شرح الدرس الرابع Identities Angles Half and Angles Double من الوحدة الحادية عشرة

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المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثاني

1	ورقة عمل الدرس الرابع Angles Half and Angles Double Identities من الوحدة الحادية عشرة
2	شرح الدرس الرابع Angles Half and Angles Double Identities من الوحدة الحادية عشرة
3	ورقة عمل الدرس الثالث Angles of Difference and Sum Identities من الوحدة الحادية عشرة

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثاني	
4	شرح الدرس الثالث Angles of Difference and Sum Identities عشرة الحادية الوحدة من
5	ورقة عمل الدرس الثاني Trigonometric Verifying Identities عشرة الحادية الوحدة من

11-4 Double-Angle and Half-Angle Identities

1 Double-Angle Identities

It is sometimes useful to have identities to find the value of a function of twice an angle or half an angle.

KeyConcept Double-Angle Identities

The following identities hold true for all values of θ.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Ex1: Given that $\sin\theta = \frac{3}{5}$, $90 < \theta < 180$, find $\sin(2\theta), \cos(2\theta), \tan(2\theta)$

in Q_2 $\Rightarrow$ $\cos\theta$ in $-ve$

to find $\cos\theta$ $\Rightarrow$ $\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(\frac{3}{5}\right)^2$

$\sqrt{\cos^2\theta} = \sqrt{\frac{16}{25}}$

$\cos\theta = \frac{-4}{5}$

① $\sin 2\theta = 2 . \sin\theta . \cos\theta$

$= 2 \times \frac{3}{5} \times \frac{-4}{5}$

$= \frac{-24}{25}$

② $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{3}{5}\right)^2$

$= \frac{7}{25}$

③ $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-24/25}{7/25} = \frac{-24}{7}$

2 Half-Angle Identities

It is sometimes useful to have identities to find the value of a function of half an angle.

KeyConcept Half-Angle Identities

The following identities hold true for all values of θ.

$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}},\ \cos\theta\neq-1$$

Ex2: Find the exact values of $\sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$., $\tan\left(\frac{\theta}{2}\right)$

$\tan\theta=\frac{5}{12};\ \pi<\theta<\frac{3\pi}{2}$

$\div 2 \quad \div 2 \quad \div 2$

$\Rightarrow \frac{\pi}{2}<\frac{\theta}{2}<\frac{3\pi}{4}$

$90<\frac{\theta}{2}<135$ in Q_2 $\Rightarrow$

$\sin\frac{\theta}{2}$: +ve

$\cos\frac{\theta}{2}$: −ve

$\tan\frac{\theta}{2}$: −ve

To find $\cos\theta$

$1+\tan^2\theta=\sec^2\theta \Rightarrow 1+\left(\frac{5}{12}\right)^2=\sec^2\theta$

$\sqrt{\frac{169}{144}}=\sqrt{\sec^2\theta}$

$\Rightarrow \sec\theta=\frac{-13}{12}$

$\Rightarrow \cos\theta=\frac{-12}{13}$

① $\sin\frac{\theta}{2}=+\sqrt{\frac{1-\left(\frac{-12}{13}\right)}{2}}=\frac{5\sqrt{26}}{26}$

② $\cos\frac{\theta}{2}=-\sqrt{\frac{1+\left(\frac{-12}{13}\right)}{2}}=\frac{-\sqrt{26}}{26}$

③ $\tan\frac{\theta}{2}=\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}=\frac{\frac{5\sqrt{26}}{26}}{\frac{-\sqrt{26}}{26}}=-5$

Ex3: **Find the exact value of each expression.**

1) $tan(15°)$ in Q1 tan is +ve

$tan\left(\frac{30}{2}\right) = +\sqrt{\frac{1-\cos 30}{1+\cos 30}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = 2-\sqrt{3}$

2 Half-Angle Identities It is sometimes useful to have identities to find the value of a function of half an angle.

KeyConcept Half-Angle Identities

The following identities hold true for all values of θ.

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}, \cos\theta \neq -1$

2) $cos(-22.5°)$

$\cos(-\theta) = \cos\theta$

$\Rightarrow \cos(-22.5) = \cos(22.5)$ in Q1 cos is +ve

$= \cos\left(\frac{45}{2}\right) = +\sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$

$=$

Ex4: **Verify that each equation is an identity.**

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$RHS = \frac{1-\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{1}-\cancel{1}+2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2\sin\theta}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$= LHS$$

$(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$(a+b)^2 = a^2 + 2ab + b^2$

$$LHS = (\sin\theta + \cos\theta)^2$$

$$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$$

$$= 1 + \sin 2\theta$$

$$= RHS$$