شكراً لتحميلك هذا الملف من موقع المناهج الإماراتية

شرح الدرس الأول rational dividing and Multiplying ريفيل السابعة الوحدة من expressions

تاريخ نشر الملف على موقع المناهج: 2023-10-07 05:02:06 | اسم المدرس: محمد زياد

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم

روابط مواد الصف الحادي عشر المتقدم على تلغرام

التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول

1	شرح الدرس الخامس Using logarithmic and exponential functions من الوحدة السادسة ريفيل
2	شرح الدرس الرابع Natural logarithms من الوحدة السادسة ريفيل
3	شرح الدرس الثالث Common logarithms من الوحدة السادسة ريفيل

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول	
4	شرح الدرس الثاني logarithms of Properties من الوحدة السادسة ريفيل
5	شرح الدرس الأول functions logarithmic and Logarithms من الوحدة السادسة ريفيل

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Lesson 7-1
Multiplying and Dividing Rational Expressions

SCAN ME

Math Garden
Telegram page

Factorising polynomials: (IMPORTANT)

1) Common factor

* $6x^2 - 4x^5 = 2x^2(3 - 2x^3)$

repeated variable

* $10x^6 - 15x^2 + 30x^3 = 5x^2(2x^4 - 3 + 6x)$

* $8y^3k^2 + 12yk^7 = 4yk^2(2y^2 + 3k^5)$

* $16y^3 - 8y^2 = 8y^2(2y - 1)$

2) Difference of two squares

$a^2 - b^2 = (a - b)(a + b)$ Rule

* $x^2 - 16 = (x - 4)(x + 4)$
  $(x)^2$ $(4)^2$

* $y^2 - 100 = (y - 10)(y + 10)$
  $(y)^2$ $(10)^2$

* $1 - 9x^2 = (1 - 3x)(1 + 3x)$
  $(1)^2$ $(3x)^2$

* $9x^2 + 16$ sum of 2 squares prime (not factorable)

Perfect squares

1	x^2
4	
9	x^4
16	
25	x^6
36	
49	
64	
81	
100	

3) Sum & Difference of two cubes

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$ Rules

Perfect cubes	
1	x^3
8	
27	x^6
64	x^9
125	

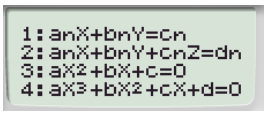
* $x^3 - 64 = (x-4)(x^2+4x+16)$
  $(x)^3$ $(4)^3$ — a, b — x^2, ab, b^2 — $(x)^2$, $(x)(4)$, $(4)^2$

* $125 + 8x^3 = (5+2x)(25-10x+4x^2)$
  $(5)^3$ $(2x)^3$ — $(5)^2$, $5(2x)$, $(2x)^2$

* $2x^4 + 54x = 2x(x^3+27) = 2x(x+3)(x^2-3x+9)$
  Common factor — sum of 2 cubes — $(x)^3$ $(3)^3$

4) Quadratic trinomial ($ax^2 + bx + c$)

* $x^2 - 2x - 8 = (x-4)(x+2)$
  $x_1 = 4$
  $x_2 = -2$



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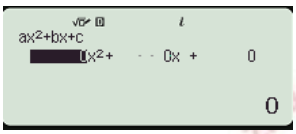

* $x^2 - 3x + 2 = (x-2)(x-1)$

* $x^2 - 6x + 9 = (x-3)(x-3)$ $x = 3$

* $2x^2 + 3x - 5 = (x-1)(2x+5)$
  $x = 1$ $x = -\frac{5}{2}$

* $4x^2 - 7x + 3 = (x-1)(4x-3)$
  $x = 1$ $x = 3/4$

Eg16 : Factorize the following polynomials

Expression	Factorization
$4x^2 - 25$ $(2x)^2 - (5)^2$	$(2x-5)(2x+5)$
$x^2 + 5x - 24$ $a=1 \quad b=5 \quad c=-24$	$(x-3)(x+8)$
$3x^3 - 6x$	$3x(x^2-2)$
$x^3 - 27$ $(x)^3 - (3)^3$	$(x-3)(x^2+3x+9)$
$2x^2 - 7x - 4$	$(x-4)(2x+1)$

Simplifying Rational expressions:

1) When no addition or subtraction included in the expressions

No factorize needed so we can simplify directly between numerator and denominator.

Ex: Simplify each expression:

a) $\dfrac{14x^2w^3}{35w^7}$ $\quad 14 \to 2 \cdot 7, \quad 35 \to 5 \cdot 7, \quad w^3 \to w^4$

$= \dfrac{2x^2}{5w^4}$

b) $\dfrac{2x^4y^2k^7}{12w^7x^3y^6k^7}$ $\quad 2 \to 1, \quad x^4 \to x, \quad 12 \to 6, \quad y^6 \to y^4$

$\dfrac{x}{6w^7y^4}$

2) When there is addition or subtraction included in the expressions

Factorization needed then simplify between numerator and denominator.

Note: To find where a given algebraic fraction is undefined, we need to find the zeroes of the denominator before the simplification

Ex: Simplify each expression, and state when the original expression is undefined. (Page 315)

1. $\frac{x(x-3)(x+6)}{x^2+x-12}$

$= \frac{x(x-3)(x+6)}{(x-3)(x+4)}$

$= \frac{x(x+6)}{x+4}$

To find where the fraction is undefined

$(x-3)(x+4) = 0$

$x-3=0 \Rightarrow x=3$

$x+4=0 \Rightarrow x=-4$

3. $\frac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$

$= \frac{(x-3)(x+3)(x+z)(x-z)}{4(x+z)(x-3)}$

$= \frac{(x+3)(x-z)}{4}$

Undefined at: $4(x+z)(x-3) = 0$

$4 \neq 0$

$x+z=0 \Rightarrow x=-z$

$x-3=0 \Rightarrow x=3$

5. $\dfrac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$

$= \dfrac{x^2(x+2)(x-4)}{6x(x-4)(x+5)}$

$= \dfrac{x(x+2)}{6(x+5)}$

undefined at:

$6x(x-4)(x+5)=0$

$x=0$, $x=4$, $x=-5$

7. $\dfrac{x^2-5x-14}{28+3x-x^2}$ → coff x^2 is negative ⇒ take -1 common factor

$= \dfrac{x^2-5x-14}{-1(-28-3x+x^2)}$

$= \dfrac{x^2-5x-14}{-1(x^2-3x-28)} = \dfrac{(x-7)(x+2)}{-1(x-7)(x+4)}$

$= \dfrac{x+2}{-(x+4)}$ (OR) $\dfrac{x+2}{-x-4}$

Undefined at : $-(x-7)(x+4)=0$

$x=7$, $x=-4$

9. $\dfrac{(x-4)(x^2+2x-48)}{(36-x^2)(x^2+4x-32)}$

$= \dfrac{(x-4)(x-6)(x+8)}{(6-x)(6+x)(x-4)(x+8)}$

$= \dfrac{-1}{(6+x)}$

undefined:

$x=6$, $x=-6$

$x=4$, $x=-8$

Note:

$\frac{x+6}{6+x} = 1$ $\quad \frac{x-6}{6-x} = -1$ $\quad \frac{x+6}{x-6}$ no simplify

$\frac{x-6}{x-6} = 1$ $\quad \frac{x-6}{-x+6} = -1$

Multiplication & Division on Rational expressions:

Key Concept • Multiplying Rational Expressions

Words: To multiply rational expressions, multiply the numerators and the denominators.

Symbols: For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ with $b \neq 0$ and $d \neq 0$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

Key Concept • Dividing Rational Expressions

Words: To divide rational expressions, multiply the dividend by the reciprocal of the divisor.

Symbols: For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ with $b \neq 0$, $c \neq 0$, and $d \neq 0$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

Note:

Simplifying is only between numerators and denominators.

Simplify each expression.

11. $\frac{3ac^3f^3}{8a^2bcf^4} \cdot \frac{12ab^2c}{18ab^3c^2f}$

$= \frac{36a^2c^4f^3b^2}{144a^3b^4c^3f^5f^2}$ (4, a, b^2)

$= \frac{c}{4ab^2f^2}$

12. $\frac{14xy^2z^3}{21w^4x^2z} \cdot \frac{7wxyz}{12w^2y^3z}$

$= \frac{98x^2y^3z^4w}{252w^6x^2z^2y^3}$ (18, w^5)

$= \frac{7z^2}{18w^5}$

13. $\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2}$

$= \frac{64a^2b^5}{35b^2c^3f^4} \cdot \frac{70abcf^2}{12a^4b^3c}$

$= \frac{4480a^3b^6cf^2}{420b^5c^4f^4a^4}$

$= \frac{32b}{3c^3f^2a}$

14. $\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2}$

$= \frac{9x^2yz}{5z^4} \cdot \frac{50xy^4z^2}{12x^4y^2}$

$= \frac{450x^3y^5z^3}{60z^4x^4y^2}$

$= \frac{15y^3}{2zx}$

Simplify each expression.

24. $\frac{y^2+8y+15}{y-6} \cdot \frac{y^2-9y+18}{y^2-9}$

$= \frac{(y+3)(y+5)}{y-6} \cdot \frac{(y-6)(y-3)}{(y-3)(y+3)}$

$= y+5$

26. $\frac{x^2+9x+20}{8x+16} \cdot \frac{4x^2+16x+16}{x^2-25}$

$8(x+2)$ ← $4(x^2+4x+4)$

$= \frac{(x+4)(x+5)}{8(x+2)} \cdot \frac{4(x+2)(x+2)}{(x-5)(x+5)}$

$= \frac{(x+4)(x+2)}{2(x-5)}$

25. $\dfrac{c^2-6c-16}{c^2-d^2} \div \dfrac{c^2-8c}{c+d}$

$= \dfrac{c^2-6c-16}{c^2-d^2} \cdot \dfrac{c+d}{c^2-8c}$

$= \dfrac{(c-8)(c+2)}{(c-d)(c+d)} \cdot \dfrac{c+d}{c(c-8)}$

$= \dfrac{c+2}{c(c-d)}$ (or) $\dfrac{c+2}{c^2-cd}$

27. $\dfrac{3a^2+6a+3}{a^2-3a-10} \div \dfrac{12a^2-12}{a^2-4}$

$= \dfrac{3a^2+6a+3}{a^2-3a-10} \cdot \dfrac{a^2-4}{12a^2-12}$

$= \dfrac{3(a^2+2a+1)}{a^2-3a-10} \cdot \dfrac{a^2-4}{12(a^2-1)} = \dfrac{3(a+1)(a+1)}{(a-5)(a+2)} \cdot \dfrac{(a-2)(a+2)}{12(a-1)(a+1)}$

$= \dfrac{(a+1)(a-2)}{4(a-5)(a-1)}$

$\dfrac{9-x^2}{x^2-4x-21} \cdot \left(\dfrac{2x^2+7x+3}{2x^2-15x+7}\right)^{-1}$

$\left(\dfrac{a}{b}\right)^{-1} = \dfrac{b}{a}$

$= \dfrac{9-x^2}{x^2-4x-21} \cdot \dfrac{2x^2-15x+7}{2x^2+7x+3}$

$x = +7$

$x = +\frac{1}{2}$

$= \dfrac{(3-x)(3+x)}{(x-7)(x+3)} \cdot \dfrac{(2x-1)(x-7)}{(2x+1)(x+3)}$

$= \dfrac{(3-x)(2x-1)}{(2x+1)(x+3)}$

Complex fractions:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

20. $\dfrac{\frac{x^2-9}{6x-12}}{\frac{x^2+10x+21}{x^2-x-2}}$

$$= \frac{x^2-9}{6x-12} \div \frac{x^2+10x+21}{x^2-x-2}$$

$$= \frac{x^2-9}{6x-12} \cdot \frac{x^2-x-2}{x^2+10x+21}$$

$$= \frac{(x-3)(x+3)}{6(x-2)} \cdot \frac{(x-2)(x+1)}{(x+7)(x+3)}$$

$$= \frac{(x-3)(x+1)}{6(x+7)}$$

22. $\dfrac{\frac{a^2-b^2}{b^3}}{\frac{b^2-ab}{a^2}}$

$$= \frac{a^2-b^2}{b^3} \div \frac{b^2-ab}{a^2}$$

$$= \frac{a^2-b^2}{b^3} \cdot \frac{a^2}{b^2-ab}$$

$$= \frac{(a-b)(a+b)}{b^3} \cdot \frac{a^2}{b(b-a)} \quad (-1)$$

$$= \frac{-a^2(a+b)}{b^4}$$