3-4 Exponential and Logarithmic Equations

Solve each equation.

1. $4^{x+7} = 8^{x+3}$

SOLUTION:

$$\begin{aligned} 4^{x+7} &= 8^{x+3} \\ (2^2)^{x+7} &= (2^3)^{x+3} \\ 2^{2x+14} &= 2^{3x+9} \\ 2x+14 &= 3x+9 \\ 5 &= x \end{aligned}$$

2. $8^{x+4} = 32^{3x}$

SOLUTION:

$$\begin{aligned} 8^{x+4} &= 32^{3x} \\ (2^3)^{x+4} &= (2^5)^{3x} \\ 2^{3x+12} &= 2^{15x} \\ 3x+12 &= 15x \\ 12 &= 12x \\ 1 &= x \end{aligned}$$

3. $49^{x+4} = 7^{18-x}$

SOLUTION:

$$\begin{aligned} 49^{x+4} &= 7^{18-x} \\ (7^2)^{x+4} &= 7^{18-x} \\ 7^{2x+8} &= 7^{18-x} \\ 2x+8 &= 18-x \\ 3x &= 10 \\ x &= \frac{10}{3} \end{aligned}$$

4. $32^{x-1} = 4^{x+5}$

SOLUTION:

$$\begin{aligned} 32^{x-1} &= 4^{x+5} \\ (2^5)^{x-1} &= (2^2)^{x+5} \\ 2^{5x-5} &= 2^{2x+10} \\ 5x-5 &= 2x+10 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

5. $\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$

SOLUTION:

$$\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$$
$$\left(\left[\frac{3}{4}\right]^2\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$$
$$\left(\frac{3}{4}\right)^{6x-4} = \left(\frac{3}{4}\right)^{5x+4}$$
$$6x - 4 = 5x + 4$$
$$x = 8$$

6. $12^{3x + 11} = 144^{2x + 7}$

SOLUTION:

$$12^{3x+11} = 144^{2x+7}$$
$$12^{3x+11} = (12^2)^{2x+7}$$
$$12^{3x+11} = 12^{4x+14}$$
$$3x + 11 = 4x + 14$$
$$-3 = x$$

7. $25^{\frac{x}{3}} = 5^{x-4}$

SOLUTION:

$$25^{\frac{x}{3}} = 5^{x-4}$$
$$(5^2)^{\frac{x}{3}} = 5^{x-4}$$
$$5^{\frac{2x}{3}} = 5^{x-4}$$
$$\frac{2x}{3} = x - 4$$
$$2x = 3x - 12$$
$$12 = x$$

8. $\left(\frac{5}{6}\right)^{4x} = \left(\frac{36}{25}\right)^{9-x}$

SOLUTION:

$$\left(\frac{5}{6}\right)^{4x} = \left(\frac{36}{25}\right)^{9-x}$$
$$\left(\frac{5}{6}\right)^{4x} = \left(\left[\frac{6}{5}\right]^{2}\right)^{9-x}$$
$$\left(\frac{5}{6}\right)^{4x} = \left(\left[\frac{5}{6}\right]^{-2}\right)^{9-x}$$
$$\left(\frac{5}{6}\right)^{4x} = \left(\frac{5}{6}\right)^{2x-18}$$
$$4x = 2x - 18$$
$$2x = -18$$
$$x = -9$$

9. **INTERNET** The number of people P in millions using two different search engines to surf the Internet t weeks after the creation of the search engine can be modeled by $P_1(t) = 1.5^{t+4}$ and $P_2(t) = 2.25^{t-3.5}$, respectively. During which week did the same number of people use each search engine?

SOLUTION:

$$P_1(t) = P_2(t)$$
$$1.5^{t+4} = 2.25^{t-3.5}$$
$$1.5^{t+4} = \left[(1.5)^2\right]^{t-3.5}$$
$$1.5^{t+4} = 1.5^{2t-7}$$
$$t + 4 = 2t - 7$$
$$11 = t$$

3-4 Exponential and Logarithmic Equations

10. **FINANCIAL LITERACY** Brandy is planning on investing $5000 and is considering two savings accounts. The first account is continuously compounded and offers a 3% interest rate. The second account is annually compounded and also offers a 3% interest rate, but the bank will match 4% of the initial investment.
a. Write an equation for the balance of each savings account at time *t* years.
b. How many years will it take for the continuously compounded account to catch up with the annually compounded savings account?
c. If Brandy plans on leaving the money in the account for 30 years, which account should she choose?

SOLUTION:

a. Use $A = Pe^{rt}$ for the continuously compounded function and use $A = P(1 + r)^t$ for the annually compounded interest function. $A = 5000e^{0.03t}$; $A = 5200(1.03)^t$
b.

$$\begin{aligned} 5000e^{0.03t} &= 5200(1.03)^t \\ e^{0.03t} &= 1.04(1.03)^t \\ \ln e^{0.03t} &= \ln\left[1.04(1.03)^t\right] \\ 0.03t &= \ln 1.04 + \ln(1.03)^t \\ 0.03t &= \ln 1.04 + t\ln 1.03 \\ 0.03t - t\ln 1.03 &= \ln 1.04 \\ t(0.03 - \ln 1.03) &= \ln 1.04 \\ t &= \frac{\ln 1.04}{0.03 - \ln 1.03} \\ t &\approx 89.0 \end{aligned}$$

c. After30 years, the annually compounded account would still have a higher balance than the other account. The annually compounded account would be the better choice.

Solve each logarithmic equation.

11. $\ln a = 4$

SOLUTION:

$$\begin{aligned} \ln a &= 4 \\ e^{\ln a} &= e^4 \\ a &= e^4 \end{aligned}$$

12. $-8 \log b = -64$

SOLUTION:

$$\begin{aligned} -8\log b &= -64 \\ \log b &= 8 \\ 10^{\log b} &= 10^8 \\ b &= 10^8 \\ b &= 100{,}000{,}000 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

13. $\ln(-2) = c$

SOLUTION:

The logarithm of a negative number provides no real solution.

14. $2 + 3 \log 3d = 5$

SOLUTION:

$$\begin{aligned} 2 + 3\log 3d &= 5 \\ 3\log 3d &= 3 \\ \log 3d &= 1 \\ 10^{\log 3d} &= 10^1 \\ 3d &= 10 \\ d &= \frac{10}{3} \end{aligned}$$

15. $14 + 20 \ln 7x = 54$

SOLUTION:

$$\begin{aligned} 14 + 20\ln 7x &= 54 \\ 20\ln 7x &= 40 \\ \ln 7x &= 2 \\ e^{\ln 7x} &= e^2 \\ 7x &= e^2 \\ x &= \frac{e^2}{7} \\ x &\approx 1.06 \end{aligned}$$

16. $100 + 500 \log_1 g = 1100$

SOLUTION:

$$\begin{aligned} 100 + 500\log_1 g &= 1100 \\ 500\log_1 g &= 1000 \\ \log_1 g &= 2 \end{aligned}$$

The expression $\log_1 g$, will always equal 1, while g can take on any value except for 0. Therefore, there is no solution.

17. $7000 \ln h = -21{,}000$

SOLUTION:

$$\begin{aligned} 7000\ln h &= -21{,}000 \\ \ln h &= -3 \\ e^{\ln h} &= e^{-3} \\ h &= e^{-3} \\ h &\approx 0.05 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

18. $-18 \log_0 j = -126$

SOLUTION:

The logarithm with a zero base provides no real solution.

19. $12{,}000 \log_2 k = 192{,}000$

SOLUTION:

$$\begin{aligned} 12{,}000 \log_2 k &= 192{,}000 \\ \log_2 k &= 16 \\ 2^{\log_2 k} &= 2^{16} \\ k &= 2^{16} \\ k &= 65{,}536 \end{aligned}$$

20. $\log_2 m^4 = 32$

SOLUTION:

$$\begin{aligned} \log_2 m^4 &= 32 \\ 4 \log_2 m &= 32 \\ \log_2 m &= 8 \\ 2^{\log_2 m} &= 2^8 \\ m &= 2^8 \\ m &= \pm 256 \end{aligned}$$

Because m is taken to an even power in the original equation, m can be positive or negative.

21. **CARS** If all other factors are equal, the higher the displacement D in liters of the air/fuel mixture of an engine, the more horsepower H it will produce. The horsepower of a naturally aspirated engine can be modeled by $H = \log_{1.003} \frac{D}{1.394}$. Find the displacement when horsepower is 200.

SOLUTION:

$$\begin{aligned} H &= \log_{1.003} \frac{D}{1.394} \\ 200 &= \log_{1.003} \frac{D}{1.394} \\ 1.003^{200} &= 1.003^{\log_{1.003} \frac{D}{1.394}} \\ 1.003^{200} &= \frac{D}{1.394} \\ 1.394(1.003^{200}) &= D \\ 2.54 &\approx D \end{aligned}$$

Solve each equation.

22. $\log_6 (x^2 + 5) = \log_6 41$

SOLUTION:

$$\begin{aligned} \log_6 (x^2 + 5) &= \log_6 41 \\ x^2 + 5 &= 41 \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

23. $\log_8 (x^2 + 11) = \log_8 92$

SOLUTION:

$$\begin{aligned} \log_8 (x^2 + 11) &= \log_8 92 \\ x^2 + 11 &= 92 \\ x^2 &= 81 \\ x &= \pm 9 \end{aligned}$$

24. $\log_9 (x^4 - 3) = \log_9 13$

SOLUTION:

$$\begin{aligned} \log_9 (x^4 - 3) &= \log_9 13 \\ x^4 - 3 &= 13 \\ x^4 &= 16 \\ x &= \pm 2 \end{aligned}$$

25. $\log_7 6x = \log_7 9 + \log_7 (x - 4)$

SOLUTION:

$$\begin{aligned} \log_7 6x &= \log_7 9 + \log_7 (x - 4) \\ \log_7 6x &= \log_7 (9x - 36) \\ 6x &= 9x - 36 \\ 36 &= 3x \\ 12 &= x \end{aligned}$$

3-4 Exponential and Logarithmic Equations

26. $\log_5 x = \log_5 (x + 6) - \log_5 4$

SOLUTION:

$$\log_5 x = \log_5 (x+6) - \log_5 4$$
$$\log_5 x = \log_5 \left(\frac{x+6}{4}\right)$$
$$x = \frac{x+6}{4}$$
$$4x = x + 6$$
$$3x = 6$$
$$x = 2$$

27. $\log_{11} 3x = \log_{11} (x + 5) - \log_{11} 2$

SOLUTION:

$$\log_{11} 3x = \log_{11} (x+5) - \log_{11} 2$$
$$\log_{11} 3x = \log_{11} \left(\frac{x+5}{2}\right)$$
$$3x = \frac{x+5}{2}$$
$$6x = x + 5$$
$$5x = 5$$
$$x = 1$$

Solve each equation. Round to the nearest hundredth.

28. $6^x = 28$

SOLUTION:

$$6^x = 28$$
$$\ln 6^x = \ln 28$$
$$x \ln 6 = \ln 28$$
$$x = \frac{\ln 28}{\ln 6}$$
$$x \approx 1.86$$

29. $1.8^x = 9.6$

SOLUTION:

$$1.8^x = 9.6$$
$$\ln 1.8^x = \ln 9.6$$
$$x \ln 1.8 = \ln 9.6$$
$$x = \frac{\ln 9.6}{\ln 1.8}$$
$$x \approx 3.85$$

30. $3e^{4x} = 45$

SOLUTION:

$$\begin{aligned} 3e^{4x} &= 45 \\ e^{4x} &= 15 \\ \ln e^{4x} &= \ln 15 \\ 4x &= \ln 15 \\ x &= \frac{\ln 15}{4} \\ x &\approx 0.68 \end{aligned}$$

31. $e^{3x + 1} = 51$

SOLUTION:

$$\begin{aligned} e^{3x+1} &= 51 \\ \ln e^{3x+1} &= \ln 51 \\ 3x + 1 &= \ln 51 \\ 3x &= \ln 51 - 1 \\ x &= \frac{\ln 51 - 1}{3} \\ x &\approx 0.98 \end{aligned}$$

32. $8^x - 1 = 3.4$

SOLUTION:

$$\begin{aligned} 8^x - 1 &= 3.4 \\ 8^x &= 4.4 \\ \ln 8^x &= \ln 4.4 \\ x \ln 8 &= \ln 4.4 \\ x &= \frac{\ln 4.4}{\ln 8} \\ x &\approx 0.71 \end{aligned}$$

33. $2e^{7x} = 84$

SOLUTION:

$$\begin{aligned} 2e^{7x} &= 84 \\ e^{7x} &= 42 \\ \ln e^{7x} &= \ln 42 \\ 7x &= \ln 42 \\ x &= \frac{\ln 42}{7} \\ x &\approx 0.53 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

34. $8.3e^{9x} = 24.9$

SOLUTION:

$$\begin{aligned} 8.3e^{9x} &= 24.9 \\ e^{9x} &= 3 \\ \ln e^{9x} &= \ln 3 \\ 9x &= \ln 3 \\ x &= \frac{\ln 3}{9} \\ x &\approx 0.12 \end{aligned}$$

35. $e^{2x} + 5 = 16$

SOLUTION:

$$\begin{aligned} e^{2x} + 5 &= 16 \\ e^{2x} &= 11 \\ \ln e^{2x} &= \ln 11 \\ 2x &= \ln 11 \\ x &= \frac{\ln 11}{2} \\ x &\approx 1.20 \end{aligned}$$

36. $2.5e^{x+4} = 14$

SOLUTION:

$$\begin{aligned} 2.5e^{x+4} &= 14 \\ e^{x+4} &= 5.6 \\ \ln e^{x+4} &= \ln 5.6 \\ x + 4 &= \ln 5.6 \\ x &= \ln 5.6 - 4 \\ x &\approx -2.28 \end{aligned}$$

37. $0.75e^{3.4x} - 0.3 = 80.1$

SOLUTION:

$$\begin{aligned} 0.75e^{3.4x} - 0.3 &= 80.1 \\ 0.75e^{3.4x} &= 80.4 \\ e^{3.4x} &= 107.2 \\ \ln e^{3.4x} &= \ln 107.2 \\ 3.4x &= \ln 107.2 \\ x &= \frac{\ln 107.2}{3.4} \\ x &\approx 1.37 \end{aligned}$$

38. **GENETICS** PCR (Polymerase Chain Reaction) is a technique commonly used in forensic labs to amplify DNA. PCR uses an enzyme to cut a designated nucleotide sequence from the DNA and then replicates the sequence. The number of identical nucleotide sequences N after t minutes can be modeled by $N(t) = 100 \cdot 1.17^t$.

a. At what time will there be 1×10^4 sequences?

b. At what time will the DNA have been amplified to 1 million sequences?

SOLUTION:

a.

$$\begin{aligned} N(t) &= 100 \cdot 1.17^t \\ 1 \cdot 10^4 &= 100 \cdot 1.17^t \\ 10,000 &= 100 \cdot 1.17^t \\ 100 &= 1.17^t \\ \ln 100 &= \ln 1.17^t \\ \ln 100 &= t \ln 1.17 \\ \frac{\ln 100}{\ln 1.17} &= t \\ 29.33 &\approx t \end{aligned}$$

b.

$$\begin{aligned} N(t) &= 100 \cdot 1.17^t \\ 1,000,000 &= 100 \cdot 1.17^t \\ 10,000 &= 1.17^t \\ \ln 10,000 &= \ln 1.17^t \\ \ln 10,000 &= t \ln 1.17 \\ \frac{\ln 10,000}{\ln 1.17} &= t \\ 58.66 &\approx t \end{aligned}$$

Solve each equation.

39. $7^{2x+1} = 3^{x+3}$

SOLUTION:

$$\begin{aligned} 7^{2x+1} &= 3^{x+3} \\ \ln 7^{2x+1} &= \ln 3^{x+3} \\ (2x+1)\ln 7 &= (x+3)\ln 3 \\ 2x\ln 7 + \ln 7 &= x\ln 3 + 3\ln 3 \\ 2x\ln 7 - x\ln 3 &= 3\ln 3 - \ln 7 \\ x(2\ln 7 - \ln 3) &= 3\ln 3 - \ln 7 \\ x &= \frac{3\ln 3 - \ln 7}{2\ln 7 - \ln 3} \\ x &\approx 0.48 \end{aligned}$$

40. $11^{x+1} = 7^{x-1}$

SOLUTION:

$$\begin{aligned}
11^{x+1} &= 7^{x-1} \\
\ln 11^{x+1} &= \ln 7^{x-1} \\
(x+1)\ln 11 &= (x-1)\ln 7 \\
x\ln 11 + \ln 11 &= x\ln 7 - \ln 7 \\
x\ln 11 - x\ln 7 &= -\ln 7 - \ln 11 \\
x(\ln 11 - \ln 7) &= -\ln 7 - \ln 11 \\
x &= \frac{-\ln 7 - \ln 11}{\ln 11 - \ln 7} \\
x &\approx -9.61
\end{aligned}$$

41. $9^{x+2} = 2^{5x-4}$

SOLUTION:

$$\begin{aligned}
9^{x+2} &= 2^{5x-4} \\
\ln 9^{x+2} &= \ln 2^{5x-4} \\
(x+2)\ln 9 &= (5x-4)\ln 2 \\
x\ln 9 + 2\ln 9 &= 5x\ln 2 - 4\ln 2 \\
x\ln 9 - 5x\ln 2 &= -4\ln 2 - 2\ln 9 \\
x(\ln 9 - 5\ln 2) &= -4\ln 2 - 2\ln 9 \\
x &= \frac{-4\ln 2 - 2\ln 9}{\ln 9 - 5\ln 2} \\
x &\approx 5.65
\end{aligned}$$

42. $4^{x-3} = 6^{2x-1}$

SOLUTION:

$$\begin{aligned}
4^{x-3} &= 6^{2x-1} \\
\ln 4^{x-3} &= \ln 6^{2x-1} \\
(x-3)\ln 4 &= (2x-1)\ln 6 \\
x\ln 4 - 3\ln 4 &= 2x\ln 6 - \ln 6 \\
x\ln 4 - 2x\ln 6 &= -\ln 6 + 3\ln 4 \\
x(\ln 4 - 2\ln 6) &= -\ln 6 + 3\ln 4 \\
x &= \frac{-\ln 6 + 3\ln 4}{\ln 4 - 2\ln 6} \\
x &\approx -1.08
\end{aligned}$$

43. $3^{4x+3} = 8^{-x+2}$

SOLUTION:

$$\begin{aligned} 3^{4x+3} &= 8^{-x+2} \\ \ln 3^{4x+3} &= \ln 8^{-x+2} \\ (4x+3)\ln 3 &= (-x+2)\ln 8 \\ 4x\ln 3 + 3\ln 3 &= -x\ln 8 + 2\ln 8 \\ 4x\ln 3 + x\ln 8 &= 2\ln 8 - 3\ln 3 \\ x(4\ln 3 + \ln 8) &= 2\ln 8 - 3\ln 3 \\ x &= \frac{2\ln 8 - 3\ln 3}{4\ln 3 + \ln 8} \\ x &\approx 0.13 \end{aligned}$$

44. $5^{3x-1} = 4^{x+1}$

SOLUTION:

$$\begin{aligned} 5^{3x-1} &= 4^{x+1} \\ \ln 5^{3x-1} &= \ln 4^{x+1} \\ (3x-1)\ln 5 &= (x+1)\ln 4 \\ 3x\ln 5 - \ln 5 &= x\ln 4 + \ln 4 \\ 3x\ln 5 - x\ln 4 &= \ln 4 + \ln 5 \\ x(3\ln 5 - \ln 4) &= \ln 4 + \ln 5 \\ x &= \frac{\ln 4 + \ln 5}{3\ln 5 - \ln 4} \\ x &\approx 0.87 \end{aligned}$$

45. $6^{x-2} = 5^{2x+3}$

SOLUTION:

$$\begin{aligned} 6^{x-2} &= 5^{2x+3} \\ \ln 6^{x-2} &= \ln 5^{2x+3} \\ (x-2)\ln 6 &= (2x+3)\ln 5 \\ x\ln 6 - 2\ln 6 &= 2x\ln 5 + 3\ln 5 \\ x\ln 6 - 2x\ln 5 &= 3\ln 5 + 2\ln 6 \\ x(\ln 6 - 2\ln 5) &= 3\ln 5 + 2\ln 6 \\ x &= \frac{3\ln 5 + 2\ln 6}{\ln 6 - 2\ln 5} \\ x &\approx -5.89 \end{aligned}$$

46. $8^{-2x-1} = 5^{-x+2}$

SOLUTION:

$$\begin{aligned} 8^{-2x-1} &= 5^{-x+2} \\ \ln 8^{-2x-1} &= \ln 5^{-x+2} \\ (-2x-1)\ln 8 &= (-x+2)\ln 5 \\ -2x\ln 8 - \ln 8 &= -x\ln 5 + 2\ln 5 \\ -2x\ln 8 + x\ln 5 &= 2\ln 5 + \ln 8 \\ x(-2\ln 8 + \ln 5) &= 2\ln 5 + \ln 8 \\ x &= \frac{2\ln 5 + \ln 8}{-2\ln 8 + \ln 5} \\ x &\approx -2.08 \end{aligned}$$

47. $2^{5x+6} = 4^{2x+1}$

SOLUTION:

$$\begin{aligned} 2^{5x+6} &= 4^{2x+1} \\ \ln 2^{5x+6} &= \ln 4^{2x+1} \\ (5x+6)\ln 2 &= (2x+1)\ln 4 \\ 5x\ln 2 + 6\ln 2 &= 2x\ln 4 + \ln 4 \\ 5x\ln 2 - 2x\ln 4 &= \ln 4 - 6\ln 2 \\ x(5\ln 2 - 2\ln 4) &= \ln 4 - 6\ln 2 \\ x &= \frac{\ln 4 - 6\ln 2}{5\ln 2 - 2\ln 4} \\ x &= -4 \end{aligned}$$

48. $6^{-x-2} = 9^{-x-1}$

SOLUTION:

$$\begin{aligned} 6^{-x-2} &= 9^{-x-1} \\ \ln 6^{-x-2} &= \ln 9^{-x-1} \\ (-x-2)\ln 6 &= (-x-1)\ln 9 \\ -x\ln 6 - 2\ln 6 &= -x\ln 9 - \ln 9 \\ -x\ln 6 + x\ln 9 &= -\ln 9 + 2\ln 6 \\ x(-\ln 6 + \ln 9) &= -\ln 9 + 2\ln 6 \\ x &= \frac{-\ln 9 + 2\ln 6}{-\ln 6 + \ln 9} \\ x &\approx 3.42 \end{aligned}$$

49. **ASTRONOMY** The brightness of two celestial bodies as seen from Earth can be compared by determining the variation in brightness between the two bodies. The variation in brightness V can be calculated by $V = 2.512^{m_f - m_b}$, where m_f is the magnitude of brightness of the fainter body and m_b is the magnitude of brightness of the brighter body.

a. The Sun has $m = -26.73$, and the full Moon has $m = -12.6$. Determine the variation in brightness between the Sun and the full Moon.
b. The variation in brightness between Mercury and Venus is 5.25. Venus has a magnitude of brightness of –3.7. Determine the magnitude of brightness of Mercury.
c. Neptune has a magnitude of brightness of 7.7, and the variation in brightness of Neptune and Jupiter is 15,856. What is the magnitude of brightness of Jupiter?

SOLUTION:

a.

$$\begin{aligned} V &= 2.512^{m_f - m_b} \\ &= 2.512^{-12.6-(-26.73)} \\ &= 2.512^{14.13} \\ &\approx 449{,}032 \end{aligned}$$

b.

$$\begin{aligned} V &= 2.512^{m_f - m_b} \\ 5.25 &= 2.512^{m_f - 5.25} \\ \ln 5.25 &= \ln 2.512^{m_f-(-3.7)} \\ \ln 5.25 &= (m_f + 3.7)\ln 2.512 \\ \frac{\ln 5.25}{\ln 2.512} &= m_f + 3.7 \\ \frac{\ln 5.25}{\ln 2.512} - 3.7 &= m_f \\ -1.9 &\approx m_f \end{aligned}$$

c.

$$\begin{aligned} V &= 2.512^{m_f - m_b} \\ 15{,}856 &= 2.512^{7.7 - m_b} \\ \ln 15{,}856 &= \ln 2.512^{7.7 - m_b} \\ \ln 15{,}856 &= (7.7 - m_b)\ln 2.512 \\ \frac{\ln 15{,}856}{\ln 2.512} &= 7.7 - m_b \\ m_b &= 7.7 - \frac{\ln 15{,}856}{\ln 2.512} \\ m_b &\approx -2.8 \end{aligned}$$

Solve each equation.

50. $e^{2x} + 3e^{x} - 130 = 0$

SOLUTION:

$$\begin{aligned} e^{2x} + 3e^{x} - 130 &= 0 \\ (e^{x} + 13)(e^{x} - 10) &= 0 \\ e^{x} + 13 &= 0 \\ e^{x} &= -13 \\ x &= \ln(-13) \\ e^{x} - 10 &= 0 \\ e^{x} &= 10 \\ x &= \ln 10 \\ x &\approx 2.30 \end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx 2.30$.

51. $e^{2x} - 15e^{x} + 56 = 0$

SOLUTION:

$$\begin{aligned} e^{2x} - 15e^{x} + 56 &= 0 \\ (e^{x} - 8)(e^{x} - 7) &= 0 \\ e^{x} - 8 &= 0 \\ e^{x} &= 8 \\ x &= \ln 8 \\ x &\approx 2.08 \\ e^{x} - 7 &= 0 \\ e^{x} &= 7 \\ x &= \ln 7 \\ x &\approx 1.95 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

52. $e^{2x} + 3e^{x} = -2$

SOLUTION:

$$\begin{aligned} e^{2x} + 3e^{x} &= -2 \\ e^{2x} + 3e^{x} + 2 &= 0 \\ (e^{x} + 1)(e^{x} + 2) &= 0 \\ e^{x} + 1 &= 0 \\ e^{x} &= -1 \\ x &= \ln(-1) \\ e^{x} + 2 &= 0 \\ e^{x} &= -2 \\ x &= \ln(-2) \end{aligned}$$

The logarithm of a negative number provides no real solution.

53. $6e^{2x} - 5e^{x} = 6$

SOLUTION:

$$\begin{aligned} 6e^{2x} - 5e^{x} &= 6 \\ 6e^{2x} - 5e^{x} - 6 &= 0 \\ (3e^{x} + 2)(2e^{x} - 3) &= 0 \\ 3e^{x} + 2 &= 0 \\ 3e^{x} &= -2 \\ e^{x} &= -\frac{2}{3} \\ x &= \ln\left(-\frac{2}{3}\right) \\ 2e^{x} - 3 &= 0 \\ 2e^{x} &= 3 \\ e^{x} &= 1.5 \\ x &= \ln 1.5 \\ x &\approx 0.41 \end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx 0.41$.

3-4 Exponential and Logarithmic Equations

54. $9e^{2x} - 3e^{x} = 6$

SOLUTION:

$$\begin{aligned} 9e^{2x} - 3e^{x} &= 6 \\ 9e^{2x} - 3e^{x} - 6 &= 0 \\ 3e^{2x} - e^{x} - 2 &= 0 \\ (3e^{x} + 2)(e^{x} - 1) &= 0 \\ 3e^{x} + 2 &= 0 \\ 3e^{x} &= -2 \\ e^{x} &= -\frac{2}{3} \\ x &= \ln\left(-\frac{2}{3}\right) \\ e^{x} - 1 &= 0 \\ e^{x} &= 1 \\ x &= \ln 1 \\ x &= 0 \end{aligned}$$

55. $8e^{4x} - 15e^{2x} + 7 = 0$

SOLUTION:

$$\begin{aligned} 8e^{4x} - 15e^{2x} + 7 &= 0 \\ (8e^{2x} - 7)(e^{2x} - 1) &= 0 \\ 8e^{2x} - 7 &= 0 \\ 8e^{2x} &= 7 \\ e^{2x} &= \frac{7}{8} \\ 2x &= \ln\frac{7}{8} \\ x &= \frac{\ln\frac{7}{8}}{2} \\ x &\approx -0.067 \\ e^{2x} - 1 &= 0 \\ e^{2x} &= 1 \\ 2x &= \ln 1 \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

56. $2e^{8x} + e^{4x} - 1 = 0$

SOLUTION:

$$\begin{aligned} 2e^{8x} + e^{4x} - 1 &= 0 \\ (2e^{4x} - 1)(e^{4x} + 1) &= 0 \\ 2e^{4x} - 1 &= 0 \\ 2e^{4x} &= 1 \\ e^{4x} &= 0.5 \\ 4x &= \ln 0.5 \\ x &= \frac{\ln 0.5}{4} \\ x &\approx -0.17 \\ e^{4x} + 1 &= 0 \\ e^{4x} &= -1 \\ 4x &= \ln(-1) \end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx -0.17$.

57. $2e^{5x} - 7e^{2x} - 15e^{-x} = 0$

SOLUTION:

$$\begin{aligned} 2e^{5x} - 7e^{2x} - 15e^{-x} &= 0 \\ 2e^{6x} - 7e^{3x} - 15 &= 0 \\ (2e^{3x} + 3)(e^{3x} - 5) &= 0 \\ 2e^{3x} + 3 &= 0 \\ 2e^{3x} &= -3 \\ e^{3x} &= -1.5 \\ 3x &= \ln(-1.5) \\ e^{3x} - 5 &= 0 \\ e^{3x} &= 5 \\ 3x &= \ln 5 \\ x &= \frac{\ln 5}{3} \\ x &\approx 0.54 \end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx 0.54$.

58. $10e^x - 15 - 45e^{-x} = 0$

SOLUTION:

$$\begin{aligned}
10e^x - 15 - 45e^{-x} &= 0\\
10e^{2x} - 15e^x - 45 &= 0\\
2e^{2x} - 3e^x - 9 &= 0\\
(2e^x + 3)(e^x - 3) &= 0\\
2e^{3x} + 3 &= 0\\
2e^{3x} &= -3\\
e^{3x} &= -1.5\\
3x &= \ln(-1.5)\\
e^x - 3 &= 0\\
e^x &= 3\\
x &= \ln 3\\
x &\approx 1.10
\end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx 1.10$.

59. $11e^x - 51 - 20e^{-x} = 0$

SOLUTION:

$$\begin{aligned}
11e^x - 51 - 20e^{-x} &= 0\\
11e^{2x} - 51e^x - 20 &= 0\\
(11e^x + 4)(e^x - 5) &= 0\\
11e^x + 4 &= 0\\
11e^x &= -4\\
e^x &= -\frac{4}{11}\\
x &= \ln\left(-\frac{4}{11}\right)\\
e^x - 5 &= 0\\
e^x &= 5\\
x &= \ln 5\\
x &\approx 1.61
\end{aligned}$$

The logarithm of a negative number provides no real solution, so $x \approx 1.61$.

3-4 Exponential and Logarithmic Equations

Solve each logarithmic equation.

60. $\ln x + \ln (x + 2) = \ln 63$

SOLUTION:

$$\begin{aligned} \ln x + \ln(x+2) &= \ln 63 \\ \ln[x(x+2)] &= \ln 63 \\ \ln(x^2+2x) &= \ln 63 \\ x^2+2x &= 63 \\ x^2+2x-63 &= 0 \\ (x+9)(x-7) &= 0 \\ x &= -9 \text{ or } 7 \end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

The logarithm of a negative number provides no real solution, so x cannot equal −9. The solution is 7.

61. $\ln x + \ln (x + 7) = \ln 18$

SOLUTION:

$$\begin{aligned} \ln x + \ln(x+7) &= \ln 18 \\ \ln[x(x+7)] &= \ln 18 \\ \ln(x^2+7x) &= \ln 18 \\ x^2+7x &= 18 \\ x^2+7x-18 &= 0 \\ (x+9)(x-2) &= 0 \\ x &= -9 \text{ or } 2 \end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

The logarithm of a negative number provides no real solution, so x cannot equal −9. The solution is 2.

62. $\ln (3x + 1) + \ln (2x - 3) = \ln 10$

SOLUTION:

$$\begin{aligned} \ln(3x+1)+\ln(2x-3) &= \ln 10 \\ \ln[(3x+1)(2x-3)] &= \ln 10 \\ (3x+1)(2x-3) &= \ln 10 \\ 6x^2-7x-3 &= 10 \\ 6x^2-7x-13 &= 0 \\ (x+1)(6x-13) &= 0 \\ x &= -1 \text{ or } \frac{13}{6} \end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

The logarithm of a negative number provides no real solution, so x cannot equal −1. The solution is $\frac{13}{6}$.

63. $\ln(x-3)+\ln(2x+3)=\ln(-4x^2)$

SOLUTION:

$$\begin{aligned}\ln(x-3)+\ln(2x+3)&=\ln(-4x^2)\\ \ln[(x-3)(2x+3)]&=\ln(-4x^2)\\ (x-3)(2x+3)&=-4x^2\\ 2x^2-3x-9&=-4x^2\\ 6x^2-3x-9&=0\\ 2x^2-x-3&=0\\ (2x-3)(x+1)&=0\\ x&=\frac{3}{2}\text{ or }-1\end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

$\frac{3}{2}-3<0$ and $-1-3<0$

The logarithm of a negative number provides no real solution, so there is no solution.

64. $\log(5x^2+4)=2\log 3x^2-\log(2x^2-1)$

SOLUTION:

$$\begin{aligned}\log(5x^2+4)&=2\log 3x^2-\log(2x^2-1)\\ \log(5x^2+4)&=\log\left(3x^2\right)^2-\log(2x^2-1)\\ \log(5x^2+4)&=\log 9x^4-\log(2x^2-1)\\ \log(5x^2+4)&=\log\frac{9x^4}{2x^2-1}\\ 5x^2+4&=\frac{9x^4}{2x^2-1}\\ (5x^2+4)(2x^2-1)&=9x^4\\ 10x^4+3x^2-4&=9x^4\\ x^4+3x^2-4&=0\\ (x^2+4)(x^2-1)&=0\\ x^2+4&=0\\ x^2&=-4\text{ (no real solution)}\\ x^2-1&=0\\ x^2&=1\\ x&=\pm1\end{aligned}$$

65. log $(x + 6)$ = log $(8x)$ − log $(3x + 2)$

SOLUTION:

$$\log(x+6) = \log 8x - \log(3x+2)$$
$$\log(x+6) = \log\frac{8x}{3x+2}$$
$$x+6 = \frac{8x}{3x+2}$$
$$(x+6)(3x+2) = 8x$$
$$3x^2 + 20x + 12 = 8x$$
$$3x^2 + 12x + 12 = 0$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)^2 = 0$$
$$x + 2 = 0$$
$$x = -2$$

Substitute into original equation to eliminate extraneous solutions.

$-2(8) < 0$

The logarithm of a negative number provides no real solution, so there is no solution.

66. $\ln(4x^2 - 3x) = \ln(16x - 12) - \ln x$

SOLUTION:

$$\ln(4x^2 - 3x) = \ln(16x - 12) - \ln x$$
$$\ln(4x^2 - 3x) = \ln\frac{16x - 12}{x}$$
$$4x^2 - 3x = \frac{16x - 12}{x}$$
$$x(4x^2 - 3x) = 16x - 12$$
$$4x^3 - 3x^2 = 16x - 12$$
$$4x^3 - 3x^2 - 16x + 12 = 0$$

Factor.

2	4	−3	−16	12
		8	10	−12
	4	5	−6	0

$$4x^3 - 3x^2 - 16x + 12 = 0$$
$$(x - 2)(4x^2 + 5x - 6) = 0$$
$$(x - 2)(x + 2)(4x - 3) = 0$$
$$x = 2, -2, or\ \frac{3}{4}$$

Substitute into original equation to eliminate extraneous solutions.

$$16(-2) - 12 = -32 - 12$$
$$= -44$$
$$16\left(\frac{3}{4}\right) - 12 = 12 - 12$$
$$= 0$$

The logarithm of a negative number provides no real solution, so $x = 2$.

67. $\ln(3x^2 - 4) + \ln(x^2 + 1) = \ln(2 - x^2)$

SOLUTION:

$$\ln(3x^2 - 4) + \ln(x^2 + 1) = \ln(2 - x^2)$$
$$\ln[(3x^2 - 4)(x^2 + 1)] = \ln(2 - x^2)$$
$$(3x^2 - 4)(x^2 + 1) = 2 - x^2$$
$$3x^4 - x^2 - 4 = 2 - x^2$$
$$3x^4 - 6 = 0$$
$$3x^4 = 6$$
$$x^4 = 2$$
$$x = \pm\sqrt[4]{2}$$
$$x \approx \pm 1.19$$

68. **SOUND** Noise-induced hearing loss (NIHL) accounts for 25% of hearing loss in the United States. Exposure to sounds of 85 decibels or higher for an extended period can cause NIHL. Recall that the decibels (dB) produced by

a sound of intensity I can be calculated by $dB = 10\log\left(\frac{I}{1\times10^{-12}}\right)$.

Intensity (W/m³)	Sound
316.227	fireworks
31.623	jet plane
3.162	ambulance
0.316	rock concert
0.032	headphones
0.003	hair dryer

Source: Dangerous Decibels

a. Which of the sounds listed in the table produce enough decibels to cause NIHL?
b. Determine the number of hair dryers that would produce the same number of decibels produced by a rock concert. Round to the nearest whole number.
c. How many jet planes would it take to produce the same number of decibels as a firework display? Round to the nearest whole number.

SOLUTION:

a.

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{316.227}{1\times10^{-12}}\right) \\ &\approx 145.0 \end{aligned}$$

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{31.623}{1\times10^{-12}}\right) \\ &\approx 135.0 \end{aligned}$$

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{3.162}{1\times10^{-12}}\right) \\ &\approx 125.0 \end{aligned}$$

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{0.316}{1\times10^{-12}}\right) \\ &\approx 115.0 \end{aligned}$$

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{0.032}{1\times10^{-12}}\right) \\ &\approx 105.0 \end{aligned}$$

$$\begin{aligned} dB &= 10\log\left(\frac{I}{1\times10^{-12}}\right) \\ &= 10\log\left(\frac{0.003}{1\times10^{-12}}\right) \\ &\approx 94.8 \end{aligned}$$

all of the objects

b.

$$\frac{0.316}{0.003} \approx 105.3$$

c.

$$\frac{316.227}{31.623} \approx 10.0$$

3-4 Exponential and Logarithmic Equations

Solve each logarithmic equation.

69. $\log_2 (2x - 6) = 3 + \log_2 x$

SOLUTION:

$$\begin{aligned}\log_2(2x-6) &= 3+\log_2 x\\ \log_2(2x-6)-\log_2 x &= 3\\ \log_2\frac{2x-6}{x} &= 3\\ \frac{2x-6}{x} &= 2^3\\ \frac{2x-6}{x} &= 8\\ 2x-6 &= 8x\\ -6 &= 6x\\ -1 &= x\end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

$$\begin{aligned}2(-1)-6 &= -8\\ -8 &< 0\end{aligned}$$

The logarithm of a negative number provides no real solution, so there is no solution.

70. $\log (3x + 2) = 1 + \log 2x$

SOLUTION:

$$\begin{aligned}\log(3x+2) &= 1+\log 2x\\ \log(3x+2)-\log 2x &= 1\\ \log\frac{3x+2}{2x} &= 1\\ \frac{3x+2}{2x} &= 10\\ 3x+2 &= 20x\\ 2 &= 17x\\ \frac{2}{17} &= x\\ 0.12 &\approx x\end{aligned}$$

3-4 Exponential and Logarithmic Equations

71. $\log x = 1 - \log (x - 3)$

SOLUTION:

$$\begin{aligned}\log x &= 1 - \log(x-3)\\ \log x + \log(x-3) &= 1\\ \log[x(x-3)] &= 1\\ \log(x^2-3x) &= 1\\ x^2-3x &= 10\\ x^2-3x-10 &= 0\\ (x-5)(x+2) &= 0\\ x &= 5 \text{ or } -2\end{aligned}$$

Substitute into original equation to eliminate extraneous solutions.

$-2 - 3 = -5$

The logarithm of a negative number provides no real solution, so $x = 5$.

72. $\log 50x = 2 + \log (2x - 3)$

SOLUTION:

$$\begin{aligned}\log 50x &= 2 + \log(2x-3)\\ \log 50x - \log(2x-3) &= 2\\ \log\frac{50x}{2x-3} &= 2\\ \frac{50x}{2x-3} &= 10^2\\ \frac{50x}{2x-3} &= 100\\ 50x &= 200x-300\\ 0 &= 150x-300\\ 300 &= 150x\\ 2 &= x\end{aligned}$$

73. $\log_9 9x - 2 = -\log_9 x$

SOLUTION:

$$\begin{aligned}\log_9 9x - 2 &= -\log_9 x\\ \log_9 9x + \log_9 x &= 2\\ \log_9[9x(x)] &= 2\\ \log_9 9x^2 &= 2\\ \log_9(3x)^2 &= 2\\ 2\log_9 3x &= 2\\ \log_9 3x &= 1\\ 3x &= 9\\ x &= 3\end{aligned}$$

3-4 Exponential and Logarithmic Equations

74. log (x – 10) = 3 + log (x – 3)

SOLUTION:

$$\log(x-10) = 3 + \log(x-3)$$
$$\log(x-10) - \log(x-3) = 3$$
$$\log\left(\frac{x-10}{x-3}\right) = 3$$
$$\frac{x-10}{x-3} = 10^3$$
$$\frac{x-10}{x-3} = 1000$$
$$x - 10 = 1000x - 3000$$
$$2990 = 999x$$
$$2.99 \approx x$$

2.99 − 10 < 0, so log (x − 10) provides no real solution.

Solve each logarithmic equation.

75. log (29,995x + 40,225) = 4 + log (3x + 4)

SOLUTION:

$$\log(29{,}995x + 40{,}225) = 4 + \log(3x+4)$$
$$\log(29{,}995x + 40{,}225) - \log(3x+4) = 4$$
$$\log\left(\frac{29{,}995x + 40{,}225}{3x+4}\right) = 4$$
$$\frac{29{,}995x + 40{,}225}{3x+4} = 10^4$$
$$\frac{29{,}995x + 40{,}225}{3x+4} = 10{,}000$$
$$29{,}995x + 40{,}225 = 30{,}000x + 40{,}000$$
$$225 = 5x$$
$$25 = x$$

76. $\log_{\frac{1}{4}}\left(\frac{1}{4}x\right) = -\log_{\frac{1}{4}}(x+8) - \frac{5}{2}$

SOLUTION:

$$\begin{aligned}
\log_{\frac{1}{4}}\left(\frac{1}{4}x\right) &= -\log_{\frac{1}{4}}(x+8) - \frac{5}{2} \\
\log_{\frac{1}{4}}\left(\frac{1}{4}x\right) + \log_{\frac{1}{4}}(x+8) &= -\frac{5}{2} \\
\log_{\frac{1}{4}}\left[\frac{1}{4}x(x+8)\right] &= -\frac{5}{2} \\
\log_{\frac{1}{4}}\left(\frac{1}{4}x^2 + 2x\right) &= -\frac{5}{2} \\
\frac{1}{4}x^2 + 2x &= \left(\frac{1}{4}\right)^{-\frac{5}{2}} \\
\frac{1}{4}x^2 + 2x &= \left(2^{-2}\right)^{-\frac{5}{2}} \\
\frac{1}{4}x^2 + 2x &= 2^5 \\
\frac{1}{4}x^2 + 2x &= 32 \\
\frac{1}{4}x^2 + 2x - 32 &= 0 \\
x^2 + 8x - 128 &= 0 \\
(x+16)(x-8) &= 0 \\
x &= 8 \text{ or } -16
\end{aligned}$$

The logarithm of a negative number provides no real solution, so $x = 8$.

77. $\log x = 3 - \log(100x + 900)$

SOLUTION:

$$\begin{aligned}
\log x &= 3 - \log(100x + 900) \\
\log x + \log(100x + 900) &= 3 \\
\log[x(100x + 900)] &= 3 \\
\log(100x^2 + 900x) &= 3 \\
100x^2 + 900x &= 10^3 \\
100x^2 + 900x &= 1000 \\
100x^2 + 900x - 1000 &= 0 \\
x^2 + 9x - 10 &= 0 \\
(x+10)(x-1) &= 0 \\
x &= -10 \text{ or } 1
\end{aligned}$$

The logarithm of a negative number provides no real solution, so $x = 1$.

3-4 Exponential and Logarithmic Equations

78. $\log_5 \frac{x2}{8} - 3 = \log_5 \frac{x}{40}$

SOLUTION:

$$\log_5 \frac{x^2}{8} - 3 = \log_5 \frac{x}{40}$$
$$\log_5 \frac{x^2}{8} - \log_5 \frac{x}{40} = 3$$
$$\log_5 \frac{\frac{x^2}{8}}{\frac{x}{40}} = 3$$
$$\frac{\frac{x^2}{8}}{\frac{x}{40}} = 5^3$$
$$\frac{x^2}{8} = 125 \cdot \frac{x}{40}$$
$$\frac{x^2}{8} = \frac{25x}{8}$$
$$x^2 = 25x$$
$$x = 25$$

79. $\log 2x + \log\left(4 - \frac{16}{x}\right) = 2\log(x-2)$

SOLUTION:

$$\log 2x + \log\left(4 - \frac{16}{x}\right) = 2\log(x-2)$$
$$\log\left[2x\left(4 - \frac{16}{x}\right)\right] = \log(x-2)^2$$
$$\log(8x - 32) = \log(x^2 - 4x + 4)$$
$$8x - 32 = x^2 - 4x + 4$$
$$0 = x^2 - 12x + 36$$
$$0 = (x-6)^2$$
$$0 = x - 6$$
$$6 = x$$

80. **TECHNOLOGY** A chain of retail computer stores opened 2 stores in its first year of operation. After 8 years of operation, the chain consisted of 206 stores.

a. Write a continuous exponential equation to model the number of stores N as a function of year of operation t. Round k to the nearest hundredth.

b. Use the model you found in part **a** to predict the number of stores in the 12th year of operation.

SOLUTION:

a. The general continuous exponential equation is $N = N_0e^{kt}$. The initial number of stores N_0 is 2. At year 1, $t = 1$ and $N = N_0 = 2$, so the equation modeling the situation is $N = N_0e^{k(t-1)}$. Use the data to solve for k.

$$\begin{aligned} N &= N_0e^{k(t-1)} \\ 206 &= 2e^{k(8-1)} \\ 206 &= 2e^{7k} \\ 103 &= e^{7k} \\ \ln 103 &= 7k \\ \frac{\ln 103}{7} &= k \\ 0.66 &\approx k \end{aligned}$$

The equation is $N = 2e^{0.66(t-1)}$.

b.

$$\begin{aligned} N &= 2e^{0.66(t-1)} \\ &= 2e^{0.66(12-1)} \\ &= 2e^{7.26} \\ &\approx 2844 \end{aligned}$$

81. **STOCK** The price per share of a coffee chain's stock was \$0.93 in a month during its first year of trading. During its fifth year of trading, the price per share of stock was \$3.52 during the same month.

a. Write a continuous exponential equation to model the price of stock P as a function of year of trading t. Round k to the nearest ten-thousandth.

b. Use the model you found in part **a** to predict the price of the stock during the ninth year of trading.

SOLUTION:

a. The general continuous exponential equation is $P = P_0e^{kt}$. The initial stock price P_0 is \$0.93 or 0.93. At year 1, $t = 1$ and $P = P_0 = 0.93$, so the equation modeling the situation is $P = P_0e^{k(t-1)}$. Use the information to solve for k.

$$\begin{aligned} P &= P_0e^{k(t-1)} \\ 3.52 &= 0.93e^{k(5-1)} \\ \frac{3.52}{0.93} &= e^{4k} \\ \ln\left(\frac{3.52}{0.93}\right) &= 4k \\ \frac{\ln\left(\frac{3.52}{0.93}\right)}{4} &= k \\ 0.3328 &\approx k \end{aligned}$$

The equation is $P = 0.93e^{0.3328(t-1)}$.

b.

$$\begin{aligned} P &= 0.93e^{0.3328(9-1)} \\ &= 0.93e^{2.6624} \\ &\approx 13.33 \end{aligned}$$

Solve each logarithmic equation.

82. $5 + 5 \log_{100} x = 20$

SOLUTION:

$$\begin{aligned} 5 + 5\log_{100} x &= 20 \\ 5\log_{100} x &= 15 \\ \log_{100} x &= 3 \\ 100^{\log_{100} x} &= 100^3 \\ x &= 1{,}000{,}000 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

83. $6 + 2\log_{e^2} x = 30$

SOLUTION:

$$\begin{aligned} 6 + 2\log_{e^2} x &= 30 \\ 2\log_{e^2} x &= 24 \\ \log_{e^2} x &= 12 \\ (e^2)^{\log_{e^2} x} &= (e^2)^{12} \\ x &= e^{24} \\ x &\approx 2.65 \times 10^{10} \end{aligned}$$

84. $5 - 4\log_{\frac{1}{2}} x = -19$

SOLUTION:

$$\begin{aligned} 5 - 4\log_{\frac{1}{2}} x &= -19 \\ -4\log_{\frac{1}{2}} x &= -24 \\ \log_{\frac{1}{2}} x &= 6 \\ \left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} x} &= \left(\frac{1}{2}\right)^6 \\ x &= \frac{1}{64} \end{aligned}$$

85. $36 + 3\log_3 x = 60$

SOLUTION:

$$\begin{aligned} 36 + 3\log_3 x &= 60 \\ 3\log_3 x &= 24 \\ \log_3 x &= 8 \\ 3^{\log_3 x} &= 3^8 \\ x &= 6561 \end{aligned}$$

86. **ACIDITY** The acidity of a substance is determined by its concentration of H^+ ions. Because the H^+ concentration of substances can vary by several orders of magnitude, the logarithmic pH scale is used to indicate acidity. pH can be calculated by pH = –log [H^+], where [H^+] is the concentration of H^+ ions in moles per liter.

Item	pH
ammonia	11.0
baking soda	8.3
human blood	7.4
water	7.0
milk	6.6
apples	3.0
lemon juice	2.0

a. Determine the H^+ concentration of baking soda.

b. How many times as acidic is milk than human blood?

c. By how many orders of magnitude is the $[H^+]$ of lemon juice greater than $[H^+]$ of ammonia?

d. How many moles of H^+ ions are in 1500 liters of human blood?

SOLUTION:

a.

$$\begin{aligned} pH &= -\log[H^+] \\ 8.3 &= -\log[H^+] \\ -8.3 &= \log[H^+] \\ 10^{-8.3} &= [H^+] \\ 5.01 \times 10^{-9} &\approx [H^+] \end{aligned}$$

b.

$$\begin{aligned} pH &= -\log[H^+] \\ 6.6 &= -\log[H^+] \\ -6.6 &= \log[H^+] \\ 10^{-6.6} &= [H^+] \\ pH &= -\log[H^+] \\ 7.4 &= -\log[H^+] \\ -7.4 &= \log[H^+] \\ 10^{-7.4} &= [H^+] \\ \frac{10^{-6.6}}{10^{-7.4}} &= 10^{0.8} \approx 6.31 \end{aligned}$$

c. A pH of 2.0 is equal to a $[H^+]$ of 10^{-2} while a pH of 11.0 is equal to a $[H^+]$ of 10^{-11}.
$-2 - (-11) = 9$

d.

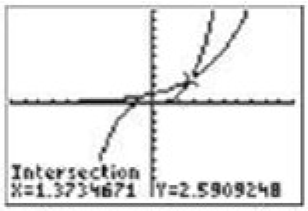
$$\begin{aligned} pH &= -\log[H^+] \\ 7.4 &= -\log[H^+] \\ -7.4 &= \log[H^+] \\ 10^{-7.4} &= [H^+] \\ 3.98 \times 10^{-8} &\approx [H^+] \\ 3.98 \times 10^{-8} \times 1500 &\approx 5.97 \times 10^{-5} \end{aligned}$$

3-4 Exponential and Logarithmic Equations

GRAPHING CALCULATOR Solve each equation algebraically, if possible. If not possible, approximate the solution to the nearest hundredth using a graphing calculator.

87. $x^3 = 2^x$

SOLUTION:



[−5, 5] scl: 0.5 by [−10, 10] scl: 1

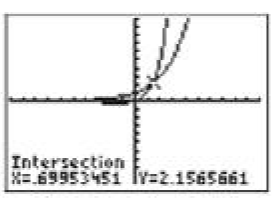
88. $\log_2 x = \log_8 x$

SOLUTION:

$$\begin{aligned} \log_2 x &= \log_8 x \\ \frac{\ln x}{\ln 2} &= \frac{\ln x}{\ln 8} \\ \ln 8 \ln x &= \ln 2 \ln x \\ \ln 8 \ln x - \ln 2 \ln x &= 0 \\ \ln x(\ln 8 - \ln 2) &= 0 \\ \ln x &= 0 \\ x &= e^0 \\ x &= 1 \end{aligned}$$

89. $3^x = x(5^x)$

SOLUTION:



[−5, 5] scl: 0.5 by [−10, 10] scl: 1

90. $\log_x 5 = \log_5 x$

SOLUTION:

$$\begin{aligned} \log_x 5 &= \log_5 x \\ \frac{\ln 5}{\ln x} &= \frac{\ln x}{\ln 5} \\ \ln x \ln x &= \ln 5 \ln 5 \\ (\ln x)^2 &= (\ln 5)^2 \\ \ln x &= \ln 5 \\ x &= 5 \end{aligned}$$

91. **RADIOACTIVITY** The isotopes phosphorous-32 and sulfur-35 both exhibit radioactive decay. The half-life of phosphorous-32 is 14.282 days. The half-life of sulfur-35 is 87.51 days.
a. Write equations to express the radioactive decay of phosphorous-32 and sulfur-35 in terms of time *t* in days and ratio *R* of remaining isotope using the general equation for radioactive decay, $A = t \cdot \frac{\ln R}{-0.693}$, where *A* is the number of days the isotope has decayed and *t* is the half-life in days.
b. At what value of *R* will sulfur-35 have been decaying 5 days longer than phosphorous-32?

SOLUTION:

a. phosphorous-32: $14.282\,\frac{ln(R)}{-0.693}$;
sulfur-35: $87.51\,\frac{ln(R)}{-0.693}$

b.

$$A_p = 14.282\frac{\ln R}{-0.693}$$
$$A_s = 87.51\frac{\ln R}{-0.693}$$
$$A_s = A_p + 5$$
$$87.51\frac{\ln R}{-0.693} = 14.282\frac{\ln R}{-0.693} + 5$$
$$-\frac{87.51}{0.693}\ln R + \frac{14.282}{0.693}\ln R = 5$$
$$\left(\frac{14.282}{0.693} - \frac{87.51}{0.693}\right)\ln R = 5$$
$$\left(-\frac{73.228}{0.693}\right)\ln R = 5$$
$$\ln R = \frac{5}{-\frac{73.228}{0.693}}$$
$$R = e^{\frac{5}{-\frac{73.228}{0.693}}}$$
$$R \approx 0.954$$

Solve each exponential inequality.

92. $2 \le 2^x \le 32$

SOLUTION:

$$
\begin{aligned}
2 &\le 2^x \le 32 \\
\ln 2 &\le \ln 2^x \le \ln 32 \\
\ln 2 &\le x \ln 2 \le \ln 32 \\
\frac{\ln 2}{\ln 2} &\le x \le \frac{\ln 32}{\ln 2} \\
1 &\le x \le \frac{\ln 2^5}{\ln 2} \\
1 &\le x \le \frac{5 \ln 2}{\ln 2} \\
1 &\le x \le 5
\end{aligned}
$$

93. $9 < 3^y < 27$

SOLUTION:

$$
\begin{aligned}
9 &< 3^y < 27 \\
\ln 9 &< \ln 3^y < \ln 27 \\
\ln 3^2 &< y \ln 3 < \ln 3^3 \\
2 \ln 3 &< y \ln 3 < 3 \ln 3 \\
2 &< y < 3
\end{aligned}
$$

94. $\frac{1}{4096} \le 8^p \le \frac{1}{64}$

SOLUTION:

$$
\begin{aligned}
\frac{1}{4096} &\le 8^p \le \frac{1}{64} \\
\ln \frac{1}{4096} &\le \ln 8^p \le \ln \frac{1}{64} \\
\ln 8^{-4} &\le p \ln 8 \le \ln 8^{-2} \\
-4 \ln 8 &\le p \ln 8 \le -2 \ln 8 \\
-4 &\le p \le -2
\end{aligned}
$$

95. $\frac{1}{2197} < 13^f \leq \frac{1}{13}$

SOLUTION:

$$\begin{aligned} \frac{1}{2197} &< 13^f \leq \frac{1}{13} \\ \ln \frac{1}{2197} &< \ln 13^f \leq \ln \frac{1}{13} \\ \ln 13^{-3} &< f \ln 13 \leq \ln 13^{-1} \\ -3 \ln 13 &< f \ln 13 \leq -\ln 13 \\ -3 &< f \leq -1 \end{aligned}$$

96. $10 < 10^d < 100{,}000$

SOLUTION:

$$\begin{aligned} 10 &< 10^d < 100{,}000 \\ \ln 10 &< \ln 10^d < \ln 100{,}000 \\ \ln 10 &< d \ln 10 < \ln 10^5 \\ \ln 10 &< d \ln 10 < 5 \ln 10 \\ 1 &< d < 5 \end{aligned}$$

97. $4000 > 5^q > 125$

SOLUTION:

$$\begin{aligned} 4000 &> 5^q > 125 \\ \ln 4000 &> \ln 5^q > \ln 125 \\ \ln 4000 &> q \ln 5 > \ln 5^3 \\ \ln 4000 &> q \ln 5 > 3 \ln 5 \\ \frac{\ln 4000}{\ln 5} &> q > 3 \\ 5.15 &> q > 3 \end{aligned}$$

98. $49 < 7^z < 1000$

SOLUTION:

$$\begin{aligned} 49 &< 7^z < 1000 \\ \ln 49 &< \ln 7^z < \ln 1000 \\ \ln 7^2 &< z \ln 7 < \ln 1000 \\ 2 \ln 7 &< z \ln 7 < \ln 1000 \\ 2 &< z < \frac{\ln 1000}{\ln 7} \\ 2 &< z < 3.55 \end{aligned}$$

3-4 Exponential and Logarithmic Equations

99. $10{,}000 < 10^a < 275{,}000$

SOLUTION:

$$\begin{aligned} 10{,}000 &< 10^a < 275{,}000 \\ \log 10{,}000 &< \log 10^a < \log 275{,}000 \\ \log 10^4 &< a < \log 275{,}000 \\ 4 &< a < 5.44 \end{aligned}$$

100. $\frac{1}{15} \geq 4^b \geq \frac{1}{64}$

SOLUTION:

$$\begin{aligned} &\frac{1}{15} \geq 4^b \geq \frac{1}{64} \\ &\ln\frac{1}{15} \geq \ln 4^b \geq \ln\frac{1}{64} \\ &\ln\frac{1}{15} \geq b\ln 4 \geq \ln 4^{-3} \\ &\ln\frac{1}{15} \geq b\ln 4 \geq -3\ln 4 \\ &\frac{\ln\frac{1}{15}}{\ln 4} \geq b \geq -3 \\ &-1.95 \geq b \geq -3 \end{aligned}$$

101. $\frac{1}{2} \geq e^c \geq \frac{1}{100}$

SOLUTION:

$$\begin{aligned} &\frac{1}{2} \geq e^c \geq \frac{1}{100} \\ &\ln\frac{1}{2} \geq \ln e^c \geq \ln\frac{1}{100} \\ &\ln\frac{1}{2} \geq c \geq \ln\frac{1}{100} \\ &-0.69 \geq c \geq -4.61 \end{aligned}$$

102. **FORENSICS** Forensic pathologists perform autopsies to determine time and cause of death. The time t in hours since death can be calculated by $t = -10\ln\left(\frac{T - R_t}{98.6 - R_t}\right)$, where T is the temperature of the body and R_t is the room temperature.

a. A forensic pathologist measures the body temperature to be 93ºF in a room that is 72ºF. What is the time of death?

b. A hospital patient passed away 4 hours ago. If the hospital has an average temperature of 75ºF, what is the body temperature?

c. A patient's temperature was 89ºF 3.5 hours after the patient passed away. Determine the room temperature.

SOLUTION:

a.

$$\begin{aligned} t &= -10\ln\left(\frac{T-R_t}{98.6-R_t}\right) \\ &= -10\ln\left(\frac{93-72}{98.6-72}\right) \\ &= -10\ln\left(\frac{21}{26.6}\right) \\ &\approx 2.36 \end{aligned}$$

b.

$$\begin{aligned} t &= -10\ln\left(\frac{T-R_t}{98.6-R_t}\right) \\ 4 &= -10\ln\left(\frac{T-75}{98.6-75}\right) \\ -0.4 &= \ln\left(\frac{T-75}{23.6}\right) \\ e^{-0.4} &= \frac{T-75}{23.6} \\ 23.6e^{-0.4} &= T-75 \\ 23.6e^{-0.4}+75 &= T \\ 90.8 &\approx T \end{aligned}$$

c.

$$\begin{aligned} t &= -10\ln\left(\frac{T-R_t}{98.6-R_t}\right) \\ 3.5 &= -10\ln\left(\frac{89-R_t}{98.6-R_t}\right) \\ -0.35 &= \ln\left(\frac{89-R_t}{98.6-R_t}\right) \\ e^{-0.35} &= \frac{89-R_t}{98.6-R_t} \\ e^{-0.35}(98.6-R_t) &= 89-R_t \\ e^{-0.35}(98.6)-R_te^{-0.35} &= 89-R_t \\ e^{-0.35}(98.6)-89 &= R_te^{-0.35}-R_t \\ e^{-0.35}(98.6)-89 &= R_t(e^{-0.35}-1) \\ \frac{e^{-0.35}(98.6)-89}{e^{-0.35}-1} &= R_t \\ 66 &\approx R_t \end{aligned}$$

103. **MEDICINE** Fifty people were treated for a virus on the same day. The virus is highly contagious, and the individuals must stay in the hospital until they have no symptoms. The number of people p who show symptoms

after t days can be modeled by $p = \dfrac{52.76}{1+0.03e0.75t}$.

a. How many show symptoms after 5 days?

b. Solve the equation for t.

c. How many days will it take until only one person shows symptoms?

SOLUTION:

a.

$$p = \frac{52.76}{1+0.03e^{0.75t}}$$
$$p = \frac{52.76}{1+0.03e^{0.75(5)}}$$
$$p \approx 23$$

b.

$$p = \frac{52.76}{1+0.03e^{0.75t}}$$
$$p(1+0.03e^{0.75t}) = 52.76$$
$$1+0.03e^{0.75t} = \frac{52.76}{p}$$
$$0.03e^{0.75t} = \frac{52.76}{p} - 1$$
$$e^{0.75t} = \frac{\frac{52.76}{p}-1}{0.03}$$
$$\ln e^{0.75t} = \ln\left(\frac{\frac{52.76}{p}-1}{0.03}\right)$$
$$0.75t = \ln\left(\frac{\frac{52.76}{p}-1}{0.03}\right)$$
$$t = \frac{\ln\left(\frac{\frac{52.76}{p}-1}{0.03}\right)}{0.75}$$

c.

$$t=\frac{\ln\left(\frac{\frac{52.76}{p}-1}{0.03}\right)}{0.75}$$
$$=\frac{\ln\left(\frac{\frac{52.76}{1}-1}{0.03}\right)}{0.75}$$
$$=\frac{\ln\left(\frac{51.76}{0.03}\right)}{0.75}$$
$$\approx 10$$

Solve each equation.

104. $27=\dfrac{12}{1-\frac{1}{2}e^{-x}}$

SOLUTION:

$$27=\frac{12}{1-\frac{1}{2}e^{-x}}$$
$$27\left(1-\frac{1}{2}e^{-x}\right)=12$$
$$27-13.5e^{-x}=12$$
$$-13.5e^{-x}=-15$$
$$e^{-x}=\frac{10}{9}$$
$$-x=\ln\frac{10}{9}$$
$$x=-\ln\frac{10}{9}$$
$$x\approx -0.11$$

3-4 Exponential and Logarithmic Equations

105. $22 = \dfrac{L}{1 + \dfrac{L-3}{3}e^{-15}}$

SOLUTION:

$$
\begin{aligned}
22 &= \frac{L}{1 + \frac{L-3}{3}e^{-15}} \\
22\left(1 + \frac{L-3}{3}e^{-15}\right) &= L \\
22 + \frac{22L - 66}{3}e^{-15} &= L \\
22 + \frac{22e^{-15}L}{3} - 22e^{-15} &= L \\
\frac{22e^{-15}L}{3} - L &= 22e^{-15} - 22 \\
L\left(\frac{22e^{-15}}{3} - 1\right) &= 22e^{-15} - 22 \\
L &= \frac{22e^{-15} - 22}{\frac{22e^{-15}}{3} - 1} \\
L &\approx 22
\end{aligned}
$$

106. $1000 = \dfrac{10,000}{1 + 19e^{-t}}$

SOLUTION:

$$
\begin{aligned}
1000 &= \frac{10,000}{1 + 19e^{-t}} \\
1000\left(1 + 19e^{-t}\right) &= 10,000 \\
1000 + 19,000e^{-t} &= 10,000 \\
19,000e^{-t} &= 9000 \\
e^{-t} &= \frac{9}{19} \\
-t &= \ln\frac{9}{19} \\
t &= -\ln\frac{9}{19} \\
t &\approx 0.75
\end{aligned}
$$

107. $300 = \frac{400}{1+3e-2k}$

SOLUTION:

$$\begin{aligned} 300 &= \frac{400}{1+3e^{-2k}} \\ 300\left(1+3e^{-2k}\right) &= 400 \\ 300+900e^{-2k} &= 400 \\ 900e^{-2k} &= 100 \\ e^{-2k} &= \frac{1}{9} \\ -2k &= \ln\frac{1}{9} \\ k &= \frac{\ln\frac{1}{9}}{-2} \\ k &\approx 1.10 \end{aligned}$$

108. $16^x + 4^x - 6 = 0$

SOLUTION:

$$\begin{aligned} 16^x + 4^x - 6 &= 0 \\ (4^x)^2 + 4^x - 6 &= 0 \\ (4^x + 3)(4^x - 2) &= 0 \\ 4^x + 3 &= 0 \\ 4^x &= -3 \text{ (no solution)} \\ 4^x - 2 &= 0 \\ 4^x &= 2 \\ (2^2)^x &= 2 \\ 2^{2x} &= 2 \\ 2x &= 1 \\ x &= 0.5 \end{aligned}$$

109. $\frac{ex + e - x}{ex - e - x} = 6$

SOLUTION:

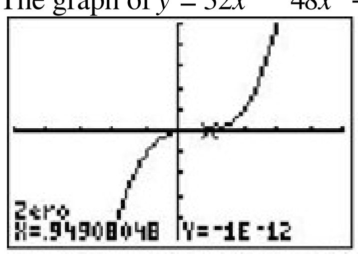
$$\frac{e^x + e^{-x}}{e^x - e^{-x}} = 6$$
$$e^x + e^{-x} = 6(e^x - e^{-x})$$
$$e^x + e^{-x} = 6e^x - 6e^{-x}$$
$$7e^{-x} = 5e^x$$
$$\frac{7}{5} = e^{2x}$$
$$\ln\frac{7}{5} = 2x$$
$$\frac{\ln\frac{7}{5}}{2} = x$$
$$0.17 \approx x$$

110. $\frac{ln(4x+2)}{ln(4x-2)} = 3$

SOLUTION:

$$\frac{\ln(4x+2)}{\ln(4x-2)} = 3$$
$$\ln(4x+2) = 3\ln(4x-2)$$
$$\ln(4x+2) = \ln(4x-2)^3$$
$$4x+2 = (4x-2)^3$$
$$4x+2 = 64x^3 - 96x^2 + 48x - 8$$
$$0 = 64x^3 - 96x^2 + 44x - 10$$
$$0 = 32x^3 - 48x^2 + 22x - 5$$

The graph of $y = 32x^3 - 48x^2 + 22x - 5$ has a zero at about 0.949.



[−5, 5] scl: 1 by [−500, 500] scl: 100

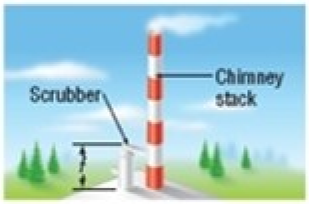
111. $\frac{ex - e - x}{ex + e - x} = \frac{1}{2}$

SOLUTION:

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{2}$$
$$2(e^x - e^{-x}) = e^x + e^{-x}$$
$$2e^x - 2e^{-x} = e^x + e^{-x}$$
$$e^x = 3e^{-x}$$
$$e^x = \frac{3}{e^x}$$
$$e^{2x} = 3$$
$$\ln e^{2x} = \ln 3$$
$$2x = \ln 3$$
$$x = \frac{\ln 3}{2}$$
$$x \approx 0.549$$

3-4 Exponential and Logarithmic Equations

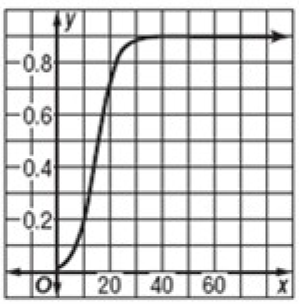
112. **POLLUTION** Some factories have added filtering systems called *scrubbers* to their smokestacks in order to reduce pollution emissions. The percent of pollution P removed after f feet of length of a particular scrubber can be modeled by $P = \frac{0.9}{1 + 70e^{-0.28f}}$.



a. Graph the percent of pollution removed as a function of scrubber length.
b. Determine the maximum percent of pollution that can be removed by the scrubber. Explain your reasoning.
c. Approximate the maximum length of scrubber that a factory should choose to use. Explain.

SOLUTION:

a.



b. Less than 90%; sample answer: As f approaches ∞, $e^{-0.28f}$ approaches 0. Therefore, P approaches $\frac{0.9}{1+0}$ or 0.9. The graph has a horizontal asymptote at 0.9, so the percent of pollution removed must be less than 90%.
c. Sample answer: The factory should choose a scrubber length of approximately 30 feet to maximize pollution reduction and minimize the materials used on the scrubber. Making the scrubber longer than 30 feet results in a minimal gain in pollution reduction.

113. **REASONING** What is the maximum number of extraneous solutions that a logarithmic equation can have? Explain your reasoning.

SOLUTION:

A logarithmic equation can have infinitely many extraneous solutions. For example, an infinite number of terms, $\ln x_1$, $\ln x_2$, $\ln x_3$, etc., can be combined to form an equation like $\ln x_1 + \ln x_2 + \ln x_3 + \ldots = n$. This equation can be simplified to $(x - a)(x - b)(x - c) \ldots = 0$, where a, b, c, … appear to be solutions but are extraneous because they cause $\ln x_1$, $\ln x_2$, $\ln x_3$, … to have no real solution.

114. **OPEN ENDED** Give an example of a logarithmic equation with infinite solutions.

SOLUTION:

Sample answer: $\log_x x^3 = 3$. This equation is true for any value of x.

115. **CHALLENGE** If an investment is made with an interest rate r compounded monthly, how long will it take for the investment to triple?

SOLUTION:

Since we are tripling the principal P, then $A = 3P$.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$3P = P\left(1+\frac{r}{12}\right)^{12t}$$
$$3 = \left(1+\frac{r}{12}\right)^{12t}$$
$$\ln 3 = \ln\left(1+\frac{r}{12}\right)^{12t}$$
$$\ln 3 = 12t\ln\left(1+\frac{r}{12}\right)$$
$$\frac{\ln 3}{12\ln\left(1+\frac{r}{12}\right)} = t$$
$$\frac{1}{12}\ln\left(3-\left[1+\frac{r}{12}\right]\right) = t$$
$$\frac{1}{12}\ln\left(2-\frac{r}{12}\right) = t$$

116. **REASONING** How can you solve an equation involving logarithmic expressions with three different bases?

SOLUTION:

Sample answer: Use the Change of Base Formula to change each logarithmic expression into a fraction. Then eliminate the denominators and use algebraic and logarithmic properties to solve.

117. **CHALLENGE** For what x values do the domains of $f(x) = \log(x^4 - x^2)$ and $g(x) = \log x + \log x + \log(x - 1) + \log(x + 1)$ differ?

SOLUTION:

$x^4 - x^2$ must be greater than 0, so the absolute value of x must be greater than 1. Therefore, $f(x)$ is defined for $x < -1$ or $x > 1$. $g(x)$ is defined only for $x > 1$.

118. ***Writing in Math*** Explain how to algebraically solve for t in $P = \dfrac{L}{1+\left(\frac{L-I}{I}\right)e^{-kt}}$.

SOLUTION:

Sample answer: Multiply each side of the equation by the denominator; then divide each side by P. Subtract 1 from each side of the equation; then divide each side by $\frac{L-I}{I}$. Take the natural log of each side to remove the exponential expression; then divide each side by $-k$.

Evaluate each logarithm.

119. $\log_8 15$

SOLUTION:

$$\log_8 15 = \frac{\ln 15}{\ln 8} \approx 1.3023$$

120. $\log_2 8$

SOLUTION:

$$\log_2 8 = \log_2 2^3 = 3$$

121. $\log_5 625$

SOLUTION:

$$\log_5 625 = \log_5 5^4 = 4$$

122. **SOUND** An equation for loudness L, in decibels, is $L = 10 \log_{10} R$, where R is the relative intensity of the sound.

Sound	Decibels
fireworks	130–190
car racing	100–130
parades	80–120
yard work	95–115
movies	90–110
concerts	75–110

a. Solve $130 = 10 \log_{10} R$ to find the relative intensity of a fireworks display with a loudness of 130 decibels.

b. Solve $75 = 10 \log_{10} R$ to find the relative intensity of a concert with a loudness of 75 decibels.

c. How many times as intense is the fireworks display as the concert? In other words, find the ratio of their intensities.

SOLUTION:

a.

$$130 = 10\log_{10} R$$
$$13 = \log_{10} R$$
$$10^{13} = R$$

b.

$$75 = 10\log_{10} R$$
$$7.5 = \log_{10} R$$
$$10^{7.5} = R$$

c. $\frac{10^{13}}{10^{7.5}} \approx 316{,}228$

For each function, (a) apply the leading term test, (b) determine the zeros, and state the multiplicity of any repeated zeros, (c) find a few additional points, and then (d) graph the function.

123. $f(x) = x^3 - 8x^2 + 7x$

SOLUTION:

a. The degree is 3, and the leading coefficient is 1. Because the degree is odd and the leading coefficient is positive, $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$.

b.

$$\begin{aligned} f(x) &= x^3 - 8x^2 + 7x \\ &= x(x^2 - 8x + 7) \\ &= x(x - 1)(x - 7) \end{aligned}$$

The zeros are 0, 1, and 7.

c. Evaluate the function for a few x-values in its domain.

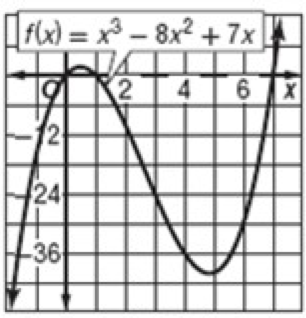
x	−1	0.5	3	8
$f(x)$	−16	1.625	−24	56

d.

Evaluate the function for several x-values in its domain.

x	−4	−2	−1	0	0.5	1	2
$f(x)$	−220	−54	−16	0	1.625	0	−10

Use these points to construct a graph.



124. $f(x) = x^3 + 6x^2 + 8x$

SOLUTION:

a. The degree is 3, and the leading coefficient is 1. Because the degree is odd and the leading coefficient is positive, $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$.

b.

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 8x \\ &= x(x^2 + 6x + 8) \\ &= x(x + 2)(x + 4) \end{aligned}$$

The zeros are 0, −2, and −4.

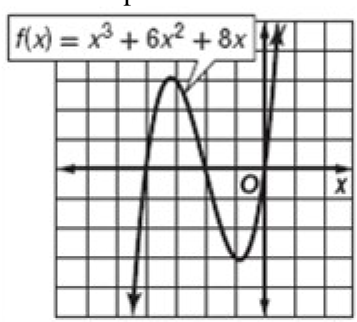
c. Evaluate the function for a few x-values in its domain.

x	−5	−3	−1	2
$f(x)$	−15	3	−3	48

d. Evaluate the function for several x-values in its domain.

x	−5	−4	−3	−2	−1	0	2
$f(x)$	15	0	3	0	−3	0	48

Use these points to construct a graph.



125. $f(x) = -x^4 + 6x^3 - 32x$

SOLUTION:

a. The degree is 4, and the leading coefficient is −1. Because the degree is even and the leading coefficient is negative, $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$.

b.

$$f(x) = -x^4 + 6x^3 - 32x$$
$$= x(-x^3 + 6x^2 - 32)$$

−2	−1	6	0	−32
		2	−16	32
	−1	8	−16	0

$-x^2 + 8x - 16 = -(x-4)^2$

$f(x) = -x(x-4)^2(x+2)$

The zeros are −2, 0, and 4. The zero of 4 has a multiplicity of 2 because $(x-4)^2$ is a factor.

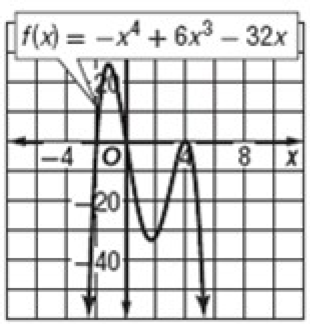
c. Evaluate the function for a few *x*-values in its domain.

x	−2.5	−1	2	4	5
f **(*x*)**	−52.8	25	−32	0	−35

d. Evaluate the function for several *x*-values in its domain.

x	−2.5	−2	−1	0	2	4	5
f **(*x*)**	−52.8	0	25	0	−32	0	−35

Use these points to construct a graph.



Solve each equation.

126. $\frac{1}{6}(12a)^{\frac{1}{3}} = 1$

SOLUTION:

$$\begin{aligned}\frac{1}{6}(12a)^{\frac{1}{3}} &= 1\\ (12a)^{\frac{1}{3}} &= 6\\ \left[(12a)^{\frac{1}{3}}\right]^3 &= 6^3\\ 12a &= 216\\ a &= 18\end{aligned}$$

127. $\sqrt[3]{x-4} = 3$

SOLUTION:

$$\begin{aligned}\sqrt[3]{x-4} &= 3\\ \left(\sqrt[3]{x-4}\right)^3 &= 3^3\\ x-4 &= 27\\ x &= 31\end{aligned}$$

128. $(3y)^{\frac{1}{3}} + 2 = 5$

SOLUTION:

$$\begin{aligned}(3y)^{\frac{1}{3}} + 2 &= 5\\ (3y)^{\frac{1}{3}} &= 3\\ \left[(3y)^{\frac{1}{3}}\right]^3 &= 3^3\\ 3y &= 27\\ y &= 9\end{aligned}$$

Use logical reasoning to determine the end behavior or limit of the function as *x* approaches infinity. Explain your reasoning.

129. $f(x) = x^{10} - x^9 + 5x^8$

SOLUTION:

∞; Sample answer: As $x \to \infty$, the leading term, x^{10} approaches infinity, so $f(x)$ will approach infinity.

3-4 Exponential and Logarithmic Equations

130. $g(x) = \frac{x2 + 5}{7 - 2x2}$

SOLUTION:

−0.5; Sample answer: As $x \to \infty$, the fraction will approach $-\frac{x2}{2x2}$, so $g(x)$ will approach $-\frac{1}{2}$ or −0.5.

131. $h(x) = |(x - 3)^2 - 1|$

SOLUTION:

∞; Sample answer: As $x \to \infty$, The value inside the absolute value symbols approaches x^2. The absolute value of x^2 is x^2, so $h(x)$ will approach infinity.

Find the variance and standard deviation of each population to the nearest tenth.

132. {48, 36, 40, 29, 45, 51, 38, 47, 39, 37}

SOLUTION:

$$\mu = \frac{48 + 36 + 40 + 29 + 45 + 51 + 38 + 47 + 39 + 37}{10} = 41$$

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{n} \approx 40$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \approx 6.3$$

133. {321, 322, 323, 324, 325, 326, 327, 328, 329, 330}

SOLUTION:

$$\mu = \frac{321 + 322 + 323 + 324 + 325 + 326 + 327 + 328 + 329 + 330}{10} = 325.5$$

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{n} \approx 8.2$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \approx 2.9$$

134. {43, 56, 78, 81, 47, 42, 34, 22, 78, 98, 38, 46, 54, 67, 58, 92, 55}

SOLUTION:

$$\mu \approx 58.2$$

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{n} \approx 424.3$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \approx 20.6$$

135. **SAT/ACT** In a movie theater, 2 boys and 3 girls are randomly seated together in a row. What is the probability that the 2 boys are seated next to each other?

A $\frac{1}{5}$

B $\frac{3}{5}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{2}{5}$

SOLUTION:

As shown in the sample space below, there are 10 possible seating arrangements. The boys are next to each other in 4 of the arrangements, so the probability is $\frac{4}{10}$ or $\frac{2}{5}$.

BBGGG
BGBGG
BGGBG
BGGGB
GBBGG
GBGBG
GBGGB
GGBBG
GGBGB
GGGBB

136. **REVIEW** Which equation is equivalent to $\log_4 \frac{1}{16} = x$?

F $\frac{14}{16} = x^4$

G $\left(\frac{1}{16}\right)^4 = x$

H $4^x = \frac{1}{16}$

J $4^{\frac{1}{16}} = x$

SOLUTION:

By converting the equation from logarithmic form to exponential form, the correct choice is H.

3-4 Exponential and Logarithmic Equations

137. If $2^4 = 3^x$, then what is the approximate value of x?

A 0.63
B 2.34
C 2.52
D 2.84

SOLUTION:

$$2^4 = 3^x$$
$$\ln 2^4 = \ln 3^x$$
$$4 \ln 2 = x \ln 3$$
$$\frac{4 \ln 2}{\ln 3} = x$$
$$2.52 \approx x$$

138. **REVIEW** The pH of a person's blood is given by pH = $6.1 + \log_{10} B - \log_{10} C$, where B is the concentration base of bicarbonate in the blood and C is the concentration of carbonic acid in the blood. Determine which substance has a pH closest to a person's blood if their ratio of bicarbonate to carbonic acid is 17.5:2.25.

Substance	pH
lemon juice	2.3
milk	6.4
baking soda	8.4
ammonia	11.9

F lemon juice
G baking soda
H milk
J ammonia

SOLUTION:

$$\begin{aligned} \text{pH} &= 6.1 + \log_{10} B - \log_{10} C \\ &= 6.1 + \log_{10} 17.5 - \log_{10} 2.25 \\ &\approx 7.0 \end{aligned}$$

6.4 is the closest to 7.0, so the correct choice is H.