3-3 Properties of Logarithms

Express each logarithm in terms of ln 2 and ln 5.

1. $\ln \frac{4}{5}$

SOLUTION:

$$\begin{aligned} \ln \frac{4}{5} &= \ln 4 - \ln 5 \\ &= \ln 2^2 - \ln 5 \\ &= 2 \ln 2 - \ln 5 \end{aligned}$$

2. $\ln 200$

SOLUTION:

$$\begin{aligned} \ln 200 &= \ln(8 \cdot 25) \\ &= \ln 8 + \ln 25 \\ &= \ln 2^3 + \ln 5^2 \\ &= 3 \ln 2 + 2 \ln 5 \end{aligned}$$

3. $\ln 80$

SOLUTION:

$$\begin{aligned} \ln 80 &= \ln(16 \cdot 5) \\ &= \ln 16 + \ln 5 \\ &= \ln 2^4 + \ln 5 \\ &= 4 \ln 2 + \ln 5 \end{aligned}$$

4. $\ln 12.5$

SOLUTION:

$$\begin{aligned} \ln 12.5 &= \ln\left(\frac{25}{2}\right) \\ &= \ln 25 - \ln 2 \\ &= \ln 5^2 - \ln 2 \\ &= 2 \ln 5 - \ln 2 \end{aligned}$$

5. $\ln \frac{0.8}{2}$

SOLUTION:

$$\begin{aligned} \ln \frac{0.8}{2} &= \ln \frac{0.4}{1} \\ &= \ln \frac{4}{10} \\ &= \ln \frac{2}{5} \\ &= \ln 2 - \ln 5 \end{aligned}$$

6. $\ln \frac{2}{5}$

SOLUTION:

$$\ln \frac{a}{b} = \ln a - \ln b$$
$$\ln \frac{2}{5} = \ln 2 - \ln 5$$

7. $\ln 2000$

SOLUTION:

$$\begin{aligned}\ln 2000 &= \ln(16 \cdot 125) \\ &= \ln 16 + \ln 125 \\ &= \ln 2^4 + \ln 5^3 \\ &= 4\ln 2 + 3\ln 5\end{aligned}$$

8. $\ln 1.6$

SOLUTION:

$$\begin{aligned}\ln 1.6 &= \ln \frac{16}{10} \\ &= \ln \frac{8}{5} \\ &= \ln 8 - \ln 5 \\ &= \ln 2^3 - \ln 5 \\ &= 3\ln 2 - \ln 5\end{aligned}$$

Express each logarithm in terms of ln 3 and ln 7.

9. $\ln 63$

SOLUTION:

$$\begin{aligned}\ln 63 &= \ln(9 \cdot 7) \\ &= \ln 9 + \ln 7 \\ &= \ln 3^2 + \ln 7 \\ &= 2\ln 3 + \ln 7\end{aligned}$$

10. $\ln \frac{49}{81}$

SOLUTION:

$$\begin{aligned}\ln \frac{49}{81} &= \ln 49 - \ln 81 \\ &= \ln 7^2 - \ln 3^4 \\ &= 2\ln 7 - 4\ln 3\end{aligned}$$

3-3 Properties of Logarithms

11. $\ln \frac{7}{9}$

SOLUTION:

$$\begin{aligned}\ln \frac{7}{9} &= \ln 7 - \ln 9 \\ &= \ln 7 - \ln 3^2 \\ &= \ln 7 - 2\ln 3\end{aligned}$$

12. $\ln 147$

SOLUTION:

$$\begin{aligned}\ln 147 &= \ln(49 \cdot 3) \\ &= \ln 49 + \ln 3 \\ &= \ln 7^2 + \ln 3 \\ &= 2\ln 7 + \ln 3\end{aligned}$$

13. $\ln 1323$

SOLUTION:

$$\begin{aligned}\ln 1323 &= \ln(49 \cdot 27) \\ &= \ln 49 + \ln 27 \\ &= \ln 7^2 + \ln 3^3 \\ &= 2\ln 7 + 3\ln 3\end{aligned}$$

14. $\ln \frac{343}{729}$

SOLUTION:

$$\begin{aligned}\ln \frac{343}{729} &= \ln 343 - \ln 729 \\ &= \ln 7^3 - \ln 3^6 \\ &= 3\ln 7 - 6\ln 3\end{aligned}$$

15. $\ln \frac{2401}{81}$

SOLUTION:

$$\begin{aligned}\ln \frac{2401}{81} &= \ln 2401 - \ln 81 \\ &= \ln 7^4 - \ln 3^4 \\ &= 4\ln 7 - 4\ln 3\end{aligned}$$

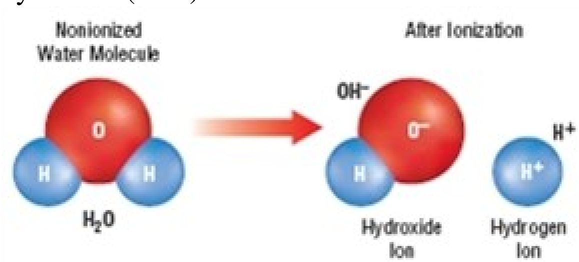
3-3 Properties of Logarithms

16. ln 1701

SOLUTION:

$$\begin{aligned}\ln 1701 &= \ln(7 \cdot 243)\\ &= \ln 7 + \ln 243\\ &= \ln 7 + \ln 3^5\\ &= \ln 7 + 5\ln 3\end{aligned}$$

17. **CHEMISTRY** The ionization constant of water K_w is the product of the concentrations of hydrogen (H^+) and hydroxide (OH^-) ions.



The formula for the ionization constant of water is $K_w = [H^+][OH^-]$, where the brackets denote concentration in moles per liter.

a. Express log K_w in terms of log $[H^+]$ and log $[OH^-]$.

b. The value of the constant K_w is 1 Í 10^{-14}. Simplify your equation from part **a** to reflect the numerical value of K_w.

c. If the concentration of hydrogen ions in a sample of water is 1 Í 10^{-9} moles per liter, what is the concentration of hydroxide ions?

SOLUTION:

a.

$$\begin{aligned}\log K_w &= \log[H^+][OH^-]\\ &= \log[H^+] + \log[OH^-]\end{aligned}$$

b.

$$\begin{aligned}\log K_w &= \log[H^+] + \log[OH^-]\\ \log(1 \times 10^{-14}) &= \log[H^+] + \log[OH^-]\\ \log 10^{-14} &= \log[H^+] + \log[OH^-]\\ -14 &= \log[H^+] + \log[OH^-]\end{aligned}$$

c.

$$\begin{aligned}-14 &= \log[H^+] + \log[OH^-]\\ -14 &= \log(1 \times 10^{-9}) + \log[OH^-]\\ -14 &= -9 + \log[OH^-]\\ -5 &= \log[OH^-]\\ 10^{-5} &= [OH^-]\end{aligned}$$

18. **TORNADOES** The distance d in miles that a tornado travels is $d = 10^{\frac{w-65}{93}}$, where w is the wind speed in miles per hour of the tornado.

a. Express w in terms of $\log d$.

b. If a tornado travels 100 miles, estimate the wind speed.

SOLUTION:

a.

$$\begin{aligned} d &= 10^{\frac{w-65}{93}} \\ \log d &= \log 10^{\frac{w-65}{93}} \\ \log d &= \frac{w-65}{93} \\ 93 \log d &= w - 65 \\ 93 \log d + 65 &= w \end{aligned}$$

b.

$$\begin{aligned} 93 \log d + 65 &= w \\ 93 \log(100) + 65 &= w \\ 93(2) + 65 &= w \\ 186 + 65 &= w \\ 251 &= w \end{aligned}$$

Evaluate each logarithm.

19. $\log_5 \sqrt[4]{25}$

SOLUTION:

$$\begin{aligned} \log_5 \sqrt[4]{25} &= \log_5 25^{\frac{1}{4}} \\ &= \frac{1}{4} \log_5 25 \\ &= \frac{1}{4}(2) \\ &= \frac{1}{2} \end{aligned}$$

20. $8 \ln e^2 - \ln e^{12}$

SOLUTION:

$$\begin{aligned} 8 \ln e^2 - \ln e^{12} &= 8(2) - 12 \\ &= 16 - 12 \\ &= 4 \end{aligned}$$

21. $9 \ln e^3 + 4 \ln e^5$

SOLUTION:

$$\begin{aligned} 9\ln e^3 + 4\ln e^5 &= 9(3) + 4(5) \\ &= 27 + 20 \\ &= 47 \end{aligned}$$

22. $\log_2 \sqrt[5]{32}$

SOLUTION:

$$\begin{aligned} \log_2 \sqrt[5]{32} &= \log_2 \sqrt[5]{2^5} \\ &= \log_2 2 \\ &= 1 \end{aligned}$$

23. $2 \log \sqrt{27}$

SOLUTION:

$$\begin{aligned} 2\log_3 \sqrt{27} &= 2\log_3 \sqrt{3^3} \\ &= 2\log_3 3^{\frac{3}{2}} \\ &= 2 \cdot \frac{3}{2} \\ &= 3 \end{aligned}$$

24. $3\log_7 \sqrt[6]{49}$

SOLUTION:

$$\begin{aligned} 3\log_7 \sqrt[6]{49} &= 3\log_7 \sqrt[6]{7^2} \\ &= 3\log_7 7^{\frac{2}{6}} \\ &= 3 \cdot \frac{2}{6} \\ &= 1 \end{aligned}$$

25. $4 \log_2 \sqrt{8}$

SOLUTION:

$$\begin{aligned} 4\log_2 \sqrt{8} &= 4\log_2 \sqrt{2^3} \\ &= 4\log_2 2^{\frac{3}{2}} \\ &= 4 \cdot \frac{3}{2} \\ &= 6 \end{aligned}$$

3-3 Properties of Logarithms

26. $50 \log_5 \sqrt{125}$

SOLUTION:

$$\begin{aligned} 50\log_5 \sqrt{125} &= 50\log_5 \sqrt{5^3} \\ &= 50\log_5 5^{\frac{3}{2}} \\ &= 50 \cdot \frac{3}{2} \\ &= 75 \end{aligned}$$

27. $\log_3 \sqrt[6]{243}$

SOLUTION:

$$\begin{aligned} \log_3 \sqrt[6]{243} &= \log_3 \sqrt[6]{3^5} \\ &= \log_3 3^{\frac{5}{6}} \\ &= \frac{5}{6} \end{aligned}$$

28. $36 \ln e^{0.5} - 4 \ln e^5$

SOLUTION:

$$\begin{aligned} 36\ln e^{0.5} - 4\ln e^5 &= 36(0.5) - 4(5) \\ &= 18 - 20 \\ &= -2 \end{aligned}$$

Expand each expression.

29. $\log_9 6x^3y^5z$

SOLUTION:

$$\begin{aligned} \log_9 6x^3y^5z &= \log_9 6 + \log_9 x^3 + \log_9 y^5 + \log_9 z \\ &= \log_9 6 + 3\log_9 x + 5\log_9 y + \log_9 z \end{aligned}$$

30. $\ln \frac{x^7}{\sqrt[3]{x+2}}$

SOLUTION:

$$\begin{aligned} \ln \frac{x^7}{\sqrt[3]{x+2}} &= \ln x^7 - \ln \sqrt[3]{x+2} \\ &= 7\ln x - \frac{1}{3}\ln(x+2) \end{aligned}$$

3-3 Properties of Logarithms

31. $\log_3 \frac{p^2 q}{\sqrt[5]{3q-1}}$

SOLUTION:

$$\begin{aligned}\log_3 \frac{p^2 q}{\sqrt[5]{3q-1}} &= \log_3 p^2 + \log_3 q - \log_3 \sqrt[5]{3q-1}\\ &= 2\log_3 p + \log_3 q - \log_3 (3q-1)^{\frac{1}{5}}\\ &= 2\log_3 p + \log_3 q - \frac{1}{5}\log_3 (3q-1)\end{aligned}$$

32. $\ln \frac{4df^5}{\sqrt[8]{1-3d}}$

SOLUTION:

$$\begin{aligned}\ln \frac{4df^5}{\sqrt[8]{1-3d}} &= \ln 4df^5 - \ln \sqrt[8]{1-3d}\\ &= \ln 4 + \ln d + \ln f^5 - \ln(1-3d)^{\frac{1}{8}}\\ &= \ln 4 + \ln d + 5\ln f - \frac{1}{8}\ln(1-3d)\end{aligned}$$

33. $\log_{11} ab^{-4}c^{12}d^7$

SOLUTION:

$$\begin{aligned}\log_{11} ab^{-4}c^{12}d^7 &= \log_{11} a + \log_{11} b^{-4} + \log_{11} c^{12} + \log_{11} d^7\\ &= \log_{11} a - 4\log_{11} b + 12\log_{11} c + 7\log_{11} d\end{aligned}$$

34. $\log_7 h^2 j^{11} k^{-5}$

SOLUTION:

$$\begin{aligned}\log_7 h^2 j^{11} k^{-5} &= \log_7 h^2 + \log_7 j^{11} + \log_7 k^{-5}\\ &= 2\log_7 h + 11\log_7 j - 5\log_7 k\end{aligned}$$

35. $\log_4 10t^2uv^{-3}$

SOLUTION:

$$\begin{aligned}\log_4 10t^2uv^{-3} &= \log_4 10 + \log_4 t^2 + \log_4 u + \log_4 v^{-3}\\ &= \log_4 10 + 2\log_4 t + \log_4 u - 3\log_4 v\end{aligned}$$

36. $\log_5 a^6b^{-3}c^4$

SOLUTION:

$$\begin{aligned}\log_5 a^6b^{-3}c^4 &= \log_5 a^6 + \log_5 b^{-3} + \log_5 c^4\\ &= 6\log_5 a - 3\log_5 b + 4\log_5 c\end{aligned}$$

3-3 Properties of Logarithms

37. $\ln \frac{3a^4b^7c}{\sqrt[4]{b-9}}$

SOLUTION:

$$\ln \frac{3a^4b^7c}{\sqrt[4]{b-9}} = \ln 3a^4b^7c - \ln \sqrt[4]{b-9}$$
$$= \ln 3 + \ln a^4 + \ln b^7 + \ln c - \ln(b-9)^{\frac{1}{4}}$$
$$= \ln 3 + 4\ln a + 7\ln b + \ln c - \frac{1}{4}\ln(b-9)$$

38. $\log_2 \frac{3x+2}{\sqrt[7]{1-5x}}$

SOLUTION:

$$\log_2 \frac{3x+2}{\sqrt[7]{1-5x}} = \log_2(3x+2) - \log_2 \sqrt[7]{1-5x}$$
$$= \log_2(3x+2) - \log_2(1-5x)^{\frac{1}{7}}$$
$$= \log_2(3x+2) - \frac{1}{7}\log_2(1-5x)$$

Condense each expression.

39. $3\log_5 x - \frac{1}{2}\log_5(6-x)$

SOLUTION:

$$3\log_5 x - \frac{1}{2}\log_5(6-x) = \log_5 x^3 - \log_5(6-x)^{\frac{1}{2}}$$
$$= \log_5 x^3 - \log_5 \sqrt{6-x}$$
$$= \log_5 \frac{x^3}{\sqrt{6-x}}$$

40. $5\log_7(2x) - \frac{1}{3}\log_7(5x+1)$

SOLUTION:

$$5\log_7(2x) - \frac{1}{3}\log_7(5x+1) = \log_7(2x)^5 - \log_7(5x+1)^{\frac{1}{3}}$$
$$= \log_7 32x^5 - \log_7 \sqrt[3]{5x+1}$$
$$= \log_7 \frac{32x^5}{\sqrt[3]{5x+1}}$$

3-3 Properties of Logarithms

41. $7 \log_3 a + \log_3 b - 2 \log_3 (8c)$

SOLUTION:

$$\begin{aligned} 7\log_3 a + \log_3 b - 2\log_3(8c) &= \log_3 a^7 + \log_3 b - \log_3(8c)^2 \\ &= \log_3 a^7 b - \log_3 64c^2 \\ &= \log_3 \frac{a^7 b}{64c^2} \end{aligned}$$

42. $4\ln(x+3) - \frac{1}{5}\ln(4x+7)$

SOLUTION:

$$\begin{aligned} 4\ln(x+3) - \frac{1}{5}\ln(4x+7) &= \ln(x+3)^4 - \ln(4x+7)^{\frac{1}{5}} \\ &= \ln(x+3)^4 - \ln\sqrt[5]{4x+7} \\ &= \ln\frac{(x+3)^4}{\sqrt[5]{4x+7}} \end{aligned}$$

43. $2 \log_8 (9x) - \log_8 (2x - 5)$

SOLUTION:

$$\begin{aligned} 2\log_8(9x) - \log_8(2x-5) &= \log_8(9x)^2 - \log_8(2x-5) \\ &= \log_8 81x^2 - \log_8(2x-5) \\ &= \log_8 \frac{81x^2}{2x-5} \end{aligned}$$

44. $\ln 13 + 7 \ln a - 11 \ln b + \ln c$

SOLUTION:

$$\begin{aligned} \ln 13 + 7\ln a - 11\ln b + \ln c &= \ln 13 + \ln a^7 - \ln b^{11} + \ln c \\ &= \ln 13a^7c - \ln b^{11} \\ &= \ln\frac{13a^7c}{b^{11}} \\ &= \ln 13a^7b^{-11}c \end{aligned}$$

45. $2 \log_6 (5a) + \log_6 b + 7 \log_6 c$

SOLUTION:

$$\begin{aligned} 2\log_6(5a) + \log_6 b + 7\log_6 c &= \log_6(5a)^2 + \log_6 b + \log_6 c^7 \\ &= \log_6 25a^2 + \log_6 b + \log_6 c^7 \\ &= \log_6 25a^2bc^7 \end{aligned}$$

3-3 Properties of Logarithms

46. $\log_2 x - \log_2 y - 3\log_2 z$

SOLUTION:

$$\begin{aligned}\log_2 x - \log_2 y - 3\log_2 z &= \log_2 x - \log_2 y - \log_2 z^3\\ &= \log_2 x - (\log_2 y + \log_2 z^3)\\ &= \log_2 x - \log_2 yz^3\\ &= \log_2 \frac{x}{yz^3}\end{aligned}$$

47. $\frac{1}{4}\ln(2a-b) - \frac{1}{5}\ln(3b+c)$

SOLUTION:

$$\begin{aligned}\frac{1}{4}\ln(2a-b) - \frac{1}{5}\ln(3b+c) &= \ln(2a-b)^{\frac{1}{4}} - \ln(3b+c)^{\frac{1}{5}}\\ &= \ln\sqrt[4]{2a-b} - \ln\sqrt[5]{3b+c}\\ &= \ln\frac{\sqrt[4]{2a-b}}{\sqrt[5]{3b+c}}\end{aligned}$$

48. $\log_3 4 - \frac{1}{2}\log_3(6x-5)$

SOLUTION:

$$\begin{aligned}\log_3 4 - \frac{1}{2}\log_3(6x-5) &= \log_3 4 - \log_3(6x-5)^{\frac{1}{2}}\\ &= \log_3 4 - \log_3\sqrt{6x-5}\\ &= \log_3\frac{4}{\sqrt{6x-5}}\end{aligned}$$

Evaluate each logarithm.

49. $\log_6 14$

SOLUTION:

$$\begin{aligned}\log_6 14 &= \frac{\ln 14}{\ln 6}\\ &\approx 1.473\end{aligned}$$

50. $\log_3 10$

SOLUTION:

$$\begin{aligned}\log_3 10 &= \frac{\ln 10}{\ln 3}\\ &\approx 2.096\end{aligned}$$

51. $\log_7 5$

SOLUTION:

$$\log_7 5 = \frac{\ln 5}{\ln 7} \approx 0.827$$

52. $\log_{128} 2$

SOLUTION:

$$\log_{128} 2 = \frac{\ln 2}{\ln 128} \approx 0.143$$

53. $\log_{12} 145$

SOLUTION:

$$\log_{12} 145 = \frac{\ln 145}{\ln 12} \approx 2.003$$

54. $\log_{22} 400$

SOLUTION:

$$\log_{22} 400 = \frac{\ln 400}{\ln 22} \approx 1.938$$

55. $\log_{100} 101$

SOLUTION:

$$\log_{100} 101 = \frac{\ln 101}{\ln 100} \approx 1.002$$

56. $\log_{\frac{1}{2}} \frac{1}{3}$

SOLUTION:

$$\log_{\frac{1}{2}} \frac{1}{3} = \frac{\ln \frac{1}{3}}{\ln \frac{1}{2}} \approx 1.585$$

57. $\log_{-2} 8$

SOLUTION:

no real solution

58. $\log_{13,000} 13$

SOLUTION:

$$\log_{13,000} 13 = \frac{\ln 13}{\ln 13,000} \approx 0.271$$

59. **COMPUTERS** Computer programs are written in sets of instructions called *algorithms*. To execute a task in a computer program, the algorithm coding in the program must be analyzed. The running time in seconds R that it takes to analyze an algorithm of n steps can be modeled by $R = \log_2 n$.

a. Determine the running time to analyze an algorithm of 240 steps.

b. To the nearest step, how many steps are in an algorithm with a running time of 8.45 seconds?

SOLUTION:

a.

$$\begin{aligned} R &= \log_2 n \\ &= \log_2 240 \\ &= \frac{\ln 240}{\ln 2} \\ &\approx 7.9 \end{aligned}$$

b.

$$\begin{aligned} R &= \log_2 n \\ 8.45 &= \log_2 n \\ 2^{8.45} &= 2^{\log_2 n} \\ 350 &\approx n \end{aligned}$$

60. **TRUCKING** Bill's Trucking Service purchased a new delivery truck for $56,000. Suppose $t = \log_{(1-r)} \frac{V}{P}$ represents the time t in years that has passed since the purchase given its initial price P, present value V, and annual rate of depreciation r.
a. If the truck's present value is $40,000 and it has depreciated at a rate of 15% per year, how much time has passed since its purchase to the nearest year?
b. If the truck's present value is $34,000 and it has depreciated at a rate of 10% per year, how much time has passed since its purchase to the nearest year?

SOLUTION:

a.

$$\begin{aligned} t &= \log_{(1-r)} \frac{V}{P} \\ &= \log_{(1-0.15)} \frac{40,000}{56,000} \\ &= \log_{0.85} \frac{5}{7} \\ &= \frac{\ln \frac{5}{7}}{\ln 0.85} \\ &\approx 2.07 \end{aligned}$$

about 2 years

b.

$$\begin{aligned} t &= \log_{(1-r)} \frac{V}{P} \\ &= \log_{(1-0.1)} \frac{34,000}{56,000} \\ &= \log_{0.9} \frac{17}{28} \\ &= \frac{\ln \frac{17}{28}}{\ln 0.9} \\ &\approx 4.73 \end{aligned}$$

about 5 years

Estimate each logarithm to the nearest whole number.

61. $\log_4 5$

SOLUTION:

$$\begin{aligned} \log_4 5 &\approx \log_4 4 \\ &\approx 1 \end{aligned}$$

62. $\log_2 13$

SOLUTION:

$$\begin{aligned}\log_2 13 &\approx \log_2 16\\ &\approx \log_2 2^4\\ &\approx 4\end{aligned}$$

63. $\log_3 10$

SOLUTION:

$$\begin{aligned}\log_3 10 &\approx \log_3 9\\ &\approx \log_3 3^2\\ &\approx 2\end{aligned}$$

64. $\log_7 400$

SOLUTION:

$$\begin{aligned}\log_7 400 &\approx \log_7 343\\ &\approx \log_7 7^3\\ &\approx 3\end{aligned}$$

65. $\log_5 \frac{1}{124}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{1}{124} &\approx \log_5 \frac{1}{125}\\ &\approx \log_5 \frac{1}{5^3}\\ &\approx \log_5 5^{-3}\\ &\approx -3\end{aligned}$$

66. $\log_{12} 177$

SOLUTION:

$$\begin{aligned}\log_{12} 177 &\approx \log_{12} 144\\ &\approx \log_{12} 12^2\\ &\approx 2\end{aligned}$$

67. $\log_{\frac{1}{5}} \frac{1}{6}$

SOLUTION:

$$\begin{aligned}\log_{\frac{1}{5}} \frac{1}{6} &\approx \log_{\frac{1}{5}} \frac{1}{5}\\ &\approx 1\end{aligned}$$

68. $\log_4 \frac{1}{165}$

SOLUTION:

$$\begin{aligned}\log_4 \frac{1}{165} &\approx \log_4 \frac{1}{256}\\ &\approx \log_4 \frac{1}{4^4}\\ &\approx \log_4 4^{-4}\\ &\approx -4\end{aligned}$$

Expand each expression.

69. $\ln \sqrt[5]{x^3(x+3)}$

SOLUTION:

$$\begin{aligned}\ln \sqrt[5]{x^3(x+3)} &= \ln\left(x^3(x+3)\right)^{\frac{1}{5}}\\ &= \frac{1}{5}\ln[x^3(x+3)]\\ &= \frac{1}{5}\ln x^3 + \frac{1}{5}\ln(x+3)\end{aligned}$$

70. $\log_5 \frac{x^2y^5}{\sqrt[3]{4x-y}}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{x^2y^5}{\sqrt[3]{4x-y}} &= \log_5 x^2y^5 - \log_5 \sqrt[3]{4x-y}\\ &= \log_5 x^2 + \log_5 y^5 - \log_5(4x-y)^{\frac{1}{3}}\\ &= 2\log_5 x + 5\log_5 y - \frac{1}{3}\log_5(4x-y)\end{aligned}$$

71. $\log_{14} \dfrac{11}{\sqrt[4]{x^5(8x-1)}}$

SOLUTION:

$$\begin{aligned}
\log_{14} \frac{11}{\sqrt[4]{x^5(8x-1)}} &= \log_{14} 11 - \log_{14} \sqrt[4]{x^5(8x-1)} \\
&= \log_{14} 11 - \log_{14}[x^5(8x-1)]^{\frac{1}{4}} \\
&= \log_{14} 11 - \frac{1}{4}\log_{14}[x^5(8x-1)] \\
&= \log_{14} 11 - \left[\frac{1}{4}\log_{14} x^5 + \frac{1}{4}\log_{14}(8x-1)\right] \\
&= \log_{14} 11 - \frac{1}{4}\log_{14} x^5 - \frac{1}{4}\log_{14}(8x-1) \\
&= \log_{14} 11 - \frac{5}{4}\log_{14} x - \frac{1}{4}\log_{14}(8x-1)
\end{aligned}$$

72. $\ln \dfrac{9x2yz3}{(y5)4}$

SOLUTION:

$$\begin{aligned}
\ln \frac{9x^2yz^3}{(y-5)^4} &= \ln 9x^2yz^3 - \ln(y-5)^4 \\
&= \ln 3^2 + \ln x^2 + \ln y + \ln z^3 - 4\ln(y-5) \\
&= 2\ln 3 + 2\ln x + \ln y + 3\ln z - 4\ln(y-5)
\end{aligned}$$

73. $\log_8 \sqrt[7]{x^3y^2(z-1)}$

SOLUTION:

$$\begin{aligned}
\log_8 \sqrt[7]{x^3y^2(z-1)} &= \log_8[x^3y^2(z-1)]^{\frac{1}{7}} \\
&= \frac{1}{7}\log_8[x^3y^2(z-1)] \\
&= \frac{1}{7}\log_8 x^3 + \frac{1}{7}\log_8 y^2 + \frac{1}{7}\log_8(z-1) \\
&= \frac{3}{7}\log_8 x + \frac{2}{7}\log_8 y + \frac{1}{7}\log_8(z-1)
\end{aligned}$$

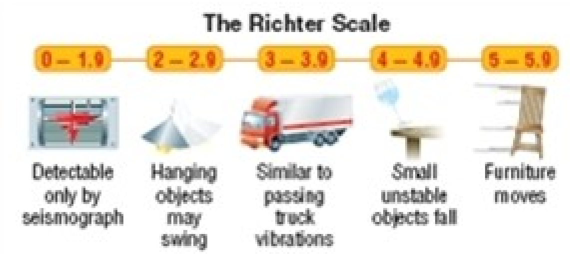
3-3 Properties of Logarithms

74. $\log_{12} \frac{5x}{\sqrt[6]{x^7(x+13)}}$

SOLUTION:

$$\begin{aligned}\log_{12} \frac{5x}{\sqrt[6]{x^7(x+13)}} &= \log_{12} 5x - \log_{12} \sqrt[6]{x^7(x+13)} \\ &= \log_{12} 5 + \log_{12} x - \log_{12}[x^7(x+13)]^{\frac{1}{6}} \\ &= \log_{12} 5 + \log_{12} x - \frac{1}{6}\log_{12}[x^7(x+13)] \\ &= \log_{12} 5 + \log_{12} x - \left[\frac{1}{6}\log_{12} x^7 + \frac{1}{6}\log_{12}(x+13)\right] \\ &= \log_{12} 5 + \log_{12} x - \left[\frac{7}{6}\log_{12} x + \frac{1}{6}\log_{12}(x+13)\right] \\ &= \log_{12} 5 + \log_{12} x - \frac{7}{6}\log_{12} x - \frac{1}{6}\log_{12}(x+13)\end{aligned}$$

75. **EARTHQUAKES** The Richter scale measures the intensity of an earthquake. The magnitude M of the seismic energy in joules E released by an earthquake can be calculated by $M = \frac{2}{3} \log \frac{E}{104.4}$.



a. Use the properties of logarithms to expand the equation.

b. What magnitude would an earthquake releasing 7.94×10^{11} joules have?

c. The 2007 Alum Rock earthquake in California released 4.47×10^{12} joules of energy. The 1964 Anchorage earthquake in Alaska measured a magnitude of 1.58×10^{18} joules of energy. How many times as great was the magnitude of the Anchorage earthquake as the magnitude of the Alum Rock earthquake?

d. Generally, earthquakes cannot be felt until they reach a magnitude of 3 on the Richter scale. How many joules of energy does an earthquake of this magnitude release?

SOLUTION:

a.

$$\begin{aligned}M &= \frac{2}{3}\log \frac{E}{10^{4.4}} \\ &= \frac{2}{3}\log E - \frac{2}{3}\log 10^{4.4} \\ &= \frac{2}{3}(\log E - \log 10^{4.4})\end{aligned}$$

b.

$$\begin{aligned} M &= \frac{2}{3}[\log(7.94\times10^{11}) - \log 10^{4.4}] \\ &= \frac{2}{3}[\log 7.94 + \log 10^{11} - \log 10^{4.4}] \\ &\approx \frac{2}{3}[0.9 + 11 - 4.4] \\ &\approx 5 \end{aligned}$$

c.

$$\begin{aligned} 3 &= \frac{2}{3}[\log E - \log 10^{4.4}] \\ 3 &= \frac{2}{3}[\log E - 4.4] \\ 3 &= \frac{2}{3}\log E - \frac{2}{3}(4.4) \\ 3 &= \frac{2}{3}\log E - \frac{8.8}{3} \\ \frac{17}{8} &= \frac{2}{3}\log E \\ 8.9 &= \log E \\ 10^{8.9} &= E \\ 7.94\times10^{8} &\approx E \end{aligned}$$

Condense each expression.

76. $\frac{3}{4}\ln x + \frac{7}{4}\ln y + \frac{5}{4}\ln z$

SOLUTION:

$$\begin{aligned} \frac{3}{4}\ln x + \frac{7}{4}\ln y + \frac{5}{4}\ln z &= \frac{1}{4}(3\ln x + 7\ln y + 5\ln z) \\ &= \frac{1}{4}(\ln x^3 + \ln y^7 + \ln z^5) \\ &= \frac{1}{4}\ln x^3y^7z^5 \\ &= \ln\left(x^3y^7z^5\right)^{\frac{1}{4}} \\ &= \ln\sqrt[4]{x^3y^7z^5} \end{aligned}$$

3-3 Properties of Logarithms

77. $\log_2 15 + 6\log_2 x - \frac{4}{3}\log_2 x - \frac{1}{3}\log_2(x+3)$

SOLUTION:

$$\begin{aligned}
&\log_2 15 + 6\log_2 x - \frac{4}{3}\log_2 x - \frac{1}{3}\log_2(x+3) \\
&= \log_2 15 + \log_2 x^6 - \frac{1}{3}[4\log_2 x + \log_2(x+3)] \\
&= \log_2 15x^6 - \frac{1}{3}[\log_2 x^4 + \log_2(x+3)] \\
&= \log_2 15x^6 - \frac{1}{3}\log_2[x^4(x+3)] \\
&= \log_2 15x^6 - \log_2[x^4(x+3)]^{\frac{1}{3}} \\
&= \log_2 15x^6 - \log_2 \sqrt[3]{x^4(x+3)} \\
&= \log_2 15x^6 - \log_2 x\sqrt[3]{x(x+3)} \\
&= \log_2 \frac{15x^6}{x\sqrt[3]{x(x+3)}} \\
&= \log_2 \frac{15x^5}{\sqrt[3]{x(x+3)}}
\end{aligned}$$

78. $\ln 14 - \frac{2}{3}\ln 3x - \frac{4}{3}\ln(4-3x)$

SOLUTION:

$$\begin{aligned}
\ln 14 - \frac{2}{3}\ln 3x - \frac{4}{3}\ln(4-3x) &= \ln 14 - \frac{1}{3}[2\ln 3x + 4\ln(4-3x)] \\
&= \ln 14 - \frac{1}{3}[\ln(3x)^2 + \ln(4-3x)^4] \\
&= \ln 14 - \frac{1}{3}[\ln 9x^2 + \ln(4-3x)^4] \\
&= \ln 14 - \frac{1}{3}[\ln 9x^2(4-3x)^4] \\
&= \ln 14 - [\ln 9x^2(4-3x)^4]^{\frac{1}{3}} \\
&= \ln 14 - \ln \sqrt[3]{9x^2(4-3x)^4} \\
&= \ln \frac{14}{\sqrt[3]{9x^2(4-3x)^4}}
\end{aligned}$$

79. $3\log_6 2x + 9\log_6 y - \frac{4}{5}\log_6 x - \frac{8}{5}\log_6 y - \frac{1}{5}\log_6 z$

SOLUTION:

$$
\begin{aligned}
&3\log_6 2x + 9\log_6 y - \frac{4}{5}\log_6 x - \frac{8}{5}\log_6 y - \frac{1}{5}\log_6 z\\
&= \log_6 (2x)^3 + \log_6 y^9 - \frac{1}{5}[4\log_6 x + 8\log_6 y + \log_6 z]\\
&= \log_6 8x^3 + \log_6 y^9 - \frac{1}{5}[\log_6 x^4 + \log_6 y^8 + \log_6 z]\\
&= \log_6 8x^3y^9 - \frac{1}{5}[\log_6 x^4y^8z]\\
&= \log_6 8x^3y^9 - [\log_6 x^4y^8z]^{\frac{1}{5}}\\
&= \log_6 8x^3y^9 - \log_6 \sqrt[5]{x^4y^8z}\\
&= \log_6 \frac{8x^3y^9}{\sqrt[5]{x^4y^8z}}\\
&= \log_6 \frac{8x^3y^9}{y\sqrt[5]{x^4y^3z}}\\
&= \log_6 \frac{8x^3y^8}{\sqrt[5]{x^4y^3z}}
\end{aligned}
$$

80. $\log_4 25 - \frac{5}{2}\log_4 x - \frac{7}{2}\log_4 y - \frac{3}{2}\log_4 (z+9)$

SOLUTION:

$$
\begin{aligned}
&\log_4 25 - \frac{5}{2}\log_4 x - \frac{7}{2}\log_4 y - \frac{3}{2}\log_4 (z+9)\\
&= \log_4 25 - \left[\frac{5}{2}\log_4 x + \frac{7}{2}\log_4 y + \frac{3}{2}\log_4 (z+9)\right]\\
&= \log_4 25 - \frac{1}{2}[5\log_4 x + 7\log_4 y + 3\log_4 (z+9)]\\
&= \log_4 25 - \frac{1}{2}\left[\log_4 x^5 + \log_4 y^7 + \log_4 (z+9)^3\right]\\
&= \log_4 25 - \frac{1}{2}\left[\log_4 x^5y^7(z+9)^3\right]\\
&= \log_4 25 - \left[\log_4 x^5y^7(z+9)^3\right]^{\frac{1}{2}}\\
&= \log_4 25 - \log_4 \sqrt{x^5y^7(z+9)^3}\\
&= \log_4 \frac{25}{\sqrt{x^5y^7(z+9)^3}}
\end{aligned}
$$

81. $\frac{5}{2}\ln x + \frac{1}{2}\ln(y+8) - 3\ln y - \ln(10-x)$

SOLUTION:

$$\begin{aligned}
&\frac{5}{2}\ln x+\frac{1}{2}\ln(y+8)-3\ln y-\ln(10-x)\\
&=\frac{1}{2}\left[5\ln x+\ln(y+8)\right]-\ln y^3-\ln(10-x)\\
&=\frac{1}{2}\left[\ln x^5+\ln(y+8)\right]-[\ln y^3+\ln(10-x)]\\
&=\frac{1}{2}\left[\ln x^5(y+8)\right]-\ln y^3(10-x)\\
&=\left[\ln x^5(y+8)\right]^{\frac{1}{2}}-\ln y^3(10-x)\\
&=\ln\sqrt{x^5(y+8)}-\ln y^3(10-x)\\
&=\ln\frac{\sqrt{x^5(y+8)}}{y^3(10-x)}
\end{aligned}$$

Use the properties of logarithms to rewrite each logarithm below in the form *a* ln 2 + *b* ln 3, where *a* and *b* are constants. Then approximate the value of each logarithm given that ln 2 ≈ 0.69 and ln 3 ≈ 1.10.

82. $\ln 4$

SOLUTION:

$$\begin{aligned}
\ln 4 &= \ln 2^2\\
&= 2\ln 2\\
&\approx 2(0.69)\\
&\approx 1.38
\end{aligned}$$

83. $\ln 48$

SOLUTION:

$$\begin{aligned}
\ln 48 &= \ln(3\cdot 16)\\
&= \ln 3+\ln 16\\
&= \ln 3+\ln 2^4\\
&= \ln 3+4\ln 2\\
&\approx 1.1+4(0.69)\\
&\approx 3.86
\end{aligned}$$

84. $\ln 162$

SOLUTION:

$$\begin{aligned}\ln 162 &= \ln(2\cdot 81)\\ &= \ln 2 + \ln 81\\ &= \ln 2 + \ln 3^4\\ &= \ln 2 + 4\ln 3\\ &\approx 0.69 + 4(1.1)\\ &\approx 5.09\end{aligned}$$

85. $\ln 216$

SOLUTION:

$$\begin{aligned}\ln 216 &= \ln(8\cdot 27)\\ &= \ln 8 + \ln 27\\ &= \ln 2^3 + \ln 3^3\\ &= 3\ln 2 + 3\ln 3\\ &\approx 3(0.69) + 3(1.1)\\ &\approx 5.37\end{aligned}$$

86. $\ln \frac{3}{2}$

SOLUTION:

$$\begin{aligned}\ln \frac{3}{2} &= \ln 3 - \ln 2\\ &\approx 1.1 - 0.69\\ &\approx 0.41\end{aligned}$$

87. $\ln \frac{4}{9}$

SOLUTION:

$$\begin{aligned}\ln \frac{4}{9} &= \ln 4 - \ln 9\\ &= \ln 2^2 - \ln 3^2\\ &= 2\ln 2 - 2\ln 3\\ &\approx 2(0.69) - 2(1.1)\\ &\approx -0.82\end{aligned}$$

88. $\ln \frac{4}{27}$

SOLUTION:

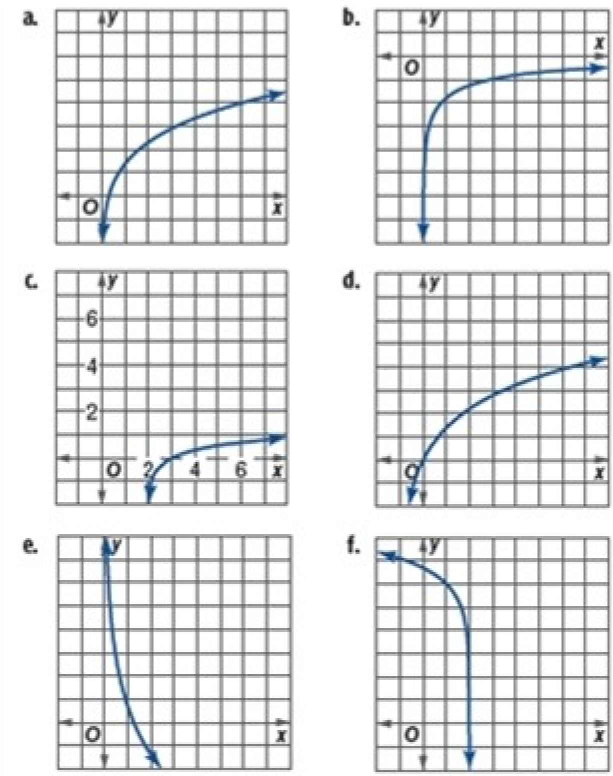
$$\begin{aligned}\ln \frac{4}{27} &= \ln 4 - \ln 27 \\ &= \ln 2^2 - \ln 3^3 \\ &= 2\ln 2 - 3\ln 3 \\ &\approx 2(0.69) - 3(1.1) \\ &\approx -1.92\end{aligned}$$

89. $\ln \frac{32}{9}$

SOLUTION:

$$\begin{aligned}\ln \frac{32}{9} &= \ln 32 - \ln 9 \\ &= \ln 2^5 - \ln 3^2 \\ &= 5\ln 2 - 2\ln 3 \\ &\approx 5(0.69) - 2(1.1) \\ &\approx 1.25\end{aligned}$$

Determine the graph that corresponds to each equation.



90. $f(x) = \ln x + \ln(x + 3)$

SOLUTION:

$$\begin{aligned} f(x) &= \ln x + \ln(x+3) \\ &= \ln x(x+3) \\ &= \ln(x^2 + 3x) \end{aligned}$$

Make a table of values.

x	0	1	2	3	4
$f(x)$	undef.	1.39	2.30	2.89	3.33

This table resembles the graphs for **a**, **c**, and **d**. However, the origin is a point on the graphs of **c** and **d**. The correct choice is **a**.

91. $f(x) = \ln x - \ln(x + 5)$

SOLUTION:

$$\begin{aligned} f(x) &= \ln x - \ln(x+5) \\ &= \ln \frac{x}{x+5} \end{aligned}$$

Make a table of values.

x	0	1	2	3	4
$f(x)$	undef.	−1.79	−1.25	−0.98	−0.69

This table resembles graph **b**.

3-3 Properties of Logarithms

92. $f(x) = 2 \ln (x + 1)$

SOLUTION:

$$\begin{aligned} f(x) &= 2\ln(x+1) \\ &= \ln(x+1)^2 \end{aligned}$$

Make a table of values.

x	0	1	2	3	4
$f(x)$	0	0.48	1.21	1.92	2.59

This table resembles graph **d**.

93. $f(x) = 0.5 \ln (x - 2)$

SOLUTION:

Make a table of values.

x	2	3	4	5	6
$f(x)$	undef.	0	0.35	0.55	0.69

This table resembles graph **c**.

94. $f(x) = \ln (2 - x) + 6$

SOLUTION:

Make a table of values.

x	−2	−1	0	1	2
$f(x)$	7.4	7.1	6.7	6	undef.

This table resembles graph **f**.

95. $f(x) = \ln 2x - 4 \ln x$

SOLUTION:

$$\begin{aligned} f(x) &= \ln 2x - 4\ln x \\ &= \ln 2x - \ln x^4 \\ &= \ln \frac{2x}{x^4} \\ &= \ln \frac{2}{x^3} \end{aligned}$$

Make a table of values.

x	0	1	2	3	4
$f(x)$	undef.	0.69	−1.39	−2.60	−3.47

This table resembles graph **e**.

Write each set of logarithmic expressions in increasing order.

96. $\log_3 \frac{12}{4}$, $\log_3 \frac{36}{3} + \log_3 4$, $\log_3 12 - 2\log_3 4$

SOLUTION:

$$\log_3 \frac{12}{4} = \log_3 3$$

$$\begin{aligned}\log_3 \frac{36}{3} + \log_3 4 &= \log_3 12 + \log_3 4 \\ &= \log_3(12 \cdot 4) \\ &= \log_3 48\end{aligned}$$

$$\begin{aligned}\log_3 12 - 2\log_3 4 &= \log_3 12 - \log_3 4^2 \\ &= \log_3 12 - \log_3 16 \\ &= \log_3 \frac{12}{16} \\ &= \log_3 \frac{3}{4}\end{aligned}$$

$$\log_3 \frac{3}{4} < \log_3 3 < \log_3 48$$

$$\log_3 12 - 2\log_3 4 < \log_3 \frac{12}{4} < \log_3 \frac{36}{3} + \log_3 4$$

97. $\log_5 55$, $\log_5 \sqrt{100}$, $3\log_5 \sqrt[3]{75}$

SOLUTION:

$$\log_5 55 = \log_5 55$$

$$\log_5 \sqrt{100} = \log_5 10$$

$$\begin{aligned}3\log_5 \sqrt[3]{75} &= \log_5 \left(\sqrt[3]{75}\right)^3 \\ &= \log_5 75\end{aligned}$$

$$\log_5 10 < \log_5 55 < \log_5 75$$

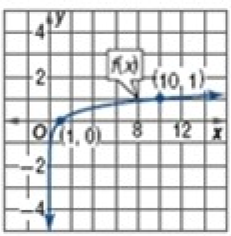
$$\log_5 \sqrt{100} < \log_5 55 < 3\log_5 \sqrt[3]{75}$$

98. **BIOLOGY** The generation time for bacteria is the time that it takes for the population to double. The generation time G can be found using $G = \frac{t}{3.3 \log_b f}$, where t is the time period, b is the number of bacteria at the beginning of the experiment, and f is the number of bacteria at the end of the experiment. The generation time for mycobacterium tuberculosis is 16 hours. How long will it take four of these bacteria to multiply into 1024 bacteria?

SOLUTION:

$$G = \frac{t}{3.3 \log_b f}$$
$$16 = \frac{t}{3.3 \log_4 1024}$$
$$16 = \frac{t}{3.3 \log_4 4^5}$$
$$16 = \frac{t}{3.3(5)}$$
$$264 = t$$

Write an equation for each graph.

99. 

SOLUTION:

Points (1, 0) and (10, 1) are located on the graph.

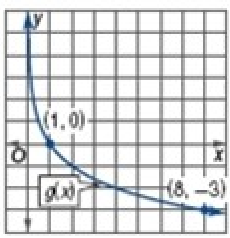
Point (1, 0) indicates that there are no translations from the parent graph.

$$f(x) = \log_b x$$
$$0 = \log_b 1$$

Use (10, 1) to identify the base.

$$f(x) = \log_b x$$
$$1 = \log_b 10$$
$$b^1 = b^{\log_b 10}$$
$$b = 10$$
$$f(x) = \log_{10} x$$

3-3 Properties of Logarithms

100. 

SOLUTION:

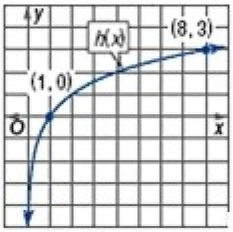
Points (1, 0) and (8, −3) are located on the graph.

Point (1, 0) indicates that there are no translations from the parent graph.

$$g(x) = \log_b x$$
$$0 = \log_b 1$$

Use (8, −3) to identify the base.

$$g(x) = \log_b x$$
$$-3 = \log_b 8$$
$$b^{-3} = b^{\log_b 8}$$
$$\frac{1}{b^3} = 8$$
$$1 = 8b^3$$
$$\frac{1}{8} = b^3$$
$$\frac{1}{2} = b$$
$$g(x) = \log_{\frac{1}{2}} x$$

101. 

SOLUTION:

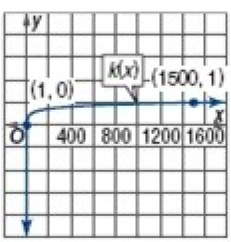
Points (1, 0) and (8, 3) are located on the graph.

Point (1, 0) indicates that there are no translations from the parent graph.

$$h(x) = \log_b x$$
$$0 = \log_b 1$$

Use (8, 3) to identify the base.

$$h(x) = \log_b x$$
$$3 = \log_b 8$$
$$b^3 = b^{\log_b 8}$$
$$b^3 = 8$$
$$b = 2$$
$$h(x) = \log_2 x$$

102. 

SOLUTION:

Points (1, 0) and (1500, 1) are located on the graph.

Point (1, 0) indicates that there are no translations from the parent graph.

$$k(x) = \log_b x$$
$$0 = \log_b 1$$

Use (1500, 1) to identify the base.

$$k(x) = \log_b x$$
$$1 = \log_b 1500$$
$$b^1 = b^{\log_b 1500}$$
$$b = 1500$$
$$k(x) = \log_{1500} x$$

103. **CHEMISTRY** pK_a is the acid dissociation constant for the acid HF, which is composed of ions H^+ and F^-. The pK_a can be calculated by $pK_a = -\log \frac{[H+][F-]}{[HF]}$, where $[H^+]$ is the concentration of H^+ ions, $[F^-]$ is the

concentration of F^{-} ions, and [HF] is the concentration of the acid solution. All of the concentrations are measured in moles per liter.

a. Use the properties of logs to expand the equation for pK_a.

b. What is the pK_a of a reaction in which $[H^+] = 0.01$ moles per liter, $[F^-] = 0.01$ moles per liter, and $[HF] = 2$ moles per liter?

c. The K_a of a substance can be calculated by $K_a = \frac{[H+][F-]}{[HF]}$. If a substance has a $pK_a = 25$, what is its K_a?

d. Aldehydes are a common functional group in organic molecules. Aldehydes have a pK_a around 17. To what K_a does this correspond?

SOLUTION:

a.

$$\begin{aligned}\mathrm{pK_a} &= -\log\frac{[H^+][F^-]}{[HF]}\\ &= -(\log[H^+] + \log[F^-] - \log[HF])\end{aligned}$$

b.

$$\begin{aligned}\mathrm{pK_a} &= -(\log[H^+] + \log[F^-] - \log[HF])\\ &= -(\log 0.01 + \log 0.01 - \log 2)\\ &= -(-2 + (-2) - \log 2)\\ &= 4 + \log 2\\ &\approx 4.30\end{aligned}$$

c.

$$\begin{aligned}\mathrm{pK_a} &= -\log\frac{[H^+][F^-]}{[HF]}\\ 25 &= -\log\frac{[H^+][F^-]}{[HF]}\\ -25 &= \log\frac{[H^+][F^-]}{[HF]}\\ 10^{-25} &= 10^{\log\frac{[H^+][F^-]}{[HF]}}\\ 10^{-25} &= \frac{[H^+][F^-]}{[HF]}\\ 10^{-25} &= \mathrm{K_a}\end{aligned}$$

d.

$$\begin{aligned} \mathrm{pK_a} &= -\log\frac{[H^+][F^-]}{[HF]} \\ 17 &= -\log\frac{[H^+][F^-]}{[HF]} \\ -17 &= \log\frac{[H^+][F^-]}{[HF]} \\ 10^{-17} &= 10^{\log\frac{[H^+][F^-]}{[HF]}} \\ 10^{-17} &= \frac{[H^+][F^-]}{[HF]} \\ 10^{-17} &= \mathrm{K_a} \end{aligned}$$

Evaluate each expression.

104. $\ln\left[\ln\left(e^{e^6}\right)\right]$

SOLUTION:

$$\begin{aligned} \ln\left[\ln\left(e^{e^6}\right)\right] &= \ln\left[e^6 \ln e\right] \\ &= \ln e^6 \\ &= 6 \end{aligned}$$

105. $10^{\log e^{\ln 4}}$

SOLUTION:

$$\begin{aligned} 10^{\log e^{\ln 4}} &= 10^{\log 4} \\ &= 4 \end{aligned}$$

106. $4\log_{17} 17^{\log_{10} 100}$

SOLUTION:

$$\begin{aligned} 4\log_{17} 17^{\log_{10} 100} &= 4\log_{17} 17^{\log_{10} 100} \\ &= 4\log_{17} 17^2 \\ &= 4\cdot 2\log_{17} 17 \\ &= 4\cdot 2 \\ &= 8 \end{aligned}$$

107. $e^{\log_4 4^{\ln 2}}$

SOLUTION:

$$\begin{aligned} e^{\log_4 4^{\ln 2}} &= e^{\ln 2} \\ &= 2 \end{aligned}$$

3-3 Properties of Logarithms

Simplify each expression.

108. $(\log_3 6)(\log_6 13)$

SOLUTION:

$$\begin{aligned}(\log_3 6)(\log_6 13) &= \frac{\log 6}{\log 3}\cdot\frac{\log 13}{\log 6}\\ &= \frac{\log 13}{\log 3}\\ &= \log_3 13\end{aligned}$$

109. $(\log_2 7)(\log_5 2)$

SOLUTION:

$$\begin{aligned}(\log_2 7)(\log_5 2) &= \frac{\log 7}{\log 2}\cdot\frac{\log 2}{\log 5}\\ &= \frac{\log 7}{\log 5}\\ &= \log_5 7\end{aligned}$$

110. $(\log_4 9) \div (\log_4 2)$

SOLUTION:

$$\begin{aligned}(\log_4 9) \div (\log_4 2) &= \frac{\log 9}{\log 4}\div\frac{\log 2}{\log 4}\\ &= \frac{\log 9}{\log 4}\cdot\frac{\log 4}{\log 2}\\ &= \frac{\log 9}{\log 2}\\ &= \log_2 9\end{aligned}$$

111. $(\log_5 12) \div (\log_8 12)$

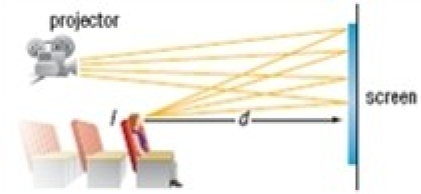
SOLUTION:

$$\begin{aligned}(\log_5 12) \div (\log_8 12) &= \frac{\log 12}{\log 5}\div\frac{\log 12}{\log 8}\\ &= \frac{\log 12}{\log 5}\cdot\frac{\log 8}{\log 12}\\ &= \frac{\log 8}{\log 5}\\ &= \log_5 8\end{aligned}$$

3-3 Properties of Logarithms

112. **MOVIES** Traditional movies are a sequence of still pictures which, if shown fast enough, give the viewer the impression of motion. If the frequency of the stills shown is too small, the moviegoer notices a flicker between each picture. Suppose the minimum frequency f at which the flicker first disappears is given by $f = K \log I$, where I the intensity of the light from the screen that reaches the viewer and K is the constant of proportionality.

a. The intensity of the light perceived by a moviegoer who sits at a distance d from the screen is given by $I = \frac{k}{d2}$, where k is a constant of proportionality. Show that $f = K(\log k - 2 \log d)$.



b. Suppose you notice the flicker from a movie projection and move to double your distance from the screen. In terms of K, how does this move affect the value of f? Explain your reasoning.

SOLUTION:

a.

$$\begin{aligned} f &= K \log I \\ &= K \log\left(\frac{k}{d^2}\right) \\ &= K(\log k - \log d^2) \\ &= K(\log k - 2\log d) \end{aligned}$$

b. Let f_1 be the minimum frequency at which the flicker first disappears when you were in your original seat at distance d from the screen. Let f_2 be the minimum frequency when your distance from the screen is $2d$. Then

$$\begin{aligned} f_1 &= K(\log k - 2\log d) \\ f_2 &= K(\log k - 2\log 2d) \\ &= K[\log k - 2(\log 2 + \log d)] \\ &= K(\log k - 2\log 2 - 2\log d) \\ &= K(\log k - \log 2^2 - 2\log d) \\ &= K(\log k - \log 4 - 2\log d) \end{aligned}$$

So the effect is $f_2 - f_1$.

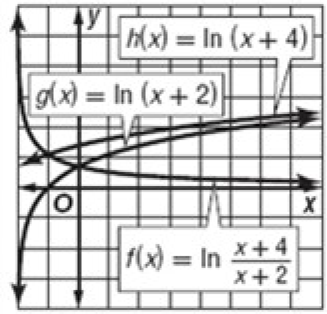
$$\begin{aligned} f_1 &= K(\log k - 2\log d) \\ f_2 - f_1 &= K(\log k - \log 4 - 2\log d) - K(\log k - 2\log 2d) \\ &= K\log k - K\log 4 - 2K\log d - K\log k + 2K\log 2d \\ &= -K\log 4 \\ &\approx -0.602K \end{aligned}$$

3-3 Properties of Logarithms

PROOF Investigate graphically and then prove each of the following properties of logarithms.

113. Quotient Property

SOLUTION:



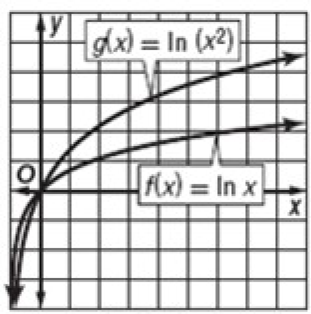
The graph of $f(x)$ represents the difference between the graphs of $h(x)$ and $g(x)$.

Let $x = \log_b m$ Let $y = \log_b n$. Then $b^x = m$ and $b^y = n$.

$\log_b\left(\frac{m}{n}\right) = \log_b\left(\frac{b^x}{b^y}\right)$	Substitution
$= \log_b(b^{x-y})$	Quotient Property
$= x - y$	Definition of logarithm
$= \log_b m - \log_b n$	Substitution

114. Power Property

SOLUTION:



All of the function values of $g(x)$ are twice the function values of $f(x)$ for each x-value in the domain.

Let $x = \log_b m$	
$b^x = m$	Definition of logarithm
$(b^x)^p = m^p$	Property of Equality of Exponents
$\log_b b^{xp} = \log_b m^p$	Property of Equality of logarithms
$xp = \log_b m^p$	Inverse Property of logarithms
$px = \log_b m^p$	Reflexive Property of Multiplication
$p \log_b m = \log_b m^p$	Substitution

3-3 Properties of Logarithms

115. **PROOF** Prove that $\log_b x = \frac{\log a x}{\log a b}$.

SOLUTION:

Let $\log_a n = x$

$a^x = n$	Definition of logarithm
$\log_b a^x = \log_b n$	One - to - One Property of logarithms
$x \log_b a = \log_b n$	Power Property of Logarithms
$x = \frac{\log_b n}{\log_b a}$	Divide each side by $\log_b a$.
$\log_a n = \frac{\log_b n}{\log_b a}$	Replace x with $\log_a n$.

3-3 Properties of Logarithms

116. **REASONING** How can the graph of $g(x) = \log_4 x$ be obtained using a transformation of the graph of $f(x) = \ln x$?

SOLUTION:

First, express $f(x) = \ln x$ and $g(x) = \log_4 x$ in similar terms.

$$\begin{aligned}\ln x &= \log_e x\\ &= \frac{\log x}{\log e}\\ &= \frac{1}{\log e}\log x\\ \log_4 x &= \frac{\log x}{\log 4}\\ &= \frac{1}{\log 4}\log x\end{aligned}$$

In order to transform the graph of $f(x)$ to the graph of $g(x)$, we need to multiply $\frac{1}{\log e}\log x$ by a value u that will yield $\frac{1}{\log 4}\log x$.

$$\begin{aligned}\left(\frac{1}{\log e}\log x\right)u &= \frac{1}{\log 4}\log x\\ u &= \frac{\frac{1}{\log 4}\log x}{\frac{1}{\log e}\log x}\\ u &= \frac{\frac{1}{\log 4}}{\frac{1}{\log e}}\\ u &= \frac{1}{\log 4}\cdot\frac{\log e}{1}\\ u &= \frac{\log e}{\log 4}\end{aligned}$$

117. **CHALLENGE** For what values of x, the set of all natural numbers, can $\ln x$ not be simplified?

SOLUTION:

$\ln x$ cannot be simplified for values of x that are prime numbers. The rest of the numbers can be factored.

3-3 Properties of Logarithms

118. **ERROR ANALYSIS** Omar and Nate expanded $\log_2\left(\frac{xy}{z}\right)^4$ using the properties of logarithms. Is either of them correct? Explain.

Omar: $4\log_2 x + 4\log_2 y - 4\log_2 z$

Nate: $2\log_4 x + 2\log_4 y - 2\log_4 z$

SOLUTION:

$$\log_2\left(\frac{xy}{z}\right)^4 = 4\log_2\left(\frac{xy}{z}\right)$$
$$= 4\log_2 x + 4\log_2 7 - 4\log_2 z$$

Omar; Nate incorrectly transposed the exponent and the base.

119. **PROOF** Use logarithmic properties to prove $\frac{\log_5(nt)^2}{\log_4\left(\frac{t}{r}\right)} = \frac{2\log n\log 4 + 2\log t\log 4}{\log 5\log t - \log 5\log r}$.

SOLUTION:

$$\frac{\log_5(nt)^2}{\log_4\frac{t}{r}} = \frac{2\log_5 nt}{\log_4\frac{t}{r}}$$
$$= \frac{2\log_5 n + \log_5 t}{\log_4 t - \log_4 r}$$
$$= \frac{2\left(\frac{\log n}{\log 5} + \frac{\log t}{\log 5}\right)}{\frac{\log t}{\log 4} - \frac{\log r}{\log 4}}$$
$$= \frac{2\left(\frac{\log n + \log t}{\log 5}\right)}{\frac{\log t - \log r}{\log 4}}$$
$$= \frac{2\log 4(\log n + \log t)}{\log 5(\log t - \log r)}$$
$$= \frac{2\log n\log 4 + 2\log t\log 4}{\log 5\log t - \log 5\log r}$$

120. ***Writing in Math*** The graph of $g(x) = \log_b x$ is actually a transformation of $f(x) = \log x$. Use the Change of Base Formula to find the transformation that relates these two graphs. Then explain the effect that different values of b have on the common logarithm graph.

SOLUTION:

Sample answer: $\log_b x = \frac{logx}{logb} = \frac{1}{logb}\log x$. Thus, a base b logarithm is a constant multiple of its corresponding common logarithm. When $b > 1$, the graph of f is expanded or compressed vertically. For example, if $b = 2$, the graph will be expanded, but when $b = 25$, the graph will be compressed. When $b < 1$, in addition to being expanded or compressed vertically, the graph is reflected in the x-axis.

3-3 Properties of Logarithms

Sketch and analyze each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

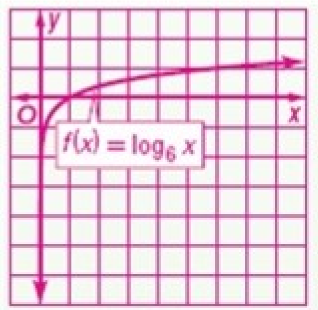
121. $f(x) = \log_6 x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	0	1	2	3	4	5	6
y	undef.	0	0.39	0.61	0.77	0.90	1

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = (0, ∞); R = (– ∞ , ∞); x-intercept: 1; asymptote: y-axis; $\lim_{x\to 0} f(x) = -\infty$ and $\lim_{x\to \infty} f(x) = \infty$; increasing on (0, ∞)

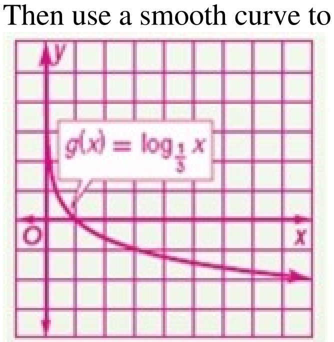
122. $g(x) = \log_{\frac{1}{3}} x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	0	1	2	3	4	5	6
y	undef.	0	0.63	−1	−1.26	−1.47	−1.63

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = (0, ∞); R = (– ∞ , ∞); x-intercept: 1; asymptote: y-axis; $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to \infty} f(x) = -\infty$; decreasing on (0, ∞)

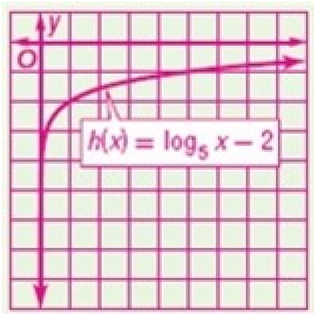
123. $h(x) = \log_5 x - 2$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	0	1	2	3	4	5	6
y	undef.	−2	−1.57	−1.32	−1.14	−1	−0.89

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing. To determine the intercept, set $h(x) = 0$.

$$\begin{aligned} h(x) &= \log_5 x - 2 \\ 0 &= \log_5 x - 2 \\ 2 &= \log_5 x \\ 5^2 &= 5^{\log_5 x} \\ 25 &= x \end{aligned}$$

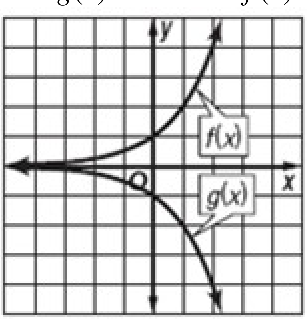
D = (0, ∞); R = (−∞ , ∞); x-intercept: 25; asymptote: y-axis; $\lim_{x \to 0} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$; increasing on (0, ∞)

Use the graph of $f(x)$ and $g(x)$ to describe the transformation that yields the graph of $g(x)$. Then sketch the graphs of $f(x)$ and $g(x)$.

124. $f(x) = 2^x$; $g(x) = -2^x$

SOLUTION:

Find $g(x)$ in terms of $f(x)$. $g(x) = -f(x)$, so the graph of $g(x)$ is the graph of $f(x)$ reflected in the x-axis.

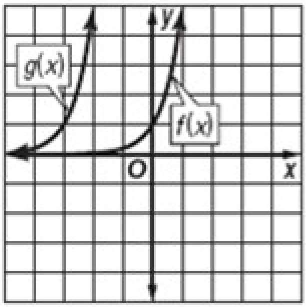


3-3 Properties of Logarithms

125. $f(x) = 5^x$; $g(x) = 5^{x+3}$

SOLUTION:

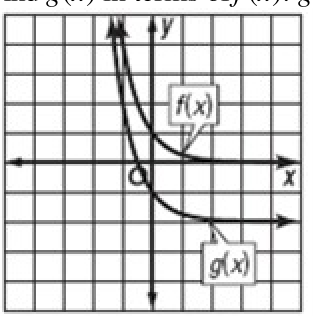
Find $g(x)$ in terms of $f(x)$. $g(x) = f(x - 3)$, so the graph of $g(x)$ is the graph of $f(x)$ translated 3 units left.



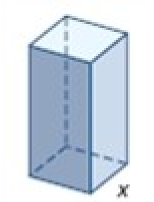
126. $f(x) = \left(\frac{1}{4}\right)^x; g(x) = \left(\frac{1}{4}\right)^x - 2$

SOLUTION:

Find $g(x)$ in terms of $f(x)$. $g(x) = f(x) - 2$, so the graph of $g(x)$ is the graph of $f(x)$ translated 2 units down.



127. **GEOMETRY** The volume of a rectangular prism with a square base is fixed at 120 cubic feet.



a. Write the surface area of the prism as a function $A(x)$ of the length of the side of the square x.
b. Graph the surface area function.
c. What happens to the surface area of the prism as the length of the side of the square approaches 0?

SOLUTION:

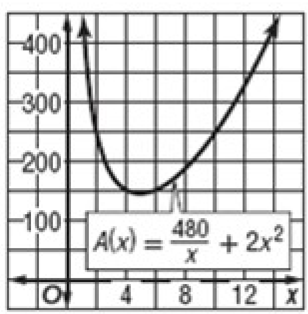
a.

$$120 = x^2 h$$

$$\frac{120}{x^2} = h$$

$$\begin{aligned} A &= 2x^2 + 4xh \\ &= 2x^2 + 4x\left(\frac{120}{x^2}\right) \\ &= 2x^2 + \frac{480}{x} \end{aligned}$$

b.



c. As x approaches 0, $\frac{480}{x}$ approaches infinity, and as indicated in the graph, $A(x)$ also approaches infinity. The surface area approaches infinity.

3-3 Properties of Logarithms

Divide using synthetic division.

128. $(x^2 - x + 4) \div (x - 2)$

SOLUTION:

$$
\begin{array}{r}
x+1 \\
x-2\overline{)x^2-x+4} \\
\underline{x^2-2x} \\
x+4 \\
\underline{x-2} \\
6
\end{array}
$$

$x + 1 + \dfrac{6}{x2}$

129. $(x^3 + x^2 - 17x + 15) \div (x + 5)$

SOLUTION:

$$
\begin{array}{r}
x^2-4x+3 \\
x+5\overline{)x^3+x^2-17x+15} \\
\underline{x^3+5x^2} \\
-4x^2-17x \\
\underline{-4x^2-20x} \\
3x+15 \\
\underline{3x+15} \\
0
\end{array}
$$

$x^2 - 4x + 3$

130. $(x^3 - x^2 + 2) \div (x + 1)$

SOLUTION:

$$
\begin{array}{r}
x^2-2x+2 \\
x+1\overline{)x^3-x^2+0x+2} \\
\underline{x^3+x^2} \\
-2x^2+0x \\
\underline{-2x^2-2x} \\
2x+2 \\
\underline{2x+2} \\
0
\end{array}
$$

$x^2 - 2x + 2$

Show that *f* and *g* are inverse functions. Then graph each function on the same graphing calculator screen.

131. $f(x) = -\frac{2}{3}x + \frac{1}{6}$

$g(x) = -\frac{3}{2}x + \frac{1}{4}$

SOLUTION:

$$f(g(x)) = -\frac{2}{3}\left(-\frac{3}{2} + \frac{1}{4}\right) + \frac{1}{6}$$

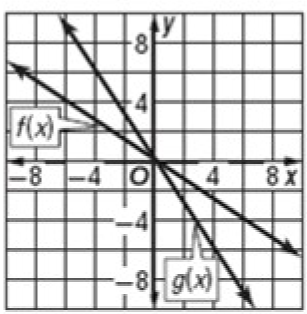
$$= x - \frac{1}{6} + \frac{1}{6}$$

$$= x;$$

$$g(f(x)) = -\frac{3}{2}\left(-\frac{2}{3} + \frac{1}{6}\right) + \frac{1}{4}$$

$$= x - \frac{1}{4} + \frac{1}{4}$$

$$= x$$



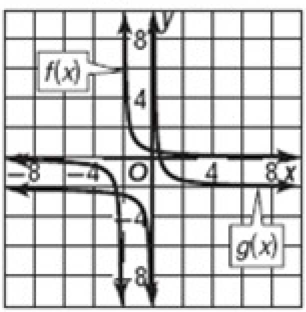
132. $f(x) = \frac{1}{x+2}$

$g(x) = \frac{1}{x} - 2$

SOLUTION:

$$\begin{aligned} f(g(x)) &= \frac{1}{\left(\frac{1}{x} - 2\right) + 2} \\ &= \frac{1}{\frac{1}{x}} \\ &= x; \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \frac{1}{\frac{1}{x+2}} - 2 \\ &= x + 2 - 2 \\ &= x \end{aligned}$$

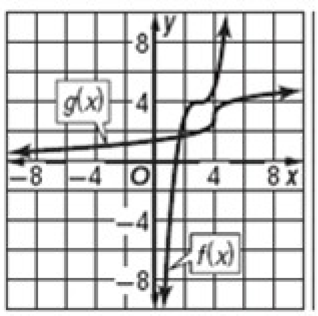


133. $f(x) = (x - 3)^3 + 4$
$g(x) = \sqrt[3]{x-4} + 3$

SOLUTION:

$$\begin{aligned} f(g(x)) &= [(\sqrt[3]{x-4} + 3) - 3]^3 + 4 \\ &= (\sqrt[3]{x-4})^3 + 4 \\ &= x - 4 + 4 \\ &= x; \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \sqrt[3]{[(x-3)^3 + 4] - 4} + 3 \\ &= \sqrt[3]{(x-3)^3} + 3 \\ &= x - 3 + 3 \\ &= x \end{aligned}$$



3-3 Properties of Logarithms

134. **SCIENCE** Specific heat is the amount of heat per unit of mass required to raise the temperature of a substance by one degree Celsius. The table lists the specific heat in Joules per gram for certain substances. The amount of heat transferred Q is given by $Q = cmT$, where c is the specific heat for a substance, m its mass, and T is the change in temperature.

Substance	Specific Heat (j/g)
aluminum	0.902
gold	0.129
mercury	0.140
iron	0.45
ice	2.03
water	4.179
air	1.01

a. Find the function for the change in temperature.
b. What is the parent graph of this function?
c. What is the relevant domain of this function?

SOLUTION:

a.

$$Q = cmT$$
$$\frac{Q}{cm} = T$$

b. The function appears to be rational. $f(x) = \frac{1}{x}$

c. D = $\{m \mid m > 0\}$. You cannot have negative mass.

135. **SAT/ACT** If $b \neq 0$, let $a \,\Delta\, b = \frac{a2}{b2}$. If $x \,\Delta\, y = 1$, then which statement must be true?

A $x = y$
B $x = -y$
C $-x = y$
D $x^2 - y^2 = 0$
E $x > 0$ and $y > 0$

SOLUTION:

$$x \,\Delta\, y = 1$$
$$\frac{x^2}{y^2} = 1$$
$$x^2 = y^2$$
$$x^2 - y^2 = 0$$

136. **REVIEW** Find the value of x for $\log_2 (9x + 5) = 2 + \log_2 (x^2 - 1)$.

F –0.4
G 0
H 1
J 3

SOLUTION:

$$\begin{aligned}\log_2(9x+5) &= 2+\log_2(x^2-1)\\ \log_2(9x+5)-\log_2(x^2-1) &= 2\\ \log_2\frac{9x+5}{(x^2-1)} &= 2\\ 2^{\log_2\frac{9x+5}{(x^2-1)}} &= 2^2\\ \frac{9x+5}{(x^2-1)} &= 4\\ 9x+5 &= 4(x^2-1)\\ 9x+5 &= 4x^2-4\\ 0 &= 4x^2-9x-9\\ 0 &= (x-3)(4x+3)\\ x &= 3 \text{ or } -\frac{3}{4}\end{aligned}$$

Since $x^2 - 1 > 0$, $x > 1$, so the correct choice is J.

137. To what is $2 \log_5 12 - \log_5 8 - 2 \log_5 3$ equal?

A $\log_5 2$
B $\log_5 3$
C $\log_5 0.5$
D 1

SOLUTION:

$$\begin{aligned}2\log_5 12-\log_5 8-2\log_5 3 &= \log_5 12^2-\log_5 8-\log_5 3^2\\ &= \log_5 144-\log_5 8-\log_5 9\\ &= \log_5\frac{144}{8}-\log_5 9\\ &= \log_5 18-\log_5 9\\ &= \log_5\frac{18}{9}\\ &= \log_5 2\end{aligned}$$

The correct choice is A.

138. **REVIEW** The weight of a bar of soap decreases by 2.5% each time it is used. If the bar of soap weighs 95 grams when it is new, what is its weight to the nearest gram after 15 uses?
F 58 g
G 59 g
H 65 g
J 93 g

SOLUTION:

If the soap decreases by 2.5% after each use, then 97.5% remains.

$95 \cdot 0.975^{15} \approx 65$

The correct choice is H.