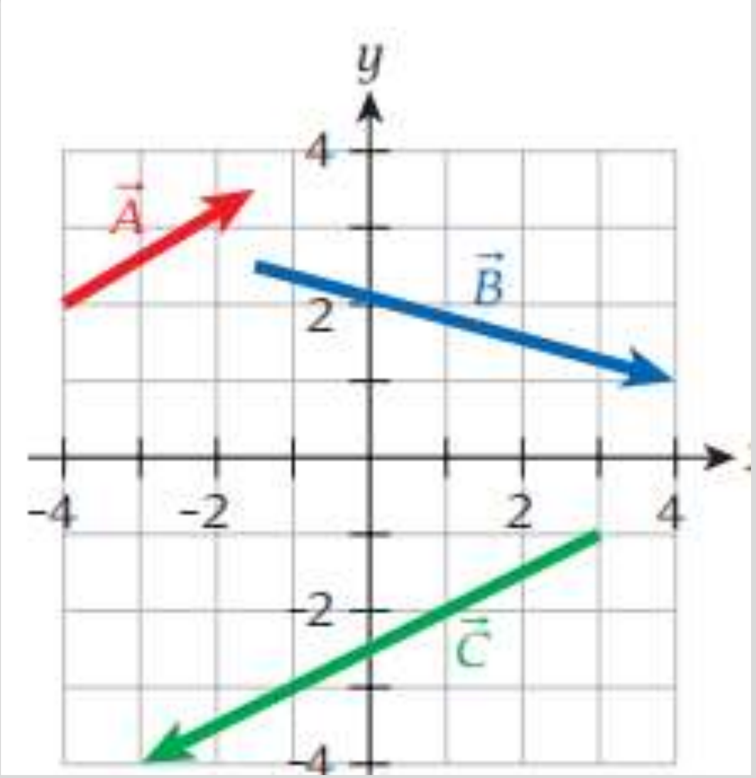
Revision 11 ADV

1	Add and subtract vectors graphically to find the resultant vectors.	Q.[1.93/1.94/1.95/1.96]	30
		Figure1.19	19

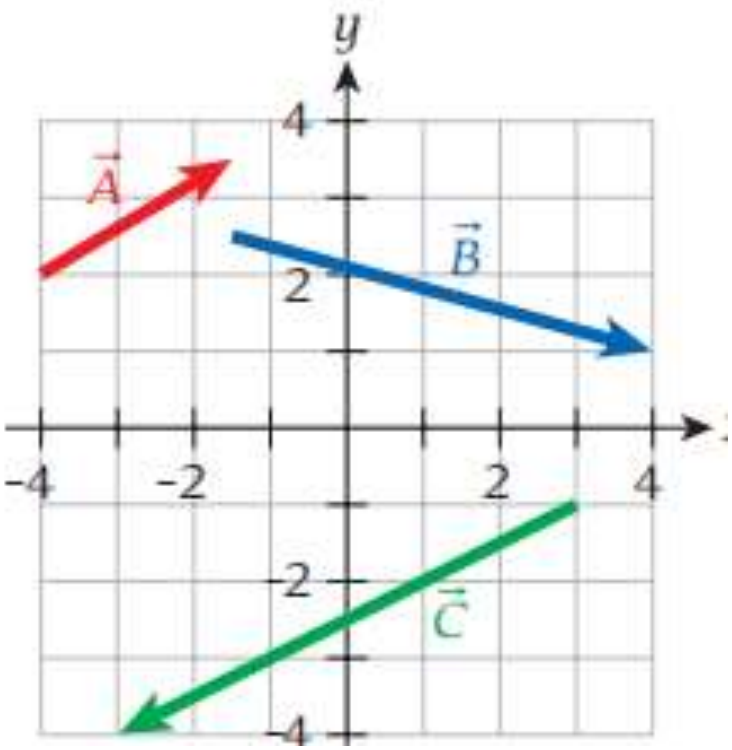
1.93 Write the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Cartesian coordinates.



CALCULATE: $\vec{A}=(-1.5-(-4))\hat{x}+(3.5-2)\hat{y}=2.5\hat{x}+1.5\hat{y}$, $\vec{B}=(4-(-1.5))\hat{x}+(1-2.5)\hat{y}=5.5\hat{x}-1.5\hat{y}$

$\vec{C}=(-3-3)\hat{x}-(4-(-1))\hat{y}=-6\hat{x}-3\hat{y}$

1.94 Calculate the length and direction of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.



RESEARCH: The length of a vector is given by the formula $\vec{L}=\sqrt{x^2+y^2}$. The direction of a vector (with respect to the x-axis) is given by $\tan\theta = y/x$.

SIMPLIFY: $\theta=\tan^{-1}\left(\frac{y}{x}\right)$

CALCULATE: $\left|\vec{A}\right|=\sqrt{(2.5)^2+(1.5)^2}=2.9,\ \theta_A=\tan^{-1}\left(\frac{1.5}{2.5}\right)=30.9638^\circ$

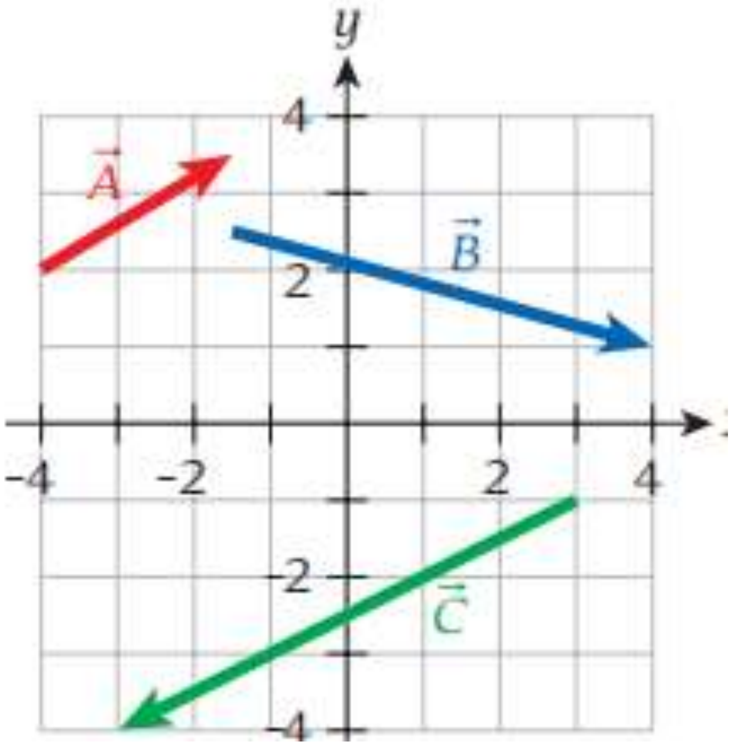
$$\left|\vec{B}\right|=\sqrt{(5.5)^2+(-1.5)^2}=5.700877,\ \theta_B=\tan^{-1}\left(\frac{-1.5}{5.5}\right)=-15.2551^\circ$$

$$\left|\vec{C}\right|=\sqrt{(-6)^2+(-3)^2}=6.7082,$$

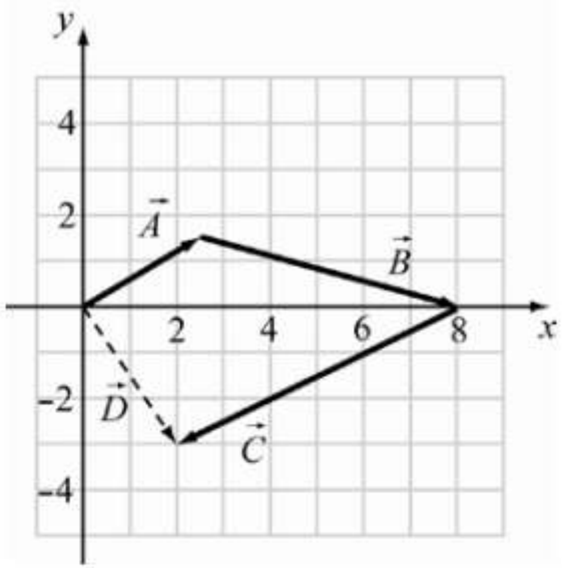
$$\theta_C=\tan^{-1}\left(\frac{-3}{-6}\right)=26.565^\circ=180^\circ+26.565^\circ=206.565^\circ$$

ROUND: The figure can reasonably be read to two significant digits, so the rounded values are $\left|\vec{A}\right|=2.9$ $\theta_A=31^\circ$, $\left|\vec{B}\right|=5.7$, $\theta_B=-15^\circ$, $\left|\vec{C}\right|=6.7$, and $\theta_C=210^\circ$.

1.95 Add the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ graphically.

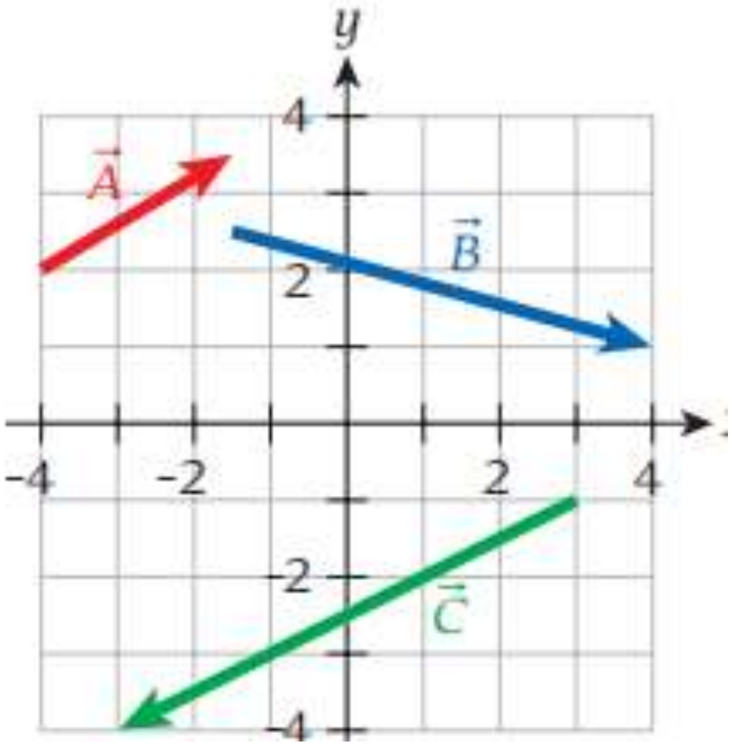


Vectors add tip to tail, $\vec{A}+\vec{B}+\vec{C}=\vec{D}$.



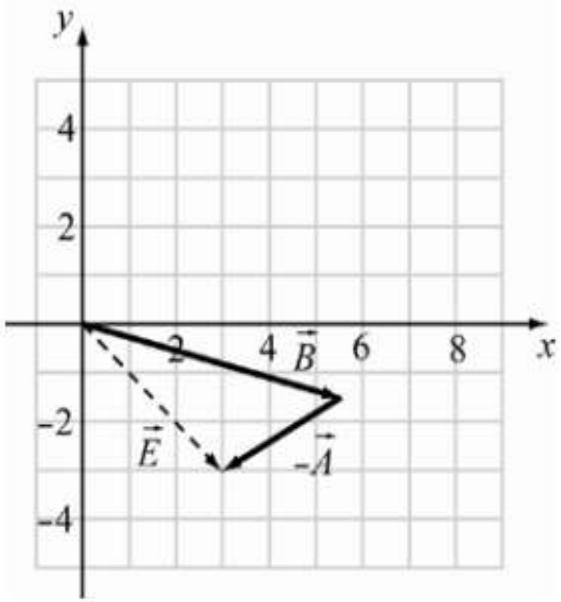
By inspecting the image, it is clear that $\vec{D}=(2,-3)$.

1.96 Determine the difference vector $\vec{E} = \vec{B} - \vec{A}$ graphically.



THINK: To subtract two vectors, reverse the direction of the vector being subtracted, and treat the operation as a sum. Denote the difference as $\vec{E} = \vec{B} - \vec{A}$.

SKETCH:



RESEARCH: $\vec{E} = \vec{B} - \vec{A} = \vec{B} + \left(-\vec{A}\right)$

SIMPLIFY: No simplification is necessary.

CALCULATE: By inspection, $\vec{E} = (3, -3)$.

If you add two real numbers, the order does not matter: $3+5=5+3$. This property is called the *commutative property of addition*. Vector addition is also commutative:

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}. \tag{1.10}$$

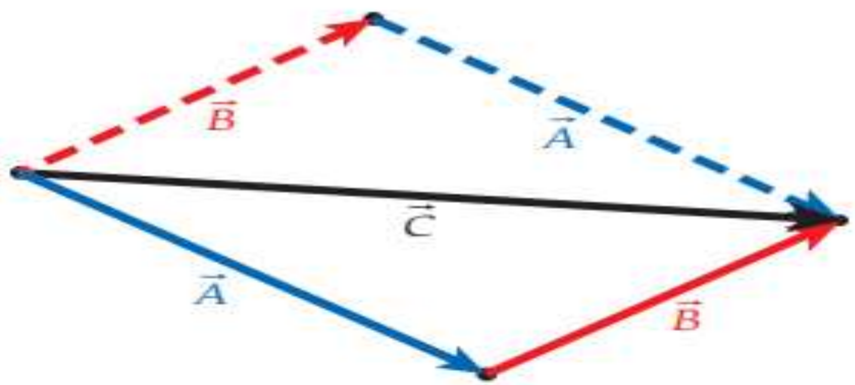


FIGURE 1.19 Commutative property of vector addition.

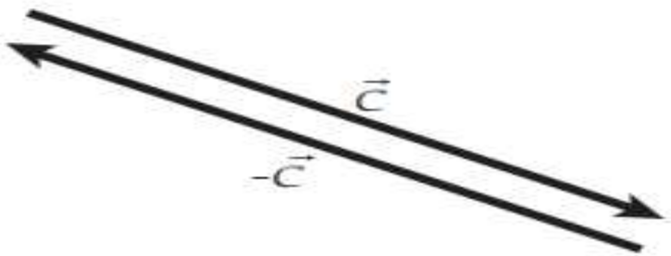


FIGURE 1.20 Inverse vector $-\vec{C}$ of a vector $\vec{C}$.

2	Find the length and direction of a two-dimensional vector from its Cartesian components.	Student Book	21
		Q.[1.67/1.77/1.101/1.99 1.100/1.101/1.102/1.103]	29-30

Figure 1.22a shows the unit vectors in two dimensions, and Figure 1.22b shows the unit vectors in three dimensions.

What is the advantage of unit vectors? We can write any vector as a sum of these unit vectors instead of using the component notation; each unit vector is multiplied by the corresponding Cartesian component of the vector:

$$\begin{aligned}\vec{A} &= (A_x, A_y, A_z)\\ &= (A_x, 0, 0) + (0, A_y, 0) + (0, 0, A_z)\\ &= A_x(1,0,0) + A_y(0,1,0) + A_z(0,0,1)\\ &= A_x\hat{x} + A_y\hat{y} + A_z\hat{z}.\end{aligned} \quad (1.18)$$

In two dimensions, we have

$$\vec{A} = A_x\hat{x} + A_y\hat{y}. \quad (1.19)$$

This unit vector representation of a general vector will be particularly useful for multiplying two vectors.

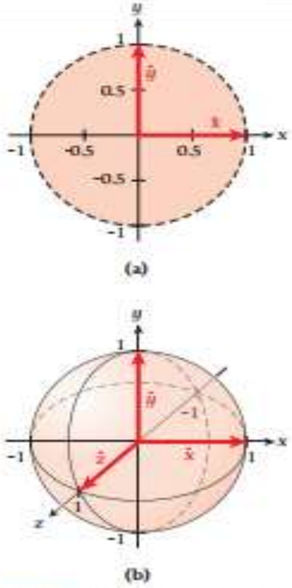


FIGURE 1.22 Cartesian unit vectors in (a) two and (b) three dimensions.

Vector Length and Direction

If we know the component representation of a vector, how can we find its length (magnitude) and the direction it is pointing in? Let's look at the most important case: a vector in two dimensions. In two dimensions, a vector $\vec{A}$ can be specified uniquely by giving the two Cartesian components, A_x and A_y. We can also specify the same vector by giving two other numbers: its length A and its angle θ with respect to the positive x-axis.

Let's take a look at Figure 1.23 to see how we can determine A and θ from A_x and A_y. Figure 1.23a shows the graphical representation of equation 1.19. The vector $\vec{A}$ is the sum of the vectors $A_x\hat{x}$ and $A_y\hat{y}$. Since the unit vectors $\hat{x}$ and $\hat{y}$ are by definition orthogonal to each other, these vectors form a 90° angle. Thus, the three vectors $\vec{A}$, $A_x\hat{x}$, and $A_y\hat{y}$ form a right triangle with side lengths A, A_x, and A_y, as shown in Figure 1.23b.

Now we can employ basic trigonometry to find θ and A. Using the Pythagorean Theorem results in

$$A = \sqrt{A_x^2 + A_y^2}. \quad (1.20)$$

We can find the angle θ from the definition of the tangent function

$$\theta = \tan^{-1}\frac{A_y}{A_x}. \quad (1.21)$$

In using equation 1.21, you must be careful that θ is in the correct quadrant. We can also invert equations 1.20 and 1.21 to obtain the Cartesian components of a vector of given length and direction:

$$A_x = A\cos\theta \quad (1.22)$$

$$A_y = A\sin\theta. \quad (1.23)$$

You will encounter these trigonometric relations again and again throughout introductory physics. If you need to refamiliarize yourself with trigonometry, consult the mathematics primer provided in Appendix A.

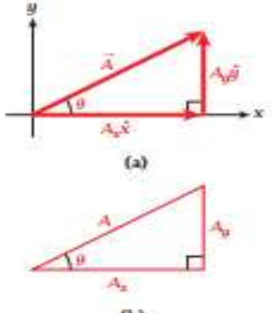


FIGURE 1.23 Length and direction of a vector. (a) Cartesian components A_x and A_y; (b) length A and angle θ.

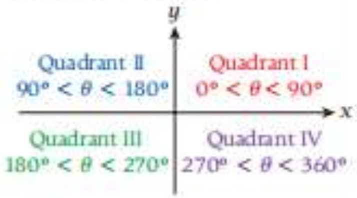
Scalar Product of Vectors

Above we saw how to multiply a vector with a scalar. Now we will define one way of multiplying a vector with a vector and obtain the **scalar product.** The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\alpha, \quad (1.24)$$

Concept Check 1.6

Into which quadrant do each of the following vectors point?



a) $A = (A_x, A_y)$ with $A_x = 1.5$ cm, $A_y = -1.0$ cm

b) a vector with length 2.3 cm and direction angle 131°

c) the inverse vector of $B = (0.5$ cm, 1.0 cm$)$

d) the sum of the unit vectors in the x- and y-directions

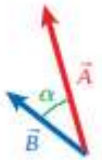


FIGURE 1.24 Two vectors $\vec{A}$ and $\vec{B}$ and the angle α between them.

where α is the angle between the vectors $\vec{A}$ and $\vec{B}$, as shown in Figure 1.24. Note the use of the larger dot (•) as the multiplication sign for the scalar product between vectors, in contrast to the smaller dot (·) that is used for the multiplication of scalars. Because of the dot, the scalar product is often referred to as the *dot product*.

If two vectors form a 90° angle, then the scalar product has the value zero. In this case, the two vectors are orthogonal to each other. The scalar product of a pair of orthogonal vectors is zero.

If $\vec{A}$ and $\vec{B}$ are given in Cartesian coordinates as $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$, then their scalar product can be shown to be equal to:

$$\vec{A} \bullet \vec{B} = (A_x, A_y, A_z) \bullet (B_x, B_y, B_z) = A_x B_x + A_y B_y + A_z B_z. \tag{1.25}$$

From equation 1.25, we can see that the scalar product has the commutative property:

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}. \tag{1.26}$$

This result is not surprising, since the commutative property also holds for the multiplication of two scalars.

For the scalar product of any vector with itself, we have, in component notation, $\vec{A} \bullet \vec{A} = A_x^2 + A_y^2 + A_z^2$. Then, from equation 1.24, we find $\vec{A} \bullet \vec{A} = |\vec{A}|\,|\vec{A}| \cos\alpha = |\vec{A}|\,|\vec{A}| = |\vec{A}|^2$ (because the angle between the vector $\vec{A}$ and itself is zero, and the cosine of that angle has the value 1). Combining these two equations, we obtain the expression for the length of a vector that was introduced in the previous subsection:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}. \tag{1.27}$$

We can also use the definition of the scalar product to compute the angle between two arbitrary vectors in three-dimensional space:

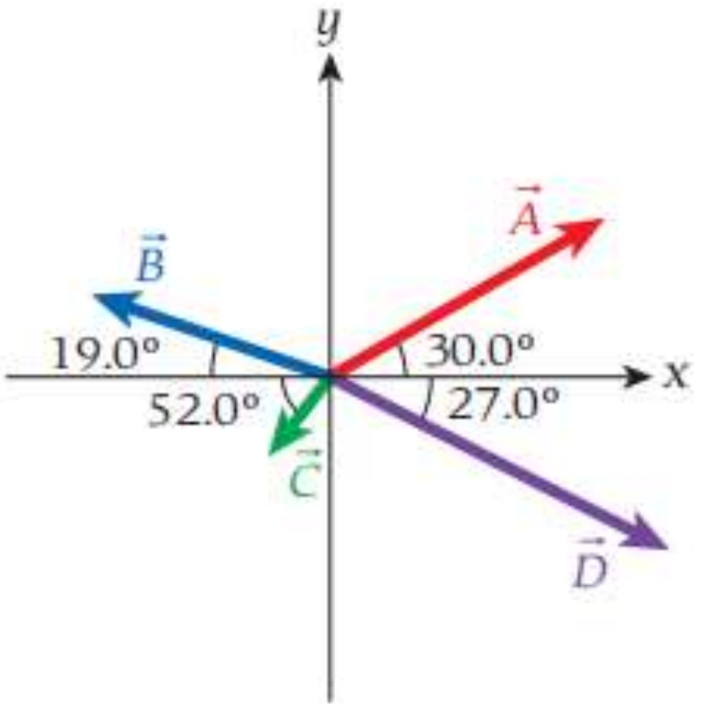
$$\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}| \cos\alpha \Rightarrow \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|} \Rightarrow \alpha = \cos^{-1}\left(\frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|}\right). \tag{1.28}$$

For the scalar product, the same distributive property that is valid for the conventional multiplication of numbers holds:

$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}. \tag{1.29}$$

The following example puts the scalar product to use.

1.67 Find the components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, if their lengths are given by $A = 75.0$, $B = 60.0$, $C = 25.0$, $D = 90.0$ and their direction angles are as shown in the figure. Write the vectors in terms of unit vectors.



SIMPLIFY: $A_x = |\vec{A}|\cos\theta_A$, $A_y = |\vec{A}|\sin\theta_A$, $B_x = |\vec{B}|\cos\theta_B$, $B_y = |\vec{B}|\sin\theta_B$, $C_x = |\vec{C}|\cos\theta_C$, $C_y = |\vec{C}|\sin\theta_C$, $D_x = |\vec{D}|\cos\theta_D$, and $D_y = |\vec{D}|\sin\theta_D$.

CALCULATE: $A_x = 75.0\cos 30.0° = 64.9519\hat{x}$, $A_y = 75.0\sin 30.0° = 37.5\hat{y}$

$B_x = 60.0\cos 161.0° = -56.73\hat{x}$, $B_y = 60.0\sin 161.0° = 19.534\hat{y}$

$C_x = 25.0\cos 232.0° = -15.3915\hat{x}$, $C_y = 25.0\sin 232.0° = -19.70027\hat{y}$

$D_x = 90.0\cos 333.0° = 80.19058\hat{x}$, $D_y = 90.0\sin 333.0° = -40.859144\hat{y}$

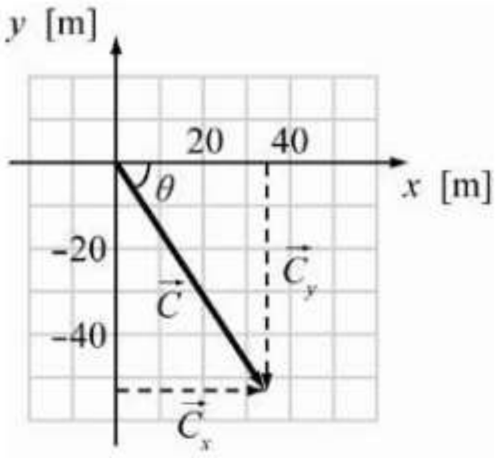
ROUND: The given values had three significant figures so the answers must be rounded to:

$\vec{A} = 65.0\hat{x} + 37.5\hat{y}$, $\vec{B} = -56.7\hat{x} + 19.5\hat{y}$, $\vec{C} = -15.4\hat{x} - 19.7\hat{y}$, $\vec{D} = 80.2\hat{x} - 40.9\hat{y}$.

1.77 A position vector has components $x = 34.6$ m and $y = -53.5$ m. Find the vector's length and angle with the x-axis.

THINK: An angle is measured counter-clockwise from the positive x-axis (0°). $\vec{C} = (34.6 \text{ m}, \; -53.5 \text{ m})$. It is also possible to measure clockwise from the positive x-axis and consider the measure to be negative.

SKETCH:



RESEARCH: $\left|\vec{C}\right| = \sqrt{C_x^{\,2} + C_y^{\,2}}, \;\; \tan\theta = \left(\dfrac{C_y}{C_x}\right)$

SIMPLIFY: $\theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right)$

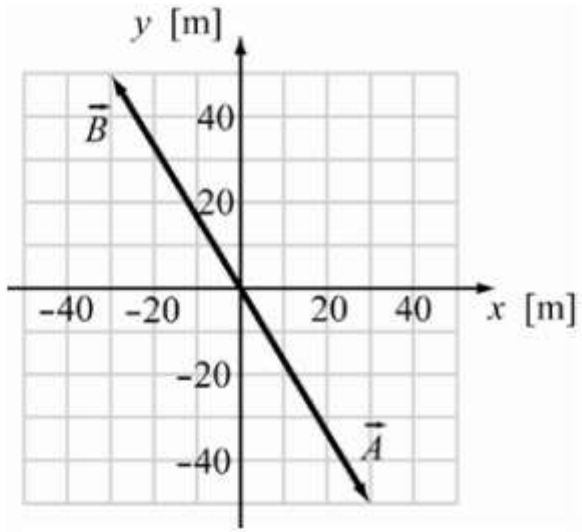
CALCULATE: $\left|\vec{C}\right| = \sqrt{(34.6 \text{ m})^2 + (-53.5 \text{ m})^2} = 63.713 \text{ m}, \;\; \theta = \tan^{-1}\left(\dfrac{-53.5 \text{ m}}{34.6 \text{ m}}\right) = -57.108°$

ROUND: $\vec{C} = 63.7$ m , $\theta = -57.1°$ or 303° (equivalent angles).

1.99 Sketch the vectors with the components $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (-30.0 \text{ m}, 50.0 \text{ m})$, and find the magnitudes of these vectors.

THINK: The two vectors are $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (-30.0 \text{ m}, 50.0 \text{ m})$. Sketch and find the magnitudes.

SKETCH:



RESEARCH: The length of a vector $\vec{C} = C_x\hat{x} + C_y\hat{y}$ is $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$.

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$

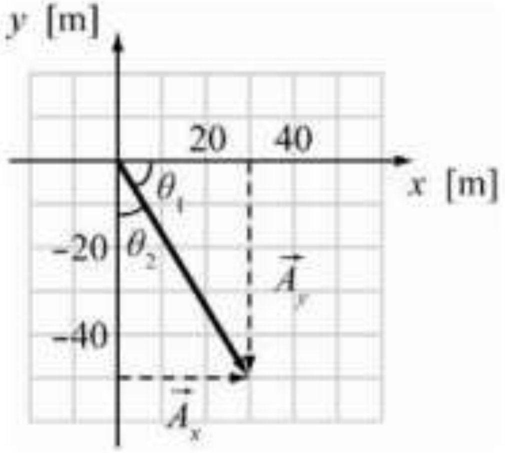
CALCULATE: $|\vec{A}| = \sqrt{(30)^2 + (-50)^2} = 58.3095 \text{ m}$, $|\vec{B}| = \sqrt{(-30)^2 + (50)^2} = 58.3095 \text{ m}$

ROUND: $|\vec{A}| = 58.3 \text{ m}$, $|\vec{B}| = 58.3 \text{ m}$

1.100 What angle does $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$ make with the positive *x*-axis? What angle does it make with the negative *y*-axis?

THINK: Use trigonometry to find the angles as indicated in the sketch below. $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$.

SKETCH:



RESEARCH: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

SIMPLIFY: $\tan\theta_1 = (A_y / A_x) \Rightarrow \theta_1 = \tan^{-1}(A_y / A_x)$, $\tan\theta_2 = (A_x / A_y) \Rightarrow \theta_2 = \tan^{-1}(A_x / A_y)$

CALCULATE: $\theta_1 = \tan^{-1}(-50/30) = -59.036°$, $\theta_2 = \tan^{-1}(30/-50) = -30.963°$

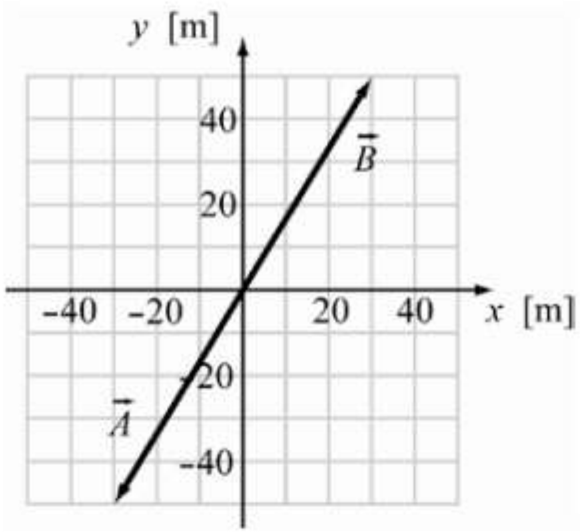
ROUND: Drop the signs of the angles and just use their size: $\theta_1 = 59.0°$, $\theta_2 = 31.0°$.

DOUBLE-CHECK: The two angles add up to 90°, which they should. The answers are reasonable.

1.101 Sketch the vectors with the components $\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$, and find the magnitudes of these vectors.

THINK: The two vectors are $\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$. Sketch and find the magnitudes.

SKETCH:



RESEARCH: $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$

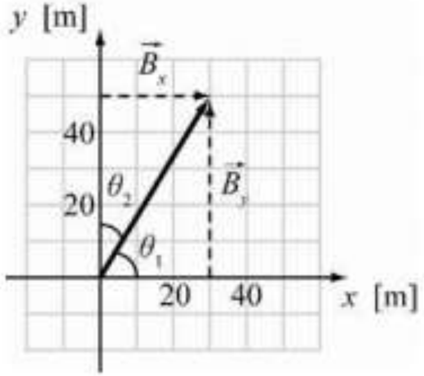
CALCULATE: $|\vec{A}| = \sqrt{(-30.0 \text{ m})^2 + (-50.0 \text{ m})^2} = 58.3095 \text{ m}$, $|\vec{B}| = \sqrt{(30.0 \text{ m})^2 + (50.0 \text{ m})^2} = 58.3095 \text{ m}$

ROUND: $|\vec{A}| = 58.3 \text{ m}$, $|\vec{B}| = 58.3 \text{ m}$

1.102 What angle does $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$ make with the positive x-axis? What angle does it make with the positive y-axis?

THINK: Using trigonometry find the angles indicated in the diagram below. The vector $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$.

SKETCH:



RESEARCH: $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

SIMPLIFY: $\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$, $\theta_1 = \tan^{-1}\left(\frac{B_y}{B_x}\right)$, $\theta_2 = \tan^{-1}\left(\frac{B_x}{B_y}\right)$

CALCULATE: $\theta_1 = \tan^{-1}\left(\frac{50.0 \text{ m}}{30.0 \text{ m}}\right) = 59.036°$, $\theta_2 = \tan^{-1}\left(\frac{30.0 \text{ m}}{50.0 \text{ m}}\right) = 30.963°$

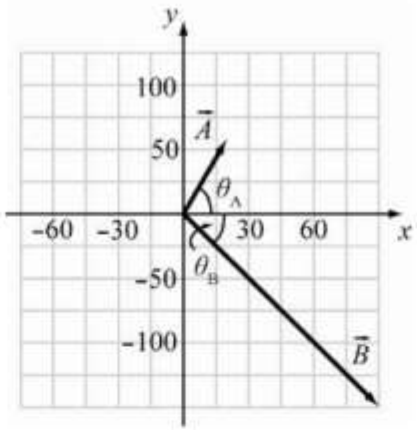
ROUND: $\theta_1 = 59.0°$, $\theta_2 = 31.0°$

DOUBLE-CHECK: The angles sum to 90°, which is expected from the sketch. Therefore, the answers are reasonable.

1.103 Find the magnitude and direction of each of the following vectors, which are given in terms of their x- and y-components: $\vec{A} = (23.0, 59.0)$, and $\vec{B} = (90.0, -150.0)$.

THINK: The two vectors are $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$. Find the magnitude and angle with respect to the positive x-axis.

SKETCH:



RESEARCH: For any vector $\vec{C} = C_x\hat{x} + C_y\hat{y}$, the magnitude is given by the formula $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, and the angle θ_C made with the x-axis is such that $\tan\theta_C = \dfrac{C_y}{C_x}$.

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$, $\theta_A = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$, $\theta_B = \tan^{-1}\left(\dfrac{B_y}{B_x}\right)$

CALCULATE: $|\vec{A}| = \sqrt{(23.0)^2 + (59.0)^2} = 63.3246$, $|\vec{B}| = \sqrt{(90.0)^2 + (-150.0)^2} = 174.9286$

$$\theta_A = \tan^{-1}\left(\frac{59.0}{23.0}\right) = 68.7026°,\ \theta_B = \tan^{-1}\left(\frac{-150.0}{90.0}\right) = -59.0362°$$

ROUND: Three significant figures: $\vec{A} = 63.3$ at $68.7°$, $\vec{B} = 175$ at $-59.0°$ or $301.0°$.

3	Find the angle between two position vectors in the cartesian coordinates	Student Book	21 , 23
		Example1.5	22
		Q.1.80	29

Scalar Product for Unit Vectors. On page 26 we introduced unit vectors in the three-dimensional Cartesian coordinate system: $\hat{x} = (1,0,0)$, $\hat{y} = (0,1,0)$, and $\hat{z} = (0,0,1)$. With our definition (1.25) of the scalar product, we find

$$\hat{x} \bullet \hat{x} = \hat{y} \bullet \hat{y} = \hat{z} \bullet \hat{z} = 1 \quad (1.30)$$

and

$$\begin{aligned} \hat{x} \bullet \hat{y} = \hat{x} \bullet \hat{z} = \hat{y} \bullet \hat{z} = 0 \\ \hat{y} \bullet \hat{x} = \hat{z} \bullet \hat{x} = \hat{z} \bullet \hat{y} = 0. \end{aligned} \quad (1.31)$$

Now we see why the unit vectors are called that: Their scalar products with themselves have the value 1. Thus, the unit vectors have length 1, or unit length, according to equation 1.27. In addition, any pair of different unit vectors has a scalar product that is zero, meaning that these vectors are orthogonal to each other. Equations 1.30 and 1.31 thus state that the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ form an orthonormal set of vectors, which makes them extremely useful for the description of physical systems.

Geometrical Interpretation of the Scalar Product. In the definition of the scalar product $\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}|\cos\alpha$ (equation 1.24), we can interpret $|\vec{A}|\cos\alpha$ as the projection of the vector $\vec{A}$ onto the vector $\vec{B}$ (Figure 1.26a). In this drawing, the line $|\vec{A}|\cos\alpha$ is rotated by 90° to show the geometrical interpretation of the scalar product as the area of a rectangle with sides $|\vec{A}|\cos\alpha$ and $|\vec{B}|$. In the same way, we can interpret $|\vec{B}|\cos\alpha$ as the projection of the vector $\vec{B}$ onto the vector $\vec{A}$ and construct a rectangle with side lengths $|\vec{B}|\cos\alpha$ and $|\vec{A}|$ (Figure 1.26b). The areas of the two yellow rectangles in Figure 1.25 are identical and are equal to the scalar product of the two vectors $\vec{A}$ and $\vec{B}$.

Finally, if we substitute from equation 1.28 for the cosine of the angle between the two vectors, the projection $|\vec{A}|\cos\alpha$ of the vector $\vec{A}$ onto the vector $\vec{B}$ can be written as

$$|\vec{A}|\cos\alpha = |\vec{A}|\frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{\vec{A} \bullet \vec{B}}{|\vec{B}|},$$

and the projection $|\vec{B}|\cos\alpha$ of the vector $\vec{B}$ onto the vector $\vec{A}$ can be expressed as

$$|\vec{B}|\cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|}.$$

Self-Test Opportunity 1.1

Show that equations 1.30 and 1.31 are correct by using equation 1.25 and the definitions of the unit vectors.

Vector Product

The **vector product** (or cross product) between two vectors $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ is defined as

$$\begin{aligned} \vec{C} &= \vec{A} \times \vec{B} \\ C_x &= A_y B_z - A_z B_y \\ C_y &= A_z B_x - A_x B_z \\ C_z &= A_x B_y - A_y B_x. \end{aligned} \quad (1.32)$$

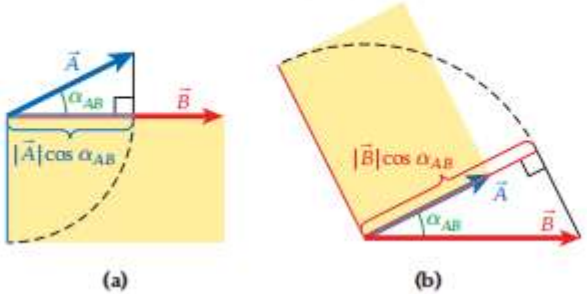


FIGURE 1.26 Geometrical interpretation of the scalar product as an area. (a) The projection of $\vec{A}$ onto $\vec{B}$. (b) The projection of $\vec{B}$ onto $\vec{A}$.

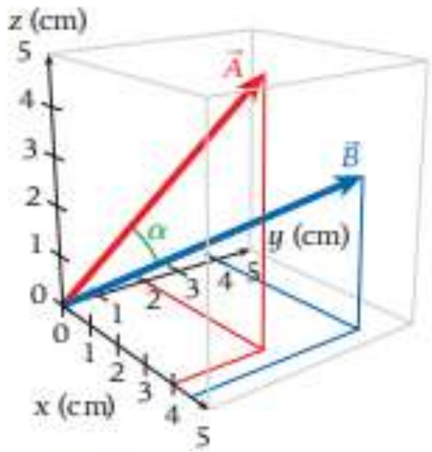


FIGURE 1.25 Calculating the angle between two position vectors.

EXAMPLE 1.5 Angle Between Two Position Vectors

PROBLEM

What is the angle α between the two position vectors shown in Figure 1.25, $\vec{A} = (4.00, 2.00, 5.00)$ cm and $\vec{B} = (4.50, 4.00, 3.00)$ cm?

SOLUTION

To solve this problem, we have to put the numbers for the components of each of the two vectors into equation 1.27 and equation 1.25 then use equation 1.28:

$$|A| = \sqrt{4.00^2 + 2.00^2 + 5.00^2} \text{ cm} = 6.71 \text{ cm}$$

$$|B| = \sqrt{4.50^2 + 4.00^2 + 3.00^2} \text{ cm} = 6.73 \text{ cm}$$

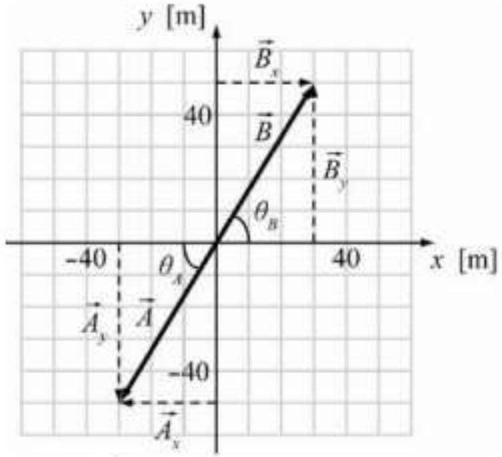
$$A \bullet B = A_x B_x + A_y B_y + A_z B_z = (4.00 \times 4.50 + 2.00 \times 4.00 + 5.00 \times 3.00) \text{ cm}^2 = 41.0 \text{ cm}^2$$

$$\alpha = \cos^{-1} \frac{41.0 \text{ cm}^2}{6.71 \text{ cm} \times 6.73 \text{ cm}} = 24.7°.$$

•**1.80** Express the vectors $\vec{A} = (A_x, A_y) = (-30.0\text{ m}, -50.0\text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0\text{ m}, 50.0\text{ m})$ by giving their magnitude and direction as measured from the positive *x*-axis.

THINK: The vectors are $\vec{A} = (A_x, A_y) = (-30.0\text{ m}, -50.0\text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0\text{ m}, 50.0\text{ m})$. Find the magnitude and angle with respect to the positive *x*-axis for each.

SKETCH:



RESEARCH: $\vec{C} = \sqrt{C_x^2 + C_y^2}$, $\tan\theta_C = \dfrac{C_y}{C_x}$

SIMPLIFY: $\overline{A} = \sqrt{A_x^2 + A_y^2}$, $\overline{B} = \sqrt{B_x^2 + B_y^2}$, $\theta_A = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$, $\theta_B = \tan^{-1}\left(\dfrac{B_y}{B_x}\right)$

CALCULATE: $\left|\overline{A}\right| = \sqrt{(-30.0\text{ m})^2 + (-50.0\text{ m})^2} = 58.3095\text{ m}$, $\left|\overline{B}\right| = \sqrt{(30.0\text{ m})^2 + (50.0\text{ m})^2} = 58.3095\text{ m}$

$$\theta_A = \tan^{-1}\left(\frac{-50.0\text{ m}}{-30.0\text{ m}}\right) = 59.036° \Rightarrow 180° + 59.036° = 239.036°$$

$$\theta_B = \tan^{-1}\left(\frac{50.0\text{ m}}{30.0\text{ m}}\right) = 59.036°$$

ROUND: $\vec{A} = 58.3$ m at 239° or $-121°$, and $\vec{B} = 58.3$ m at 59.0°.

4	I. Multiply a vector with a scalar. II. Add or subtract vectors using Cartesian components.	Student Book	20
		Q.[1.76/1.79/1.105/1.106]	29

Multiplication of a Vector with a Scalar

What is $\vec{A}+\vec{A}+\vec{A}$? If your answer to this question is $3\vec{A}$, you already understand multiplying a vector with a scalar. The vector that results from multiplying the vector $\vec{A}$ with the scalar 3 is a vector that points in the same direction as the original vector $\vec{A}$ but is 3 times as long.

Multiplication of a vector with an arbitrary positive scalar–that is, a positive number–results in another vector that points in the same direction but has a magnitude that is the product of the magnitude of the original vector and the value of the scalar. Multiplication of a vector by a negative scalar results in a vector pointing in the opposite direction to the original with a magnitude that is the product of the magnitude of the original vector and the magnitude of the scalar.

Again, the component notation is useful. For the multiplication of a vector $\vec{A}$ with a scalar s, we obtain:

$$\vec{E}=s\vec{A}=s(A_x,A_y,A_z)=(sA_x,sA_y,sA_z). \quad (1.15)$$

In other words, each component of the vector $\vec{A}$ is multiplied by the scalar in order to arrive at the components of the product vector:

$$\begin{aligned} E_x &= sA_x \\ E_y &= sA_y \\ E_z &= sA_z. \end{aligned} \quad (1.16)$$

Unit Vectors

There is a set of special vectors that make much of the math associated with vectors easier. Called **unit vectors,** they are vectors of magnitude 1 directed along the main coordinate axes of the coordinate system. In two dimensions, these vectors point in the positive x-direction and the positive y-direction. In three dimensions, a third unit vector points in the positive z-direction. In order to distinguish these as unit vectors, we give them the symbols $\hat{x}$, $\hat{y}$, and $\hat{z}$. Their component representation is

$$\begin{aligned} \hat{x} &= (1,0,0) \\ \hat{y} &= (0,1,0) \\ \hat{z} &= (0,0,1). \end{aligned} \quad (1.17)$$

1.76 Find the vector $\vec{C}$ that satisfies the equation $3\hat{x} + 6\hat{y} - 10\hat{z} + \vec{C} = -7\hat{x} + 14\hat{y}$.

$3\hat{x}+6\hat{y}-10\hat{z}+\vec{C}=-7\hat{x}+14\hat{y}$, $\quad \vec{C}=(-7\hat{x}-3\hat{x})+(14\hat{y}-6\hat{y})+10\hat{z}=-10\hat{x}+8\hat{y}+10\hat{z}$

•1.79 Find the magnitude and direction of (a) $9\vec{B} - 3\vec{A}$ and (b) $-5\vec{A} + 8\vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

RESEARCH: $\vec{C}=(C_x,C_y)$, $C_i = nA_i + mB_i$, $|\vec{C}|=\sqrt{C_x^2+C_y^2}$, $\tan\theta_C = \dfrac{C_y}{C_x}$

SIMPLIFY:

(a) Since $n=-3$ and $m=9$, $C_x=-3A_x+9B_x$ and $C_y=-3A_y+9B_y$. Also, $\theta_C=\tan^{-1}(C_y/C_x)$.

(b) Since $n=-5$ and $m=8$, $C_x=-5A_x+8B_x$ and $C_y=-5A_y+8B_y$. Also, $\theta_C=\tan^{-1}(C_y/C_x)$.

CALCULATE:

(a) $C_x=-3(23.0)+9(90.0)=741.0$, $C_y=-3(59.0)+9(-150)=-1527.0$

$\vec{A}=(A_x,A_y)=(-30.0\text{ m},-50.0\text{ m})$

(b) $=(30.0\text{ m},50.0\text{ m})$. $|\vec{C}|=\sqrt{(605.0)^2+(-1495.0)^2}=1612.78$

$$\theta_C=\tan^{-1}\left(\frac{-1495.0}{605.0}\right)=-67.97^\circ$$

ROUND:

(a) $\vec{C}=1.70\cdot 10^3$ at -64.1° or 296°

(b) $\vec{C}=1.61\cdot 10^3$ at -68.0° or 292°

1.105 Find the magnitude and direction of $-5\vec{A}+\vec{B}$, where $\vec{A}=(23.0, 59.0)$, $\vec{B}=(90.0, -150.0)$.

SIMPLIFY: $C_x=-5A_x+B_x$, $C_y=-5A_y+B_y$, $\left|\vec{C}\right|=\sqrt{C_x^{\,2}+C_y^{\,2}}$, $\theta_C=\tan^{-1}\left(\frac{C_y}{C_x}\right)$

CALCULATE: $C_x=-5(23.0)+90.0=-25.0$, $C_y=-5(59.0)+(-150)=-445.0$

$$\left|\vec{C}\right|=\sqrt{(-25.0)^2+(-445.0)^2}=445.702$$

$$\theta_C=\tan^{-1}\left(\frac{-445.0}{-25.0}\right)=86.785^\circ \Rightarrow 180^\circ+86.785^\circ=266.785^\circ$$

ROUND: $\overline{C}=446$ at 267° or -93.2°

1.106 Find the magnitude and direction of $-7\vec{B}+3\vec{A}$, where $\vec{A}=(23.0, 59.0)$, $\vec{B}=(90.0, -150.0)$.

CALCULATE: $C_x=3(23.0)-7(90.0)=-561.0$, $C_y=3(59.0)-7(-150)=1227.0$

$$\left|\vec{C}\right|=\sqrt{(-561.0)^2+(1227.0)^2}=1349.17$$

$$\theta_C=\tan^{-1}\left(\frac{1227.0}{-561.0}\right)=-65.43^\circ \Rightarrow 180^\circ-65.43^\circ=114.57^\circ$$

ROUND: $\left|\vec{C}\right|=1.35\cdot10^3$ at 115°

5	Differentiate between distance and displacement.	Solved Problem 2.1 Q.[2.29/2.30]	35 61

SOLVED PROBLEM 2.1 Trip Segments

The distance between Des Moines and Iowa City is 170.5 km along Interstate 80, and as you can see from the map (Figure 2.4), the route is a straight line to a good approximation. Approximately halfway between the two cities, where I80 crosses highway US63, is the city of Malcom, 89.9 km from Des Moines.

FIGURE 2.4 Route I80 between Des Moines and Iowa City.

PROBLEM

If we drive from Malcom to Des Moines and then go to Iowa City, what are the total distance and total displacement for this trip?

SOLUTION

THINK Distance and displacement are not identical. If the trip consisted of one segment in one direction, the distance would just be the absolute value of the displacement, according to equation 2.4. However, this trip is composed of segments with a direction change, so we need to be careful. We'll treat each segment individually and then add up the segments in the end.

SKETCH Because I80 is almost a straight line, it is sufficient to draw a straight horizontal line and make this our coordinate axis. We enter the positions of the three cities as x_I (Iowa City), x_M (Malcom), and x_D (Des Moines). We always have the freedom to define the origin of our coordinate system, so we elect to put it at Des Moines, thus setting $x_D = 0$. As is conventional, we define the positive direction to the right, in the eastward direction. See Figure 2.5.

We also draw arrows for the displacements of the two segments of the trip. We represent segment 1 from Malcom to Des Moines by a red arrow, and segment 2 from Des Moines to Iowa City by a blue arrow. Finally, we draw a diagram for the total trip as the sum of the two trips.

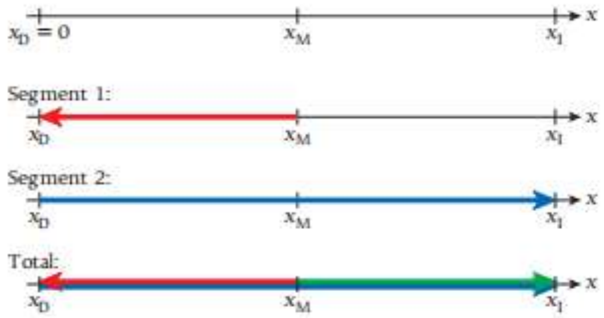


FIGURE 2.5 Coordinate system and trip segments for the Malcom to Des Moines to Iowa City trip.

RESEARCH With our assignment of $x_D = 0$, Des Moines is the origin of the coordinate system. According to the information given us, Malcom is then at $x_M = +89.9$ km and Iowa City is at $x_I = +170.5$ km. Note that we write a plus sign in front of the numbers for x_M and x_I to remind us that these are components of position vectors and can have positive or negative values.

For the first segment, the *displacement* is given by

$$\Delta x_1 = x_D - x_M.$$

Thus, the distance driven for this segment is

$$\ell_1 = |\Delta x_1| = |x_D - x_M|.$$

In the same way, the displacement and distance for the second segment are

$$\Delta x_2 = x_I - x_D$$
$$\ell_2 = |\Delta x_2| = |x_I - x_D|.$$

– Continued

For the sum of the two segments, the total trip, we use simple addition to find the displacement,

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2,$$

and the total distance,

$$\ell_{\text{total}} = \ell_1 + \ell_2.$$

SIMPLIFY We can simplify the equation for the total displacement a little bit by inserting the expressions for the displacements for the two segments:

$$\begin{aligned}\Delta x_{\text{total}} &= \Delta x_1 + \Delta x_2 \\ &= (x_D - x_M) + (x_I - x_D) \\ &= x_I - x_M.\end{aligned}$$

This is an interesting result–for the total displacement of the entire trip, it does not matter at all that we went to Des Moines. All that matters is where the trip started and where it ended. The total displacement is a result of a one-dimensional vector addition, as indicated in the bottom part of Figure 2.5 by the green arrow.

CALCULATE Now we can insert the numbers for the positions of the three cities in our coordinate system. We then obtain for the net displacement in our trip

$$\Delta x_{\text{total}} = x_I - x_M = (+170.5\text{ km}) - (+89.9\text{ km}) = +80.6\text{ km}.$$

For the total distance driven, we get

$$\ell_{\text{total}} = |89.9\text{ km}| + |170.5\text{ km}| = 260.4\text{ km}.$$

(Remember, the distance between Des Moines and Malcom, or Δx_1, and that between Des Moines and Iowa City, or Δx_2, were given in the problem; so we do not have to calculate them again from the differences in the position vectors of the cities.)

ROUND The numbers for the distances were initially given to a tenth of a kilometer. Since our entire calculation only amounted to adding or subtracting these numbers, it is not surprising that we end up with numbers that are also accurate to a tenth of a kilometer. No further rounding is needed.

2.29 A car travels north at 30.0 m/s for 10.0 min. It then travels south at 40.0 m/s for 20.0 min. What are the total distance the car travels and its displacement?

RESEARCH: The distance is equal to the product of velocity and time. The distance traveled is $p = v_1 t_1 + v_2 t_2$ and the displacement is the distance between where you start and where you finish, $d = v_1 t_1 - v_2 t_2$.

CALCULATE: $p = v_1 t_1 + v_2 t_2 = (30.\text{ m/s})(6.00 \cdot 10^2\text{ s}) + (40.\text{ m/s})(1.20 \cdot 10^3\text{ s}) = 66{,}000.\text{ m}$

$d = v_1 t_1 - v_2 t_2 = (30.\text{ m/s})(6.00 \cdot 10^2\text{ s}) - (40.\text{ m/s})(1.20 \cdot 10^3\text{ s}) = -30{,}000.\text{ m}$

ROUND: The total distance traveled is 66.0 km, and the displacement is 30.0 km in southern direction.

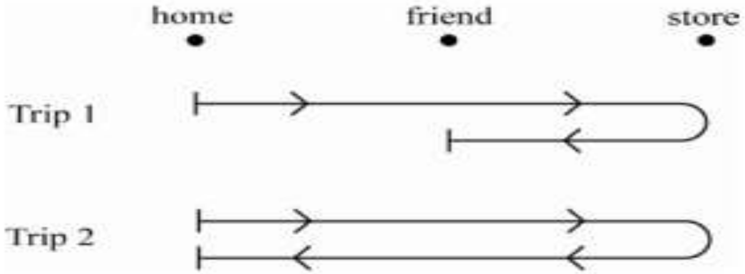
2.30 You ride your bike along a straight line from your house to a store 1000. m away. On your way back, you stop at a friend's house which is halfway between your house and the store.

a) What is your displacement?

b) What is the distance you have traveled?

c) After talking to your friend, you continue to your house. When you arrive back at your house, what is your displacement?

d) What is the total distance you have traveled?



RESEARCH: displacement (d) = final position − initial position
distance traveled = distance of path taken

SIMPLIFY:

(a) $d = \frac{1}{2}l - 0 = \frac{1}{2}l$

(b) $p = l + \frac{1}{2}l = \frac{3}{2}l$

(c) $d = 0 - 0 = 0$

(d) $p = l + l = 2l$

CALCULATE:

(a) $d = \frac{1}{2}l = \frac{1}{2}(1000.\text{ m}) = 500.0\text{ m}$

(b) $p = \frac{3}{2}l = \frac{3}{2}(1000.\text{ m}) = 1500.\text{ m}$

(c) $d = 0\text{ m}$

(d) $p = 2l = 2(1000.\text{ m}) = 2000.\text{ m}$

6	Interpret motion of an object from its position-time graph.	Figure(2.18/2.19)	47/49

SOLVED PROBLEM 2.3 Racing with a Head Start

Sultan has a new Dodge Charger with a Hemi engine and has challenged Khaled, who owns a tuned VW GTI, to a race at a local track. Khaled knows that Sultan's Charger is rated to go from 0 to 100 kph in 5.3 s, whereas his VW needs 7.0 s. Khaled asks for a head start and Sultan agrees to give him exactly 1.0 s.

PROBLEM

How far down the track is Khaled before Sultan gets to start the race? At what time does Sultan catch Khaled? How far away from the start are they when this happens? (Assume constant acceleration for each car during the race.)

SOLUTION

THINK This race is a good example of one-dimensional motion with constant acceleration. The temptation is to look over the kinematical equations 2.23 and see which one we can apply. However, we have to be a bit more careful here, because the time delay between Khaled's start and Sultan's start adds a complication. In fact, if you try to solve this problem using the kinematical equations directly, you will not get the right answer. Instead, this problem requires some careful definition of the time coordinates for each car.

SKETCH For our sketch, we plot time on the horizontal axis and position on the vertical axis. Both cars move with constant acceleration from a standing start, so we expect simple parabolas for their paths in this diagram.

Since Sultan's car has the greater acceleration, its parabola (blue curve in Figure 2.18) has the larger curvature and thus the steeper rise. Therefore, it is clear that Sultan will catch Khaled at some point, but it is not yet clear where this point is.

- Continued

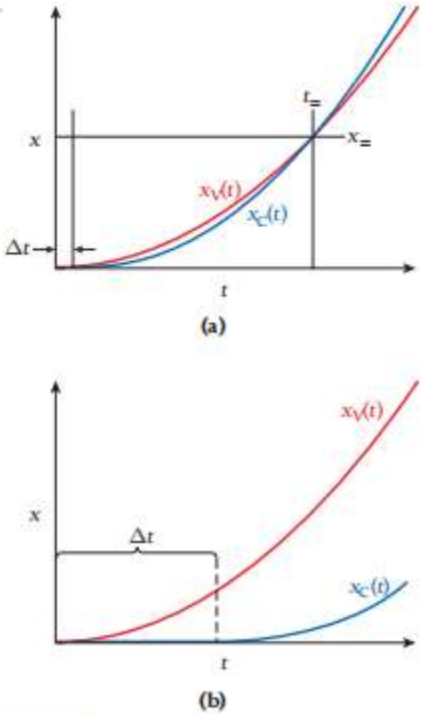


FIGURE 2.18 Position versus time for the race between Sultan and Khaled. Part (a) shows the entire duration of the race, and part (b) shows a magnified view of the start.

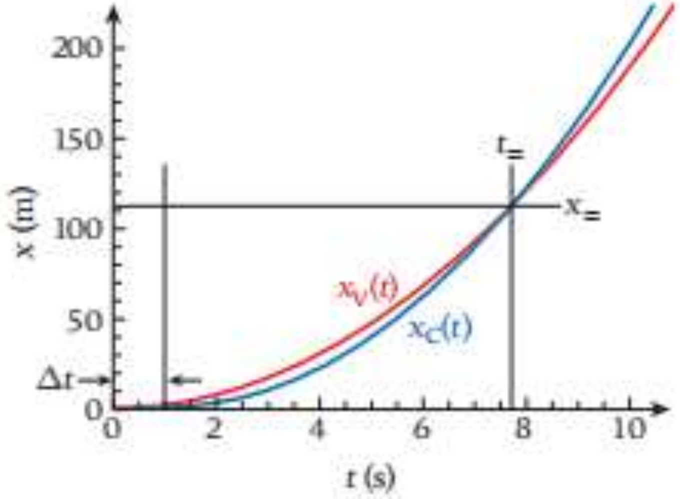


FIGURE 2.19 Plot of the parameters and equations of motion for the race between Sultan and Khaled.

7	Determine a particle's instantaneous acceleration given its position as a function of time $\left[a_x = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)\right]$	Student Book	40
		Q.[2.34/2.35/2.37]	61

$$a_x = \frac{d}{dt}v_x = \frac{d}{dt}\left(\frac{d}{dt}x\right) = \frac{d^2}{dt^2}x.$$

2.34 The position of a particle moving along the x-axis is given by $x = (11 + 14t - 2.0t^2)$, where t is in seconds and x is in meters. What is the average velocity during the time interval from $t = 1.0$ s to $t = 4.0$ s?

SIMPLIFY: $\bar{v} = \frac{x(t_2)-x(t_1)}{t_2-t_1} = \frac{(11+14t_2-2.0t_2^2)-(11+14t_1-2.0t_1^2)}{t_2-t_1} = \frac{14(t_2-t_1)-2.0(t_2^2-t_1^2)}{t_2-t_1}$

CALCULATE: $\bar{v} = \frac{14(4.0\text{ s}-1.0\text{ s})-2.0((4.0\text{ s})^2-(1.0\text{ s})^2)}{4.0\text{ s}-1.0\text{ s}} = 4.0\text{ m/s}$

•**2.35** The position of a particle moving along the x-axis is given by $x = 3.0t^2 - 2.0t^3$, where x is in meters and t is in seconds. What is the position of the particle when it achieves its maximum speed in the positive x-direction?

$$v(t) = \frac{d}{dt}x(t),\ a(t) = \frac{d}{dt}v(t).$$

Find the places where the acceleration is zero. The maximum speed will be the maximum of the speeds at the places where the acceleration is zero.

SIMPLIFY: $v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(3.0t^2-2.0t^3) = 2\cdot 3.0t^{2-1} - 3\cdot 2.0t^{3-1} = 6.0t - 6.0t^2$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(6.0t-6.0t^2) = 6.0t^{1-1} - 2\cdot 6.0t^{2-1} = 6.0 - 12t$$

CALCULATE: Solving for the value of t where a is zero:

$$0 = 6.0 - 12t \Rightarrow 6.0 = 12t \Rightarrow t = 0.50\text{ s}$$

This time can now be used to solve for the position:

$$x(0.50) = 3.0(0.50)^2 - 2.0(0.50)^3 = 0.500\text{ m}$$

Since there is only one place where the acceleration is zero, the maximum speed in the positive x-direction must occur here.

•2.37 The position of an object as a function of time is given as $x = At^3 + Bt^2 + Ct + D$. The constants are $A = 2.10\ \text{m/s}^3$, $B = 1.00\ \text{m/s}^2$, $C = -4.10\ \text{m/s}$, and $D = 3.00\ \text{m}$.

a) What is the velocity of the object at $t = 10.0$ s?

b) At what time(s) is the object at rest?

c) What is the acceleration of the object at $t = 0.50$ s?

d) Plot the acceleration as a function of time for the time interval from $t = -10.0$ s to $t = 10.0$ s.

RESEARCH:

(a) The velocity is given by the time derivative of the positive function $v(t) = \frac{d}{dt}x(t)$.

(b) To find the time when the object is at rest, set the velocity to zero, and solve for t. This is a q

equation of the form $ax^2 + bx + c = 0$, whose solution is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(c) The acceleration is given by the time derivative of the velocity: $a(t) = \frac{d}{dt}v(t)$.

(d) The equation for acceleration found in part (c) can be used to plot the graph of the function.

(a) $v(t = 10.0\ s) = 3(2.10\ \text{m/s}^3)(10.0\ \text{s})^2 + 2(1.00\ \text{m/s}^2)(10.0\ \text{s}) - 4.10\ \text{m/s} = 645.9\ \text{m/s}$

(b) $t = \frac{-2(1.00\ \text{m/s}^2) \pm \sqrt{4(1.00\ \text{m/s}^2)^2 - 12(2.10\ \text{m/s}^3)(-4.10\ \text{m/s})}}{6(2.1\ 0\text{m/s}^3)}$

$= 0.6634553\ \text{s}, -0.9809156\ \text{s}$

(c) $a(t = 0.50\ \text{s}) = 6(2.10\ \text{m/s}^3)(0.50\ \text{s}) + 2(1.00\ \text{m/s}^2) = 8.30\ \text{m/s}^2$

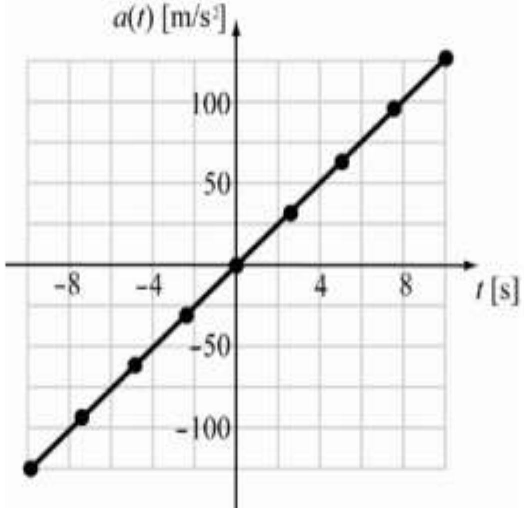
(d) The acceleration function, $a(t) = 6At + 2B$, can be used to compute the acceleration for time steps of 2.5 s. For example:

$$a(t = -2.5\ \text{s}) = 6(2.10\ \text{m/s}^3)(-2.5\ \text{s}) + 2(1.00\ \text{m/s}^2) = -29.5\ \text{m/s}^2$$

The result is given in the following table.

t [s]	–10.0	–7.5	–5.0	–2.5	0.0	2.5	5.0	7.5	10.0
a [m/s^2]	–124.0	–92.5	–61.0	–29.5	2.0	33.5	65.0	96.5	128.0

These values are used to plot the function.



8	I. Differentiate between average and instantaneous velocity. II. Calculate the average velocity/average speed.	Example2.2 Q.[2.31/2.32/2.33]	39 61

EXAMPLE 2.2 Speed and Velocity

Suppose a swimmer completes the first 50 m of the 100-m freestyle in 38.2 s. Once she reaches the far side of the 50-m-long pool, she turns around and swims back to the start in 42.5 s.

PROBLEM

What are the swimmer's average velocity and average speed for (a) the leg from the start to the far side of the pool, (b) the return leg, and (c) the total lap?

SOLUTION

We start by defining our coordinate system, as shown in Figure 2.9. The positive x-axis points toward the bottom of the page.

(a) *First leg of the swim:*
The swimmer starts at $x_1 = 0$ and swims to $x_2 = 50$ m. It takes her $\Delta t = 38.2$ s to accomplish this leg. Her average velocity for leg 1 then, according to our definition, is

$$\bar{v}_{x1} = \frac{x_2 - x_1}{\Delta t} = \frac{50\text{ m} - 0\text{ m}}{38.2\text{ s}} = \frac{50}{38.2}\text{ m/s} = 1.31\text{ m/s}.$$

Her average speed is the distance divided by time interval, which, in this case, is the same as the absolute value of her average velocity, or $|\bar{v}_{x1}| = 1.31$ m/s.

(b) *Second leg of the swim:*
We use the same coordinate system for leg 2 as for leg 1. This choice means that the swimmer starts at $x_1 = 50$ m and finishes at $x_2 = 0$, and it takes $\Delta t = 42.5$ s to complete this leg. Her average velocity for this leg is

$$\bar{v}_{x2} = \frac{x_2 - x_1}{\Delta t} = \frac{0\text{ m} - 50\text{ m}}{42.5\text{ s}} = \frac{-50}{42.5}\text{ m/s} = -1.18\text{ m/s}.$$

Note the negative sign for the average velocity for this leg. The average speed is again the absolute magnitude of the average velocity, or $|\bar{v}_{x2}| = |-1.18\text{ m/s}| = 1.18$ m/s.

(c) *The entire lap:*
We can find the average velocity in two ways, demonstrating that they result in the same answer. First, because the swimmer started at $x_1 = 0$ and finished at $x_2 = 0$, the difference is 0. Thus, the net displacement is 0, and consequently the average velocity is also 0.

We can also find the average velocity for the whole lap by taking the time-weighted sum of the components of the average velocities of the individual legs:

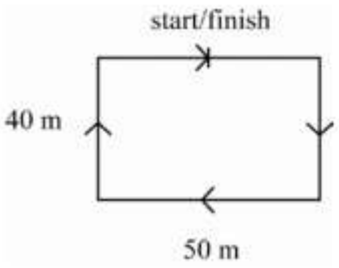
$$\bar{v}_x = \frac{\bar{v}_{x1}\cdot\Delta t_1 + \bar{v}_{x2}\cdot\Delta t_2}{\Delta t_1 + \Delta t_2} = \frac{(1.31\text{ m/s})(38.2\text{ s}) + (-1.18\text{ m/s})(42.5\text{ s})}{(38.2\text{ s}) + (42.5\text{ s})} = 0.$$

What do we find for the average speed? The average speed, according to our definition, is the total distance divided by the total time. The total distance is 100 m and the total time is 38.2 s plus 42.5 s, or 80.7 s. Thus,

$$\bar{v} = \frac{\ell}{\Delta t} = \frac{100\text{ m}}{80.7\text{ s}} = 1.24\text{ m/s}.$$

We can also use the time-weighted sum of the average speeds, leading to the same result. Note that the average speed for the entire lap is between that for leg 1 and that for leg 2. It is not exactly halfway between these two values, but is closer to the lower value because the swimmer spent more time completing leg 2.

2.31 Running on a 50-m by 40-m rectangular track, you complete one lap in 100 s. What is your average velocity for the lap?



RESEARCH: average velocity = $\dfrac{\text{final position} - \text{initial position}}{\text{time}}$

SIMPLIFY: $\bar{v} = \dfrac{x_f - x_i}{t}$

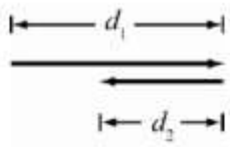
CALCULATE: $\bar{v} = \dfrac{0\text{ m} - 0\text{ m}}{100\text{ s}} = 0\text{ m/s}$

2.32 An electron moves in the positive x-direction a distance of 2.42 m in 2.91×10^{-8} s, bounces off a moving proton, and then moves in the opposite direction a distance of 1.69 m in 3.43×10^{-8} s.

a) What is the average velocity of the electron over the entire time interval?

b) What is the average speed of the electron over the entire time interval?

SKETCH:



RESEARCH:

(a) average velocity = $\dfrac{\text{final position} - \text{initial position}}{\text{time}}$

(b) speed = $\dfrac{\text{total distance traveled}}{\text{time}}$

SIMPLIFY:

(a) $\bar{v} = \dfrac{d_1 - d_2}{t_1 + t_2}$

(b) $s = \dfrac{d_1 + d_2}{t_1 + t_2}$

CALCULATE:

(a) $\bar{v} = \dfrac{d_1 - d_2}{t_1 + t_2} = \dfrac{2.42\text{ m} - 1.69\text{ m}}{2.91 \cdot 10^{-8}\text{ s} + 3.43 \cdot 10^{-8}\text{ s}} = 11{,}514{,}195\text{ m/s}$

(b) $s = \dfrac{d_1 + d_2}{t_1 + t_2} = \dfrac{2.42\text{ m} + 1.69\text{ m}}{2.91 \cdot 10^{-8}\text{ s} + 3.43 \cdot 10^{-8}\text{ s}} = 64{,}826{,}498\text{ m/s}$

ROUND:

(a) $\bar{v} = 1.15 \cdot 10^{7}\text{ m/s}$

(b) $s = 6.48 \cdot 10^{7}\text{ m/s}$

2.33 The graph describes the position of a particle in one dimension as a function of time.

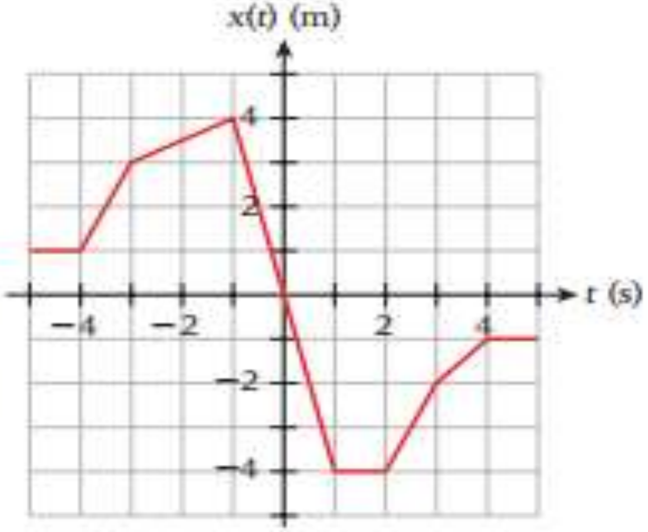
a) In which time interval does the particle have its maximum speed? What is that speed?

b) What is the average velocity in the time interval between −5 s and +5 s?

c) What is the average speed in the time interval between −5 s and +5 s?

d) What is the ratio of the velocity in the interval between 2 s and 3 s to the velocity in the interval between 3 s and 4 s?

e) At what time(s) is the particle's velocity zero?



RESEARCH: The velocity is given by the slope on a distance versus time graph. A steeper slope means a greater speed.

$$\text{average velocity} = \frac{\text{final position} - \text{initial position}}{\text{time}}, \quad \text{speed} = \frac{\text{total distance traveled}}{\text{time}}$$

(a) The largest speed is where the slope is the steepest.

(b) The average velocity is the total displacement over the time interval.

(c) The average speed is the total distance traveled over the time interval.

(d) The ratio of the velocities is $v_1 : v_2$.

(e) A velocity of zero is indicated by a slope that is horizontal.

SIMPLIFY:

(a) The largest speed is given by the steepest slope occurring between –1 s and +1 s.

$$s = \frac{|x(t_2) - x(t_1)|}{t_2 - t_1}, \text{ with } t_2 = 1 \text{ s and } t_1 = -1 \text{ s.}$$

(b) The average velocity is given by the total displacement over the time interval.

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \text{ with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s.}$$

(c) In order to calculate the speed in the interval –5 s to 5 s, the path must first be determined. The path is given by starting at 1 m, going to 4 m, then turning around to move to –4 m and finishing at –1 m. So the total distance traveled is

$$\begin{aligned} p &= |(4 \text{ m} - 1 \text{ m})| + |((-4 \text{ m}) - 4 \text{ m})| + |(-1 \text{ m} - (-4 \text{ m}))| \\ &= 3 \text{ m} + 8 \text{ m} + 3 \text{ m} \\ &= 14 \text{ m} \end{aligned}$$

This path can be used to find the speed of the particle in this time interval.

$$s = \frac{p}{t_2 - t_1}, \text{ with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s.}$$

(d) The first velocity is given by $v_1 = \dfrac{x(t_3) - x(t_2)}{t_3 - t_2}$ and the second by $v_2 = \dfrac{x(t_4) - x(t_3)}{t_4 - t_3}$,

(e) The velocity is zero in the regions 1 s to 2 s, −5 s to −4 s, and 4 s to 5 s.

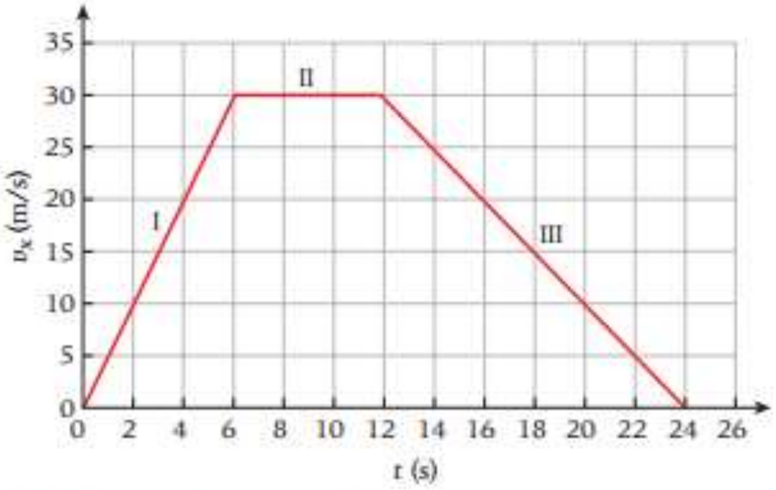
CALCULATE:

(a) $s = \dfrac{|-4 \text{ m} - 4 \text{ m}|}{1 \text{ s} - (-1 \text{ s})} = 4.0 \text{ m/s}$

(b) $\bar{v} = \dfrac{-1 \text{ m} - 1 \text{ m}}{5 \text{ s} - (-5 \text{ s})} = -0.20 \text{ m/s}$

9	**Determine a particle's change in position by finding the area under the curve on a graph of velocity versus time.**	Q.[2.42/2.52]	61/62

2.42 A fellow student found in the performance data for his new car the velocity-versus-time graph shown in the figure.



a) Find the average acceleration of the car during each of the segments I, II, and III.

b) What is the total distance traveled by the car from $t = 0$ s to $t = 24$ s?

RESEARCH:

(a) The acceleration is given by the slope of a velocity versus time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

(b) The displacement is the sum of the areas of two triangles and one rectangle. Recall the area formulas:

$$\text{area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

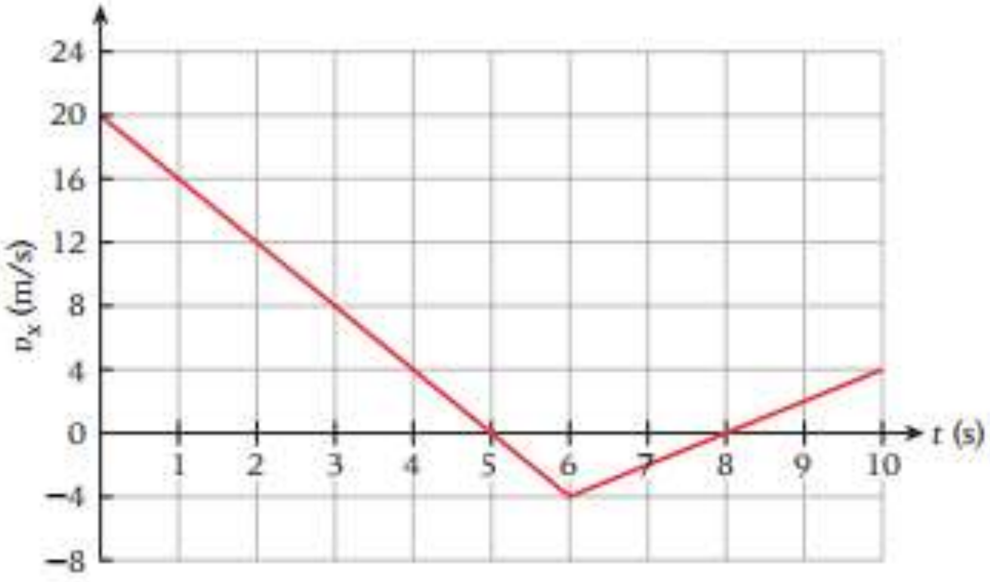
$$\text{area of a rectangle} = \text{base} \times \text{height}$$

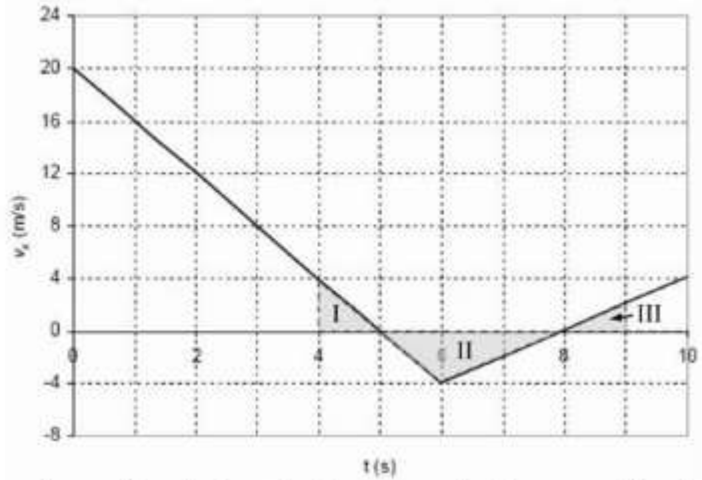
(a) $a_{\text{I}} = \dfrac{30.0 \text{ m/s} - 0 \text{ m/s}}{6.0 \text{ s} - 0 \text{ s}} = 5.0 \text{ m/s}^2, \quad a_{\text{II}} = \dfrac{30.0 \text{ m/s} - 30.0 \text{ m/s}}{12.0 \text{ s} - 6.0 \text{ s}} = 0.0 \text{ m/s}^2,$

$$a_{\text{III}} = \frac{0.0 \text{ m/s} - 30.0 \text{ m/s}}{24.0 \text{ s} - 12.0 \text{ s}} = -2.50 \text{ m/s}^2$$

(b) $x = \frac{1}{2}(30.0 \text{ m/s})(6.0 \text{ s} - 0.0 \text{ s}) + (30.0 \text{ m/s})(12.0 \text{ s} - 6.0 \text{ s}) + \frac{1}{2}(30.0 \text{ m/s})(24.0 \text{ s} - 12.0 \text{ s}) = 450.0 \text{ m}$

•**2.52** A car is moving along the x-axis and its velocity, v_x, varies with time as shown in the figure. What is the displacement, Dx, of the car from $t = 4$ s to $t = 9$ s?





RESEARCH: The change in position is given by the area under the curve of a velocity versus time graph. Note that it is hard to read the value of the velocity at $t = 9.0$ s. This difficulty can be overcome by finding the slope of the line for this section. Using the slope, the velocity during this time can be determined: $\Delta x = \text{Area}$, $m = \dfrac{\text{rise}}{\text{run}}$. Let A_{I} be the area of region I, let A_{II} be the area of region II, and let A_{III} be the area of region III.

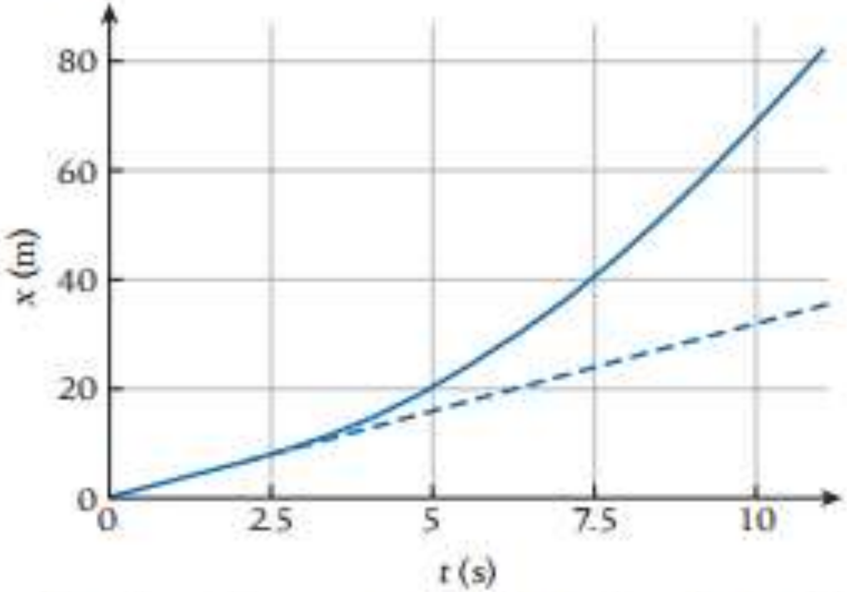
SIMPLIFY: $\Delta x = A_{\text{I}} + A_{\text{II}} + A_{\text{III}}$

CALCULATE: $m = \dfrac{4.0 \text{ m/s} - (-4.0 \text{ m/s})}{10.0 \text{ s} - 6.0 \text{ s}} = 2.0 \text{ m/s}^2$

$$\Delta x = \frac{1}{2}(4.0 \text{ m/s})(5.0 \text{ s} - 4.0 \text{ s}) + \frac{1}{2}(-4.0 \text{ m/s})(8.0 \text{ s} - 5.0 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s})(9.0 \text{ s} - 8.0 \text{ s}) = -3.0 \text{ m}$$

ROUND: $\Delta x = -3.0$ m

2.26 A car moves along a road with a constant velocity. Starting at time $t = 2.5$ s, the driver accelerates with constant acceleration. The resulting position of the car as a function of time is shown by the blue curve in the figure.

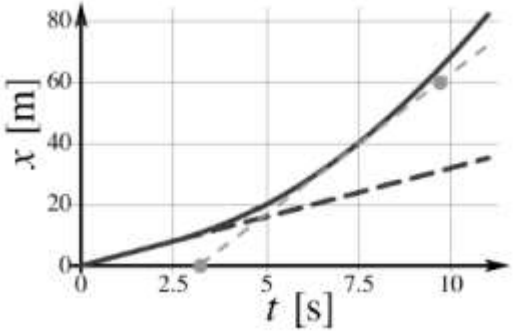


a) What is the value of the constant velocity of the car before 2.5 s? (*Hint:* The dashed blue line is the path the car would take in the absence of the acceleration.)

b) What is the velocity of the car at $t = 7.5$ s? Use a graphical technique (i.e., draw a slope).

c) What is the value of the constant acceleration?

Velocity can be estimated by computing the slope of a curve in a distance versus time plot.



Velocity is defined by $v = \Delta x / \Delta t$. If acceleration is constant, then $a = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$. (a) Estimate the slope of the dashed blue line. Pick two points: it is more accurate to pick a point that coincides with horizontal lines of the grid. Choosing points $t = 0$ s, $x = 0$ m and $t = 6.25$ s, $x = 20$ m:

$$v = \frac{20.\text{ m} - 0\text{ m}}{6.25\text{ s} - 0\text{ s}} = 3.2\text{ m/s}$$

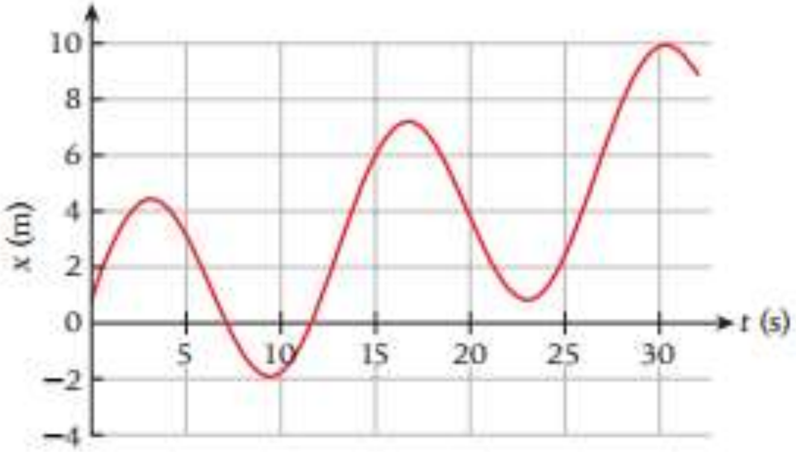
(b) Examine the sketch. There is a tangent to the curve at $t = 7.5$ s. Pick two points on the line. Choosing points: $t = 3.4$ s, $x = 0$ m and $t = 9.8$ s, $x = 60$ m:

$$v = \frac{60.\text{ m} - 0\text{ m}}{9.8\text{ s} - 3.4\text{ s}} = 9.4\text{ m/s}$$

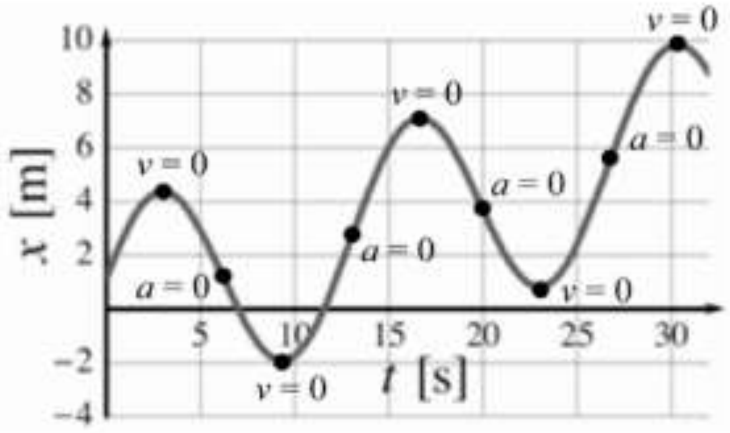
(c) From (a), $v = 3.2$ m/s at $t = 2.5$ s and from (b), $v = 9.4$ m/s at $t = 7.5$ s. From the definition of constant acceleration,

$$a = \frac{9.4\text{ m/s} - 3.2\text{ m/s}}{7.5\text{ s} - 2.5\text{ s}} = \frac{6.2\text{ m/s}}{5.0\text{ s}} = 1.2\text{ m/s}^2.$$

2.76 On the graph of position as a function of time, mark the points where the velocity is zero and the points where the acceleration is zero.

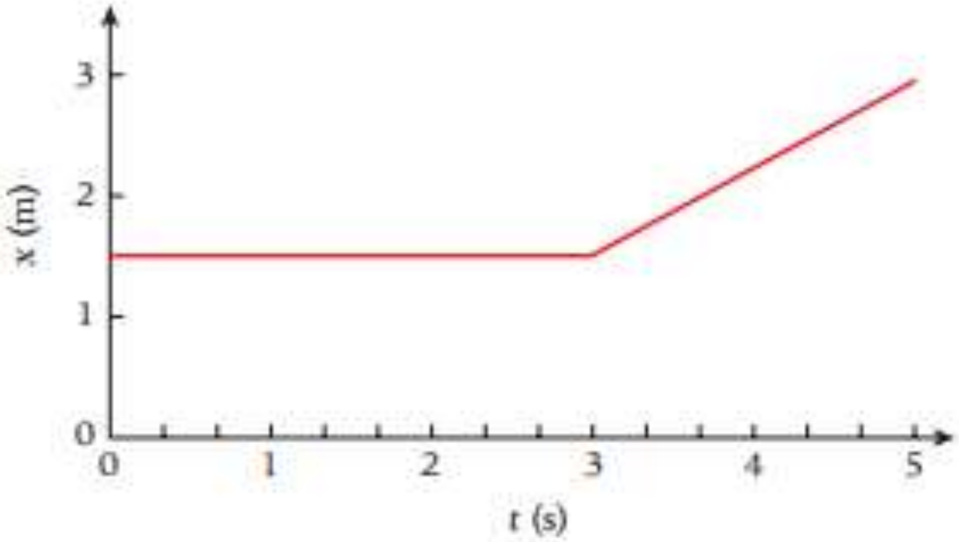


Velocity is the slope of the position versus time graph. Therefore, $v = 0$ at the local maxima and minima. Acceleration is the slope of the velocity versus time graph. On a position versus time graph, acceleration, a is zero at inflection points on the curve that are not maxima or minima, i.e. $a = 0$ as the slope of x vs. t approaches a constant value over some non-zero time interval, Δt :



11	I. Solve problems related to graphical integration in motion analysis. II. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration	Q.[2.12/2.26/2.76]	59 60/63

2.12 The figure describes the position of an object as a function of time. Which one of the following statements is true?



a) The position of the object is constant.

b) The velocity of the object is constant.

c) The object moves in the positive x-direction until $t = 3$ s, and then the object is at rest.

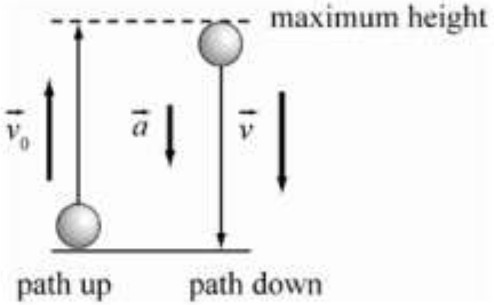
d) The object's position is constant until $t = 3$ s, and then the object begins to move in the positive x-direction.

e) The object moves in the positive x-direction from $t = 0$ to $t = 3$ s and then moves in the negative x-direction from $t = 3$ s to $t = 5$ s.

2.66 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?

THINK: I know that $v_0 = 26.4$ m/s and $a = -g = -9.81 \text{ m/s}^2$. I want to find t_{total}. Note that once the ball gets back to the starting point, $v = -26.4$ m/s, or $v = -v_0$.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $t = \dfrac{v - v_0}{a} = \dfrac{-v_0 - v_0}{-g} = \dfrac{2v_0}{g}$

CALCULATE: $t = \dfrac{2(26.4 \text{ m/s})}{9.81 \text{ m/s}^2} = 5.38226$ s

ROUND: Since all the values given have three significant digits, $t = 5.38$ s.

2.67 A stone is thrown upward, from ground level, with an initial velocity of 10.0 m/s.

a) What is the velocity of the stone after 0.50 s?

b) How high above ground level is the stone after 0.50 s?

SIMPLIFY:

(a) $v = v_0 - gt$

(b) $h = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2$

CALCULATE:

(a) $v = 10.0\text{ m/s} - (9.81\text{ m/s}^2)(0.50\text{ s})$

$= 10.0\text{ m/s} - 4.905\text{m/s}$

$= 5.095\text{ m/s}$

(b) $h = (10.0\text{ m/s})(0.50\text{ s}) - \frac{1}{2}(9.81\text{ m/s}^2)(0.50\text{ s})^2$

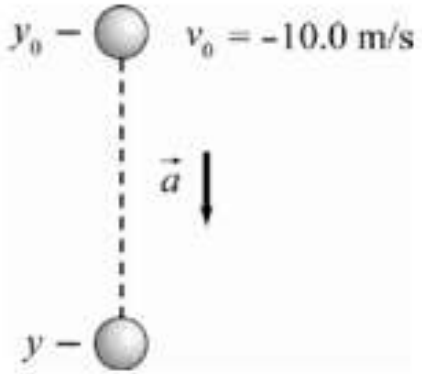
$= 5.0\text{ m} - 1.226\text{ m}$

$= 3.774\text{ m}$

2.68 A stone is thrown downward with an initial velocity of 10.0 m/s. The acceleration of the stone is constant and has the value of the free-fall acceleration, 9.81 m/s^2. What is the velocity of the stone after 0.500 s?

THINK: I know that $v_0 = -10.0 \text{ m/s}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find v at $t = 0.500$ s.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $v = v_0 - gt$

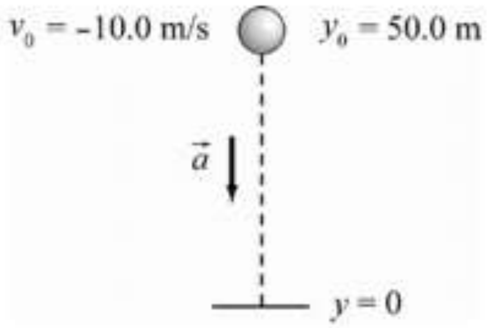
CALCULATE: $v = -10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.500 \text{ s}) = -10.0 \text{ m/s} - 4.905 \text{ m/s} = -14.905 \text{ m/s}$

ROUND: Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = -14.9$ m/s.

2.69 A ball is thrown directly downward, with an initial speed of 10.0 m/s, from a height of 50.0 m. After what time interval does the ball strike the ground?

THINK: Take "downward" to be along the negative y-axis. I know that $v_0 = -10.0\text{ m/s}$, $\Delta y = -50.0\text{ m}$, and $a = -g = -9.81\text{ m/s}^2$. I want to find t, the time when the ball reaches the ground.

SKETCH:



RESEARCH: $\Delta y = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: $\frac{1}{2}at^2 + v_0 t - \Delta y = 0$. This is a quadratic equation. Solving for t:

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-\Delta y)}}{2\left(\frac{1}{2}a\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{-g}$$

CALCULATE: $t = \dfrac{-(-10.0\text{ m/s}) \pm \sqrt{(-10.0\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(-50.0\text{ m})}}{-9.81\text{ m/s}^2}$

$= -4.3709\text{ s},\ 2.3322\text{ s}$

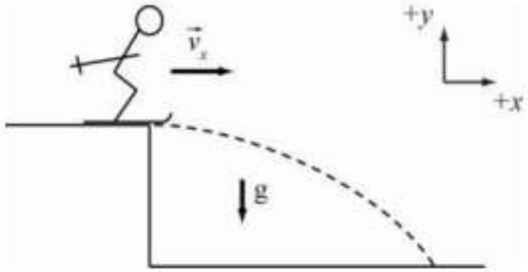
The time interval has to be positive, so $t = 2.3322$ s.

13	I. Define maximum height, range of a projectile and time of flight. II. Calculate the maximum height, range of a projectile and the time of flight for a projectile	Q.[3.41/3.43/3.47]	87

3.41 A skier launches off a ski jump with a horizontal velocity of 30.0 m/s (and no vertical velocity component). What are the magnitudes of the horizontal and vertical components of her velocity the instant before she lands 2.00 s later?

THINK: Ignoring air resistance, the skier's horizontal velocity will remain unchanged, while her vertical velocity is influenced solely by gravity. $v_x = 30.0$ m/s , $g = 9.81$ m/s^2 and $t = 2.00$ s.

SKETCH:



RESEARCH: $v_{ix} = v_{fx}$ and $v_{fy} = v_{iy} + at$.

SIMPLIFY: $v_{fy} = 0 - gt = -gt$

CALCULATE: $v_{fx} = 30.0$ m/s and $v_{fy} = -(9.81 \text{ m/s}^2)(2.00 \text{ s}) = -19.62$ m/s.

ROUND: $|v_{fx}| = 30.0$ m/s and $|v_{fy}| = 19.6$ m/s.

3.43 A football is kicked with an initial speed of 27.5 m/s and a launch angle of 56.7°. What is its hang time (the time until it hits the ground again)?

THINK: Assume the ball starts on the ground so that the initial and final heights are the same. The initial velocity of the ball is $v_i = 27.5 \text{ m/s}$, with $\theta = 56.7°$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:

RESEARCH: $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$ and $v_{iy} = v_i \sin\theta$.

SIMPLIFY: $0 = v_i(\sin\theta)t - \frac{1}{2}gt^2 \Rightarrow v_i \sin\theta = \frac{1}{2}gt \Rightarrow t = \frac{2v_i \sin\theta}{g}$

CALCULATE: $t = \frac{2(27.5 \text{ m/s})\sin(56.7°)}{9.81 \text{ m/s}^2} = 4.6860 \text{ s}$

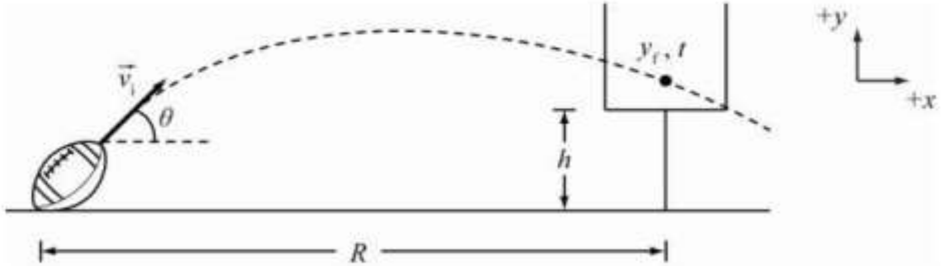
ROUND: Rounding to three significant figures, $t = 4.69$ s.

•3.47 A football player kicks a ball with a speed of 22.4 m/s at an angle of 49.0° above the horizontal from a distance of 39.0 m from the goal-post.

a) By how much does the ball clear or fall short of clearing the crossbar of the goalpost if that bar is 3.05 m high?

b) What is the vertical velocity of the ball at the time it reaches the goalpost?

SKETCH:



RESEARCH: $R = v_{ix}t$; $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$; $v_{fy} = v_{iy} + at$; $v_{ix} = v_i\cos\theta$; and $v_{iy} = v_i\sin\theta$.

SIMPLIFY:

(a) $t = \frac{R}{v_i\cos\theta}$ and $y_f = (v_i\sin\theta)t - \frac{1}{2}gt^2 = R\tan\theta - \frac{gR^2}{2v_i^2\cos^2\theta}$.

In order to compare the height of the ball to the height of the goal post, subtract h from both sides of the equation,

$$\Delta h = y_f - h = -h + R\tan\theta - \frac{gR^2}{2v_i^2\cos^2\theta}.$$

(b) $v_{fy} = v_i\sin\theta - \frac{gR}{v_i\cos\theta}$

CALCULATE:

(a) $\Delta h = -3.05 \text{ m} + (39.0 \text{ m})\tan(49.0°) - \frac{(9.81 \text{ m/s}^2)(39.0 \text{ m})^2}{2(22.4 \text{ m/s})^2\cos^2(49.0°)} = 7.2693 \text{ m}$

(b) $v_{fy} = (22.4 \text{ m/s})\sin(49.0°) - \frac{(9.81 \text{ m/s}^2)(39.0 \text{ m})}{(22.4 \text{ m/s})\cos(49.0°)} = -9.1286 \text{ m/s}$

ROUND: Round to the appropriate three significant figures:

(a) The ball clears the post by 7.27 m.

(b) The ball is heading downward at 9.13 m/s.

14	Determine direction and magnitude of a frictional force	Student Book	108-110
		Example 4.6 Q.4.57	110 124

EXAMPLE 4.6 Realistic Snowboarding

Let's reconsider the snowboarding situation from Solved Problem 4.1, but now include friction. A snowboarder moves down a slope for which $\theta = 22°$. Suppose the coefficient of kinetic friction between his board and the snow is 0.21, and his velocity, which is along the direction of the slope, is measured as 8.3 m/s at a given instant.

PROBLEM 1

Assuming a constant slope, what will be the speed of the snowboarder along the direction of the slope when he is 100 m farther down the slope?

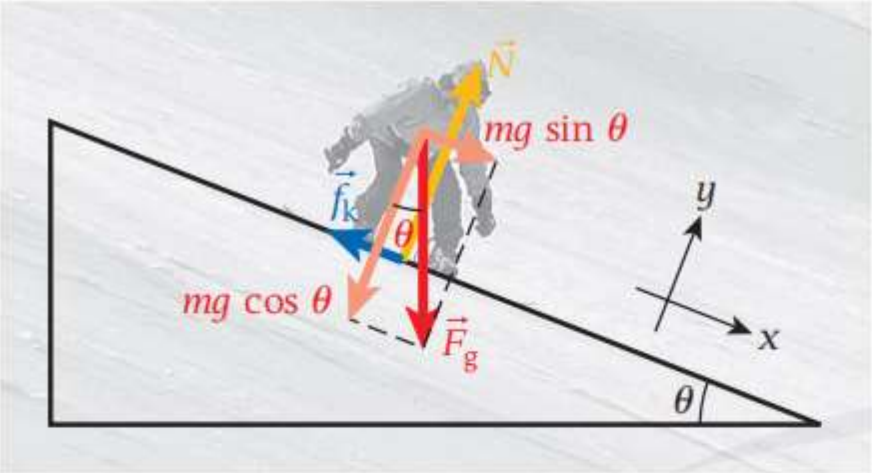


FIGURE 4.19 Free-body diagram of a snowboarder, including the friction force.

$$mg\sin\theta - \mu_k mg\cos\theta = ma_x \Rightarrow$$

$$a_x = g(\sin\theta - \mu_k \cos\theta).$$

$$a \equiv a_x = (9.81 \text{ m/s}^2)(\sin 22° - 0.21\cos 22°) = 1.76 \text{ m/s}^2.$$

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 8.3$ m/s and $x - x_0 = 100$ m, we calculate the final speed:

$$v = \sqrt{v_0^2 + 2a(x - x_0)}$$

$$= \sqrt{(8.3 \text{ m/s})^2 + 2(1.76 \text{ m/s}^2)(100 \text{ m})}$$

$$= 20.5 \text{ m/s}.$$

PROBLEM 2

How long does it take the snowboarder to reach this speed?

SOLUTION 2

Since we now know the acceleration and the final speed and were given the initial speed, we use

$$v = v_0 + at \Rightarrow t = \frac{v - v_0}{a} = \frac{(20.5 - 8.3)\ \text{m/s}}{1.76\ \text{m/s}^2} = 6.9\ \text{s}.$$

PROBLEM 3

Given the same coefficient of friction, what would the angle of the slope have to be for the snowboarder to glide with constant velocity?

SOLUTION 3

Motion with constant velocity implies zero acceleration. We have already derived an equation for the acceleration as a function of the slope angle. We set this expression equal to zero and solve the resulting equation for the angle θ:

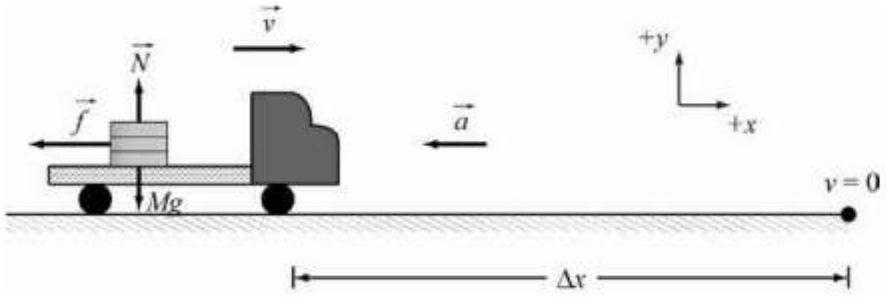
$$\begin{aligned} a_x &= g(\sin\theta - \mu_k \cos\theta) = 0 \\ &\Rightarrow \sin\theta = \mu_k \cos\theta \\ &\Rightarrow \tan\theta = \mu_k \\ &\Rightarrow \theta = \tan^{-1} \mu_k. \end{aligned}$$

Because $\mu_k = 0.21$ was given, the angle is $\theta = \tan^{-1} 0.21 = 12°$. With a steeper slope, the snowboarder will accelerate, and with a shallower slope, the snowboarder will slow down until he comes to a stop.

4.57 An engine block of mass M is on the flatbed of a pickup truck that is traveling in a straight line down a level road with an initial speed of 30.0 m/s. The coefficient of static friction between the block and the bed is $\mu_s = 0.540$. Find the minimum distance in which the truck can come to a stop without the engine block sliding toward the cab.

THINK: The initial speed of the truck is $v = 30.0$ m/s. The final speed of the truck is $v = 0$. The mass of the block is M. The coefficient of static friction is $\mu_s = 0.540$. Determine the minimum distance, Δx the truck can travel while stopping without causing the block to slide.

SKETCH:



RESEARCH: The minimum stopping distance occurs at the maximum acceleration the truck can undergo without causing the block to slide. Use the equation $v^2 = v_0^2 + 2a\Delta x$ to determine Δx. The acceleration is found from balancing the forces in the horizontal direction acting on the block.

SIMPLIFY: For the block, when it is just about to slide, $F_{\text{net},x} = -f_{\text{s,max}}$. Then, $Ma_{\text{net},x} = -\mu_s N = -\mu_s Mg$ $\Rightarrow a_{\text{net},x} = -\mu_s g$. Since the block and the truck remain in contact, they form a single system with the same acceleration. With $v = 0$,

$$0 = v_0^2 + 2a_{\text{net},x}\Delta x \Rightarrow \Delta x = \frac{-v_0^2}{2a_{\text{net},x}} = \frac{-v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2\mu_s g}.$$

CALCULATE: $\Delta x = \dfrac{(30.0\text{ m/s})^2}{2(0.540)(9.81\text{ m/s}^2)} = 84.95\text{ m}$

ROUND: To three significant figures, the result should be written $\Delta x = 84.9$ m.

15	Realize that in two or three dimensions, an acceleration vector arises if an object's velocity vector changes in magnitude or direction.	Student Book	68

3.2 Velocity and Acceleration in Two or Three Dimensions

The most striking difference between velocity along a line and velocity in two or more dimensions is that the latter can change direction even in cases where the speed stays constant. Because acceleration is defined as a change in velocity–any change in velocity–divided by a time interval, there can be acceleration even when the magnitude of the velocity does not change.

Consider, for example, a particle moving in two dimensions (that is, in a plane). At time t_1, the particle has velocity $\vec{v}_1$, and at a later time t_2, the particle has velocity $\vec{v}_2$. The change in velocity of the particle is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. The average acceleration, $\vec{a}_{ave}$, for the time interval $\Delta t = t_2 - t_1$ is given by

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}. \quad (3.7)$$

Figure 3.4 shows three different cases for the change in velocity of a particle moving in two dimensions over a given time interval. Figure 3.4a shows the initial and final velocities of the particle having the same direction, but the magnitude of the final velocity is greater than the magnitude of the initial velocity. The resulting change in velocity and the average acceleration are in the same direction as the velocities. Figure 3.4b again shows the initial and final velocities pointing in the same direction, but the magnitude of the final velocity is less than the magnitude of the initial velocity. The resulting change in velocity and the average acceleration are in the opposite direction from the velocities. Figure 3.4c illustrates the case when the initial and final velocities have the same magnitude but the direction of the final velocity vector is different from the direction of the initial velocity vector. Even though the magnitudes of the initial and final velocity vectors are the same, the change in velocity and the average acceleration are not zero and can be in a direction not obviously related to the initial or final velocity directions.

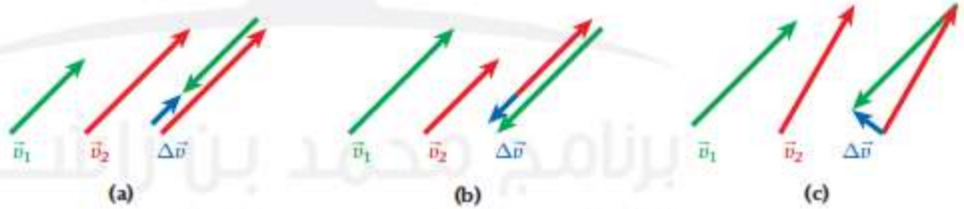
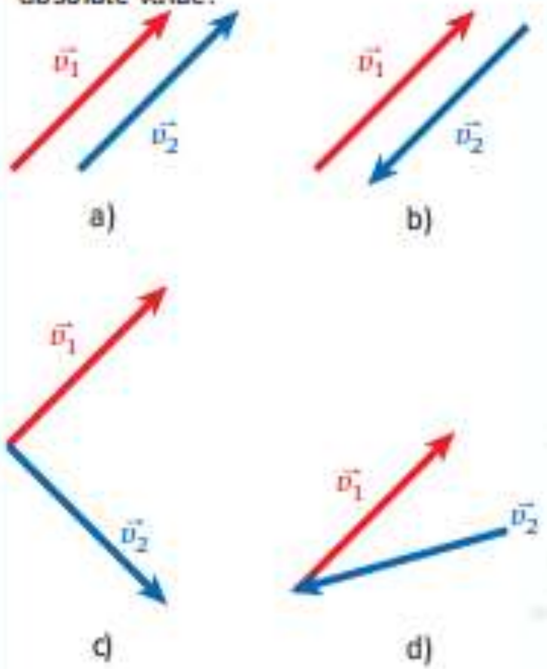


FIGURE 3.4 At time t_1, a particle has a velocity $\vec{v}_1$. At a later time t_2, the particle has a velocity $\vec{v}_2$. The average acceleration is given by $\vec{a}_{ave} = \Delta\vec{v}/\Delta t = (\vec{v}_2 - \vec{v}_1)/(t_2 - t_1)$. (a) A time interval corresponding to $|\vec{v}_2| > |\vec{v}_1|$, with $\vec{v}_2$ and $\vec{v}_1$ in the same direction. (b) A time interval corresponding to $|\vec{v}_2| < |\vec{v}_1|$, with $\vec{v}_2$ and $\vec{v}_1$ in the same direction. (c) A time interval with $|\vec{v}_2| = |\vec{v}_1|$, but with $\vec{v}_2$ in a different direction from $\vec{v}_1$.

In general, an acceleration vector arises if an object's velocity vector changes in magnitude or direction. Any time an object travels along a curved path, in two or three dimensions, it must have acceleration. We will examine the components of acceleration in more detail in Chapter 9, when we discuss circular motion.

In all of the cases shown below, the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ have the same length. In which case does $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ have the *largest* absolute value?



e) All of the cases are identical.

Concept Check 3.2

In all of the cases shown in Concept Check 3.1, the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ have the same length. In which case does the acceleration $\vec{a} = \Delta\vec{v} / \Delta t$ have the *smallest* absolute value?

Paper exam

16	I. Add or subtract vectors using Cartesian components. II. Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis	Solved Problem1.3	24-25
		Q.(1.93/1.94/1.95/1.67 1.68/1.104)	29/30

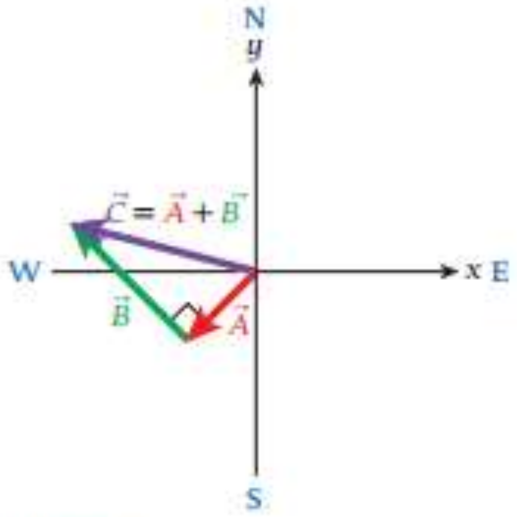


FIGURE 1.28 Hike with a 90° turn.

SOLVED PROBLEM 1.3 Hiking

PROBLEM

You are hiking in the Florida Everglades heading southwest from your base camp, for 1.72 km. You reach a river that is too deep to cross; so you make a 90° right turn and hike another 3.12 km to a bridge. How far away are you from your base camp?

SOLUTION

THINK If you are hiking, you are moving in a two-dimensional plane: the surface of Earth (because the Everglades are flat). Thus, we can use two-dimensional vectors to characterize the various segments of the hike. Making one straight-line hike, then performing a turn, followed by another straight-line hike amounts to a problem of vector addition that is asking for the length of the resultant vector.

SKETCH Figure 1.28 presents a coordinate system in which the y-axis points north and the x-axis points east, as is conventional. The first portion of the hike, in the southwestern direction, is indicated by the vector $\vec{A}$, and the second portion by the vector $\vec{B}$. The figure also shows the resultant vector, $\vec{C} = \vec{A} + \vec{B}$, for which we want to determine the length.

RESEARCH If you have drawn the sketch with sufficient accuracy, making the lengths of the vectors in your drawing to be proportional to the lengths of the segments of the hike (as was done in Figure 1.28), then you can measure the length of the vector $\vec{C}$ to determine the distance from your base camp at the end of the second segment of the hike. However, the given distances are specified to three significant digits, so the answer should also have three significant digits. Thus, we cannot rely on the graphical method but must use the component method of vector addition.

In order to calculate the components of the vectors, we need to know their angles relative to the positive x-axis. For the vector $\vec{A}$, which points southwest, this angle is $\theta_A = 225°$, as shown in Figure 1.29. The vector $\vec{B}$ has an angle of 90° relative to $\vec{A}$, and thus $\theta_B = 135°$ relative to the positive x-axis. To make this point clearer, the starting point of $\vec{B}$ has been moved to the origin of the coordinate system in Figure 1.29. (Remember: We can move vectors around at will. As long as we leave the direction and length of a vector the same, the vector remains unchanged.)

Now we have everything in place to start our calculation. We have the lengths and directions of both vectors, allowing us to calculate their Cartesian components. Then, we will add their components to calculate the components of the vector $\vec{C}$, from which we can calculate the length of this vector.

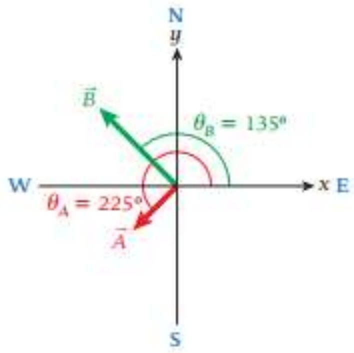


FIGURE 1.29 Angles of the two hike segments.

SIMPLIFY The components of the vector $\vec{C}$ are:

$$C_x = A_x + B_x = A\cos\theta_A + B\cos\theta_B$$
$$C_y = A_y + B_y = A\sin\theta_A + B\sin\theta_B.$$

Thus, the length of the vector $\vec{C}$ is (compare with equation 1.20)

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{(A\cos\theta_A + B\cos\theta_B)^2 + (A\sin\theta_A + B\sin\theta_B)^2}.$$

CALCULATE Now all that is left is to put in the numbers to obtain the vector length:

$$C = \sqrt{((1.72\text{ km})\cos 225° + (3.12\text{ km})\cos 135°)^2 + ((1.72\text{ km})\sin 225° + (3.12\text{ km})\sin 135°)^2}$$
$$= \sqrt{\left(1.72\times(-\sqrt{1/2}) + 3.12\times(-\sqrt{1/2})\right)^2 + \left((1.72\times(-\sqrt{1/2}) + 3.12\times\sqrt{1/2}\right)^2}\text{ km.}$$

Entering these numbers into a calculator, we obtain:

$$C = 3.562695609\text{ km.}$$

ROUND Because the initial distances were given to three significant figures, our final answer should also have (at most) the same precision. Rounding to three significant figures yields our final answer:

$$C = 3.56\text{ km.}$$

•**1.68** Use the components of the vectors from Problem 1.67 to find

a) the sum $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ in terms of its components

b) the magnitude and direction of the sum $\vec{A}-\vec{B}+\vec{D}$

RESEARCH:

(a) The resultant vector is $\vec{V}=\vec{A}+\vec{B}+\vec{C}+\vec{D}$.

(b) The magnitude of a vector is $\left|\vec{V}\right|=\sqrt{(V_x)^2+(V_y)^2}$. The direction of the vector $\vec{V}$ is

$\theta_V=\tan^{-1}\left(V_y/V_x\right)$.

SIMPLIFY:

(a) $\vec{A}+\vec{B}+\vec{C}+\vec{D}=(A_x+B_x+C_x+D_x)\hat{x}+(A_y+B_y+C_y+D_y)\hat{y}$

(b) $\left|\vec{V}\right|=\left|\vec{A}-\vec{B}+\vec{D}\right|=\sqrt{(A_x-B_x+D_x)^2+(A_y-B_y+D_y)^2}$

$$\theta_V=\tan^{-1}\left(\frac{A_y-B_y+D_y}{A_x-B_x+D_x}\right)$$

CALCULATE:

(a) $\vec{A}+\vec{B}+\vec{C}+\vec{D}=(65.0-56.7-15.4+80.2)\hat{x}+(37.5+19.5-19.7-40.9)\hat{y}$

$=73.1\hat{x}-3.6\hat{y}$

(b) $\left|\vec{A}-\vec{B}+\vec{D}\right|=\sqrt{(65.0-(-56.7)+80.2)^2+(37.5-19.7-40.9)^2}=203.217$

$$\theta_V=\tan^{-1}\left[\frac{37.5-19.7.-40.9}{65.0-(-56.7)+80.2}\right]=-6.5270^\circ$$

1.104 Find the magnitude and direction of $-\vec{A} + \vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

RESEARCH: $\vec{C} = (C_x, C_y)$, $C_i = nA_i + mB_i$ with $n = -1$ and $m = +1$, $\left|\vec{C}\right| = \sqrt{C_x^2 + C_y^2}$, $\tan\theta_C = \dfrac{C_y}{C_x}$.

SIMPLIFY: $C_x = -A_x + B_x$, $C_y = -A_y + B_y$, $\theta_C = \tan^{-1}\left(\dfrac{C_y}{C_x}\right)$

CALCULATE: $C_x = -23.0 + 90.0 = 67.0$, $C_y = -59.0 + (-150) = -209.0$,

$\left|\vec{C}\right| = \sqrt{(67.0)^2 + (-209.0)^2} = 219.477$, and $\theta_C = \tan^{-1}\left(\dfrac{-209.0}{67.0}\right) = -72.225°$.

ROUND: $\vec{C} = 219$ at $-72.2°$ or $288°$

17	I. Given a particle's position vector as a function of time, Determine its instantaneous velocity vector. II. Calculate the components of a velocity vector by the time derivative of the position vector.	Example2.1	38
		Q [2.13/2.14/2.15/2.16 76]	60/61 63

EXAMPLE 2.1 Time Dependence of Velocity

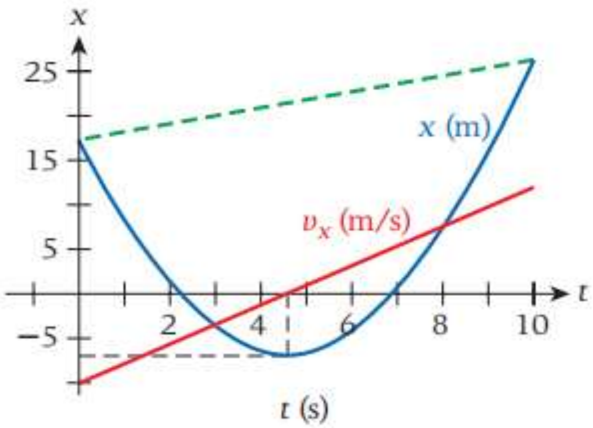


FIGURE 2.7 Graph of the position x and velocity v_x as a function of the time t. The slope of the dashed line represents the average velocity for the time interval from 0 to 10 s.

PROBLEM

During the time interval from 0.0 to 10.0 s, the position vector of a car on a road is given by $x(t) = a + bt + ct^2$, with $a = 17.2$ m, $b = -10.1$ m/s, and $c = 1.10$ m/s^2. What is the car's velocity as a function of time? What is the car's average velocity during this interval?

SOLUTION

According to the definition of velocity in equation 2.6, we simply take the time derivative of the position vector function to arrive at our solution:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(a + bt + ct^2) = b + 2ct = -10.1 \text{ m/s} + 2 \cdot (1.10 \text{ m/s}^2)t.$$

It is instructive to graph this solution. In Figure 2.7, the position as a function of time is shown in blue, and the velocity as a function of time is shown in red. Initially, the velocity has a value of -10.1 m/s, and at $t = 10$ s, the velocity has a value of $+11.9$ m/s.

Note that the velocity is initially negative, is zero at 4.59 s (indicated by the vertical dashed line in Figure 2.7), and then is positive after 4.59 s. At $t = 4.59$ s, the position graph $x(t)$ shows an extremum (a minimum in this case), just as expected from calculus, since

$$\frac{dx}{dt} = b + 2ct_0 = 0 \Rightarrow t_0 = -\frac{b}{2c} = -\frac{-10.1 \text{ m/s}}{2.20 \text{ m/s}^2} = 4.59 \text{ s}.$$

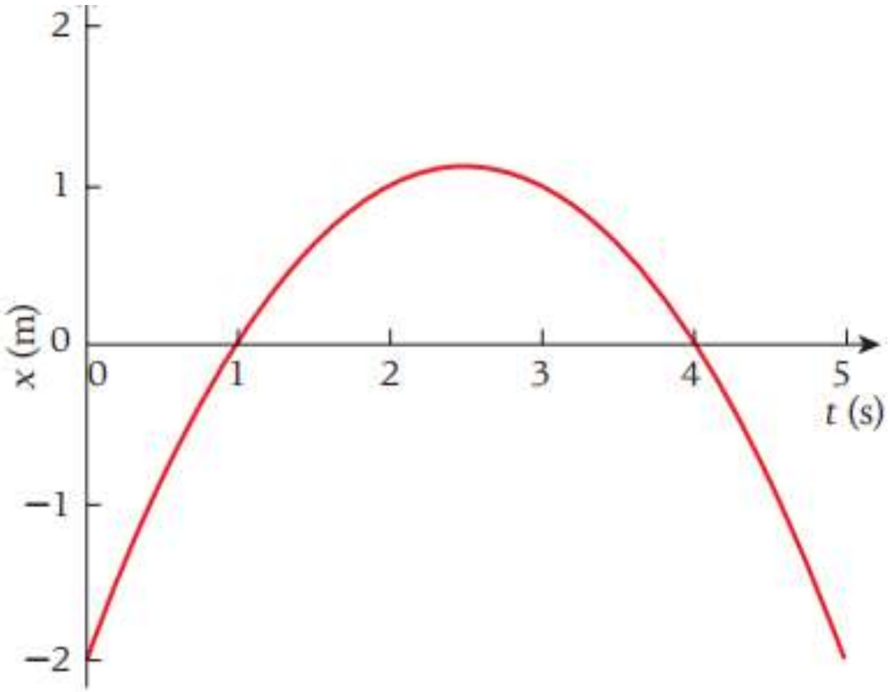
From the definition of average velocity, we know that to determine the average velocity during a time interval, we need to subtract the position at the beginning of the interval from the position at the end of the interval. By inserting $t = 0$ and $t = 10$ s into the equation for the position vector as a function of time, we obtain $x(t = 0) = 17.2$ m and $x(t = 10 \text{ s}) = 26.2$ m. Therefore,

$$\Delta x = x(t = 10) - x(t = 0) = 26.2 \text{ m} - 17.2 \text{ m} = 9.0 \text{ m}.$$

We then obtain for the average velocity over this time interval:

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{9.0 \text{ m}}{10 \text{ s}} = 0.90 \text{ m/s}.$$

'he slope of the green dashed line in Figure 2.7 is the average velocity over this time interval.



This figure describes the position of an object as a function of time. Refer to it to answer Questions 2.13-2.16.

2.13 Which one of the following statements is true at $t = 1$ s?

a) The x-component of the velocity of the object is zero.

b) The x-component of the acceleration of the object is zero.

The x-component of the velocity of the object is positive.

d) The x-component of the velocity of the object is negative.

2.14 Which one of the following statements is true at $t = 4$ s?

a) The x-component of the velocity of the object is zero.

b) The x-component of the acceleration of the object is zero.

c) The x-component of the velocity of the object is positive.

The x-component of the velocity of the object is negative.

2.15 Which one of the following statements is true at $t = 2.5$ s?

The x-component of the velocity of the object is zero.

b) The x-component of the acceleration of the object is zero.

c) The x-component of the velocity of the object is positive.

d) The x-component of the velocity of the object is negative.

2.16 Which one of the following statements is true at $t = 2.5$ s?

a) The x-component of the acceleration of the object is zero.

b) The x-component of the acceleration of the object is positive.

The x-component of the acceleration of the object is negative.

d) The acceleration of the object at that time can't be determined from the figure.

18	I. Describe the motion of an object under free fall. II. Interpret motion graphs for objects under free fall. III. Apply the constant-acceleration equations to free-fall motion	Solved Problem 2.5 Example/2.5	52/53/54
		Q [2.66/2.67/2.68/ 2.69/2.70/2.72/2.73]	63

EXAMPLE 2.5 Reaction Time

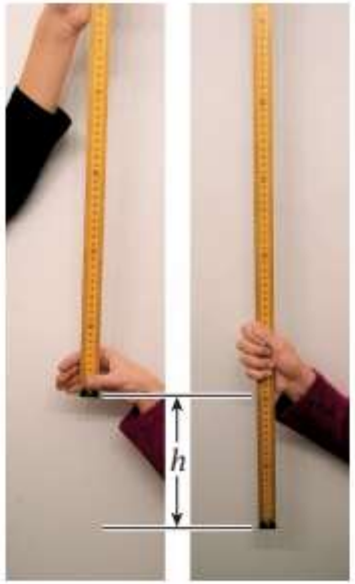


FIGURE 2.25 Simple experiment to measure reaction time.

It takes time for a person to react to any external stimulus. For example, at the beginning of a 100-m dash in a track-and-field meet, a gun is fired by the starter. A slight time delay occurs before the runners come out of the starting blocks, due to their nonzero reaction time. In fact, it counts as a false start if a runner leaves the blocks less than 0.1 s after the gun is fired. Any shorter time indicates that the runner has "jumped the gun."

There is a simple test, shown in Figure 2.25, that you can perform to determine your reaction time. Your partner holds a meter stick, and you get ready to catch it when your partner releases it, as shown in the left frame of the figure. From the distance h that the meter stick falls after it is released until you grab it (shown in the right frame), you can determine your reaction time.

PROBLEM

If the meter stick falls 0.20 m before you catch it, what is your reaction time?

SOLUTION

This situation is a free-fall scenario. For these problems, the solution invariably comes from one of equations 2.25. The problem we want to solve here involves the time as an unknown.

We are given the displacement, $h = y_0 - y$. We also know that the initial velocity of the meter stick is zero because it is released from rest. We can use kinematical equation 2.25(i): $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$. With $h = y_0 - y$ and $v_0 = 0$, this equation becomes

$$
\begin{aligned}
&y = y_0 - \tfrac{1}{2} g t^2 \\
&\Rightarrow h = \tfrac{1}{2} g t^2 \\
&\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 0.20 \text{ m}}{9.81 \text{ m/s}^2}} = 0.20 \text{ s.}
\end{aligned}
$$

Your reaction time was 0.20 s. This reaction time is typical. For comparison, when Usain Bolt established a world record of 9.69 s for the 100-m dash in August 2008, his reaction time was measured to be 0.165 s. (One year later Bolt ran the 100 m in 9.58 s, which is the current world record.)

SOLVED PROBLEM 2.5 | Melon Drop

Suppose you decide to drop a melon from rest from the first observation platform of the Eiffel Tower. The initial height h from which the melon is released is 58.3 m above the head of your French friend Khaled, who is standing on the ground right below you. At the same instant you release the melon, Khaled shoots an arrow straight up with an initial velocity of 25.1 m/s. (Of course, Khaled makes sure the area around him is cleared and gets out of the way quickly after he shoots his arrow.)

PROBLEM

(a) How long after you drop the melon will the arrow hit it? (b) At what height above Khaled's head does this collision occur?

SOLUTION

THINK At first sight, this problem looks complicated. We will solve it using the full set of steps and then examine a shortcut we could have taken. Obviously, the dropped melon is in free fall. However, because the arrow is shot straight up, the arrow is also in free fall, only with an upward initial velocity.

SKETCH We set up our coordinate system with the y-axis pointing vertically up, as is conventional, and we locate the origin of the coordinate system at Khaled's head (Figure 2.26). Thus, the arrow is released from an initial position $y = 0$, and the melon from $y = h$.

RESEARCH We use the subscripts m for "melon" and a for "arrow." We start with the general free-fall equation, $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$, and use the initial conditions given for the melon ($v_{y0} = 0$, $y_0 = h = 58.3$ m) and for the arrow ($v_{y0} \equiv v_{a0} = 25.1$ m/s, $y_0 = 0$) to set up the two equations of free-fall motion:

$$y_m(t) = h - \tfrac{1}{2}gt^2$$

$$y_a(t) = v_{a0}t - \tfrac{1}{2}gt^2.$$

The key insight is that at t_c, the moment when the melon and arrow collide, their coordinates are identical:

$$y_a(t_c) = y_m(t_c).$$

SIMPLIFY Inserting t_c into the two equations of motion and setting them equal results in

Concept Check 2.9

If the reaction time of person B determined with the meter stick method is twice as long as that of person A, then the displacement h_B measured for person B in terms of the displacement h_A for person A is

a) $h_B = 2h_A$.

b) $h_B = \frac{1}{2}h_A$.

c) $h_B = \sqrt{2}h_A$.

d) $h_B = 4h_A$.

e) $h_B = \sqrt{\frac{1}{2}}h_A$.

FIGURE 2.26 The melon drop (melon and person are not drawn to scale!).

$$h - \tfrac{1}{2}gt_c^2 = v_{a0}t_c - \tfrac{1}{2}gt_c^2 \Rightarrow$$
$$h = v_{a0}t_c \Rightarrow$$
$$t_c = \frac{h}{v_{a0}}.$$

We can now insert this value for the time of collision in either of the two free-fall equations and obtain the height above Khaled's head at which the collision occurs. We select the equation for the melon:

$$y_m(t_c) = h - \tfrac{1}{2}gt_c^2.$$

CALCULATE (a) All that is left to do is to insert the numbers given for the height of release of the melon and the initial velocity of the arrow, which results in

$$t_c = \frac{58.3 \text{ m}}{25.1 \text{ m/s}} = 2.32271 \text{ s}$$

for the time of impact.
(b) Using the number we obtained for the time, we find the position at which the collision occurs:

$$y_m(t_c) = 58.3 \text{ m} - \tfrac{1}{2}(9.81 \text{ m/s}^2)(2.32271 \text{ s})^2 = 31.8376 \text{ m}.$$

ROUND Since the initial values of the release height and the arrow velocity were given to three significant figures, we have to limit our final answers to three digits. Thus, the arrow will hit the melon after 2.32 s, and this will occur at a position 31.8 m above Khaled's head.

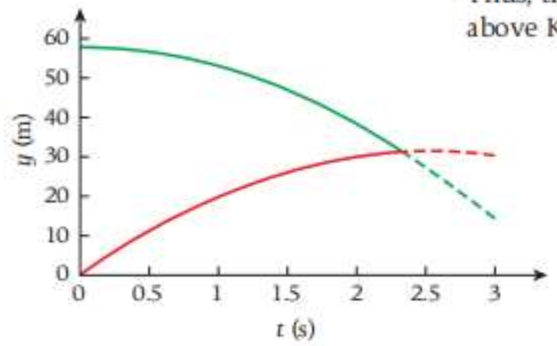


FIGURE 2.27 Position as a function of time for the arrow (red curve) and the melon (green curve).

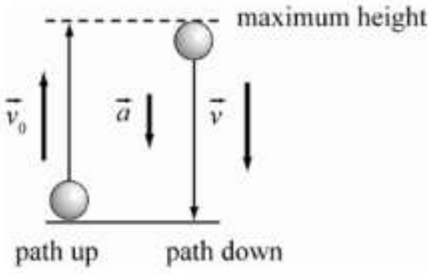
DOUBLE-CHECK Could we have obtained the answers in an easier way? Yes, if we had realized that both melon and arrow fall under the influence of the same gravitational acceleration, and thus their free-fall motion does not influence the distance between them. This means that the time it takes them to meet is simply the initial distance between them divided by their initial velocity difference. With this realization, we could have written $t = h/v_{a0}$ right away and been done. However, thinking in terms of relative motion in this way takes some practice, and we will return to it in more detail in the next chapter.

Figure 2.27 shows the complete graph of the positions of arrow and melon as functions of time. The dashed portions of both graphs indicate where the arrow and melon would have gone, had they not collided.

2.66 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?

THINK: I know that $v_0 = 26.4$ m/s and $a = -g = -9.81$ m/s^2. I want to find t_{total}. Note that once the ball gets back to the starting point, $v = -26.4$ m/s, or $v = -v_0$.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $t = \dfrac{v - v_0}{a} = \dfrac{-v_0 - v_0}{-g} = \dfrac{2v_0}{g}$

CALCULATE: $t = \dfrac{2(26.4 \text{ m/s})}{9.81 \text{ m/s}^2} = 5.38226$ s

ROUND: Since all the values given have three significant digits, $t = 5.38$ s.

2.67 A stone is thrown upward, from ground level, with an initial velocity of 10.0 m/s.

a) What is the velocity of the stone after 0.50 s?

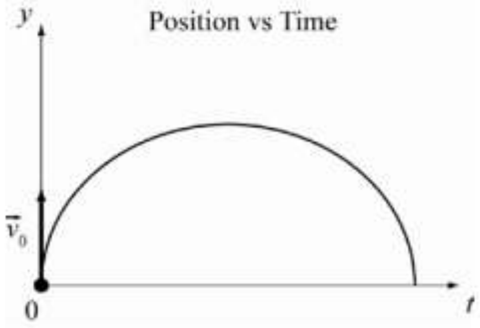
b) How high above ground level is the stone after 0.50 s?

THINK: I know that $v_0 = 10.0$ m/s, $a = -g = -9.81$ m/s^2, and $y_0 = 0$ m.

(a) I want to find the velocity v at $t = 0.50$ s.

(b) I want to find the height h of the stone at $t = 0.50$ s.

SKETCH:



RESEARCH:

(a) $v = v_0 + at$

(b) $\Delta y = v_0 t + \frac{1}{2}at^2$ and $\Delta y = h$

SIMPLIFY:

(a) $v = v_0 - gt$

(b) $h = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2$

CALCULATE:

(a) $v = 10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.50 \text{ s})$

$= 10.0 \text{ m/s} - 4.905\text{m/s}$

$= 5.095 \text{ m/s}$

(b) $h = (10.0 \text{ m/s})(0.50 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.50 \text{ s})^2$

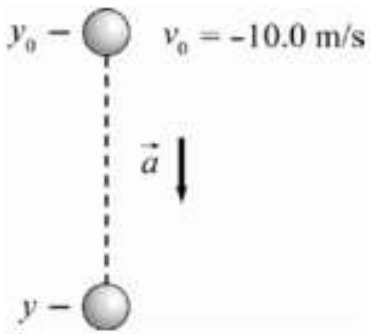
$= 5.0 \text{ m} - 1.226 \text{ m}$

$= 3.774 \text{ m}$

2.68 A stone is thrown downward with an initial velocity of 10.0 m/s. The acceleration of the stone is constant and has the value of the free-fall acceleration, 9.81 m/s^2. What is the velocity of the stone after 0.500 s?

THINK: I know that $v_0 = -10.0$ m/s, and $a = -g = -9.81$ m/s^2. I want to find v at $t = 0.500$ s.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $v = v_0 - gt$

CALCULATE: $v = -10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.500 \text{ s}) = -10.0 \text{ m/s} - 4.905 \text{ m/s} = -14.905 \text{ m/s}$

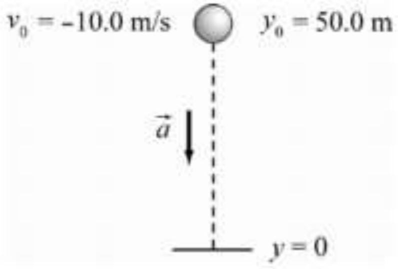
ROUND: Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = -14.9$ m/s.

DOUBLE-CHECK: A negative v indicates that the stone is (still) falling downward. This makes sense, since the stone was thrown downward.

2.69 A ball is thrown directly downward, with an initial speed of 10.0 m/s, from a height of 50.0 m. After what time interval does the ball strike the ground?

THINK: Take "downward" to be along the negative y-axis. I know that $v_0 = -10.0$ m/s, $\Delta y = -50.0$ m, and $a = -g = -9.81$ m/s^2. I want to find t, the time when the ball reaches the ground.

SKETCH:



RESEARCH: $\Delta y = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: $\frac{1}{2}at^2 + v_0 t - \Delta y = 0$. This is a quadratic equation. Solving for t:

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-\Delta y)}}{2\left(\frac{1}{2}a\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{-g}$$

CALCULATE: $t = \dfrac{-(-10.0 \text{ m/s}) \pm \sqrt{(-10.0 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-50.0 \text{ m})}}{-9.81 \text{ m/s}^2}$

$= -4.3709$ s, 2.3322 s

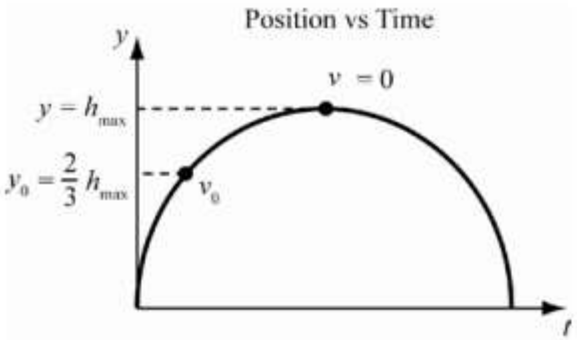
The time interval has to be positive, so $t = 2.3322$ s.

ROUND: All original quantities are precise to three significant digits, therefore $t = 2.33$ s.

2.70 An object is thrown vertically upward and has a speed of 20.0 m/s when it reaches two thirds of its maximum height above the launch point. Determine its maximum height.

THINK: I know that $v_0 = 20.0$ m/s, $y_0 = (2/3)h_{max}$, and $a = -g = -9.81$ m/s^2. I want to find h_{max}. Note that when $y = h_{max}$, the velocity is $v = 0$.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: $v^2 = v_0^2 - 2g\left(h_{max} - \frac{2}{3}h_{max}\right) \Rightarrow v^2 - v_0^2 = -2g\left(\frac{1}{3}h_{max}\right) \Rightarrow h_{max} = -\frac{3\left(v^2 - v_0^2\right)}{2g} \Rightarrow h_{max} = \frac{3v_0^2}{2g}$

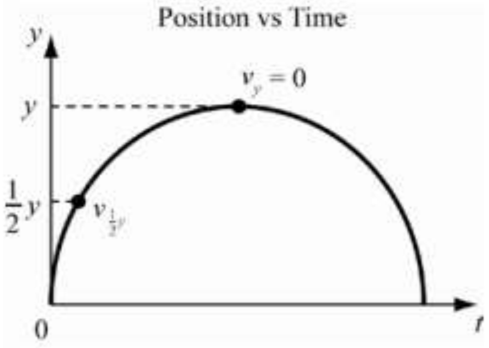
CALCULATE: $h_{max} = \frac{3(20.0 \text{ m/s})^2}{2\left(9.81 \text{ m/s}^2\right)} = 61.16$ m

ROUND: $h_{max} = 61.2$ m

2.71 What is the velocity at the midway point of a ball able to reach a height y when thrown upward with an initial velocity v_0?

THINK: I know the final height is y and the initial velocity is v_0. The velocity at this height is zero: $v_y = 0$. Also, $a_y = -g$. I want to know the velocity at half of the final height, $v_{\frac{1}{2}y}$. Assume $y_0 = 0$.

SKETCH:



RESEARCH: $v_y^2 = v_0^2 + 2a(y - y_0)$

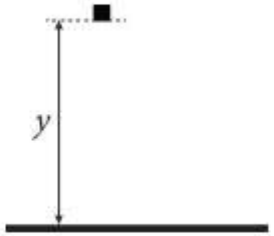
SIMPLIFY: The initial velocity, v_0, is $v_0 = \sqrt{v_y^2 - 2a(y - y_0)} = \sqrt{2gy}$. Then $v_{\frac{1}{2}y}$, in terms of the maximum height y, is

$$\left(v_{\frac{1}{2}y}\right)^2 = v_0^2 + 2a\left(\left(\frac{1}{2}y\right) - y_0\right) \Rightarrow v_{\frac{1}{2}y}^2 = \left(\sqrt{2gy}\right)^2 - 2g\left(\frac{1}{2}y\right) \Rightarrow v_{\frac{1}{2}y}^2 = 2gy - gy \Rightarrow v_{\frac{1}{2}y} = \sqrt{gy}$$

2.72 On August 2, 1971, Astronaut David Scott, while standing on the surface of the Moon, dropped a 1.3-kg hammer and a 0.030-kg falcon feather from a height of 1.6 m. Both objects hit the Moon's surface 1.4 s after being released. What is the acceleration due to gravity on the surface of the Moon?

THINK: The acceleration of an object due to gravity on the surface of the Moon is independent of the mass of the object.

SKETCH:



RESEARCH: We can use $y=\frac{1}{2}gt^2$, where y is the distance the objects fall, t is the time it takes for the objects to fall, and g is the acceleration of gravity on the Moon.

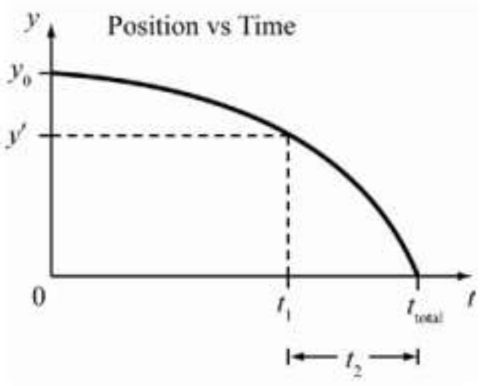
SIMPLIFY: We can solve our equation for g: $y=\frac{1}{2}gt^2 \Rightarrow g=\frac{2y}{t^2}$.

CALCULATE: $g=\frac{2y}{t^2}=\frac{2(1.6 \text{ m})}{(1.4 \text{ s})^2}=1.6327 \text{ m/s}^2$.

ROUND: The values given are all accurate to two significant digits, so the answer is given two by two significant digits: $g = 1.6 \text{ m/s}^2$.

•2.73 An object is thrown vertically and has an upward velocity of 25 m/s when it reaches one fourth of its maximum height above its launch point. What is the initial (launch) speed of the object?

SKETCH:



RESEARCH: We will use the formula $t=\sqrt{2h/g}$ from Example 2.5. If you look at the sketch, you see that $t_{\text{total}}=\sqrt{2h_{\text{total}}/g}=\sqrt{2y_0/g}$ and that $t_1=\sqrt{2h_1/g}=\sqrt{2(y_0-y')/g}$.

SIMPLIFY: Solving for the time difference gives:

$$t_2=t_{\text{total}}-t_1=\sqrt{2y_0/g}-\sqrt{2(y_0-y')/g}$$

CALCULATE: $t_2=\sqrt{2(63.17\text{ m})/(9.81\text{ m/s}^2)}-\sqrt{2(63.17\text{ m}-40.95\text{ m})/(9.81\text{ m/s}^2)}$

$=1.4603\text{ s}$

ROUND: We round to $t_2=1.46$ s., because g has three significant figures.

19	**I. Calculate the launch velocity, launch angel from given data for an instant during the flight.** **II. Calculate the components as well as net velocity of a projectile at any point of its trajectory.** **III. Calculate the particle's position, displacement, and velocity at a given instant during the flight given the launch velocity.**	**Q.[3.41/3.46/3.47/3.48]**	**87**

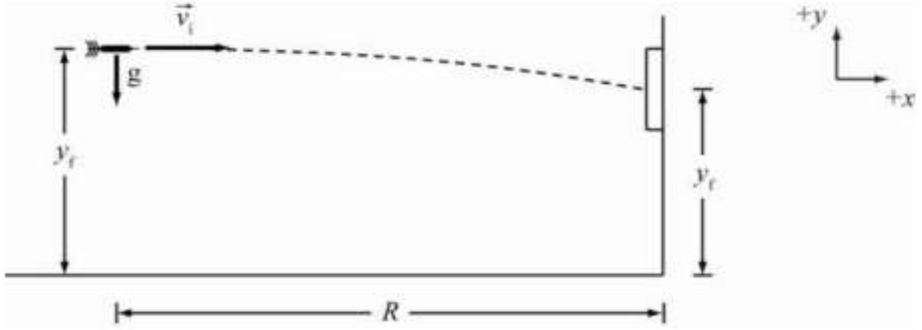
3.46 You are practicing throwing darts in your room. You st from the wall on which the board hangs. The dart leaves you a horizontal velocity at a point 2.00 m above the ground. Th the board at a point 1.65 m from the ground. Calculate:

a) the time of flight of the dart;

b) the initial speed of the dart;

c) the velocity of the dart when it hits the board.

SKETCH:



RESEARCH:

(a) $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$ and $v_{iy} = 0$ and $a = -g$.

(b) $v_{ix} = v_{fx}$ and $R = v_{ix}t$.

(c) $\overline{v_f} = v_{fx}\hat{x} + v_{fy}\hat{y}$; $v_{fy} = v_{iy} + at$; and $\left|\overline{v_f}\right| = \sqrt{v_{fx}^2 + v_{fy}^2}$.

SIMPLIFY:

(a) $y_f - y_i = 0 - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{-2(y_f - y_i)}{g}}$

(b) $v_i = v_{ix} = \frac{R}{t}$

(c) $v_{fy} = -gt \Rightarrow \overline{v_f} = v_i\hat{x} - gt\hat{y} \Rightarrow \left|\overline{v_f}\right| = \sqrt{v_i^2 + (-gt)^2}$

CALCULATE:

(a) $t = \sqrt{\frac{-2(1.65\text{ m} - 2.00\text{ m})}{9.81\text{ m/s}^2}} = 0.26712\text{ s}$

(b) $v_i = \frac{3.0\text{ m}}{0.26712\text{ s}} = 11.231\text{ m/s}$

(c) $\left|\overline{v_f}\right| = \sqrt{(11.231\text{ m/s})^2 + \left((-9.81\text{ m/s}^2)(0.26712\text{ s})\right)^2} = 11.532\text{ m/s}$

ROUND: Rounding to two significant figures, $t = 0.27\text{ s}$, $v_i = 11\text{ m/s}$ and $\left|\overline{v_f}\right| = 12\text{ m/s}$.

•**3.48** An object fired at an angle of 35.0° above the horizontal takes 1.50 s to travel the last 15.0 m of its vertical distance and the last 10.0 m of its horizontal distance. With what speed was the object launched? (Note: The problem does not specify that the initial and final elevation of the object are the same!)

THINK: Since the time of the last portion of the flight and the horizontal displacement during that time are given, the x component of the initial velocity can be determined, because the horizontal velocity component remains constant throughout the flight. The initial velocity can then be determined, because we also know the initial angle of $\theta = 35.0°$. The vertical displacement of the projectile during the last flight phase is also given. However, since we do not know the relative altitude of the beginning and end of the trajectory, the vertical displacement provides no useful information and is thus a distractor, which we can and should ignore.

SKETCH: A sketch is not needed in this case.

RESEARCH: $v_{ix} = v_{fx} = v_i \cos\theta$; $v_{ix} = \dfrac{d}{\Delta t}$

SIMPLIFY: $v_i \cos\theta = \dfrac{d}{\Delta t} \Rightarrow v_i = \dfrac{d}{\cos\theta \Delta t}$.

CALCULATE: $v_i = (10.0 \text{ m})/\left[\cos(35.0°)(1.50 \text{ s})\right] = 8.138497 \text{ m/s}$

ROUND: Rounding to three significant figures, $v_i = 8.14$ m/s.

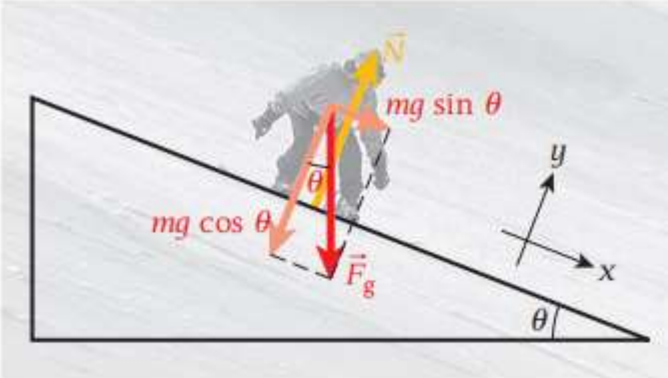
DOUBLE-CHECK: This speed is reasonable.

20	I. Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction. II. Distinguish between friction in a static situation and a kinetic situation. III. Relate the magnitude of static or dynamic frictional forces to the magnitude of the normal force through the coefficient of static or kinetic friction. IV. Describe an object in static equilibrium and dynamic equilibrium	Solved problem 4.1	104
		Example/4.6 Q.4.48 Q.4.81	110 123 125

OLVED PROBLEM 4.1 Snowboarding

PROBLEM

A snowboarder (mass 72.9 kg, height 1.79 m) glides down a slope with an angle of 22° with respect to the horizontal (Figure 4.15a). If we can neglect friction, what is his acceleration?



$$F_{g,x} = F_g \sin\theta = mg\sin\theta$$
$$F_{g,y} = -F_g \cos\theta = -mg\cos\theta.$$

$$F_{g,y} + N = 0 \Rightarrow$$
$$-mg\cos\theta + N = 0 \Rightarrow$$
$$N = mg\cos\theta.$$

$$F_{g,x} = mg\sin\theta = ma_x \Rightarrow$$
$$a_x = g\sin\theta.$$

$$\vec{a} = (g\sin\theta)\hat{x}.$$

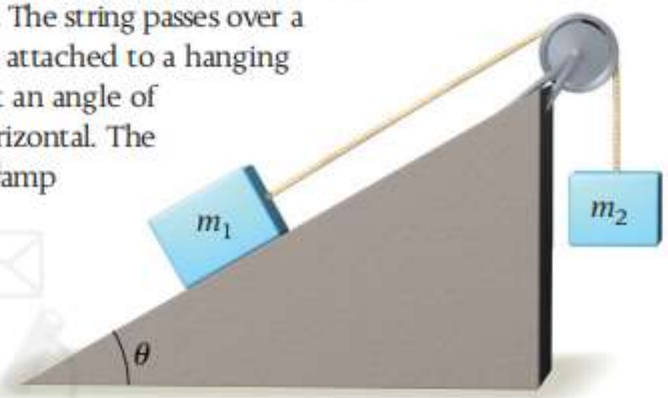
CALCULATE Putting in the given value for the angle leads to

$$a_x = (9.81 \text{ m/s}^2)(\sin 22°) = 3.67489 \text{ m/s}^2.$$

ROUND Because the angle of the slope was given to only two-digit accuracy, it makes no sense to give our result to a greater precision. The final answer is

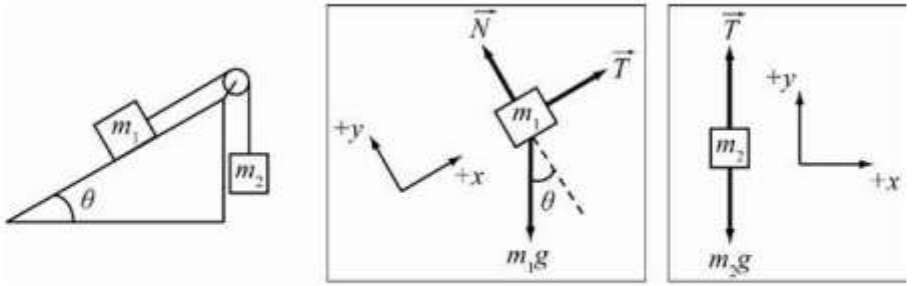
$$a_x = 3.7 \text{ m/s}^2.$$

•**4.48** A mass, $m_1 = 20.0$ kg, on a frictionless ramp is attached to a light string. The string passes over a frictionless pulley and is attached to a hanging mass, m_2. The ramp is at an angle of $\theta = 30.0°$ above the horizontal. The mass m_1 moves up the ramp uniformly (at constant speed). Find the value of m_2.



THINK: The mass, $m_1 = 20.0$ kg. The ramp angle is $\theta = 30.0°$. The acceleration of the masses is $a_1 = a_2 = 0$.

SKETCH:



RESEARCH: For m_2, $F_{\text{net},y} = T - m_2 g = 0$. Determine T from the sum of forces on m_1.

SIMPLIFY: For m_1, $F_{\text{net},x} = T - F_{g1,x} = 0$, $T - m_1 g \sin\theta = 0$, and $T = m_1 g \sin\theta$. Then for m_2, $T = m_2 g \Rightarrow m_2 = T/g = m_1 \sin\theta$.

CALCULATE: $m_2 = (20.0 \text{ kg})\sin(30.0°) = 10.0 \text{ kg}$

ROUND: $m_2 = 10.0$ kg.

DOUBLE-CHECK: m_2 is the same order of magnitude as m_1.

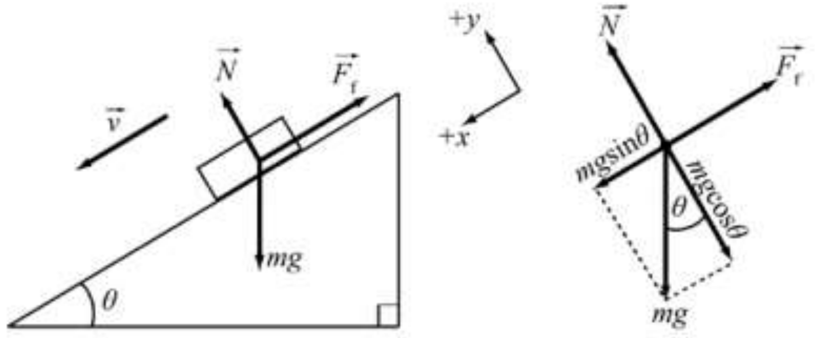
•**4.81** A block of mass 5.00 kg is sliding at a constant velocity down an inclined plane that makes an angle of 37.0° with respect to the horizontal.

a) What is the friction force?

b) What is the coefficient of kinetic friction?

THINK: A block of mass, $m = 5.00$ kg is sliding down an inclined plane of angle $\theta = 37.0°$ at a constant velocity ($a = 0$). Determine the frictional force and the coefficient of kinetic friction.

SKETCH:



RESEARCH: There is no acceleration in any direction, $a_x = a_y = 0$. The force of friction is given by $F_f = \mu_k N$. Using Newton's second law: $\sum F_y = ma_y = 0$ and $a_y = 0$, so $N - mg\cos\theta = 0$ $\Rightarrow N = mg\cos\theta$. Also, $\sum F_x = ma_x = 0$ and $a_x = 0$, so $mg\sin\theta - F_f = 0 \Rightarrow F_f = mg\sin\theta$.

SIMPLIFY: $\mu_k N = mg\sin\theta \Rightarrow \mu_k mg\cos\theta = mg\sin\theta \Rightarrow \mu_k = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

CALCULATE:

(a) $F_f = (5.00 \text{ kg})(9.81 \text{ m/s}^2)\sin 37.0° = 29.519 \text{ N}$

(b) $\mu_k = \tan(37.0°) = 0.75355$

ROUND: Rounding to three significant figures,

(a) $F_f = 29.5.$ N and

(b) $\mu_k = 0.754$.