# شرح الدرس الثالث Sum and Difference of Angles Identities من الوحدة الحادية عشرة




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| المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثاني | |
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# 11-3 Sum and Difference of Angles Identities



## KeyConcept Sum and Difference Identities

**Sum Identities**

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

**Difference Identities**

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

## Famous angles

| Degree | 30° | 45° | 60° | 0° | 90° | 180° | 270° | 360° |
|---|---|---|---|---|---|---|---|---|
| Radian | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |

| | 30° | 60° | 45° | 0° or 360° | 90° | 180° | 270° |
|---|---|---|---|---|---|---|---|
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | 0 | 1 | 0 | -1 |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | 0 | -1 | 0 |
| $\tan\theta$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | 1 | 0 | undefined | 0 | undefined |

**We can create a combination between two of the previous angles to find any trigonometric function for a non-famous angle.**

**Ex1:** Find the exact value of each expression.

1) $cos(105°)$

$\cos(60+45) = \cos 60 . \cos 45 - \sin 60 . \sin 45$

$= \left(\frac{1}{2} \times \frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

2) $sin(210°)$

$\sin(30+180) = \sin 30 . \cos 180 + \cos 30 . \sin 180$

$= \left(\frac{1}{2} \times -1\right) + \left(\frac{\sqrt{3}}{2} \times 0\right)$

$= -\frac{1}{2}$

3) $tan(-45°)$

dont use 90 or 270 in tan(a+b) or tan(a-b)

$\tan(0-45)$

$= \frac{\tan 0 - \tan 45}{1 + (\tan 0 \times \tan 45)}$

$= \frac{0-1}{1+(0 \times 1)} = -1$

OR

$\tan(-\theta) = -\tan\theta$

$\Rightarrow \tan(-45) = -\tan 45$

$= -1$

## Verifying identities:

**Ex2:** Verify that each equation is an identity.

**8.** $\sin(90^\circ + \theta) = \cos\theta$

$LHS = \sin(90 + \theta)$

$= \sin 90 \cdot \cos\theta + \cos 90 \sin\theta$

$= (1 \cdot \cos\theta) + (0 \times \cancel{\sin\theta})^{0} = \cos\theta = RHS$

**9.** $\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$

$LHS = \cos\left(\frac{3\pi}{2} - \theta\right) = \cos\left(\frac{3\pi}{2}\right) \cdot \cos\theta + \sin\left(\frac{3\pi}{2}\right) \cdot \sin\theta$

$= (0 \times \cancel{\cos\theta})^{0} + (-1 \times \sin\theta)$

$= -\sin\theta = RHS$

**10.** $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$ 90° we cannot use it

$LHS = \tan(\theta + \pi/2) = \frac{\sin(\theta + \pi/2)}{\cos(\theta + \pi/2)}$

$= \frac{\sin\theta \cdot \cos \pi/2 + \cos\theta \sin \pi/2}{\cos\theta \cdot \cos \pi/2 - \sin\theta \sin \pi/2}$

$= \frac{(\cancel{\sin\theta \times 0})^{0} + (\cos\theta \times 1)}{(\cancel{\cos\theta \times 0})_{0} - (\sin\theta \times 1)} = \frac{\cos\theta}{-\sin\theta}$

$= -\cot\theta = RHS$

**11.** $\sin(\theta + \pi) = -\sin\theta$

$LHS = \sin(\theta + \pi)$

$= \sin\theta \times \cos\pi + \cos\theta \cdot \sin\pi$

$= (\sin\theta \times -1) + (\cancel{\cos\theta \times 0})^{0}$

$= -\sin\theta$

$$\sin(A+B) = \frac{\tan A + \tan B}{\sec A \sec B}$$

$$RHS = \frac{\tan A + \tan B}{\sec A \cdot \sec B} = \frac{\frac{\sin A \cdot \cos B}{\cos A \cdot \cos B} + \frac{\sin B \cdot \cos A}{\cos B \cdot \cos A}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$$

$$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cancel{\cos B \cos A}}}{\frac{1}{\cancel{\cos A \cos B}}}$$

$$\frac{\frac{a}{\cancel{b}}}{\frac{c}{\cancel{b}}} = \frac{a}{c}$$

$$= \sin A \cos B + \cos A \sin B$$

$$= \sin(A+B) = LHS$$

**KeyConcept** Sum and Difference Identities

| Sum Identities | Difference Identities |
|---|---|
| • $\sin(A+B) = \sin A \cos B + \cos A \sin B$ | • $\sin(A-B) = \sin A \cos B - \cos A \sin B$ |
| • $\cos(A+B) = \cos A \cos B - \sin A \sin B$ | • $\cos(A-B) = \cos A \cos B + \sin A \sin B$ |
| • $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ | • $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ |

$$\sec(A-B) = \frac{\sec A \sec B}{1 + \tan A \tan B}$$

$$RHS = \frac{\sec A \sec B}{1 + \tan A \tan B}$$

$$= \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \left(\frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}\right)}$$

$$= \frac{\frac{1}{\cos A \cos B}}{\frac{1 \cdot \cos A \cos B}{1 \cdot \cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\frac{1}{\cancel{\cos A \cos B}}}{\frac{\cos A \cos B + \sin A \sin B}{\cancel{\cos A \cos B}}}$$

$$= \frac{1}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{1}{\cos(A-B)}$$

$$= \sec(A-B)$$

$$= LHS$$