شرح الدرس الرابع Natural logarithms من الوحدة السادسة ريفيل

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول	
5	أوراق عمل لدروس الوحدة السادسة

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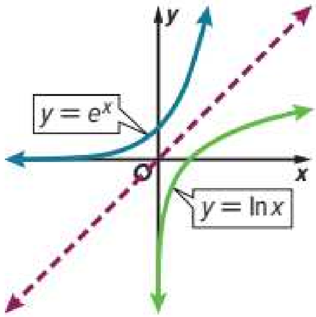
Lesson 6-4

Natural Logarithms

An exponential function with base e, written as $y = e^x$, is called a **natural base exponential function.** Recall that e is an irrational number with an approximate value of 2.7182818...

The inverse of a natural base exponential function is a **natural logarithm,** which can be written as $\log_e x$, but is more often abbreviated as $\ln x$. Every point on the graph of $y = e^x$ is reflected in the line $y = x$, resulting in the graph of $y = \ln x$.



You can write equivalent exponential equations by using the definition of a natural logarithm.

$\ln 6 = x \rightarrow \log_e 6 = x \rightarrow e^x = 6$

Ex: Write the following:

a) $\ln(x+1) = 5$ **in exponential form**

$e^5 = x+1$

b) $e^{3x^2} = 10$ **in Logarithmic form**

$\ln 10 = 3x^2$

Ex: Write the following as a single logarithm

a) $2\ln 8 - \ln k + \ln 3$

$\frac{a}{b} \cdot \frac{2}{1}$

$= \ln 8^2 - \ln k + \ln 3$

$= \ln 64 - \ln k + \ln 3$

$= \ln\left(\frac{64}{k}\right) + \ln 3$

$= \ln\left(\frac{64}{k} \cdot \frac{3}{1}\right) = \ln\left(\frac{192}{k}\right)$

b) $\ln 4 + \ln k - 2\ln 3$

$= \ln 4 + \ln k - \ln 3^2$

$= (\ln 4 + \ln k) - \ln 9$

$= \ln(4k) - \ln(9) = \ln\left(\frac{4k}{9}\right)$

Ex: Solve each equation. Round to the nearest ten-thousandth

a) $\frac{\cancel{2}e^{x-5}}{\cancel{2}} = \frac{10}{2}$

$e^{x-5} = 5$

take ln to both sides

$\boxed{\begin{array}{l} \cancel{\ln e}^{x} = \ln y \Rightarrow x = \ln y \\ \cancel{\ln}x = y \Rightarrow x = e^{y} \\ \cancel{e} \qquad\ \ e \end{array}}$

$\cancel{\ln e}^{x-5} = \ln 5$

$\cancel{\ln}x = b \quad e^{b} = x$

$\cancel{e} \qquad e$

$x - 5 = \ln 5$

$x = \ln 5 + 5 = 6.6094$

b) $3e^{2x+1} - \cancel{2} = 10$

$\quad +\cancel{2} \quad +2$

$\frac{\cancel{3}e^{2x+1}}{\cancel{3}} = \frac{12}{3}$

$\cancel{\ln e}^{2x+1} = \ln 4$

$2x + 1 = \ln 4$

$\quad -1 \quad -1$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{\ln(4) - 1}{2} \Rightarrow x = \frac{\ln 4 - 1}{2} = 0.1931$

c) $\cancel{\ln}(2x - 6) = 4$

$\ \ \ \cancel{e} \qquad\qquad\quad e$

$2x - 6 = e^{4}$

$\quad +6 \quad +6$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{e^{4}+6}{2} \Rightarrow x = \frac{e^{4}+6}{2} \approx 30.2991$

Check:

$\ln\left(\cancel{2}\left(\frac{e^{4}\cancel{+6}}{\cancel{2}}\right)\cancel{-6}\right) \stackrel{?}{=} 4$

$\cancel{\ln e}^{4} = 4$

$4 = 4$ ✓

d) $ln(2x) + ln(5x) = 1$

$\ln(2x \cdot 5x) = 1$

$\cancel{\ln}(10x^2) = 1$

$\cancel{e} \qquad e$

$\Rightarrow \frac{\cancel{10}x^2}{\cancel{10}} = \frac{e^1}{10} \Rightarrow \sqrt{x^2} = \sqrt{\frac{e}{10}} \Rightarrow x = \mp\sqrt{\frac{e}{10}} = \mp 0.5214$

Check

$x = -0.5214$

$\ln(2(-0.5214)) + \ln(5(-0.5214)) \stackrel{?}{=} 1$

undefined

extraneous solution

$x = 0.5214$

$\ln(2(0.5214)) + \ln(5(0.5214)) \stackrel{?}{=} 1$

$1 = 1$ ✓

Ex: USE A MODEL **Monique wants to invest \$4000 in a savings account that pays 3.4% annual interest compounded continuously. The formula $A = Pe^{rt}$ is used to find the amount in the account, where A is the amount in the account after t years, P is the principal amount invested, and r is the annual interest rate.**

a. What is the balance of Monique's account after 5 years?

$P = 4000$, $r = 3.4\% = 0.034$, $t = 5$

$A = Pe^{rt}$

$A = 4000 \cdot e^{0.034(5)} = 4741.2194\ \$$

b. Suppose Monique wants to wait until there is at least \$6000 in her account before withdrawing any money. How long must she keep her money in the savings account?

$A \geq 6000$

$\frac{\cancel{4000} \cdot e^{0.034t}}{\cancel{4000}} \geq \frac{6000}{4000}$

$\cancel{\ln} e^{0.034t} \geq \ln 1.5$

$\frac{0.\cancel{034}t}{\cancel{0.034}} \geq \frac{\ln 1.5}{0.034} \Rightarrow t \geq 11.925$ years