

شكراً لتحميلك هذا الملف من موقع المناهج الإماراتية



شرح الدرس الخامس Equations Trigonometric Solving من الوحدة الحادية عشرة

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← رياضيات ← الفصل الثاني ← الملف

تاريخ نشر الملف على موقع المناهج: 07:45:36 2024-01-13 | اسم المدرس: محمد زياد

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



روابط مواد الصف الحادي عشر المتقدم على تلغرام

[الرياضيات](#)

[اللغة الانجليزية](#)

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المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثاني

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المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثاني

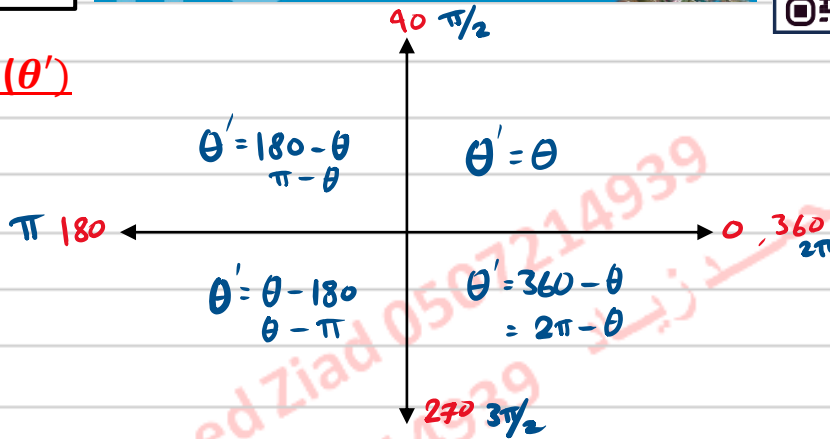
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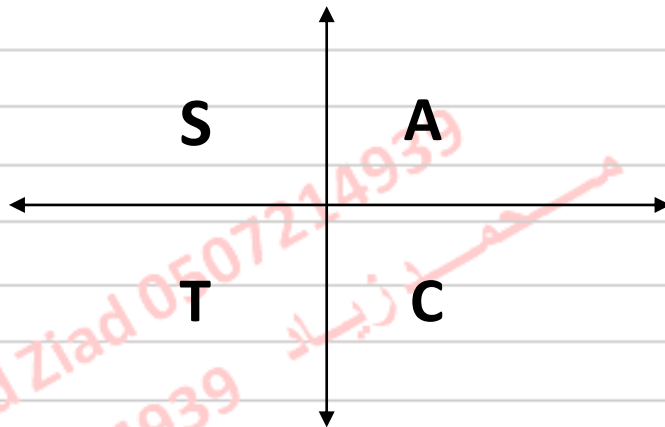
5



Reference angle (θ')



Trigonometric functions signs:



Quadrantal angles :

	0	90	180	270	360
Sin	0	1	0	-1	0
Cos	1	0	-1	0	1
tan	0	Un	0	Un	0

To find general solutions add $360k$ (or) $2\pi k$, $k = 0, \pm 1, \pm 2, \dots$

Ex: Solve the following equation :

$$1) 2\cos\theta + 1 = 0$$

$$\frac{2\cos\theta}{2} = \frac{-1}{2}$$

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta' = \cos^{-1}\left(+\frac{1}{2}\right) = 60^\circ$$

Q₂ & Q₃ cos is -ve

$$Q_2: \theta' = 180 - \theta \Rightarrow 60 = 180 - \theta \Rightarrow \theta_1 = 180 - 60 = 120^\circ$$

$$Q_3: \theta' = \theta - 180 \Rightarrow 60 = \theta - 180 \Rightarrow \theta_2 = 60 + 180 = 240^\circ$$

General Solutions: $120^\circ + 360k$, $240 + 360k$

To find θ' use positive values only

$$2) \tan^2\theta + 5 = 6$$

$$\sqrt{\tan^2\theta} = \sqrt{1} \Rightarrow \tan\theta = \pm 1$$

eq① $\tan\theta = 1$ Q₁ & Q₃

$$\theta' = \tan^{-1}(1) = 45^\circ$$

$$Q_1: \theta' = \theta \Rightarrow \theta_1 = 45^\circ$$

$$Q_3: \theta' = \theta - 180 \Rightarrow \theta_2 = 225^\circ$$
$$45 = \theta - 180$$

eq②

$$\tan\theta = -1$$
 Q₂ & Q₄

$$\theta' = \tan^{-1}(1) = 45^\circ$$

$$Q_2: \theta' = 180 - \theta$$

$$45 = 180 - \theta \Rightarrow \theta_3 = 135^\circ$$

$$Q_4: \theta' = 360 - \theta$$

$$45 = 360 - \theta \Rightarrow \theta_4 = 315^\circ$$

Solutions: $45 + 360k$, $225 + 360k$, $135 + 360k$, $315 + 360k$

3) $\csc^2 \theta - 2 \csc \theta = 0$ in Degree

$$\csc \theta (\csc \theta - 2) = 0$$

$$\csc \theta = 0$$

↓

$$\frac{1}{\sin \theta} = \frac{0}{1}$$

$$\sin \theta = \frac{1}{0}$$

No solution

$$\csc \theta - 2 = 0$$

$$\csc \theta = 2$$

$$\frac{1}{\sin \theta} = \frac{2}{1}$$

$$\sin \theta = \frac{1}{2} \quad Q_1 \text{ \& } Q_2$$

$$\theta' = \sin^{-1}(\frac{1}{2}) = 30^\circ$$

$$Q_1: \theta' = \theta \Rightarrow \theta_1 = 30^\circ$$

$$Q_2: \theta' = 180 - \theta$$

$$30 = 180 - \theta \Rightarrow \theta_2 = 150^\circ$$

Solutions: $30 + 360k, 150 + 360k$

4) $2 \sin^2 \theta - \cos \theta - 1 = 0$ in Radian

We have to write the equation in term of cos

$$2(1 - \cos^2 \theta) - \cos \theta - 1 = 0$$

$$2 - 2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$[1 - 2 \cos^2 \theta - \cos \theta = 0] \times -1$$

$$2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

$$x + 1 = 0$$

$$x = -1$$

$$\cos \theta = \frac{1}{2} \quad Q_1 \text{ \& } Q_4 \quad \text{or}$$

$$\theta' = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$$

$$Q_1: \theta' = \theta \Rightarrow \theta_1 = \frac{\pi}{3}$$

$$\cos \theta = -1$$

$$\theta_3 = \pi$$

$$Q_4: \theta' = 2\pi - \theta$$

$$\frac{\pi}{3} = 2\pi - \theta \Rightarrow \theta_2 = \frac{5\pi}{3}$$

Solutions:

$$\frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k, \pi + 2\pi k$$

5) $\cos(2\theta)\sin(\theta) = 1$ in Radian

$$(1 - 2\sin^2\theta) \cdot \sin\theta = 1$$

$$\sin\theta - 2\sin^3\theta = 1$$

$$-2\sin^3\theta + \sin\theta - 1 = 0$$

$$-2x^3 + x - 1 = 0 \quad \text{mode 5 4}$$

$$\Rightarrow x = -1 \rightarrow \text{direct from calc}$$

$$\Rightarrow \sin\theta = -1$$

$$\Rightarrow \theta = \frac{3\pi}{2}$$

General Solution : $\frac{3\pi}{2} + 2\pi k$

6) $2\cos(\theta)\sin(\theta) - \sqrt{3}\sin(\theta) = 0$ in Radian

$$\sin\theta(2\cos\theta - \sqrt{3}) = 0$$

$$\sin\theta = 0$$

$$\downarrow$$

$$\theta_1 = 0$$

$$\theta_2 = \pi$$

$$2\cos\theta - \sqrt{3} = 0$$

$$\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \quad Q_1 \text{ \& } Q_4$$

$$\theta' = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$Q_1: \theta' = \theta \Rightarrow \theta_3 = \frac{\pi}{6}$$

$$Q_4: \theta' = 2\pi - \theta$$

$$\frac{\pi}{6} = 2\pi - \theta \Rightarrow \theta_4 = \frac{11\pi}{6}$$

General solutions:

$$0 + 2\pi k, \pi + 2\pi k, \frac{\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$$

(or) $\rightarrow k\pi$

$$7) \cos(2\theta) + 4\cos(\theta) = -3 \quad 0 \leq \theta \leq 360^\circ \rightarrow$$

↓

$$2\cos^2\theta - 1 + 4\cos\theta = -3$$

$$2\cos^2\theta - 1 + 4\cos\theta + 3 = 0$$

$$2\cos^2\theta + 4\cos\theta + 2 = 0$$

$$\cos^2\theta + 2\cos\theta + 1 = 0$$

$$(\cos\theta + 1)(\cos\theta + 1) = 0$$

$$\downarrow$$

$$\cos\theta + 1 = 0$$

$$\cos\theta = -1$$

$$\Rightarrow \theta = 180^\circ$$

interval is given
no need to
write general
solution

$$8) 4\sin^2\theta - 5 = -3, \quad 0 \leq \theta \leq 360^\circ$$

$$\frac{4\sin^2\theta}{4} = \frac{2}{4}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow \sin\theta = \pm \frac{\sqrt{2}}{2}$$

eq①

eq②

$$\sin\theta = \frac{\sqrt{2}}{2} \quad Q_1 \& Q_2$$

$$\theta' = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$Q_1: \theta' = \theta \Rightarrow \theta = 45^\circ$$

$$Q_2: \theta' = 180 - \theta$$

$$45 = 180 - \theta$$

$$\theta_2 = 135^\circ$$

$$\sin\theta = -\frac{\sqrt{2}}{2} \quad Q_3 \& Q_4$$

$$\theta' = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$Q_3: \theta' = \theta - 180$$

$$45 = \theta - 180$$

$$\theta_3 = 225^\circ$$

$$Q_4: \theta' = 360 - \theta$$

$$45 = 360 - \theta$$

$$\theta_4 = 315^\circ$$

$$9) \sin\theta + \cos\theta = 1, \quad 0 \leq \theta \leq 360^\circ$$

$$(\sin\theta + \cos\theta)^2 = (1)^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1$$

$$(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = 1$$

$$\cancel{1} + 2\sin\theta\cos\theta = \cancel{1}$$

$$2\sin\theta \cdot \cos\theta = 0$$

$$\frac{2\sin\theta}{2} = \frac{0}{2}$$

$$\sin\theta = 0$$

$$\theta = 0, 180, 360$$

$$\cos\theta = 0$$

$$\theta = 90, 270$$

When Squaring both sides \Rightarrow Check is compulsory

Check

$$\sin\theta + \cos\theta = 1$$

$$\theta = 0 \quad \sin 0 + \cos 0 = 1 \quad \checkmark$$

$$\theta = 90 \quad \sin 90 + \cos 90 = 1 \quad \checkmark$$

$$\theta = 180 \quad \sin 180 + \cos 180 = -1 \quad \times$$

$$\theta = 270 \quad \sin 270 + \cos 270 = -1 \quad \times$$

$$\theta = 360 \quad \sin 360 + \cos 360 = 1 \quad \checkmark$$

180, 270 extraneous solutions

$$10) \cos\left(\frac{\theta}{2}\right) - \sin(\theta) = 0 \quad 0 \leq \theta \leq 360$$

$$\cos\left(\frac{\theta}{2}\right) - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$\cos\frac{\theta}{2} \cdot (1 - 2\sin\frac{\theta}{2}) = 0$$

$$\cos\frac{\theta}{2} = 0$$

$$2 \times \frac{\theta}{2} = 90 \times 2 \text{ \& } 2 \times \frac{\theta}{2} = 270 \times 2$$

$$\theta = 180 \in [0, 360], \theta = 540 \notin [0, 360]$$

$$1 - 2\sin\frac{\theta}{2} = 0$$

$$\frac{-2\sin\left(\frac{\theta}{2}\right)}{2} = \frac{-1}{2}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{2} \text{ in } Q_1 \text{ \& } Q_2$$

$$\theta' = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$Q_1: 2 \times \frac{\theta}{2} = 30 \times 2 \Rightarrow \theta = 60 \in [0, 360]$$

$$Q_2: 30 \overset{\theta' = 180 - \theta}{=} 180 - \frac{\theta}{2}$$

$$2 \times \frac{\theta}{2} = 150 \times 2$$

$$\theta = 300 \in [0, 360]$$

Solutions : $180^\circ, 60^\circ, 300^\circ$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 4\theta = 2 \sin 2\theta \cos 2\theta$$

$$\sin 8\theta = 2 \sin 4\theta \cos 4\theta$$

$$\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$11) \sin(2\theta) = \frac{1}{2}, \quad 0 \leq \theta \leq 360$$

↓
sin is +ve in Q₁ & Q₂

$$\theta' = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$Q_1: \frac{2\theta}{2} = \frac{30^\circ}{2}, \quad \frac{2\theta}{2} = \frac{30 + 360}{2}$$

$$\theta = 15^\circ$$

$$\theta = 195^\circ$$

$$Q_2: 2\theta = 180 - 30$$

$$\frac{2\theta}{2} = \frac{150}{2}$$

$$\theta = 75^\circ$$

$$\frac{2\theta}{2} = \frac{150 + 360}{2}$$

$$\theta = 255^\circ$$

Solutions: $15^\circ, 75^\circ, 195^\circ, 255^\circ$