شرح الدرس الخامس Using exponential and logarithmic functions من الوحدة السادسة ريفيل

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Key Concept • Continuous Exponential Growth

Exponential growth can be modeled by the function

$$f(x) = ae^{kt},$$

where a is the initial value, t is time in years, and k is a constant representing the rate of continuous growth.

The Continuous Exponential Growth equation $y = ae^{kt}$ is often used to represent population growth and is the same as the continuously compounded interest formula.

Continuously Compounded Interest	Continuous Exponential Growth
$A = Pe^{rt}$	$y = ae^{kt}$
P = initial amount	a = initial population
A = amount at time t	y = population at time t
r = interest rate	k = rate of continuous growth

Learn Using Logarithms to Solve Exponential Decay Problems

Exponential functions with base e are frequently used by researchers and scientists to represent situations involving continuous decay.

Key Concept • Continuous Exponential Decay

Exponential decay can be modeled by the function

$$f(t) = ae^{-kt},$$

where a is the initial value, t is time in years, and k is a positive constant representing the rate of continuous decay.

1. POPULATION In 2000, the world population was estimated to be 6.124 billion people. In 2005, it was 6.515 billion.

a. Write an exponential growth equation to represent the population y in billions t years after 2000.

$a = 6.124$, $k = ?$

$f(t) = a \cdot e^{kt}$ after 5 years $\Rightarrow f = 6.515$

$f(t) = 6.124\, e^{kt}$, but $f(5) = 6.515$

$$\frac{6.515}{6.124} = \frac{\cancel{6.124} \cdot e^{k(5)}}{\cancel{6.124}}$$

$$\cancel{\ln} \cancel{e}^{5k} = \ln 1.064$$

$$\frac{\cancel{5}k}{\cancel{5}} = \frac{\ln 1.064}{5} \Rightarrow k = 0.0124$$

$\Rightarrow$ Equation : $\boxed{f(t) = 6.124\, e^{0.0124t}}$

b. Use the equation to predict the year in which the world population reached 7.5 billion people.

$y(t) \Rightarrow t = ?$

$$\frac{7.5}{6.124} = \frac{\cancel{6.124}\, e^{0.0124t}}{\cancel{6.124}}$$

e

$$\cancel{\ln} \cancel{e}^{0.0124t} = \ln 1.22$$

$$\frac{0.0124t}{0.0124} = \frac{\ln(1.22)}{0.0124}$$

$t = 16.36$ years

this will happen in 2016

2. CONSUMER AWARENESS Jason wants to buy a new HD television but he thinks that if he waits, the quality of HD televisions will improve. The television he wants to buy costs \$2500 now, and based on pricing trends, Jason thinks that the price will increase by 4% each year.

a. Write an exponential growth equation to represent the price y of a new HD television t years from now.

$a = 2500$, $k = r = 0.04$

$f(t) = a \cdot e^{kt}$

$f(t) = 2500 \cdot e^{0.04t}$

b. Use the equation to predict when a new HD television will cost \$3000. → $f(t)$

$3000 = 2500 \cdot e^{0.04t}$

c. Jason decides to wait to buy a new television and saves his money. He puts \$2200 in a savings account with 4.7% annual interest compounded continuously. Determine when the amount in his savings will exceed the cost of a new television.

Save

$y(t) = P \cdot e^{rt} = 2200\, e^{0.047t}$

Tv price

$y(t) = a \cdot e^{kt} = 2500 \cdot e^{0.04t}$

Save > Tv price

$\frac{2200 e^{0.047t}}{e^{0.04t}} > \frac{2500 \cancel{e^{0.04t}}}{\cancel{e^{0.04t}}}$

divide by $e^{0.04t}$

$0.047t - 0.04t = 0.007t$

$\frac{\cancel{2200}\, e^{0.007t}}{2200} > \frac{2500}{2200}$

$\cancel{\ln e}^{0.007t} > \ln 1.14$

$\frac{0.\cancel{007}t}{0.\cancel{007}} > \frac{\ln(1.14)}{0.007}$

$t > 18.72$

After half lifetime t the remaining amount will be half of the original

If half lifetime t_1 is given then to find k sub $0.5a = ae^{-k.t_1}$

3. REASONING A radioactive substance has a half-life of 32 years. find k

a. Determine the value of k and the equation of decay for this radioactive substance.

After 32 years the remaining amount will be half of a

$$0.5a = ae^{-k.t_1}$$

$$\frac{0.5\cancel{a}}{\cancel{a}} = \frac{\cancel{a}.e^{-k(32)}}{\cancel{a}}$$

divide by a

$$\cancel{\ln} e^{-32k} = \ln 0.5$$

$$\frac{-\cancel{32}k}{-\cancel{32}} = \frac{\ln 0.5}{-32}$$

$$k = 0.0217$$

b. How much of a 5-gram sample of the radioactive substance should be left after 100 years?

$\underbrace{\text{5-gram}}_{a}$, $\underbrace{\text{100 years}}_{t}$

$$y(t) = a.e^{-kt}$$

$$y(100) = 5.e^{-0.0217(100)}$$

$$= 0.571\ g$$

4. CARBON DATING Carbon-14 has a decay constant k of 0.00012. Use this information to determine the age of the objects based on the amount of Carbon-14.

age: t = ?

a. a fossil that has lost 95% of its Carbon-14

remainig = 100% − 95% = 5%

$$y(t) = a.e^{-kt}$$

$$0.05a = a.e^{-kt}$$

$$\frac{0.05\cancel{a}}{\cancel{a}} = \frac{\cancel{a} \cdot e^{-0.00012t}}{\cancel{a}}$$

$$\cancel{\ln e}^{-0.00012t} = \ln 0.05$$

$$\frac{-0.\cancel{00012}t}{-0.\cancel{00012}} = \frac{\ln 0.05}{-0.00012}$$

$$t = 24964.4356 \text{ years}$$

b. an animal skeleton that has 95% of its Carbon-14 remaining

$$\frac{0.95a}{\cancel{a}} = \frac{\cancel{a} \cdot e^{-0.00012t}}{a}$$

$$\cancel{\ln e}^{-0.00012t} = \ln 0.95$$

$$\frac{-0.\cancel{00012}t}{-0.\cancel{00012}} = \frac{\ln 0.95}{-0.00012}$$

$$t = 427.44 \text{ year}$$

7. USE A MODEL Consider a certain bacteria which is undergoing continuous exponential growth.

a. If there are $\underbrace{\text{80 cells}}_{a}$ initially and $\underbrace{675}_{y(30)}$ cells after $\underbrace{\text{30 minutes}}_{t}$, determine the value of *k*.

$$y(t) = a e^{kt}$$

$$\frac{675}{80} = \frac{\cancel{80} \cdot e^{k(30)}}{\cancel{80}}$$

$$\cancel{\ln e}^{30k} = \ln 8.4375$$

$$\frac{30k}{30} = \frac{\ln(8.4375)}{30}$$

$$\Rightarrow k = 0.0711$$

b. When will the bacteria reach a population of $\underbrace{6000}_{y(t)}$ cells?

$$y(t) = a \cdot e^{kt}$$

$$\frac{6000}{80} = \frac{\cancel{80} \cdot e^{0.0711t}}{\cancel{80}}$$

$$\cancel{\ln} \cancel{e}^{0.0711t} = \ln 75$$

$$\frac{\cancel{0.0711}t}{\cancel{0.0711}} = \frac{\ln 75}{0.0711}$$

$$t = 60.72 \text{ min}$$

9. DEPRECIATION A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS is now worth \$8600. If the value of the system depreciates at a rate of 6.2% annually, how many years ago did Abu buy the GPS system?

depreciates → decay   6.2% → k

$a = 12500$, $y(t) = 8600$ $k = 0.062$

$$y(t) = a \cdot e^{-kt}$$

$$\frac{8600}{12500} = \frac{\cancel{12500} \cdot e^{-0.062t}}{\cancel{12500}}$$

$$\cancel{\ln} \cancel{e}^{-0.062t} = \ln 0.688$$

$$\frac{-\cancel{0.062}t}{-\cancel{0.062}} = \frac{\ln(0.688)}{-0.062}$$

$$t = 6.032 \text{ years}$$