise	Page
15	Find inverses of relations.	1 to 16	171+172

M4L2: Inverse Relations and Functions.

For each polygon, find the inverse of the relation. Then, graph both the original relation and its inverse.

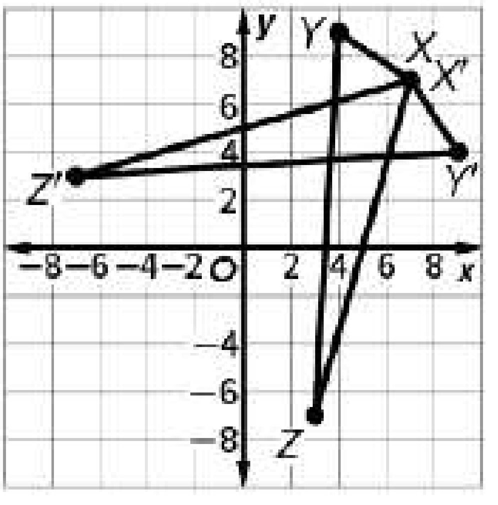
1. △ MNP with vertices at {(−8, 6), (6, −2), (4, −6)}

{(6, –8), (–2, 6), (–6, 4)}



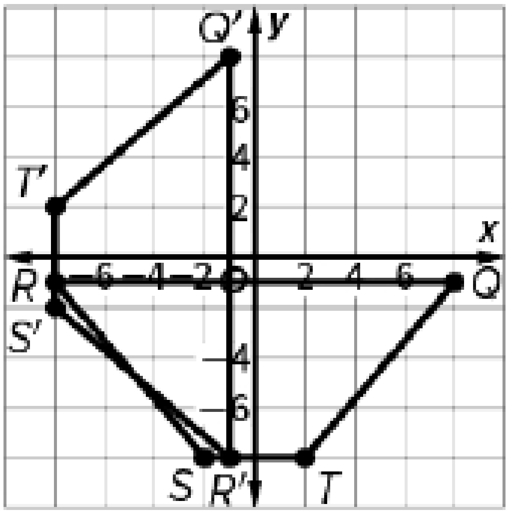
2. △ XYZ with vertices at {(7, 7), (4, 9), (3, −7)}

{(7, 7), (9, 4), (–7, 3)}



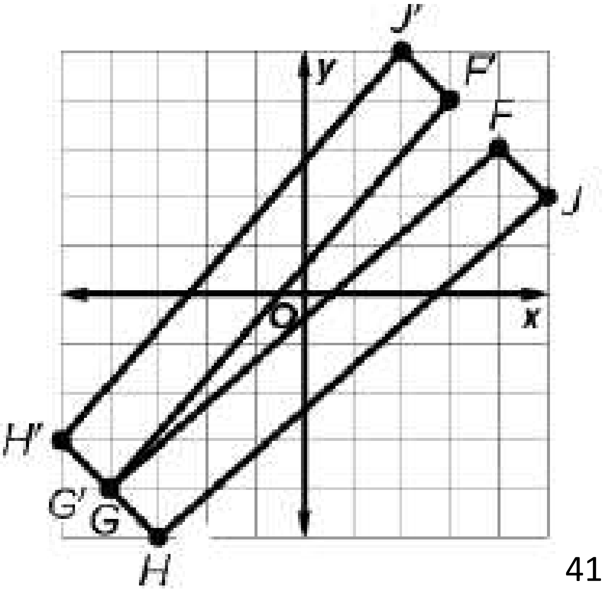
3. trapezoid *QRST* with vertices {(8, −1), (−8, −1), (−2, −8), (2, −8)}

{(–1, 8), (–1, –8), (–8, –2), (–8, 2)}



4. quadrilateral *FGHJ* with vertices at {(4, 3), (−4, −4), (−3, −5), (5, 2)}

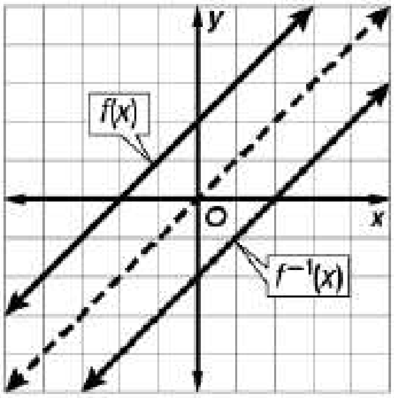
{(3, 4), (–4, –4), (–5, –3), (2, 5)}



Find the inverse of each function. Then graph the function and its inverse. If necessary, restrict the domain of the inverse so that it is a function.

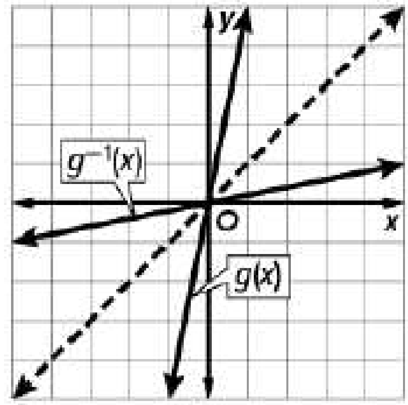
5. $f(x) = x + 2$

$$y = x + 2$$
$$x = y + 2$$
$$x - 2 = y$$
$$f^{-1}(x) = x - 2$$



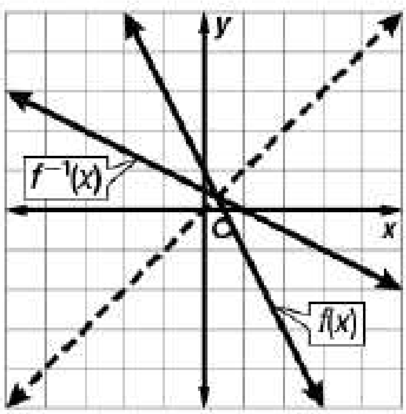
6. $g(x) = 5x$

$$y = 5x$$
$$x = 5y$$
$$\frac{x}{5} = y$$
$$g^{-1}(x) = \frac{x}{5}$$



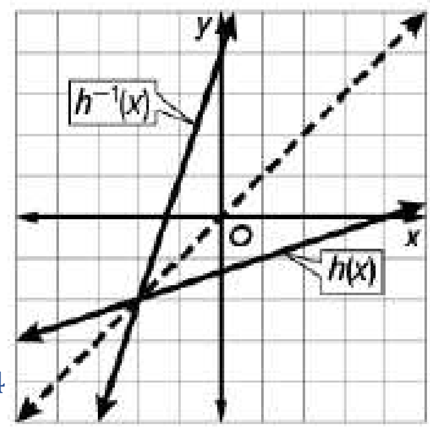
7. $f(x) = -2x + 1$

$$y = -2x + 1$$
$$x = -2y + 1$$
$$x - 1 = -2y$$
$$\frac{x-1}{-2} = y$$
$$f^{-1}(x) = \frac{-1}{2}x + \frac{1}{2}$$



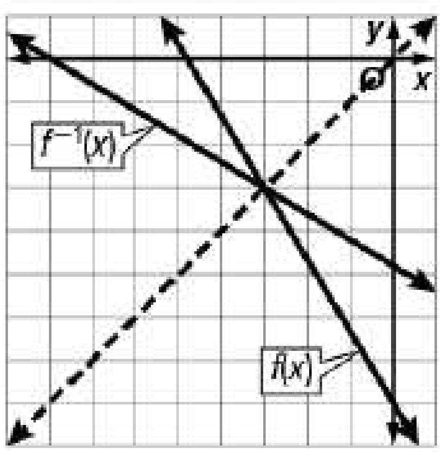
8. $h(x) = \frac{x-4}{3}$

$$y = \frac{x-4}{3}$$
$$x = \frac{y-4}{3}$$
$$3x = y - 4$$
$$3x + 4 = y$$
$$h^{-1}(x) = 3x + 4$$



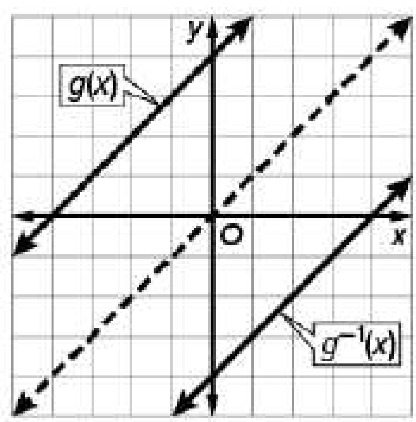
9. $f(x) = -\frac{5}{3}x - 8$

$$y = -\frac{5}{3}x - 8$$
$$x = -\frac{5}{3}y - 8$$
$$x + 8 = -\frac{5}{3}y$$
$$-\frac{3}{5}(x + 8) = y$$
$$f^{-1}(x) = -\frac{3}{5}x - \frac{24}{5}$$



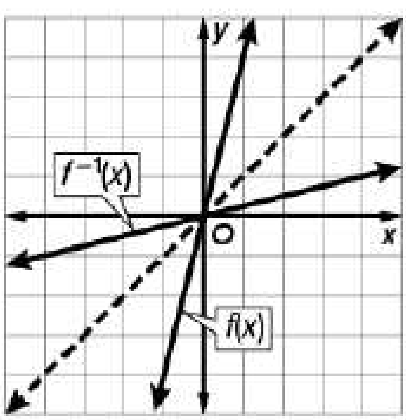
10. $g(x) = x + 4$

$$y = x + 4$$
$$x = y + 4$$
$$x - 4 = y$$
$$g^{-1}(x) = x - 4$$



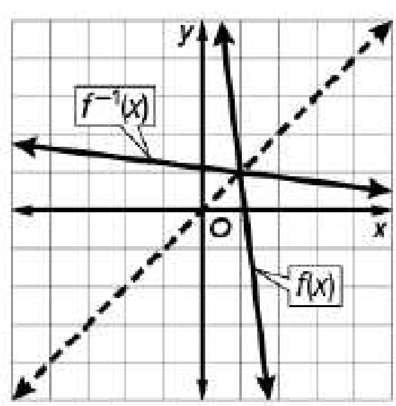
11. $f(x) = 4x$

$$y = 4x$$
$$x = 4y$$
$$\frac{x}{4} = y$$
$$f^{-1}(x) = \frac{1}{4}x$$



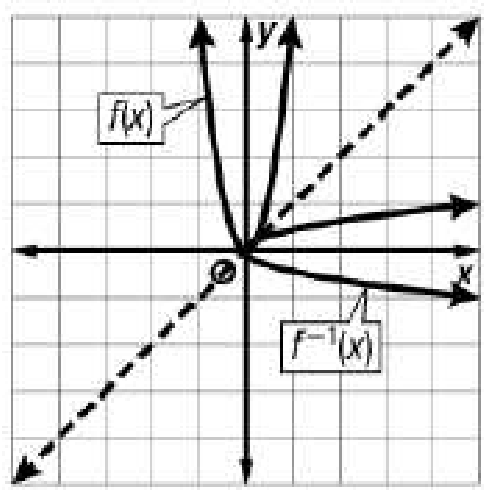
12. $f(x) = -8x + 9$

$$y = -8x + 9$$
$$x = -8y + 9$$
$$x - 9 = -8$$
$$\frac{x-9}{-8} = y$$
$$f^{-1}(x) = \frac{-1}{8}x + \frac{9}{8}$$



13. $f(x) = 5x^2$

$$y = 5x^2$$
$$x = 5y^2$$
$$\frac{x}{5} = y^2$$
$$y = \mp\sqrt{\frac{x}{5}}$$
$$y = \mp\frac{\sqrt{x}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$y = \mp\frac{\sqrt{5x}}{5}$$
$$f^{-1}(x) = \mp\frac{\sqrt{5x}}{5}$$

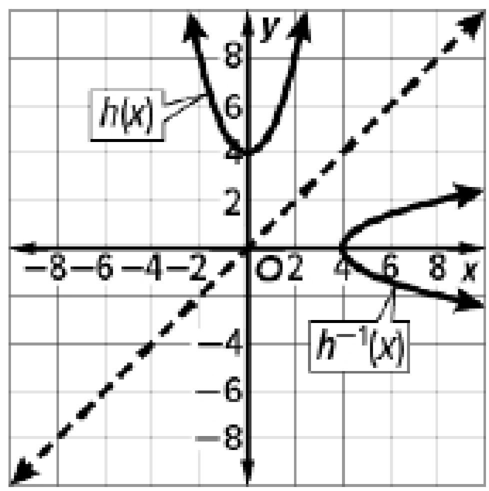


If the domain of $f(x)$ is restricted to $(-\infty, 0]$, $f^{-1}(x) = -\frac{\sqrt{5x}}{5}$

If the domain of $f(x)$ is restricted to $[0, \infty)$, $f^{-1}(x) = \frac{\sqrt{5x}}{5}$

14. $h(x) = x^2 + 4$

$$y = x^2 + 4$$
$$x = y^2 + 4$$
$$x - 4 = y^2$$
$$y = \pm\sqrt{x - 4}$$
$$h^{-1}(x) = \pm\sqrt{x - 4}$$



If the domain of $f(x)$ is restricted to $(-\infty, 0]$, $h^{-1}(x) = -\sqrt{x - 4}$

If the domain of $f(x)$ is restricted to $[0, \infty)$, $h^{-1}(x) = \sqrt{x - 4}$

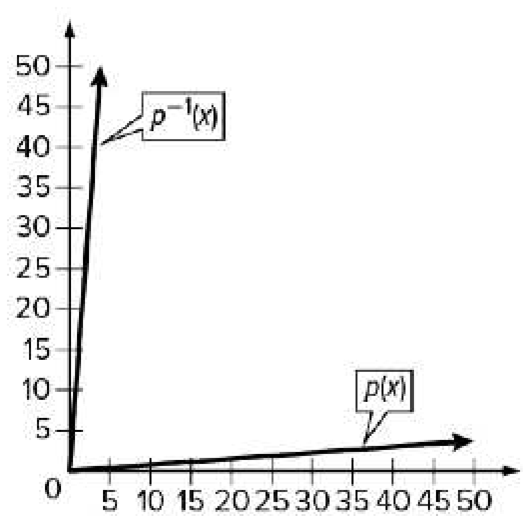
15. **WEIGHT** The formula to convert weight in pounds to stones is $p(x) = \frac{x}{14}$, where x is the weight in pounds.

a. Find the inverse of $p(x)$ and describe its meaning.

b. Graph $p(x)$ and $p^{-1}(x)$. Use the graph to find the weight in pounds of a dog that weighs about 2.5 stones.

a) $$y = \frac{x}{4}$$
$$x = \frac{y}{4}$$
$$4x = y$$
$$P^{-1}(x) = 4x$$

The inverse converts stones to pounds, where x is the weight in stones.



b) Look at $P^{-1}(x)$. When $x = 2.5$, $P^{-1}(2.5) = 35$.
So, the weight in pounds of a dog that weighs about 2.5 stones is about 35 pounds.

16. **CRYPTOGRAPHY** DeAndre is designing a code to send secret messages. He assigns each letter of the alphabet to a number, where A = 1, B = 2, C = 3, and so on. Then he uses $c(x) = 4x - 9$ to create the secret code.

a. Find the inverse of $c(x)$ and describe its meaning.

b. Make tables of $c(x)$ and $c^{-1}(x)$. Use the table to decipher the message: 15, 75, 47, 3, 71, 27, 51, 47, 67.

a) $y = 4x - 9$

$x = 4y - 9$

$x + 9 = 4y$

$\frac{x+9}{4} = y$

$c^{-1}(x) = \frac{1}{4}x + \frac{9}{4}$

The inverse can be used to convert the secret code to the original message.

b)

$c^{-1}(x)$	Letter	$c^{-1}(x)$	Letter	$c^{-1}(x)$	Letter
–5	A = 1	31	J = 10	67	S = 19
–1	B = 2	35	K = 11	71	T = 20
3	C = 3	39	L = 12	75	U = 21
7	D = 4	43	M = 13	79	V = 22
11	E = 5	47	N = 15	83	W = 23
15	F = 6	51	O = 15	87	X = 24
19	G = 7	55	P = 16	91	X = 25
23	H = 8	59	Q = 17	95	Z = 26
27	I = 9	63	R = 18		

Using the table, so the message is FUNCTIONS.