تجميعة أسئلة صفحات الكتاب وفق الهيكل الوزاري ريفيل

مؤسسة الإمارات للتعليم المدرسي
EMIRATES SCHOOLS ESTABLISHMENT

GOOD LUCK

Grade 11 Advanced

5510 - Fatima Al Zahraa Cycle 3 School

Term 2 – (2023/2024)

End of Term 2 Exam Coverage

20 Questions

Teacher Dana Kamal Abdel Jabbar

LESSON 11-1 Trigonometric Identities

KeyConcept Basic Trigonometric Identities

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos\theta \neq 0$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sin\theta \neq 0$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$, $\csc\theta \neq 0$

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta \neq 0$

$\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta \neq 0$

$\csc\theta = \frac{1}{\sin\theta}$, $\sin\theta \neq 0$

$\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \neq 0$

$\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta \neq 0$

Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $\cot^2\theta + 1 = \csc^2\theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

The negative angle identities are sometimes called *odd-even* identities.

1	Use trigonometric identities to simplify expressions	Exercises (28-33)	G10Adv(T3) P773

Simplify each expression.

28. $\dfrac{1 - \sin^2 \theta}{\sin^2 \theta}$

29. $\tan \theta \csc \theta$

30. $\dfrac{1}{\sin^2 \theta} - \dfrac{\cos^2 \theta}{\sin^2 \theta}$

1	Use trigonometric identities to simplify expressions	Exercises (28-33)	G10Adv(T3) P773

Simplify each expression.

31. $2(\csc^2 \theta - \cot^2 \theta)$

32. $(1 + \sin \theta)(1 - \sin \theta)$

33. $2 - 2 \sin^2 \theta$

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LESSON 11-3 Sum and Difference of Angles Identities

KeyConcept Sum and Difference Identities

Sum Identities

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

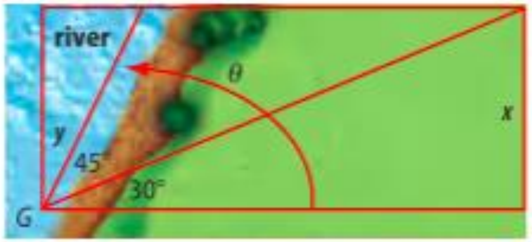
Difference Identities

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

2	Find values of sine and cosine by using sum and difference identities	Example 2	G10Adv (T3) P783

Real-World Example 2 Sum and Difference of Angles Identities

A geologist measures the angle between one side of a rectangular lot and the line from her position to the opposite corner of the lot as 30°. She then measures the angle between that line and the line to the point on the property where a river crosses as 45°. She stands 100 meters from the opposite corner of the property. How far is she from the point at which the river crosses the property line?



3	Find values of sine and cosine by using sum and difference identities	Exercises (12-17) & (24-29)	G10Adv (T3) P785

Lesson 11-3 | Sum and Difference of Angles Identities

Find the exact value of each expression.

12. $\sin 165^\circ$

13. $\cos 135^\circ$

14. $\cos \frac{7\pi}{12}$

15. $\sin \frac{\pi}{12}$

16. $\tan 195^\circ$

17. $\cos\left(-\frac{\pi}{12}\right)$

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LESSON 11-4 Double-Angle and Half-Angle Identities

KeyConcept Double-Angle Identities

The following identities hold true for all values of θ.

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

PRECISION Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

1. $\sin \theta = \frac{1}{4}$; $0^\circ < \theta < 90^\circ$

2. $\sin \theta = \frac{4}{5}$; $90^\circ < \theta < 180^\circ$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

PRECISION Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

3. $\cos \theta = -\frac{5}{13}$; $\frac{\pi}{2} < \theta < \pi$

4. $\cos \theta = \frac{3}{5}$; $270^\circ < \theta < 360^\circ$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

PRECISION Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

5. $\tan \theta = -\frac{8}{15}$; $90^\circ < \theta < 180^\circ$

6. $\tan \theta = \frac{5}{12}$; $\pi < \theta < \frac{3\pi}{2}$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

12. $\sin \theta = \frac{2}{3}$; $90^\circ < \theta < 180^\circ$

13. $\sin \theta = -\frac{15}{17}$; $\pi < \theta < \frac{3\pi}{2}$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

Lesson 11-4 | Double-Angle and Half-Angle Identities

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

14. $\cos \theta = \frac{3}{5}; \frac{3\pi}{2} < \theta < 2\pi$

15. $\cos \theta = \frac{1}{5}; 270^\circ < \theta < 360^\circ$

4	Find values of sine and cosine by using half-angle identities	Exercises (1-6) & (12-17)	G10Adv (T3) P793

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$.

16. $\tan \theta = \frac{4}{3}$; $180^\circ < \theta < 270^\circ$

17. $\tan \theta = -2$; $\frac{\pi}{2} < \theta < \pi$

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LESSON 5-2 Matrix Multiplication, Inverses, and Determinants

1 Multiply Matrices The three basic matrix operations are matrix addition, scalar multiplication, and matrix multiplication. You have seen that adding matrices is similar to adding real numbers, and multiplying a matrix by a scalar is similar to multiplying real numbers.

Matrix Addition

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}$$

Scalar Multiplication

$$k\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{bmatrix}$$

LESSON 5-2 Matrix Multiplication, Inverses, and Determinants

Matrix multiplication has no operational counterpart in the real number system. To multiply matrix A by matrix B, the number of columns in A *must* be equal to the number of rows in B. This can be determined by examining the dimensions of A and B. If it exists, product matrix AB has the same number of rows as A and the same number of columns as B.

matrix A	$\cdot$	matrix B	$=$	AB
3×2		2×4		3×4

Equal

Dimensions of AB

KeyConcept Matrix Multiplication

Words If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix obtained by adding the products of the entries of a row in A to the corresponding entries of a column in B.

Symbols If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix in which

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ir}b_{rj}.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ir} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rj} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots & & \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & \cdots & \vdots & & \\ c_{m1} & c_{m2} & & c_{mj} & \cdots & c_{mn} \end{bmatrix}$$

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*. (Example 5)

27. $A = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix}$

28. $A = \begin{bmatrix} -4 & 8 \\ 1 & -2 \end{bmatrix}$

5	Find inverses of 2 × 2 and 3 × 3 matrices	Exercises (27-34)	P291

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*. (Example 5)

29. $A = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

30. $A = \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}$

5	Find inverses of 2 × 2 and 3 × 3 matrices	Exercises (27-34)	P291

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*. (Example 5)

31. $A = \begin{bmatrix} -1 & -1 & -3 \\ 3 & 6 & 4 \\ 2 & 1 & 8 \end{bmatrix}$

32. $A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 3 & 5 \\ 6 & -1 & -4 \end{bmatrix}$

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*. (Example 5)

33. $A = \begin{bmatrix} 5 & 2 & -1 \\ 4 & 7 & -3 \\ 1 & -5 & 2 \end{bmatrix}$

34. $A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix}$

6	Multiply matrices	Exercises (1-8)	P291

Lesson 5-2 | Matrix Multiplication, Inverses, and Determinants

Find AB and BA, if possible. (Example 1)

1. $A = \begin{bmatrix} 8 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$

6	Multiply matrices	Exercises (1-8)	P291

Find AB and BA, if possible. (Example 1)

3. $A = \begin{bmatrix} 3 & -5 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 0 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 1 & -10 & 9 \end{bmatrix}$

Find AB and BA, if possible. (Example 1)

5. $A = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$

Find AB and BA, if possible. (Example 1)

7. $A = \begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$

8. $A = \begin{bmatrix} 6 & -9 & 10 \\ 4 & 3 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -8 \\ 3 & -9 \\ -2 & 5 \\ 4 & 1 \end{bmatrix}$

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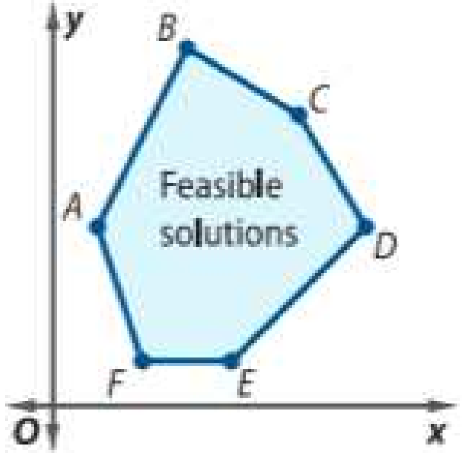
LESSON 5-5 Linear Optimization

KeyConcept Vertex Theorem for Optimization

Words If a linear programming problem can be optimized, an optimal value will occur at one of the vertices of the region representing the set of feasible solutions.

Example The maximum or minimum value of $f(x, y) = ax + by + c$ over the set of feasible solutions graphed occurs at point A, B, C, D, E, or F.



7	Use linear programming to solve applications	Exercises (1-8)	P318

Find the maximum and minimum values of the objective function $f(x, y)$ and for what values of x and y they occur, subject to the given constraints. (Example 1)

1. $f(x, y) = 3x + y$

$y \leq 2x + 1$

$x + 2y \leq 12$

$1 \leq y \leq 3$

2. $f(x, y) = -x + 4y$

$y \leq x + 4$

$y \geq -x + 3$

$1 \leq x \leq 4$

7	Use linear programming to solve applications	Exercises (1-8)	P318

Find the maximum and minimum values of the objective function $f(x, y)$ and for what values of x and y they occur, subject to the given constraints. (Example 1)

3. $f(x, y) = x - y$

$x + 2y \leq 6$

$2x - y \leq 7$

$x \geq -2$

$y \geq -3$

4. $f(x, y) = 3x - 5y$

$x \geq 0, y \geq 0$

$x + 2y \leq 6$

$2y - x \leq 2$

$x + y \leq 5$

Find the maximum and minimum values of the objective function $f(x, y)$ and for what values of x and y they occur, subject to the given constraints. (Example 1)

5. $f(x, y) = 3x - 2y$

$y \leq x + 3$

$1 \leq x \leq 5$

$y \geq 2$

6. $f(x, y) = 3y + x$

$4y \leq x + 8$

$2y \geq 3x - 6$

$2x + 2y \geq 4$

7	Use linear programming to solve applications	Exercises (1-8)	P318

Find the maximum and minimum values of the objective function $f(x, y)$ and for what values of x and y they occur, subject to the given constraints. (Example 1)

7. $f(x, y) = x - 4y$

$x \geq 2,\ y \geq 1$

$x - 2y \geq -4$

$2x - y \leq 7$

$x + y \leq 8$

8. $f(x, y) = x - y$

$3x - 2y \geq -7$

$x + 6y \geq -9$

$5x + y \leq 13,\ x - 3y \geq -7$

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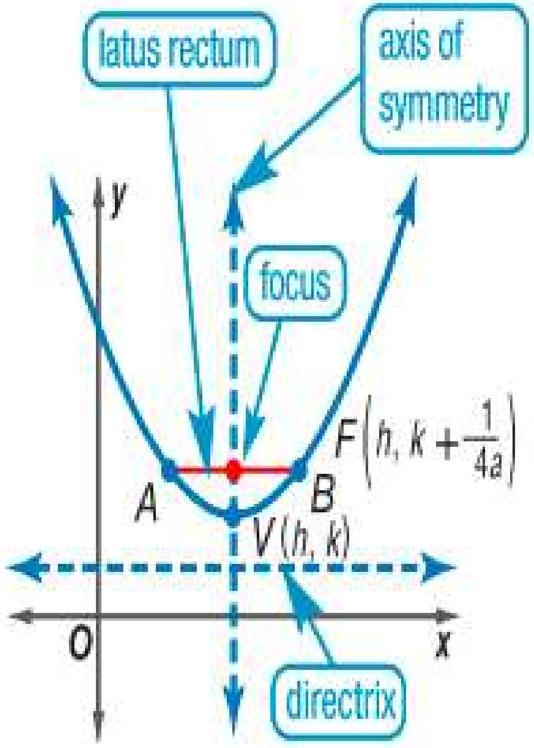
LESSON 6-1 Parabolas

KeyConcept Equations of Parabolas

Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Length of Latus Rectum	$\left\lvert\frac{1}{a}\right\rvert$ units	$\left\lvert\frac{1}{a}\right\rvert$ units



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LESSON 6-1 Parabolas

The **standard form** of the equation of a parabola with vertex (h, k) and axis of symmetry $x = h$ is $y = a(x - h)^2 + k$.

- If $a > 0$, k is the minimum value of the related function and the parabola opens upward.
- If $a < 0$, k is the maximum value of the related function and the parabola opens downward.

An equation of a parabola in the form $y = ax^2 + bx + c$ is the **general form**. Any equation in general form can be written in standard form. The shape of a parabola and the distance between the focus and directrix depend on the value of a in the equation.

8	Write equations of parabolas in standard form	Exercises (1-4)	P335

Lesson 6-1 | Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. $y = 2x^2 - 24x + 40$

2. $y = 3x^2 - 6x - 4$

8	Write equations of parabolas in standard form	Exercises (1-4)	P335

Lesson 6-1 | Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

3. $x = y^2 - 8y - 11$

4. $x + 3y^2 + 12y = 18$

9	Graph parabolas	Exercises (26-31)	P335

Write an equation for each parabola described below. Then graph the equation.

26. vertex (0, 1), focus (0, 4)

27. vertex (1, 8), directrix $y = 3$

9	Graph parabolas	Exercises (26-31)	P335

Lesson 6-1 | Parabolas

Write an equation for each parabola described below. Then graph the equation.

28. focus $(-2, -4)$, directrix $x = -6$

29. focus $(2, 4)$, directrix $x = 10$

9	Graph parabolas	Exercises (26-31)	P335

Lesson 6-1 | Parabolas

Write an equation for each parabola described below. Then graph the equation.

30. vertex $(-6, 0)$, directrix $x = 2$

31. vertex $(9, 6)$, focus $(9, 5)$

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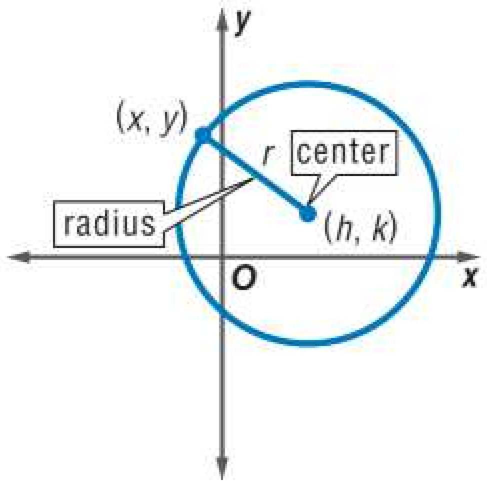
LESSON 6-2 Circles

1 Equations of Circles

A **circle** is the set of all points in a plane that are equidistant from a given point in the plane, called the **center**. Any segment with endpoints at the center and a point on the circle is a **radius** of the circle.

Assume that (x, y) are the coordinates of a point on the circle at the right. The center is at (h, k), and the radius is r. You can find an equation of the circle by using the Distance Formula.



$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$	Distance Formula
$\sqrt{(x - h)^2 + (y - k)^2} = r$	$(x_1, y_1) = (h, k)$, $(x_2, y_2) = (x, y)$, $d = r$
$(x - h)^2 + (y - k)^2 = r^2$	Square each side.

KeyConcept Equations of Circles

Standard Form of Equation	$x^2 + y^2 = r^2$	$(x - h)^2 + (y - k)^2 = r^2$
Center	(0, 0)	(h, k)
Radius	r	r

10	Graph circles	Exercises (47-50)	P343

47. **SPACE** A satellite is in a circular orbit 25,000 miles above Earth.

a. Write an equation for the orbit of this satellite if the origin is at the center of Earth. Use 8000 miles as the diameter of Earth.

b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

10	Graph circles	Exercises (47-50)	P343

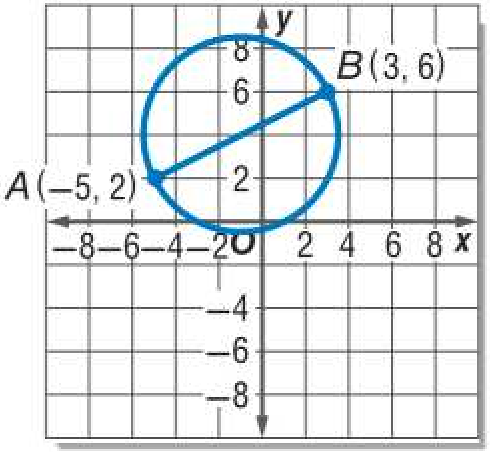
48. **SENSE-MAKING** Suppose an unobstructed radio station broadcast could travel 120 kilometers. Assume the station is centered at the origin.

a. Write an equation to represent the boundary of the broadcast area with the origin as the center.

b. If the transmission tower is relocated 40 kilometers east and 10 kilometers south of the current location, and an increased signal will transmit signals an additional 80 kilometers, what is an equation to represent the new broadcast area?

10	Graph circles	Exercises (47-50)	P343

49. GEOMETRY Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.



a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

c. Graph the circles from parts a and b on the same coordinate plane.

10	Graph circles	Exercises (47-50)	P343

50. **EARTHQUAKES** A stadium is located about 35 kilometers west and 40 kilometers north of a city. Suppose an earthquake occurs with its epicenter about 55 kilometers from the stadium. Assume that the origin of a coordinate plane is located at the center of the city. Write an equation for the set of points that could be the epicenter of the earthquake.

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LESSON 6-4 Hyperbolas

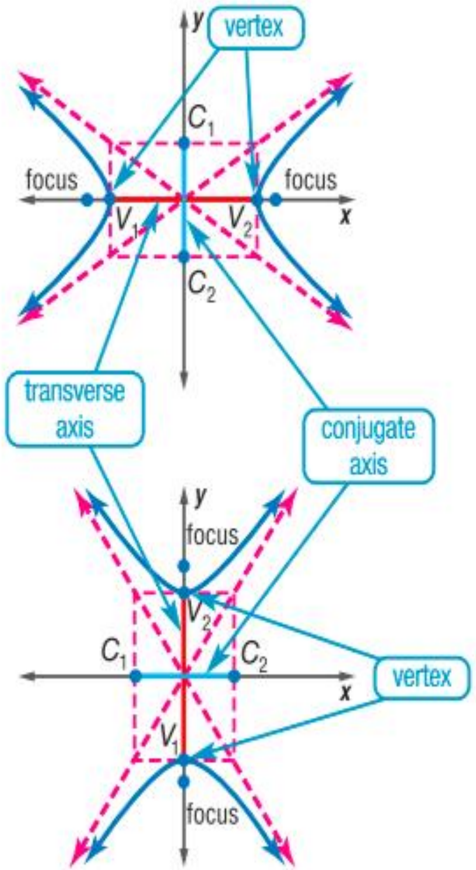
1 Equations of Hyperbolas

Similar to an ellipse, a **hyperbola** is the set of all points in a plane such that the absolute value of the differences of the distances from the foci is constant.

Every hyperbola has two axes of symmetry, the **transverse axis** and the **conjugate axis**. The axes are perpendicular at the center of the hyperbola.

The **foci** of a hyperbola always lie on the transverse axis. The **vertices** are the endpoints of the transverse axis. The **co-vertices** are the endpoints of the conjugate axis.

As a hyperbola recedes from the center, both halves approach asymptotes.



KeyConcept Equations of Hyperbolas Centered at the Origin

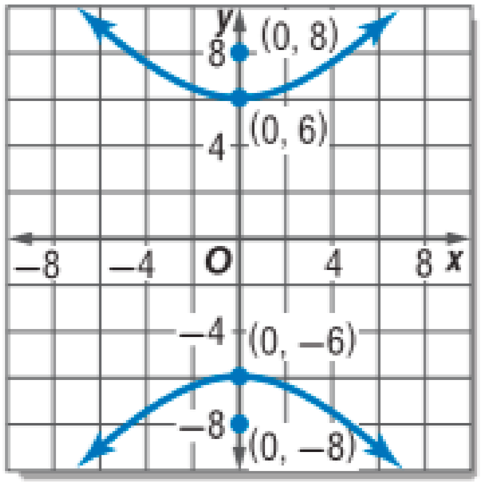
Standard Form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(\pm c, 0)$	$(0, \pm c)$
Length of Transverse Axis	$2a$ units	$2a$ units
Length of Conjugate Axis	$2b$ units	$2b$ units
Equations of Asymptotes	$y = \pm \frac{b}{a}x$	$y = \pm \frac{a}{b}x$

As with ellipses, there are several important relationships among the parts of hyperbolas.

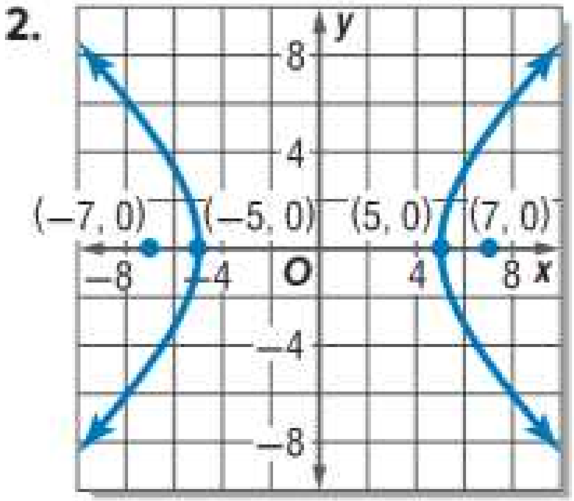
- There are two axes of symmetry.
- The values of a, b, and c are related by the equation $c^2 = a^2 + b^2$.

Write an equation for each hyperbola.

1.



2.



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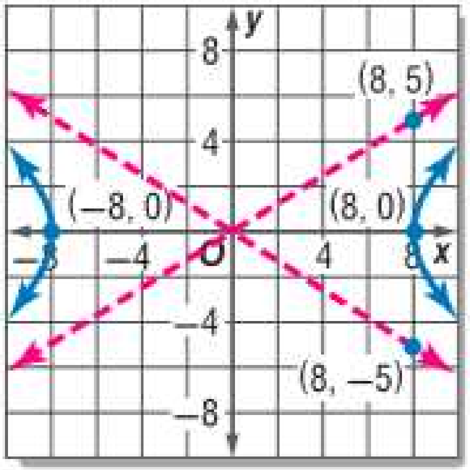
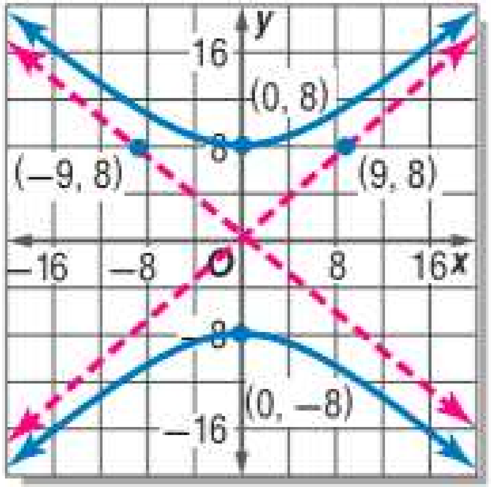
Lesson 6-4 | Hyperbolas

Write an equation for each hyperbola.

3.



4.



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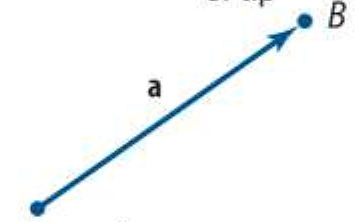
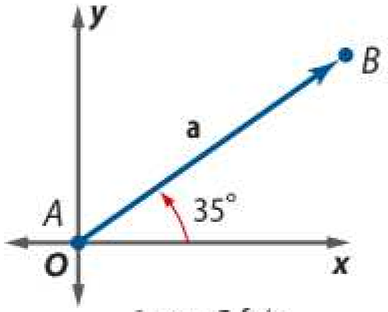
LESSON 7-1 Introduction to Vectors

A vector can be represented geometrically by a directed line segment, or arrow diagram, that shows both magnitude and direction. Consider the directed line segment with an **initial point** *A* (also known as the *tail*) and **terminal point** *B* (also known as the *head* or *tip*) shown. This vector is denoted by $\overrightarrow{AB}$, $\vec{a}$, or **a**.



If a vector has its initial point at the origin, it is in **standard position**. The **direction** of a vector is the directed angle between the vector and the horizontal line that could be used to represent the positive *x*-axis. The direction of **a** is 35°.



The length of the line segment represents, and is proportional to, the **magnitude** of the vector. If the scale of the arrow diagram for **a** is 1 cm = 5 ft/s, then the magnitude of **a**, denoted |**a**|, is 2.6 × 5 or 13 feet per second.

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LESSON 7-1 Introduction to Vectors

Two or more vectors with a sum that is a vector **r** are called **components** of **r**. While components can have any direction, it is often useful to express or *resolve* a vector into two perpendicular components. The **rectangular components** of a vector are horizontal and vertical.

In the diagram, the force **r** exerted to pull the wagon can be thought of as the sum of a horizontal component force **x** that moves the wagon forward and a vertical component force **y** that pulls the wagon upward.

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12	Solve vector problems and resolve vectors into their rectangular components	Exercises (38-44)	P417

Lesson 7-1 | Introduction to Vectors

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components. (Example 6)

38. $2\frac{1}{8}$ centimeters at 310° to the horizontal

39. 1.5 centimeters at a bearing of N49°E

12	Solve vector problems and resolve vectors into their rectangular components	Exercises (38-44)	P417

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components. (Example 6)

40. 3.2 centimeters per hour at a bearing of S78°W

41. $\frac{3}{4}$ centimeter per minute at a bearing of 255°

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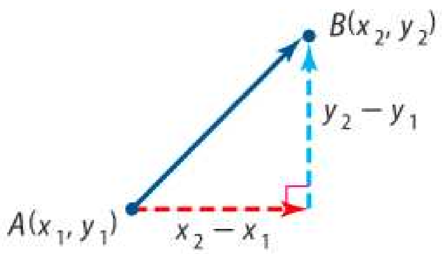
LESSON 7-2 Vectors in the Coordinate Plane

KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$



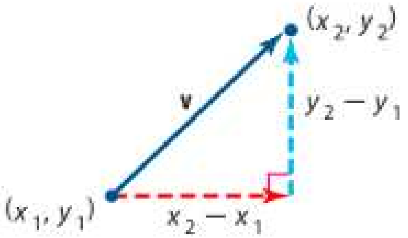
The magnitude of a vector in the coordinate plane is found by using the Distance Formula.

KeyConcept Magnitude of a Vector in the Coordinate Plane

If **v** is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2),then the magnitude of **v** is given by

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

If **v** has a component form of $\langle a, b \rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.



13	Represent and operate with vectors in the coordinate plane	Exercises (1-10)	P425

Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. (Examples 1 and 2)

1. $A(-3, 1), B(4, 5)$

2. $A(2, -7), B(-6, 9)$

3. $A(10, -2), B(3, -5)$

4. $A(-2, 7), B(-9, -1)$

13	Represent and operate with vectors in the coordinate plane	Exercises (1-10)	P425

Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. (Examples 1 and 2)

5. $A(-5, -4), B(8, -2)$

6. $A(-2, 6), B(1, 10)$

7. $A(2.5, -3), B(-4, 1.5)$

8. $A(-4.3, 1.8), B(9.4, -6.2)$

13	Represent and operate with vectors in the coordinate plane	Exercises (1-10)	P425

Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. (Examples 1 and 2)

9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

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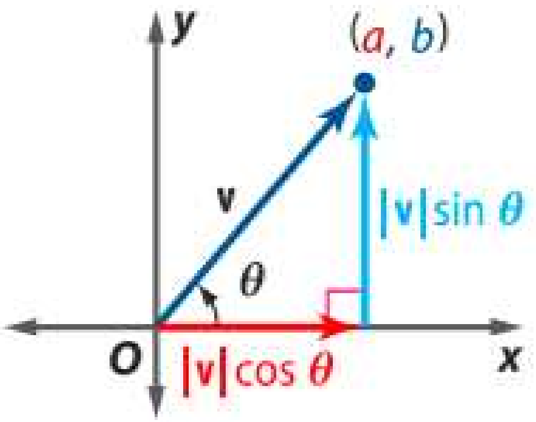
LESSON 7-2 Vectors in the Coordinate Plane

A way to specify the direction of a vector $\mathbf{v} = \langle a, b \rangle$ is to state the direction angle θ that $\mathbf{v}$ makes with the positive x-axis. From Figure 7.2.5, it follows that $\mathbf{v}$ can be written in component form or as a linear combination of $\mathbf{i}$ and $\mathbf{j}$ using the magnitude and direction angle of the vector.



$\mathbf{v} = \langle a, b \rangle$ — Component form

$= \langle |\mathbf{v}| \cos \theta, |\mathbf{v}| \sin \theta \rangle$ — Substitution

$= |\mathbf{v}| (\cos \theta)\mathbf{i} + |\mathbf{v}| (\sin \theta)\mathbf{j}$ — Linear combination of i and j

14	Write a vector as a linear combination of unit vectors	Exercises (38-43)	P425

Find the component form of v with the given magnitude and direction angle. (Example 6)

38. $|\mathbf{v}| = 12, \theta = 60^\circ$

39. $|\mathbf{v}| = 4, \theta = 135^\circ$

14	Write a vector as a linear combination of unit vectors	Exercises (38-43)	P425

Find the component form of v with the given magnitude and direction angle. (Example 6)

40. $|\mathbf{v}| = 6,\ \theta = 240°$

41. $|\mathbf{v}| = 16,\ \theta = 330°$

14	Write a vector as a linear combination of unit vectors	Exercises (38-43)	P425

Find the component form of v with the given magnitude and direction angle. (Example 6)

42. $|\mathbf{v}| = 28, \theta = 273^\circ$

43. $|\mathbf{v}| = 15, \theta = 125^\circ$

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LESSON 7-4 Vectors in Three-Dimensional Space

KeyConcept Vector Operations in Space

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, and any scalar k, then

Vector Addition $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

Vector Subtraction $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

Scalar Multiplication $k\mathbf{a} = \langle ka_1, ka_2, ka_3 \rangle$

15	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Find each of the following for $\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$. (Example 5)

36. $6\mathbf{a} - 7\mathbf{b} + 8\mathbf{c}$

37. $7\mathbf{a} - 5\mathbf{b}$

15	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Find each of the following for $\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$. (Example 5)

38. $2\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$

39. $6\mathbf{b} + 4\mathbf{c} - 4\mathbf{a}$

15	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Find each of the following for $\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$. (Example 5)

40. $8\mathbf{a} - 5\mathbf{b} - \mathbf{c}$

41. $-6\mathbf{a} + \mathbf{b} + 7\mathbf{c}$

15	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Find each of the following for $\mathbf{x} = -9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{y} = 6\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$, and $\mathbf{z} = -2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. (Example 5)

42. $7\mathbf{x} + 6\mathbf{y}$

43. $3\mathbf{x} - 5\mathbf{y} + 3\mathbf{z}$

Find each of the following for $\mathbf{x} = -9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{y} = 6\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$, and $\mathbf{z} = -2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. (Example 5)

44. $4\mathbf{x} + 3\mathbf{y} + 2\mathbf{z}$

45. $-8\mathbf{x} - 2\mathbf{y} + 5\mathbf{z}$

15	Express vectors algebraically and operate with vectors in space	Exercises (36-47)	P442

Find each of the following for $\mathbf{x} = -9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{y} = 6\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$, and $\mathbf{z} = -2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. (Example 5)

46. $-6\mathbf{y} - 9\mathbf{z}$

47. $-\mathbf{x} - 4\mathbf{y} - \mathbf{z}$

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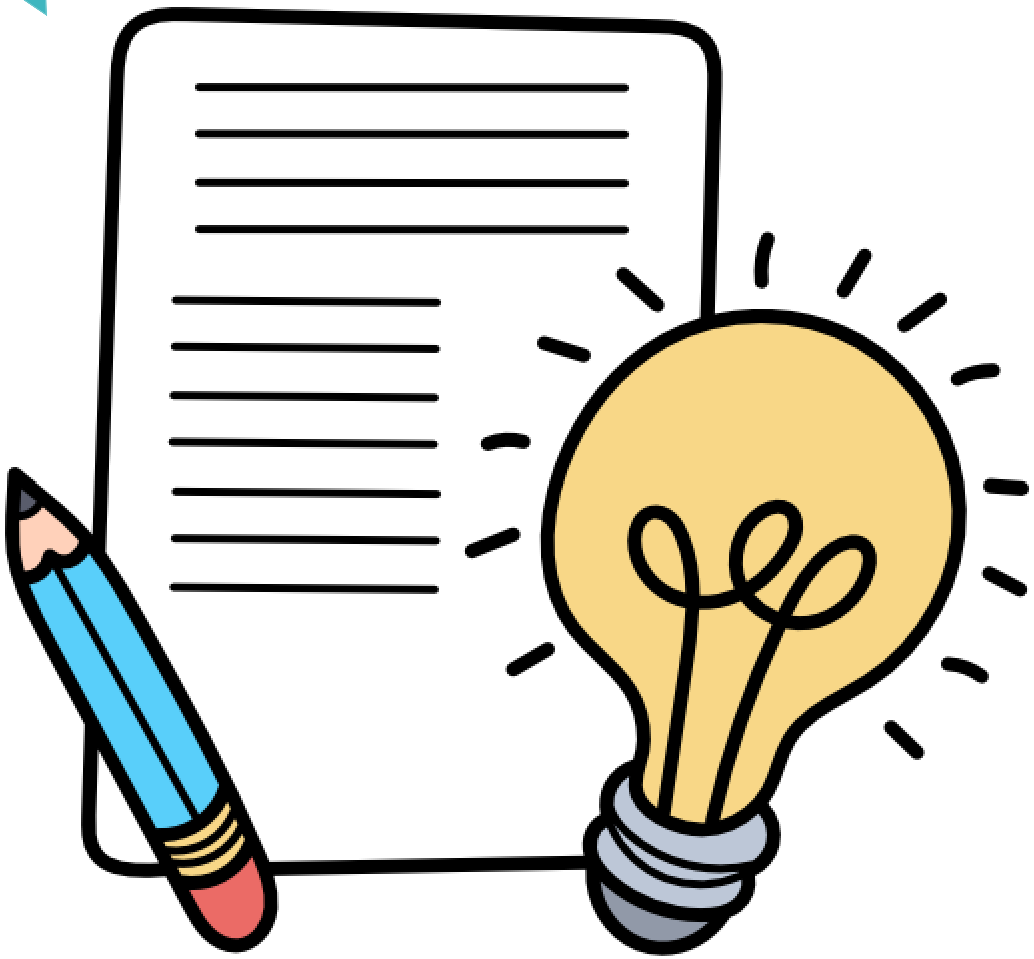

5 Free Response Questions

Lessons :

11.2 _ 11.5 _ 5.3 _ 6.3 _7.3

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LESSON 11-2 Verifying Trigonometric Identities

2 Transform Each Side of an Equation Sometimes it is easier to transform each side of an equation separately into a common form. The following suggestions may be helpful as you verify trigonometric identities.

KeyConcept Suggestions for Verifying Identities

- Substitute one or more basic trigonometric identities to simplify the expression.
- Factor or multiply as necessary. You may have to multiply both the numerator and denominator by the same trigonometric expression.
- Write each side of the identity in terms of sine and cosine only. Then simplify each side as much as possible.
- The properties of equality do not apply to identities as with equations. Do not perform operations to the quantities on each side of an unverified identity.

Example 3 Verify by Transforming Each Side

Verify that $1 - \tan^4 \theta = 2 \sec^2 \theta - \sec^4 \theta$ is an identity.

$1 - \tan^4 \theta \stackrel{?}{=} 2 \sec^2 \theta - \sec^4 \theta$	Original equation
$(1 - \tan^2 \theta)(1 + \tan^2 \theta) \stackrel{?}{=} \sec^2 \theta\,(2 - \sec^2 \theta)$	Factor each side.
$[1 - (\sec^2 \theta - 1)] \sec^2 \theta \stackrel{?}{=} (2 - \sec^2 \theta) \sec^2 \theta$	$1 + \tan^2 \theta = \sec^2 \theta$
$(2 - \sec^2 \theta) \sec^2 \theta = (2 - \sec^2 \theta) \sec^2 \theta$ ✓	Simplify.

Example 3 Verify that each equation is an identity.

19. $\sec\theta - \tan\theta = \frac{1 - \sin\theta}{\cos\theta}$

20. $\frac{1 + \tan\theta}{\sin\theta + \cos\theta} = \sec\theta$

Example 3 Verify that each equation is an identity.

21. $\sec \theta \csc \theta = \tan \theta + \cot \theta$

22. $\sin \theta + \cos \theta = \frac{2 \sin^2 \theta - 1}{\sin \theta - \cos \theta}$

Example 3 **Verify that each equation is an identity.**

23. $(\sin\theta + \cos\theta)^2 = \dfrac{2 + \sec\theta\csc\theta}{\sec\theta\csc\theta}$

24. $\dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$

16	Verify trigonometric identities by transforming each side of an equation into the same form	Exercises (19-32)	P779

Example 3 Verify that each equation is an identity.

25. $\csc \theta - 1 = \dfrac{\cot^2 \theta}{\csc \theta + 1}$

26. $\cos \theta \cot \theta = \csc \theta - \sin \theta$

Example 3 Verify that each equation is an identity.

27. $\sin \theta \cos \theta \tan \theta + \cos^2 \theta = 1$

28. $(\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Example 3 Verify that each equation is an identity.

29. $\csc^2 \theta = \cot^2 \theta + \sin \theta \csc \theta$

30. $\dfrac{\sec \theta - \csc \theta}{\csc \theta \sec \theta} = \sin \theta - \cos \theta$

Example 3 Verify that each equation is an identity.

31. $\sin^2\theta + \cos^2\theta = \sec^2\theta - \tan^2\theta$

32. $\sec\theta - \cos\theta = \tan\theta\sin\theta$

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LESSON 11-5

Solving Trigonometric Equations

Examples 4–5 **Solve each equation.**

45. $2\sin^2\theta = 3\sin\theta + 2$

46. $2\cos^2\theta + 3\sin\theta = 3$

Examples 4–5 **Solve each equation.**

47. $\sin^2 \theta + \cos 2\theta = \cos \theta$

48. $2 \cos^2 \theta = -\cos \theta$

Examples 4–5 **Solve each equation.**

49. SENSE-MAKING Due to ocean tides, the depth y in meters of the River Thames in London varies as a sine function of x, the hour of the day. On a certain day that function was $y = 3 \sin\left[\frac{\pi}{6}(x - 4)\right] + 8$, where $x = 0, 1, 2, \ldots, 24$ corresponds to 12:00 midnight, 1:00 A.M., 2:00 A.M., ..., 12:00 midnight the next night.

a. What is the maximum depth of the River Thames on that day?

b. At what times does the maximum depth occur?

Solve each equation if θ is measured in radians.

50. $(\cos\theta)(\sin 2\theta) - 2\sin\theta + 2 = 0$

51. $2\sin^2\theta + \left(\sqrt{2} - 1\right)\sin\theta = \frac{\sqrt{2}}{2}$

Solve each equation if θ is measured in degrees.

52. $\sin 2\theta + \frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta + \cos\theta$

53. $1 - \sin^2\theta - \cos\theta = \frac{3}{4}$

Solve each equation.

54. $2 \sin \theta = \sin 2\theta$

55. $\cos \theta \tan \theta - 2 \cos^2 \theta = -1$

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LESSON 5-3 Solving Linear Systems using Inverses and Cramer's Rule

Gabriel Cramer
(1704 – 1752)

No. of equations = No. of unknowns

Cramer's Rule only works if the system has a unique solution.

$$D \neq 0$$

Cramer's Rule for Two Equations

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \text{ has solutions } x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D},\ y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D}, \text{ where } D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Therefore, the determinant of a 2×2 matrix provides a test for determining if the matrix is invertible.

Notice that the determinant of a 2×2 matrix is the difference of the product of the two diagonals of the matrix.

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

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LESSON 5-3 Solving Linear Systems using Inverses and Cramer's Rule

Recall how to find determinant of a 3 × 3 matrix:
You may use any of the two methods-

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

OR

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

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LESSON 5-3 Solving Linear Systems using Inverses and Cramer's Rule

2 Use Cramer's Rule

Another method for solving square systems, known as **Cramer's Rule**, uses determinants instead of row reduction or inverse matrices.

Consider the following 2 × 2 system.

$ax + by = e$

$cx + dy = f$

Use the elimination method to solve for x.

$$\begin{array}{lrl} \text{Multiply by } d. \rightarrow & adx + bdy & = ed \\ \text{Multiply by } -b. \rightarrow & (+)\quad -bcx - bdy & = -fb \\ & (ad - bc)x & = ed - fb \end{array}$$

So, $x = \dfrac{ed - fb}{ad - bc}$.

Similarly, it can be shown that $y = \dfrac{af - ce}{ad - bc}$. You should recognize the denominator of each fraction as the determinant of the system's coefficient matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Both the numerator and denominator of each solution can be expressed using determinants.

$$x = \frac{ed - fb}{ad - bc} = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{|A_x|}{|A|} \qquad y = \frac{af - ce}{ad - bc} = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{|A_y|}{|A|}$$

Notice that numerators $|A_x|$ and $|A_y|$ are the determinants of the matrices formed by replacing the coefficients of x or y, respectively, in the coefficient matrix with the column of constant terms $\begin{matrix} e \\ f \end{matrix}$ from the original system $\left[\begin{array}{cc|c} a & b & e \\ c & d & f \end{array}\right]$.

Cramer's Rule can be generalized to systems of n equations in n variables.

KeyConcept Cramer's Rule

Let A be the coefficient matrix of a system of n linear equations in n variables given by $AX = B$. If $\det(A) \neq 0$, then the unique solution of the system is given by

$$x_1 = \frac{|A_1|}{|A|}, x_2 = \frac{|A_2|}{|A|}, x_3 = \frac{|A_3|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|},$$

where A_i is obtained by replacing the ith column of A with the column of constant terms B. If $\det(A) = 0$, then $AX = B$ has either no solution or infinitely many solutions.

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

11. $-3x + y = 4$
$2x + y = -6$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

12. $2x + 3y = 4$
$5x + 6y = 5$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

13. $5x + 4y = 7$

$-x - 4y = -3$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

14. $4x + \frac{1}{3}y = 8$

$3x + y = 6$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

15. $2x - y + z = 1$
$x + 2y - 4z = 3$
$4x + 3y - 7z = -8$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

16. $x + y + z = 12$
$6x - 2y - z = 16$
$3x + 4y + 2z = 28$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

17. $x + 2y = 12$
$3y - 4z = 25$
$x + 6y + z = 20$

18	Solve systems of linear equations by using Cramer's rule	Exercises (11-18)	P300

Lesson 5-3 | Solving Linear Systems using Inverses and Cramer's Rule

Use Cramer's Rule to find the solution of each system of linear equations, if a unique solution exists. (Examples 3 and 4)

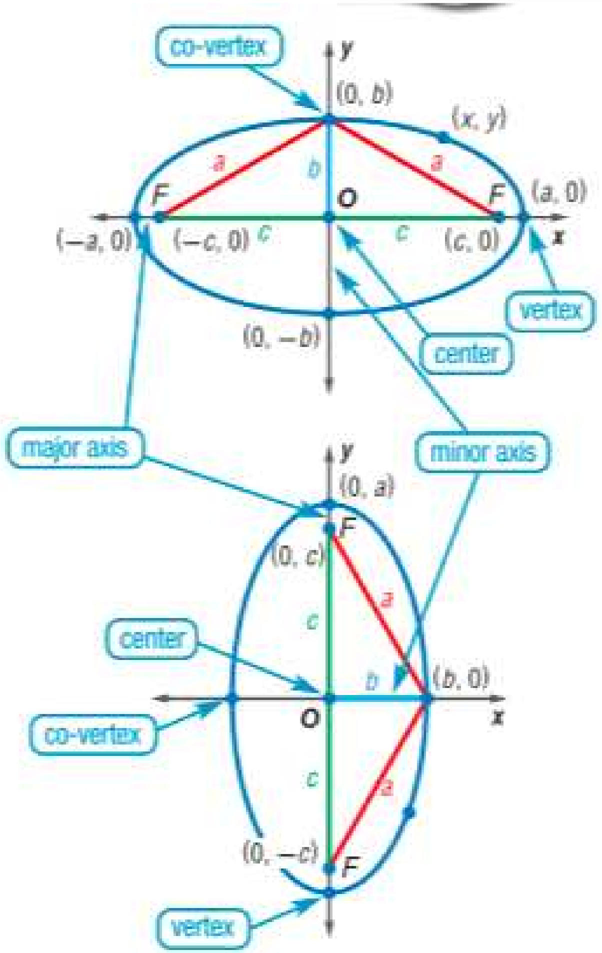
18. $9x + 7y = -30$
$8y + 5z = 11$
$-3x + 10z = 73$

LESSON 6-3 Ellipses

1 Equations of Ellipses An **ellipse** is the set of all points in a plane such that the sum of the distances from two fixed points is constant. These two points are called the **foci** of the ellipse.

Every ellipse has two axes of symmetry, the **major axis** and the **minor axis**. The axes are perpendicular at the **center** of the ellipse.

The foci of an ellipse always lie on the major axis. The endpoints of the major axis are the **vertices** of the ellipse and the endpoints of the minor axis are the **co-vertices** of the ellipse.



KeyConcept Equations of Ellipses Centered at the Origin

Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

There are several important relationships among the many parts of an ellipse.

- The length of the major axis, $2a$ units, equals the sum of the distances from the foci to any point on the ellipse.
- The values of a, b, and c are related by the equation $c^2 = a^2 - b^2$.
- The distance from a focus to either co-vertex is a units.

19	Graph equations of ellipses	Exercises (24-31)	P352

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

24. $\frac{(x-3)^2}{36} + \frac{(y-2)^2}{128} = 1$

25. $\frac{(x+6)^2}{50} + \frac{(y-3)^2}{72} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

26. $\frac{x^2}{27} + \frac{(y-5)^2}{64} = 1$

27. $\frac{(x+4)^2}{16} + \frac{y^2}{75} = 1$

19	Graph equations of ellipses	Exercises (24-31)	P352

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

28. $3x^2 + y^2 - 6x - 8y - 5 = 0$

29. $3x^2 + 4y^2 - 18x + 24y + 3 = 0$

19	Graph equations of ellipses	Exercises (24-31)	P352

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

30. $7x^2 + y^2 - 56x + 6y + 93 = 0$

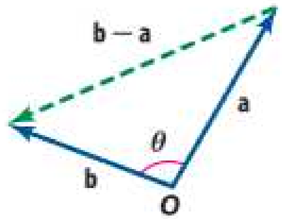
31. $3x^2 + 2y^2 + 12x - 20y + 14 = 0$

LESSON 7-3 Dot Products and Vector Projections

KeyConcept Angle Between Two Vectors

If θ is the angle between nonzero vectors **a** and **b**, then

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}.$$



Proof

Consider the triangle determined by **a**, **b**, and **b** − **a** in the figure above.

$\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta = \lvert\mathbf{b}-\mathbf{a}\rvert^2$	Law of Cosines
$\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta = (\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})$	$\lvert\mathbf{u}\rvert^2 = \mathbf{u}\cdot\mathbf{u}$
$\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta = \mathbf{b}\cdot\mathbf{b} - \mathbf{b}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{a}$	Distributive Property for Dot Products
$\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta = \lvert\mathbf{b}\rvert^2 - 2\mathbf{a}\cdot\mathbf{b} + \lvert\mathbf{a}\rvert^2$	$\mathbf{u}\cdot\mathbf{u} = \lvert\mathbf{u}\rvert^2$
$-2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta = -2\mathbf{a}\cdot\mathbf{b}$	Subtract $\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2$ from each side.
$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert}$	Divide each side by $-2\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert$.

20	Find the dot product of two vectors and use the dot product to find the angle between them	Exercises (16-23)	P434

Find the angle θ between u and v to the nearest tenth of a degree. (Example 3)

16. $\mathbf{u} = \langle 0, -5 \rangle, \mathbf{v} = \langle 1, -4 \rangle$

17. $\mathbf{u} = \langle 7, 10 \rangle, \mathbf{v} = \langle 4, -4 \rangle$

18. $\mathbf{u} = \langle -2, 4 \rangle, \mathbf{v} = \langle 2, -10 \rangle$

19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}, \mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

20	Find the dot product of two vectors and use the dot product to find the angle between them	Exercises (16-23)	P434

Find the angle θ between u and v to the nearest tenth of a degree. (Example 3)

20. $\mathbf{u} = \langle -9, 0 \rangle, \mathbf{v} = \langle -1, -1 \rangle$

21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

22. $\mathbf{u} = \langle 6, 0 \rangle, \mathbf{v} = \langle -10, 8 \rangle$

23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}, \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

Dear Students ,

Please make sure to study all the required lessons and check all resources uploaded on LMS. Keep practicing and don't hesitate to ask any question !