تجميعة أسئلة نهائية وفق الهيكل الوزاري منهج ريفيل

إعداد: أمل الحسبان

Simplify each expression.

28. $\dfrac{1 - \sin^2 \theta}{\sin^2 \theta}$

29. $\tan \theta \csc \theta$

30. $\dfrac{1}{\sin^2 \theta} - \dfrac{\cos^2 \theta}{\sin^2 \theta}$

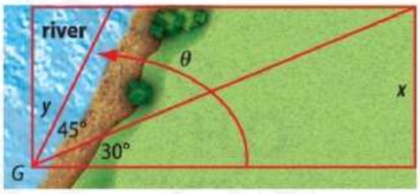
31. $2(\csc^2 \theta - \cot^2 \theta)$

32. $(1 + \sin \theta)(1 - \sin \theta)$

33. $2 - 2 \sin^2 \theta$

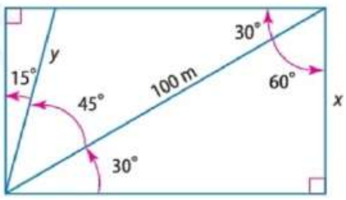
Real-World Example 2 Sum and Difference of Angles Identities

A geologist measures the angle between one side of a rectangular lot and the line from her position to the opposite corner of the lot as 30°. She then measures the angle between that line and the line to the point on the property where a river crosses as 45°. She stands 100 meters from the opposite corner of the property. How far is she from the point at which the river crosses the property line?



Understand The question asks for the distance between the geologist and the point where the river crosses the property line, or y.

Plan Draw a picture that labels all the things that you know from the information given.



Solve Solve for x.

$\sin 30^\circ = \frac{x}{100}$ Definition of sine

$x = 100 \sin 30^\circ$

$x = 50$ Since the lot is rectangular, opposite sides are equal.

Now look at the triangle on the far left and solve for y.

$\cos 15^\circ = \frac{50}{y}$ Definition of cosine

$\cos (45^\circ - 30^\circ) = \frac{50}{y}$ $15 = 45 - 30$

$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{50}{y}$ Difference identity

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{50}{y}$ Evaluate.

$\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{50}{y}$ Simplify.

$(\sqrt{6} + \sqrt{2})y = 200$ Cross products

$y = \frac{200}{(\sqrt{6} + \sqrt{2})} \cdot \frac{(\sqrt{6} - \sqrt{2})}{(\sqrt{6} - \sqrt{2})}$

$y = 50(\sqrt{6} - \sqrt{2})$

$y = 50\sqrt{6} - 50\sqrt{2}$ or about 51.8

The geologist is about 51.8 meters from the point where the river crosses the property line.

Check Use a calculator to find the Arccos of $\frac{50}{51.8} \approx 15^\circ$. ✓

Guided Practice

2. The harmonic motion of an object can be described by $x = 4 \cos \left(2\pi t - \frac{\pi}{4}\right)$, where x is distance from the equilibrium point in centimeters and t is time in minutes. Find the exact distance from the equilibrium point at 45 seconds.

1 **Find the exact value of each expression.**

12. $\sin 165°$ **13.** $\cos 135°$ **14.** $\cos \frac{7\pi}{12}$

15. $\sin \frac{\pi}{12}$ **16.** $\tan 195°$ **17.** $\cos\left(-\frac{\pi}{12}\right)$

Find the exact value of each expression.

24. $\tan 165°$ **25.** $\sec 1275°$ **26.** $\sin 735°$

27. $\tan \frac{23\pi}{12}$ **28.** $\csc \frac{5\pi}{12}$ **29.** $\cot \frac{113\pi}{12}$

1–3 PRECISION **Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.**

1. $\sin \theta = \frac{1}{4};\ 0° < \theta < 90°$ **2.** $\sin \theta = \frac{4}{5};\ 90° < \theta < 180°$

3. $\cos \theta = -\frac{5}{13};\ \frac{\pi}{2} < \theta < \pi$ **4.** $\cos \theta = \frac{3}{5};\ 270° < \theta < 360°$

5. $\tan \theta = -\frac{8}{15};\ 90° < \theta < 180°$ **6.** $\tan \theta = \frac{5}{12};\ \pi < \theta < \frac{3\pi}{2}$

1–3 **Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.**

12. $\sin \theta = \frac{2}{3};\ 90° < \theta < 180°$ **13.** $\sin \theta = -\frac{15}{17};\ \pi < \theta < \frac{3\pi}{2}$

14. $\cos \theta = \frac{3}{5};\ \frac{3\pi}{2} < \theta < 2\pi$ **15.** $\cos \theta = \frac{1}{5};\ 270° < \theta < 360°$

16. $\tan \theta = \frac{4}{3};\ 180° < \theta < 270°$ **17.** $\tan \theta = -2;\ \frac{\pi}{2} < \theta < \pi$

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*. (Example 5)

27. $A = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix}$

28. $A = \begin{bmatrix} -4 & 8 \\ 1 & -2 \end{bmatrix}$

29. $A = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

30. $A = \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}$

31. $A = \begin{bmatrix} -1 & -1 & -3 \\ 3 & 6 & 4 \\ 2 & 1 & 8 \end{bmatrix}$

32. $A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 3 & 5 \\ 6 & -1 & -4 \end{bmatrix}$

33. $A = \begin{bmatrix} 5 & 2 & -1 \\ 4 & 7 & -3 \\ 1 & -5 & 2 \end{bmatrix}$

34. $A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix}$

Find *AB* and *BA*, if possible. (Example 1)

1. $A = \begin{bmatrix} 8 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & -5 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 0 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 1 & -10 & 9 \end{bmatrix}$

5. $A = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$

8. $A = \begin{bmatrix} 6 & -9 & 10 \\ 4 & 3 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -8 \\ 3 & -9 \\ -2 & 5 \\ 4 & 1 \end{bmatrix}$

Find the maximum and minimum values of the objective function $f(x, y)$ and for what values of x and y they occur, subject to the given constraints. (Example 1)

1. $f(x, y) = 3x + y$
$y \leq 2x + 1$
$x + 2y \leq 12$
$1 \leq y \leq 3$

2. $f(x, y) = -x + 4y$
$y \leq x + 4$
$y \geq -x + 3$
$1 \leq x \leq 4$

3. $f(x, y) = x - y$
$x + 2y \leq 6$
$2x - y \leq 7$
$x \geq -2$
$y \geq -3$

4. $f(x, y) = 3x - 5y$
$x \geq 0, y \geq 0$
$x + 2y \leq 6$
$2y - x \leq 2$
$x + y \leq 5$

5. $f(x, y) = 3x - 2y$
$y \leq x + 3$
$1 \leq x \leq 5$
$y \geq 2$

6. $f(x, y) = 3y + x$
$4y \leq x + 8$
$2y \geq 3x - 6$
$2x + 2y \geq 4$

7. $f(x, y) = x - 4y$
$x \geq 2, y \geq 1$
$x - 2y \geq -4$
$2x - y \leq 7$
$x + y \leq 8$

8. $f(x, y) = x - y$
$3x - 2y \geq -7$
$x + 6y \geq -9$
$5x + y \leq 13, x - 3y \geq -7$

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. $y = 2x^2 - 24x + 40$ **2.** $y = 3x^2 - 6x - 4$

3. $x = y^2 - 8y - 11$

4. $x + 3y^2 + 12y = 18$

47. SPACE A satellite is in a circular orbit 25,000 miles above Earth.

a. Write an equation for the orbit of this satellite if the origin is at the center of Earth. Use 8000 miles as the diameter of Earth.

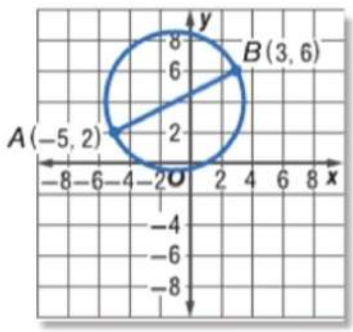
b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

48. SENSE-MAKING Suppose an unobstructed radio station broadcast could travel 120 kilometers. Assume the station is centered at the origin.

a. Write an equation to represent the boundary of the broadcast area with the origin as the center.

b. If the transmission tower is relocated 40 kilometers east and 10 kilometers south of the current location, and an increased signal will transmit signals an additional 80 kilometers, what is an equation to represent the new broadcast area?

49. GEOMETRY Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.



a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

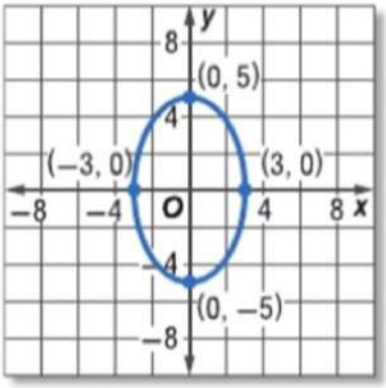
b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

c. Graph the circles from parts a and b on the same coordinate plane.

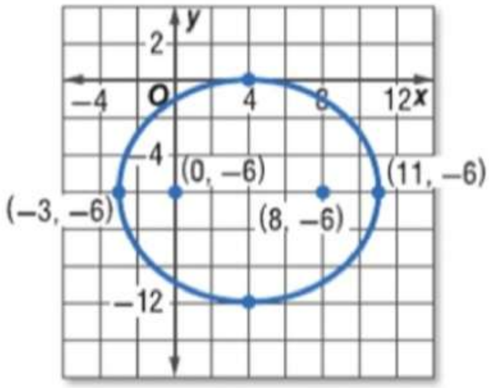
50. EARTHQUAKES A stadium is located about 35 kilometers west and 40 kilometers north of a city. Suppose an earthquake occurs with its epicenter about 55 kilometers from the stadium. Assume that the origin of a coordinate plane is located at the center of the city. Write an equation for the set of points that could be the epicenter of the earthquake.

e 1 **Write an equation for each ellipse.**

1.



2.



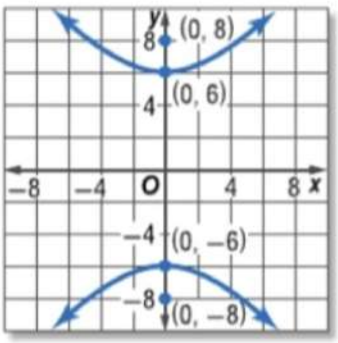
e 2 **Write an equation for an ellipse that satisfies each set of conditions.**

3. vertices at $(-2, -6)$ and $(-2, 4)$, co-vertices at $(-5, -1)$ and $(1, -1)$

4. vertices at $(-2, 5)$ and $(14, 5)$, co-vertices at $(6, 1)$ and $(6, 9)$

amples 1–2 **Write an equation for each hyperbola.**

1.



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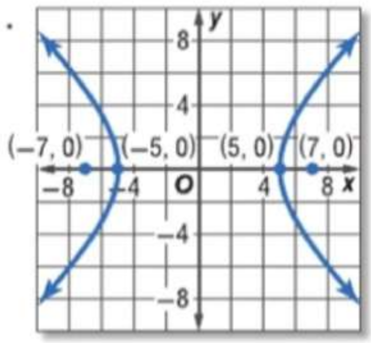

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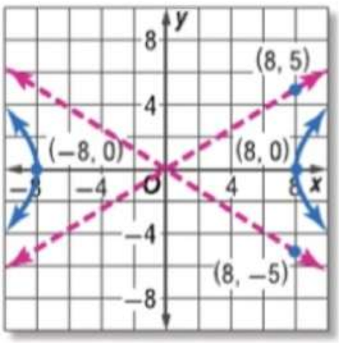

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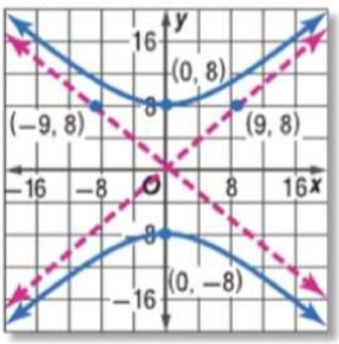
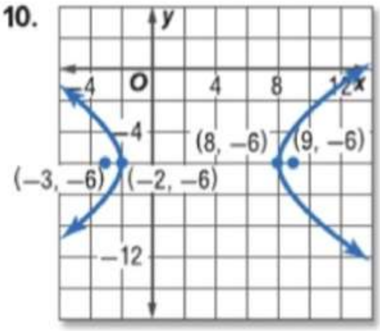
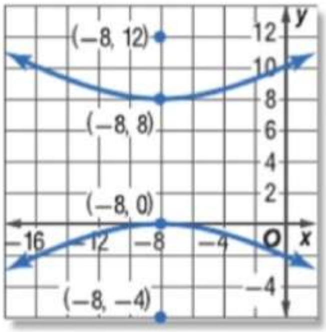


ples 1–2 **Write an equation for each hyperbola.**

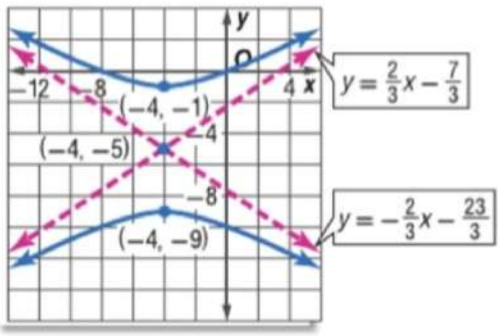
10.



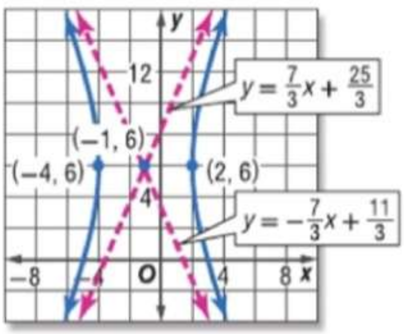
11.



12.



1 **3.**



the heading velocity and other forces are combined.

Real-World Example 5 Use Vectors to Solve Navigation Problems

AVIATION An airplane is flying with an airspeed of 310 knots on a heading of 050°. If a 78-knot wind is blowing from a true heading of 125°, determine the speed and direction of the plane relative to the ground.

Step 1 Draw a diagram to represent the heading and wind velocities (Figure 7.1.4). Translate the wind vector as shown in Figure 7.1.5, and use the triangle method to obtain the resultant vector representing the plane's ground velocity **g**. In the triangle formed by these vectors (Figure 7.1.6), $\gamma = 125^\circ - 50^\circ$ or 75°.

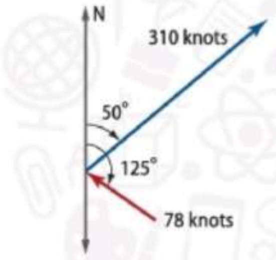


Figure 7.1.4

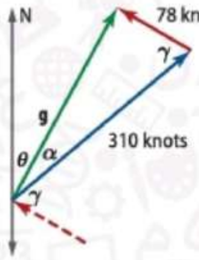


Figure 7.1.5

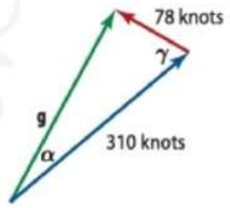


Figure 7.1.6

Step 2 Use the Law of Cosines to find $|\mathbf{g}|$, the plane's speed relative to the ground.

$c^2 = a^2 + b^2 - 2ab\cos\gamma$ — Law of Cosines

$|\mathbf{g}|^2 = 78^2 + 310^2 - 2(78)(310)\cos 75^\circ$ — $c = |\mathbf{g}|$, $a = 78$, $b = 310$, and $\gamma = 75^\circ$

$|\mathbf{g}| = \sqrt{78^2 + 310^2 - 2(78)(310)\cos 75^\circ}$ — Take the positive square root of each side.

≈ 299.4 — Simplify.

The ground speed of the plane is about 299.4 knots.

Step 3 The heading of the resultant **g** is represented by angle θ, as shown in Figure 7.1.5. To find θ, first calculate α using the Law of Sines.

$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$ — Law of Sines

$\frac{\sin\alpha}{78} = \frac{\sin 75^\circ}{299.4}$ — $c = |\mathbf{g}|$ or 299.4, $a = 78$, and $\gamma = 75^\circ$

$\sin\alpha = \frac{78\sin 75^\circ}{299.4}$ — Solve for $\sin\alpha$.

$\alpha = \sin^{-1}\frac{78\sin 75^\circ}{299.4}$ — Apply the inverse sine function.

$\approx 14.6^\circ$ — Simplify.

The measure of θ is $50^\circ - \alpha$, which is $50^\circ - 14.6^\circ$ or 35.4°.

Therefore, the speed of the plane relative to the ground is about 299.4 knots at about 035°.

Guided Practice

5. **SWIMMING** Ali rows due east at a speed of 3.5 feet per second across a river directly toward the opposite bank. At the same time, the current of the river is carrying him due south at a rate of 2 feet per second. Find Ali's speed and direction relative to the shore.

الجزء الورقي

Use the dot product to find the magnitude of the given vector. (Example 2)

10. $\mathbf{m} = \langle -3, 11 \rangle$

11. $\mathbf{r} = \langle -9, -4 \rangle$

12. $\mathbf{n} = \langle 6, 12 \rangle$

13. $\mathbf{v} = \langle 1, -18 \rangle$

14. $\mathbf{p} = \langle -7, -2 \rangle$

15. $\mathbf{t} = \langle 23, -16 \rangle$

Find the projection of u onto v. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v. (Examples 4 and 5)

25. $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j}, \mathbf{v} = -5\mathbf{i} + 2\mathbf{j}$

26. $\mathbf{u} = \langle 5, 7 \rangle, \mathbf{v} = \langle -4, 4 \rangle$

27. $\mathbf{u} = \langle 8, 2 \rangle, \mathbf{v} = \langle -4, 1 \rangle$

28. $\mathbf{u} = 6\mathbf{i} + \mathbf{j}, \mathbf{v} = -3\mathbf{i} + 9\mathbf{j}$

29. $\mathbf{u} = \langle 2, 4 \rangle, \mathbf{v} = \langle -3, 8 \rangle$

30. $\mathbf{u} = \langle -5, 9 \rangle, \mathbf{v} = \langle 6, 4 \rangle$

31. $\mathbf{u} = 5\mathbf{i} - 8\mathbf{j}, \mathbf{v} = 6\mathbf{i} - 4\mathbf{j}$

32. $\mathbf{u} = -2\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 9\mathbf{i} + 7\mathbf{j}$

Find each of the following for $\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$. (Example 5)

36. $6\mathbf{a} - 7\mathbf{b} + 8\mathbf{c}$

37. $7\mathbf{a} - 5\mathbf{b}$

38. $2\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$

39. $6\mathbf{b} + 4\mathbf{c} - 4\mathbf{a}$

40. $8\mathbf{a} - 5\mathbf{b} - \mathbf{c}$

41. $-6\mathbf{a} + \mathbf{b} + 7\mathbf{c}$

Find the angle θ between vectors u and v to the nearest tenth of a degree. (Example 2)

10. $\mathbf{u} = \langle 3, -2, 2 \rangle$, $\mathbf{v} = \langle 1, 4, -7 \rangle$

11. $\mathbf{u} = \langle 6, -5, 1 \rangle$, $\mathbf{v} = \langle -8, -9, 5 \rangle$

12. $\mathbf{u} = \langle -8, 1, 12 \rangle$, $\mathbf{v} = \langle -6, 4, 2 \rangle$

13. $\mathbf{u} = \langle 10, 0, -8 \rangle$, $\mathbf{v} = \langle 3, -1, -12 \rangle$

14. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$

15. $\mathbf{u} = -6\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = -4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

Write an equation for each parabola described below. Then graph the equation.

26. vertex (0, 1), focus (0, 4) **27.** vertex (1, 8), directrix $y = 3$

28. focus (−2, −4), directrix $x = -6$ **29.** focus (2, 4), directrix $x = 10$

30. vertex (−6, 0), directrix $x = 2$ **31.** vertex (9, 6), focus (9, 5)

Determine whether A and B are inverse matrices. (Example 4)

19. $A = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

20. $A = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$

21. $A = \begin{bmatrix} -5 & 3 \\ 6 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$

22. $A = \begin{bmatrix} -8 & 4 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

23. $A = \begin{bmatrix} 9 & 2 \\ 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 5 & -9 \end{bmatrix}$

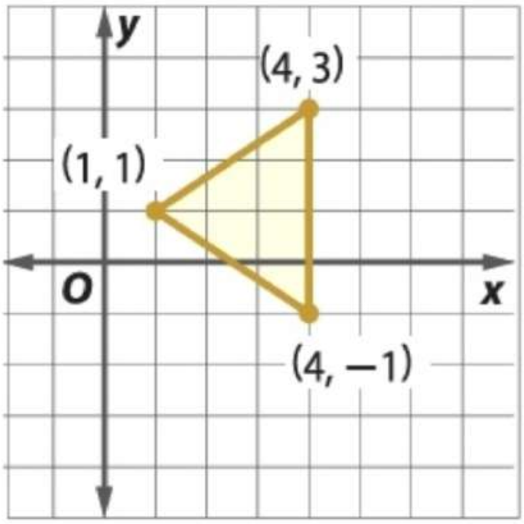
24. $A = \begin{bmatrix} 7 & 5 \\ -6 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -5 \\ 6 & 7 \end{bmatrix}$

25. $A = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$

26. $A = \begin{bmatrix} 9 & -7 \\ 8 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -6 \\ 4 & 10 \end{bmatrix}$

Find the area A of each triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3), by using $A = \frac{1}{2}|\det(X)|$, where X is $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$.

45.



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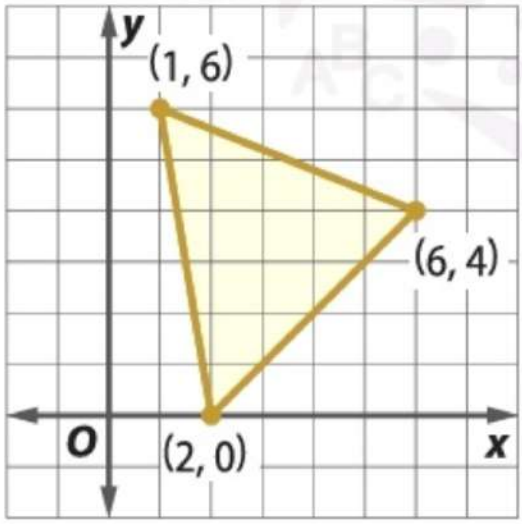

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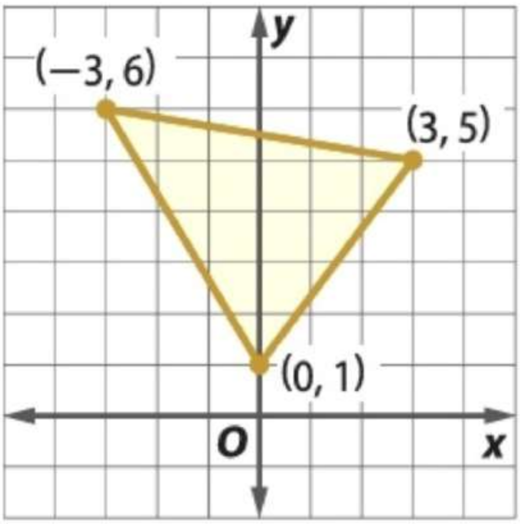

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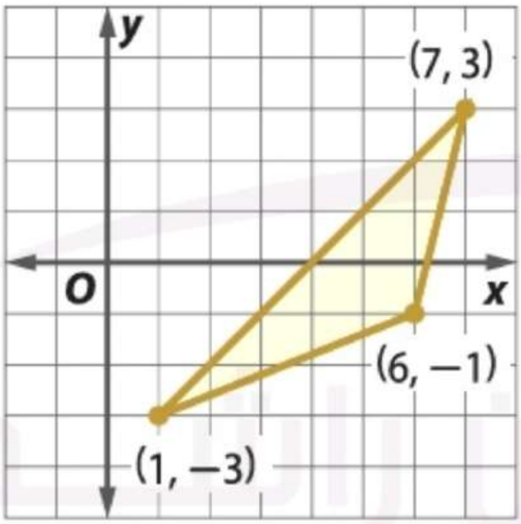

Examples 4–5 **Solve each equation.**

45. $2 \sin^2 \theta = 3 \sin \theta + 2$

46. $2 \cos^2 \theta + 3 \sin \theta = 3$

47. $\sin^2 \theta + \cos 2\theta = \cos \theta$

48. $2 \cos^2 \theta = -\cos \theta$

49. **SENSE-MAKING** Due to ocean tides, the depth y in meters of the River Thames in London varies as a sine function of x, the hour of the day. On a certain day that function was $y = 3 \sin\left[\frac{\pi}{6}(x - 4)\right] + 8$, where $x = 0, 1, 2, \ldots, 24$ corresponds to 12:00 midnight, 1:00 A.M., 2:00 A.M., ..., 12:00 midnight the next night.

a. What is the maximum depth of the River Thames on that day?

b. At what times does the maximum depth occur?

Solve each equation if θ is measured in radians.

50. $(\cos \theta)(\sin 2\theta) - 2 \sin \theta + 2 = 0$

51. $2 \sin^2 \theta + (\sqrt{2} - 1) \sin \theta = \frac{\sqrt{2}}{2}$

Solve each equation if θ is measured in degrees.

52. $\sin 2\theta + \frac{\sqrt{3}}{2} = \sqrt{3} \sin \theta + \cos \theta$

53. $1 - \sin^2 \theta - \cos \theta = \frac{3}{4}$

Solve each equation.

54. $2 \sin \theta = \sin 2\theta$

55. $\cos \theta \tan \theta - 2 \cos^2 \theta = -1$

Verify that each equation is an identity.

19. $\sec\theta - \tan\theta = \dfrac{1-\sin\theta}{\cos\theta}$

20. $\dfrac{1+\tan\theta}{\sin\theta+\cos\theta} = \sec\theta$

21. $\sec\theta\csc\theta = \tan\theta + \cot\theta$

22. $\sin\theta + \cos\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta - \cos\theta}$

23. $(\sin\theta + \cos\theta)^2 = \dfrac{2 + \sec\theta\csc\theta}{\sec\theta\csc\theta}$

24. $\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

25. $\csc\theta - 1 = \dfrac{\cot^2\theta}{\csc\theta + 1}$

26. $\cos\theta\cot\theta = \csc\theta - \sin\theta$

27. $\sin\theta\cos\theta\tan\theta + \cos^2\theta = 1$

28. $(\csc\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

29. $\csc^2\theta = \cot^2\theta + \sin\theta\csc\theta$

30. $\dfrac{\sec\theta - \csc\theta}{\csc\theta\sec\theta} = \sin\theta - \cos\theta$

31. $\sin^2\theta + \cos^2\theta = \sec^2\theta - \tan^2\theta$

32. $\sec\theta - \cos\theta = \tan\theta\sin\theta$