شرح الدرس الرابع Graphing rational functions من الوحدة السابعة

موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ رياضيات ⇦ الفصل الأول ⇦ الملف

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Learn Graphing Rational Functions with Vertical and Horizontal Asymptotes

A **rational function** has an equation of the form $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x) \neq 0$.

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.
  1. If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
  2. If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
  3. If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

Ex1: For the following functions find the vertical and horizontal asymptotes.

1) $f(x) = \frac{x^2-25}{2x^2+6x}$

factorise $f(x) = \frac{(x-5)(x+5)}{2x(x+3)}$ No simplification

① Vertical asy : $\frac{2x}{2} = \frac{0}{2}$ $\boxed{x=0}$; $x+3=0$ $\boxed{x=-3}$

② Hor asy : deg num = deg deno

$\Rightarrow y = \frac{\text{lead}}{\text{lead}} = \frac{1}{2} \Rightarrow \boxed{y = \frac{1}{2}}$

2) $f(x) = \frac{3x^2+8}{x^3-4x} = \frac{3x^2+8}{x(x^2-4)} = \frac{3x^2+8}{x(x-2)(x+2)}$ No simplify

① VA : $\boxed{x=0}$, $x-2=0$ $\boxed{x=2}$, $x+2=0$ $\boxed{x=-2}$

② H.A : deg(num) < deg(deno)

$\Rightarrow \boxed{y=0}$

Learn Graphing Rational Functions with Oblique Asymptotes

An **oblique asymptote**, or slant asymptote, is neither horizontal nor vertical.

Key Concept • Oblique Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

Ex2: **For the following functions find the oblique asymptote.**

1) $f(x) = \frac{8x^3-25}{2x^2+6x} = \frac{8x^3-25}{2x(x+3)}$ No simplify

$\frac{8x^3}{2x^2} = 4x$

$\frac{-24x^2}{2x^2} = -12$

$4x - 12 \rightarrow$ slant

$$\begin{array}{r|l} 2x^2+6x & 8x^3 - 25 \\ & \ominus\, 8x^3 \ominus 24x^2 \\ \hline & -24x^2 - 25 \\ & \oplus\, 24x^2 \oplus 72x \\ \hline & 72x - 25 \end{array}$$

Slant asy $\Rightarrow \boxed{y = 4x - 12}$

2) $f(x) = \frac{3x^4+5x}{x^3-2}$

$\frac{3x^4}{x^3}$ x

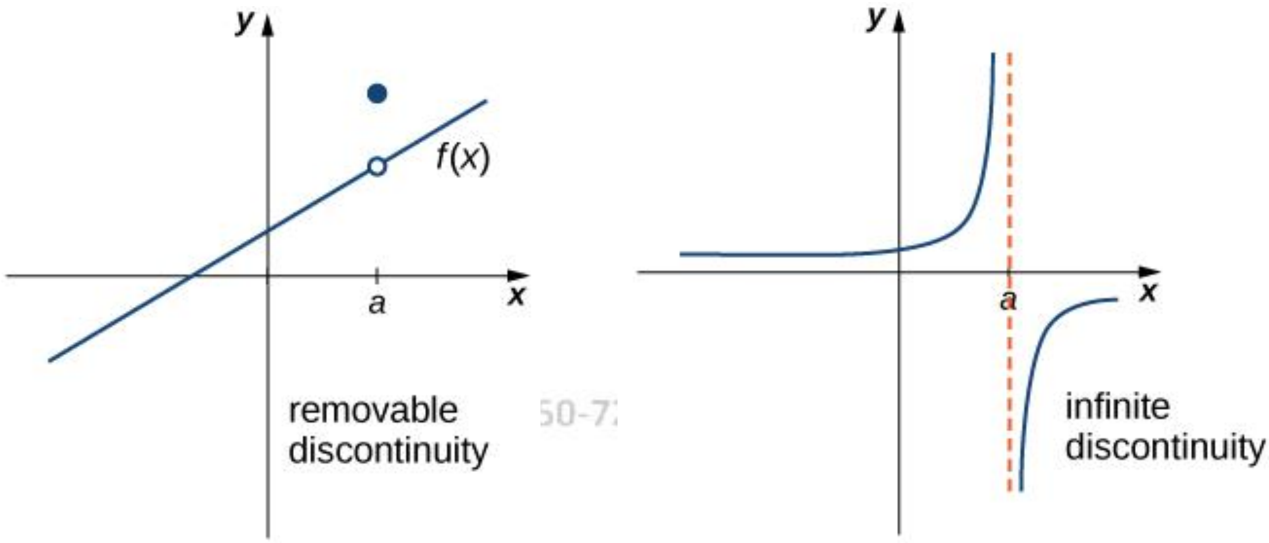
$$\begin{array}{r|l} & 3x \\ \hline x^3-2 & 3x^4+5x \\ & -(3x^4-6x) \\ \hline & 11x \end{array}$$

oblique asy : $y = 3x$

<u>Discontinuity of Rational function:</u>

1) Hole (Removable): at zeros of denominator if the factor can be simplified from both numerator and denominator

2) Infinite (Non-Removable): at zeros of denominator after simplifications



<u>Ex3:</u> Find the point of discontinuity and classify its types.

$f(x) = \frac{2x+6}{x^2-9} = \frac{2\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}$

non-simplified

$x-3=0$

$\boxed{x=3}$

Infinite non-removable (vertical asymptote)

simplified

$x+3=0$

$\boxed{x=-3}$

Hole (Removable)

Ex4: Graph each function

a) $f(x) = \dfrac{x^4-16}{x^2-1}$ $\quad \dfrac{(x^2)^2-4^2}{(x)^2-(1)^2}$ $= \dfrac{(x^2-4)(x^2+4)}{(x-1)(x+1)} = \dfrac{(x-2)(x+2)(x^2+4)}{(x-1)(x+1)}$ $\quad (x)^2-(2)^2$

No simplify

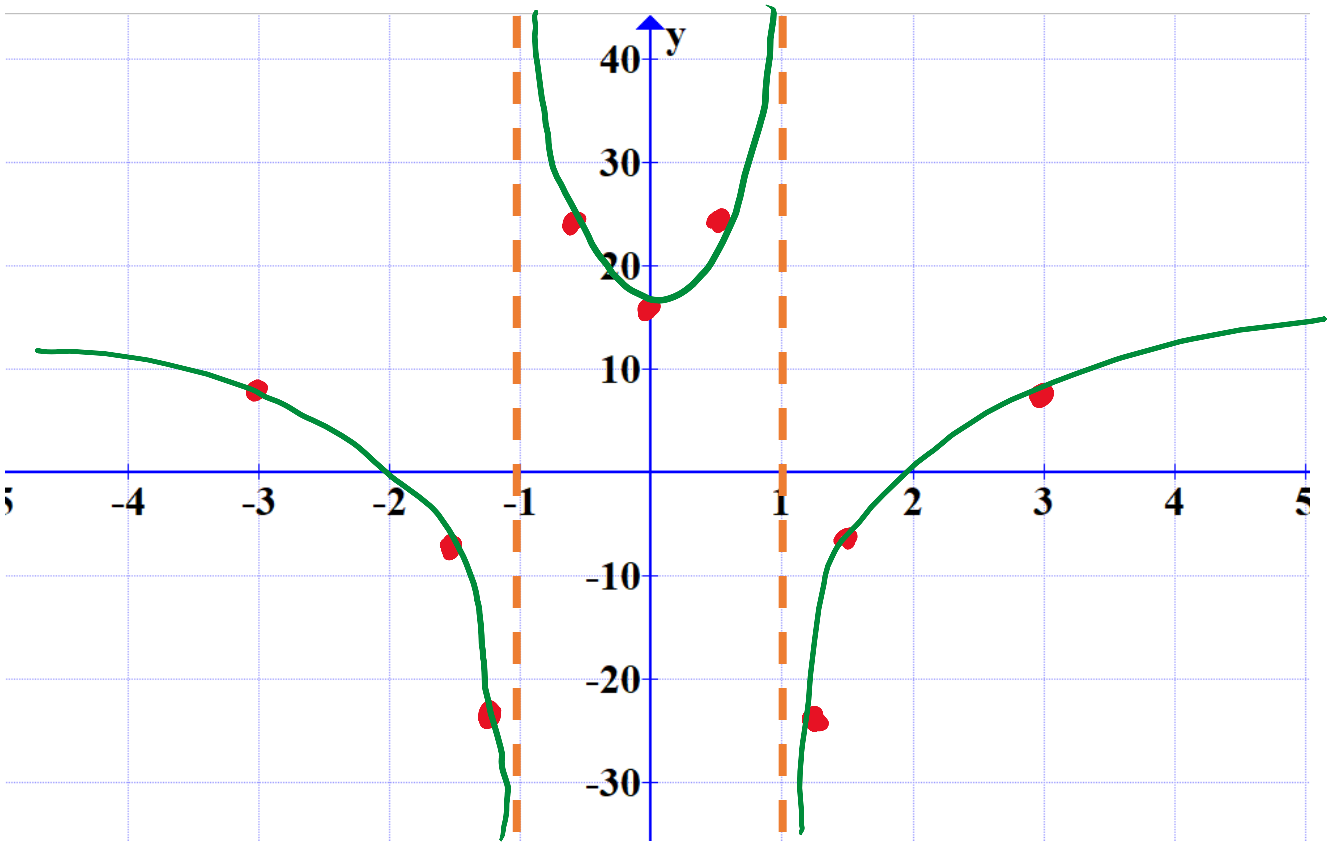
$x-1 \overset{\to}{=} 0$ $\quad \boxed{x=1}$

$x+1 \overset{\to}{=} 0$ $\quad \boxed{x=-1}$

Vertical asymptotes

- deg(num) > deg(deno) ⇒ No H.A
- deg(num) − deg(den) ≠ 1 ⇒ No oblique asy

x	-3	-1.5	-1.25	-1	-0.5	0	0.5	1	1.25	1.5	3
$f(x)$	8.1	-8.8	-24.1	/////	21.3	16	21.3	/////	-24.1	-8.8	8.1



b) $f(x) = \frac{x^2+4x+3}{x^2-x-2} = \frac{(\cancel{x+1})(x+3)}{(x-2)(\cancel{x+1})}$

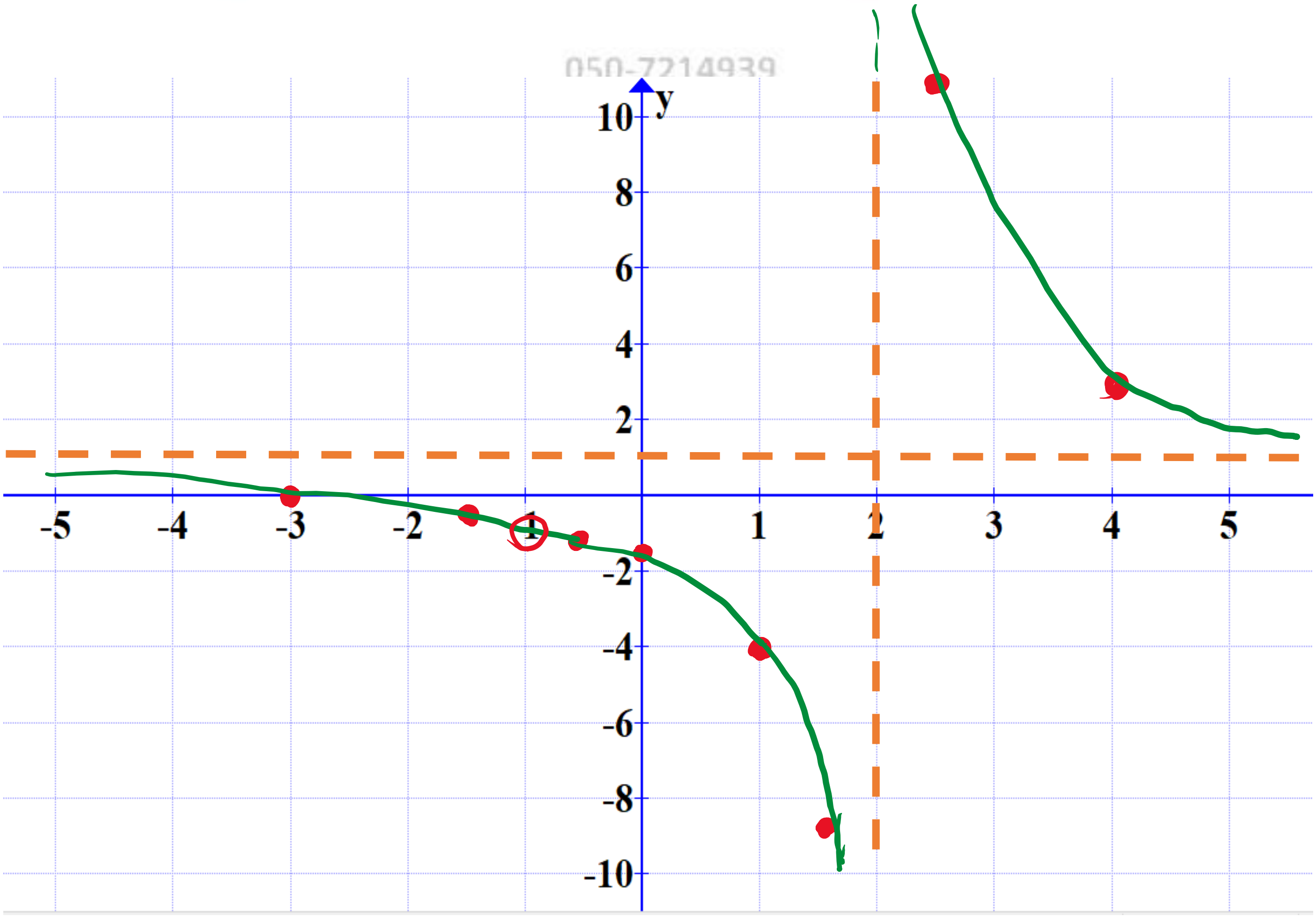
$x-2 \ne 0$ $\boxed{x=2}$ V.A

$x+1 = 0$ $\boxed{x=-1}$ Hole

- deg(num) = deg(den) $\Rightarrow$ H.A $y = \frac{\text{lead}}{\text{lead}} = \frac{1}{1}$ $\boxed{y=1}$

- deg(num) − deg(den) $\ne 1 \Rightarrow$ No oblique asy

x	−3	−1.5	−1	−0.5	0	1	1.5	2	2.25	2.5	4
f(x)	0	−0.5		−1	−1.5	−4	−9		21	11	3.5
			Hole					VA			



c) $f(x) = \frac{x^3+1}{x^2-1} = \frac{\cancel{(x+1)}(x^2-x+1)}{(x-1)\cancel{(x+1)}}$

$x-1 = 0$

$\boxed{x=1}$ VA

$x+1 = 0$

$\boxed{x=-1}$ Hole

- deg(num) > deg(den) ⇒ No H.A
- deg(num) − deg(den) = 1 ⇒ there exist an oblique asy

$$\begin{array}{r|l} & x \\ \hline x^2-1 & x^3+1 \\ & \ominus x^3 \oplus x \\ \hline & x+1 \end{array}$$

oblique : $\boxed{y=x}$

x	−3	−2	−1	−0.5	0	0.5	0.75	1	1.25	1.5	2
f(x)	−3.3	−2.3	////	−1.2	−1	−1.5	−3.3	////	5.3	3.5	3