Ghayathi Common School

وزارة التربية والتعليم
MINISTRY OF EDUCATION

EOT 11A Revision
2024/2025
Physics (BRIDGE)
101 C

By :- Kholoud Khaleel

MCQ Electronic Exam

1	[1] Represent a point in one, two and three dimensional space in terms of its Cartesian coordinates. [2] Represent a vector in terms of its components in Cartesian coordinates- in two, and three-dimensional space.	STUDENT TEXTBOOK	18
		FIGURE 1.15 FIGURE 1.25	18 22

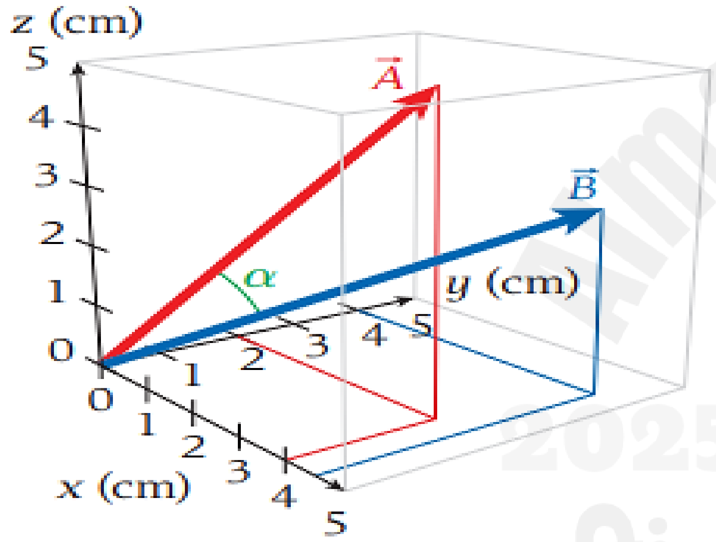


FIGURE 1.25 Calculating the angle between two position vectors.

$\vec{A} = (4.00, 2.00, 5.00)$ cm and $\vec{B} = (4.50, 4.00, 3.00)$ cm?

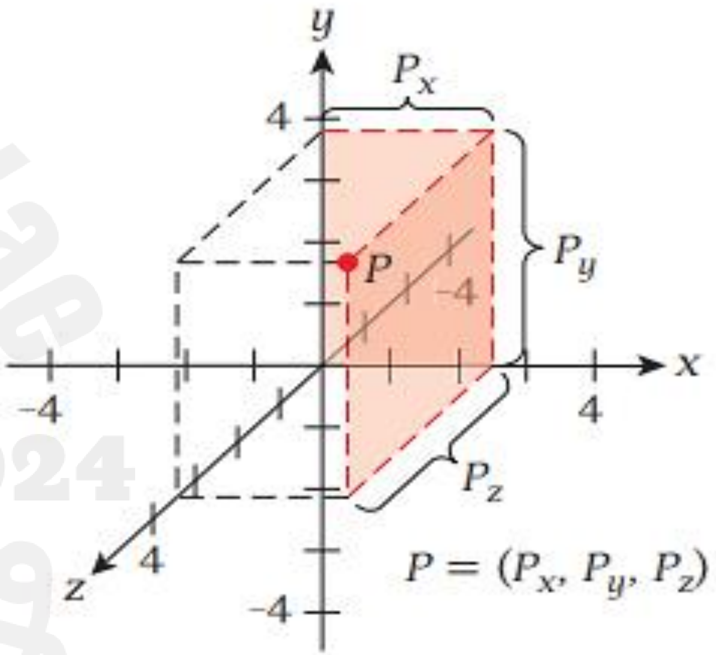


FIGURE 1.15 Representation of a point *P* in a three-dimensional space in terms of its Cartesian coordinates.

2	Find the length and direction of a two-dimensional vector from its Cartesian components.	STUDENT TEXTBOOK	21
		Q. [1.99/1.100/1.102/1.104]	30

Vector Length and Direction

If we know the component representation of a vector, how can we find its length (magnitude) and the direction it is pointing in? Let's look at the most important case: a vector in two dimensions. In two dimensions, a vector $\vec{A}$ can be specified uniquely by giving the two Cartesian components, A_x and A_y. We can also specify the same vector by giving two other numbers: its length A and its angle θ with respect to the positive x-axis.

Let's take a look at Figure 1.23 to see how we can determine A and θ from A_x and A_y. Figure 1.23a shows the graphical representation of equation 1.19. The vector $\vec{A}$ is the sum of the vectors $A_x\hat{x}$ and $A_y\hat{y}$. Since the unit vectors $\hat{x}$ and $\hat{y}$ are by definition orthogonal to each other, these vectors form a 90° angle. Thus, the three vectors $\vec{A}$, $A_x\hat{x}$, and $A_y\hat{y}$ form a right triangle with side lengths A, A_x, and A_y, as shown in Figure 1.23b.

Now we can employ basic trigonometry to find θ and A. Using the Pythagorean Theorem results in

$$A = \sqrt{A_x^2 + A_y^2}. \tag{1.20}$$

We can find the angle θ from the definition of the tangent function

$$\theta = \tan^{-1}\frac{A_y}{A_x}. \tag{1.21}$$

In using equation 1.21, you must be careful that θ is in the correct quadrant. We can also invert equations 1.20 and 1.21 to obtain the Cartesian components of a vector of given length and direction:

$$A_x = A\cos\theta \tag{1.22}$$

$$A_y = A\sin\theta. \tag{1.23}$$

You will encounter these trigonometric relations again and again throughout introductory physics. If you need to refamiliarize yourself with trigonometry, consult the mathematics primer provided in Appendix A.

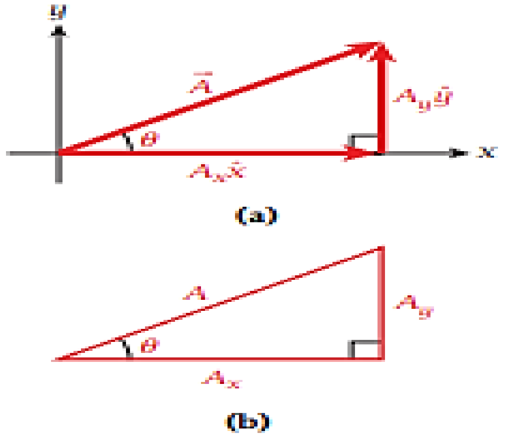


FIGURE 1.23 Length and direction of a vector. (a) Cartesian components A_x and A_y; (b) length A and angle θ.

2	Find the length and direction of a two-dimensional vector from its Cartesian components.	STUDENT TEXTBOOK	21
		Q. [1.99/1.100/1.102/1.104]	30

1.99 Sketch the vectors with the components A = (Ax, Ay) = (30.0 m, -50.0 m) and B = (Bx, By) = (-30.0 m, 50.0 m), and find the magnitudes of these vectors

1.100 What angle does A = (Ax, Ay) = (30.0 m, -50.0 m) make with the positive x-axis? What angle does it make with the negative y-axis?

2	Find the length and direction of a two-dimensional vector from its Cartesian components.	STUDENT TEXTBOOK	21
		Q. [1.99/1.100/1.102/1.104]	30

1.102 What angle does B = (Bx, By) = (30.0 m, 50.0 m) make with the positive x-axis?
What angle does it make with the positive y-axis?

1.104 Find the magnitude and direction of – A + B , where A = (23.0, 59.0), B = (90.0, -150.0).

3	Find the angle between two position vectors in the cartesian coordinates.	STUDENT TEXTBOOK	21,23
		EXAMPLE 1.5 / Q.1.80/Q1.103	22 / 29 /30

Scalar Product of Vectors

Above we saw how to multiply a vector with a scalar. Now we will define one way of multiplying a vector with a vector and obtain the **scalar product.** The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right| \left|\vec{B}\right| \cos\alpha, \tag{1.24}$$

Scalar Product for Unit Vectors. On page 26 we introduced unit vectors in the three-dimensional Cartesian coordinate system: $\hat{x} = (1,0,0)$, $\hat{y} = (0,1,0)$, and $\hat{z} = (0,0,1)$. With our definition (1.25) of the scalar product, we find

$$\hat{x} \bullet \hat{x} = \hat{y} \bullet \hat{y} = \hat{z} \bullet \hat{z} = 1 \tag{1.30}$$

and

$$\begin{aligned} \hat{x} \bullet \hat{y} = \hat{x} \bullet \hat{z} = \hat{y} \bullet \hat{z} = 0 \\ \hat{y} \bullet \hat{x} = \hat{z} \bullet \hat{x} = \hat{z} \bullet \hat{y} = 0. \end{aligned} \tag{1.31}$$

Now we see why the unit vectors are called that: Their scalar products with themselves have the value 1. Thus, the unit vectors have length 1, or unit length, according to equation 1.27. In addition, any pair of different unit vectors has a scalar product that is zero, meaning that these vectors are orthogonal to each other. Equations 1.30 and 1.31 thus state that the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ form an orthonormal set of vectors, which makes them extremely useful for the description of physical systems.

Self-Test Opportunity 1.1

Show that equations 1.30 and 1.31 are correct by using equation 1.25 and the definitions of the unit vectors.

3	Find the angle between two position vectors in the cartesian coordinates.	STUDENT TEXTBOOK	21,23
		EXAMPLE 1.5 / Q.1.80/Q1.103	22 / 29 /30

$$\vec{A} \bullet \vec{B} = (A_x, A_y, A_z) \bullet (B_x, B_y, B_z) = A_x B_x + A_y B_y + A_z B_z .$$

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}.$$

$$\vec{A} \bullet \vec{B} = |\vec{A}| \; |\vec{B}| \cos\alpha \Rightarrow \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|} \Rightarrow \alpha = \cos^{-1}\left(\frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|}\right).$$

Geometrical Interpretation of the Scalar Product. In the definition of the scalar product $\vec{A} \bullet \vec{B} = |\vec{A}| \; |\vec{B}| \cos\alpha$ (equation 1.24), we can interpret $|\vec{A}| \cos\alpha$ as the projection of the vector $\vec{A}$ onto the vector $\vec{B}$ (Figure 1.26a). In this drawing, the line $|\vec{A}| \cos\alpha$ is rotated by 90° to show the geometrical interpretation of the scalar product as the area of a rectangle with sides $|\vec{A}| \cos\alpha$ and $|\vec{B}|$. In the same way, we can interpret $|\vec{B}| \cos\alpha$ as the projection of the vector $\vec{B}$ onto the vector $\vec{A}$ and construct a rectangle with side lengths $|\vec{B}| \cos\alpha$ and $|\vec{A}|$ (Figure 1.26b). The areas of the two yellow rectangles in Figure 1.25 are identical and are equal to the scalar product of the two vectors $\vec{A}$ and $\vec{B}$.

Finally, if we substitute from equation 1.28 for the cosine of the angle between the two vectors, the projection $|\vec{A}| \cos\alpha$ of the vector $\vec{A}$ onto the vector $\vec{B}$ can be written as

$$|\vec{A}| \cos\alpha = |\vec{A}| \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|} = \frac{\vec{A} \bullet \vec{B}}{|\vec{B}|},$$

and the projection $|\vec{B}| \cos\alpha$ of the vector $\vec{B}$ onto the vector $\vec{A}$ can be expressed as

$$|\vec{B}| \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|}.$$

3	Find the angle between two position vectors in the cartesian coordinates.	STUDENT TEXTBOOK	21,23
		EXAMPLE 1.5 / Q.1.80/Q1.103	22 / 29 /30

Vector Product

The **vector product** (or cross product) between two vectors $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ is defined as

$$\vec{C} = \vec{A} \times \vec{B}$$

$$C_x = A_y B_z - A_z B_y$$

$$C_y = A_z B_x - A_x B_z \qquad (1.32)$$

$$C_z = A_x B_y - A_y B_x.$$

$$\hat{x} \times \hat{y} = \hat{z}$$

$$\hat{y} \times \hat{z} = \hat{x}$$

$$\hat{z} \times \hat{x} = \hat{y}.$$

$$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta.$$

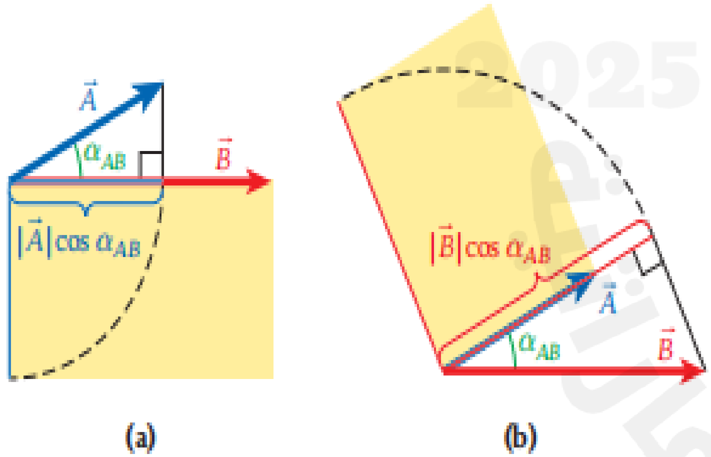


FIGURE 1.26 Geometrical interpretation of the scalar product as an area. (a) The projection of $\vec{A}$ onto $\vec{B}$. (b) The projection of $\vec{B}$ onto $\vec{A}$.

3	Find the angle between two position vectors in the cartesian coordinates.	STUDENT TEXTBOOK	21,23
		EXAMPLE 1.5 / Q.1.80/Q1.103	22 / 29 /30

EXAMPLE 1.5 Angle Between Two Position Vectors

PROBLEM

What is the angle α between the two position vectors shown in Figure 1.25, $\vec{A} = (4.00, 2.00, 5.00)$ cm and $\vec{B} = (4.50, 4.00, 3.00)$ cm?

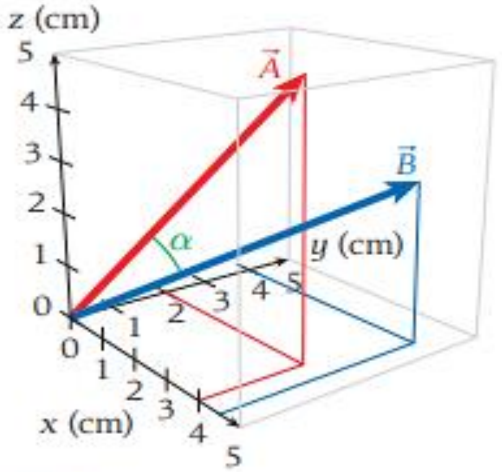


FIGURE 1.25 Calculating the angle between two position vectors.

3	Find the angle between two position vectors in the cartesian coordinates.	STUDENT TEXTBOOK	21,23
		EXAMPLE 1.5 / Q.1.80/Q1.103	22 / 29 /30

1.80 Express the vectors A = (Ax, Ay) = (-30.0 m, -50.0 m) and B = (Bx, By) = (30.0 m, 50.0 m) by giving their magnitude and direction as measured from the positive x-axis

1.103 Find the magnitude and direction of each of the following vectors, which are given in terms of their x- and y-components: A = (23.0, 59.0), and B = (90.0, -150.0)

4	[1] Multiply a vector with a scalar. [2] Add or subtract vectors using Cartesian components.	STUDENT TEXTBOOK	20
		Q. [1.76/1.79/1.105/1.106]	29

Multiplication of a Vector with a Scalar

What is $\vec{A}+\vec{A}+\vec{A}$? If your answer to this question is $3\vec{A}$, you already understand multiplying a vector with a scalar. The vector that results from multiplying the vector $\vec{A}$ with the scalar 3 is a vector that points in the same direction as the original vector $\vec{A}$ but is 3 times as long.

Multiplication of a vector with an arbitrary positive scalar—that is, a positive number—results in another vector that points in the same direction but has a magnitude that is the product of the magnitude of the original vector and the value of the scalar. Multiplication of a vector by a negative scalar results in a vector pointing in the opposite direction to the original with a magnitude that is the product of the magnitude of the original vector and the magnitude of the scalar.

Again, the component notation is useful. For the multiplication of a vector $\vec{A}$ with a scalar s, we obtain:

$$\vec{E}=s\vec{A}=s(A_x,A_y,A_z)=(sA_x,sA_y,sA_z). \tag{1.15}$$

In other words, each component of the vector $\vec{A}$ is multiplied by the scalar in order to arrive at the components of the product vector:

$$\begin{aligned} E_x &= sA_x \\ E_y &= sA_y \\ E_z &= sA_z. \end{aligned} \tag{1.16}$$

Unit Vectors

$$\hat{x}=(1,0,0)$$
$$\hat{y}=(0,1,0)$$
$$\hat{z}=(0,0,1).$$

4	[1] Multiply a vector with a scalar. [2] Add or subtract vectors using Cartesian components.	STUDENT TEXTBOOK	20
		Q. [1.76/1.79/1.105/1.106]	29

1.76 Find the **vector C** that satisfies the equation $3\hat{x} + 6\hat{y} - 10\hat{z} + \vec{C} = -7\hat{x} + 14\hat{y}$

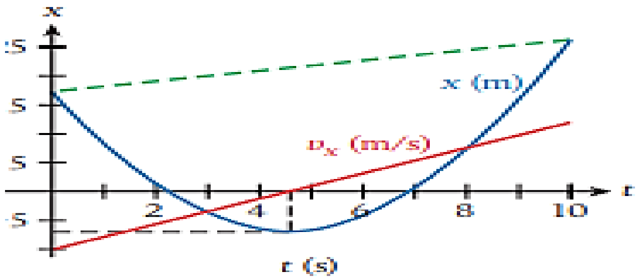
•1.79 Find the magnitude and direction of (a) $9\vec{B} - 3\vec{A}$ and (b) $-5\vec{A} + 8\vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

4	[1] Multiply a vector with a scalar. [2] Add or subtract vectors using Cartesian components.	STUDENT TEXTBOOK	20
		Q. [1.76/1.79/1.105/1.106]	29

1.105 Find the magnitude and direction of $-5\vec{A} + \vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

1.106 Find the magnitude and direction of $-7\vec{B} + 3\vec{A}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

5	[1] Calculate the speed as the magnitude of instantaneous velocity. [2] Calculate the average speed & average velocity. [3] Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.	STUDENT TEXTBOOK	36-39
		EXAMPLE (2.1 , 2.2) Q. [2.31/2.32/2.33]	38,39 61



IGURE 2.7 Graph of the position x and elocity v_x as a function of the time t. The ope of the dashed line represents the verage velocity for the time interval from to 10 s.

EXAMPLE 2.1 Time Dependence of Velocity

PROBLEM

During the time interval from 0.0 to 10.0 s, the position vector of a car on a road is given by $x(t) = a + bt + ct^2$, with $a = 17.2$ m, $b = -10.1$ m/s, and $c = 1.10$ m/s^2. What is the car's velocity as a function of time? What is the car's average velocity during this interval?

SOLUTION

According to the definition of velocity in equation 2.6, we simply take the time derivative of the position vector function to arrive at our solution:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(a + bt + ct^2) = b + 2ct = -10.1 \text{ m/s} + 2 \cdot (1.10 \text{ m/s}^2)t.$$

It is instructive to graph this solution. In Figure 2.7, the position as a function of time is shown in blue, and the velocity as a function of time is shown in red. Initially, the velocity has a value of -10.1 m/s, and at $t = 10$ s, the velocity has a value of $+11.9$ m/s.

Note that the velocity is initially negative, is zero at 4.59 s (indicated by the vertical dashed line in Figure 2.7), and then is positive after 4.59 s. At $t = 4.59$ s, the position graph $x(t)$ shows an extremum (a minimum in this case), just as expected from calculus, since

$$\frac{dx}{dt} = b + 2ct_0 = 0 \Rightarrow t_0 = -\frac{b}{2c} = -\frac{-10.1 \text{ m/s}}{2.20 \text{ m/s}^2} = 4.59 \text{ s}.$$

From the definition of average velocity, we know that to determine the average velocity during a time interval, we need to subtract the position at the beginning of the interval from the position at the end of the interval. By inserting $t = 0$ and $t = 10$ s into the equation for the position vector as a function of time, we obtain $x(t = 0) = 17.2$ m and $x(t = 10 \text{ s}) = 26.2$ m. Therefore,

$$\Delta x = x(t = 10) - x(t = 0) = 26.2 \text{ m} - 17.2 \text{ m} = 9.0 \text{ m}.$$

We then obtain for the average velocity over this time interval:

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{9.0 \text{ m}}{10 \text{ s}} = 0.90 \text{ m/s}.$$

The slope of the green dashed line in Figure 2.7 is the average velocity over this time interval.

5	[1] Calculate the speed as the magnitude of instantaneous velocity. [2] Calculate the average speed & average velocity. [3] Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.	STUDENT TEXTBOOK	36-39
		EXAMPLE (2.1 , 2.2) Q. [2.31/2.32/2.33]	38,39 61

EXAMPLE 2.2 Speed and Velocity

Suppose a swimmer completes the first 50 m of the 100-m freestyle in 38.2 s. Once she reaches the far side of the 50-m-long pool, she turns around and swims back to the start in 42.5 s.

PROBLEM

What are the swimmer's average velocity and average speed for (a) the leg from the start to the far side of the pool, (b) the return leg, and (c) the total lap?

Concept Check 2.3

The speedometer in your car shows

a) average speed.

b) instantaneous speed.

c) average displacement.

d) instantaneous displacement.

FIGURE 2.9 Choosing an x-axis in a swimming pool.

5	[1] Calculate the speed as the magnitude of instantaneous velocity. [2] Calculate the average speed & average velocity. [3] Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.	STUDENT TEXTBOOK	36-39
		EXAMPLE (2.1 , 2.2) Q. [2.31/2.32/2.33]	38,39 61

2.31 Running on a 50-m by 40-m rectangular track, you complete one lap in 100 s. What is your average velocity for the lap?

2.32 An electron moves in the positive x-direction a distance of 2.42 m in 2.91 × 10–8 s, bounces off a moving proton, and then moves in the opposite direction a distance of 1.69 m in 3.43 × 10–8 s.

a) What is the average velocity of the electron over the entire time interval?

b) What is the average speed of the electron over the entire time interval?

5	[1] Calculate the speed as the magnitude of instantaneous velocity. [2] Calculate the average speed & average velocity. [3] Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.	STUDENT TEXTBOOK	36-39
		EXAMPLE (2.1 , 2.2) Q. [2.31/2.32/2.33]	38,39 61

2.33 The graph describes the position of a particle in one dimension as a function of time.

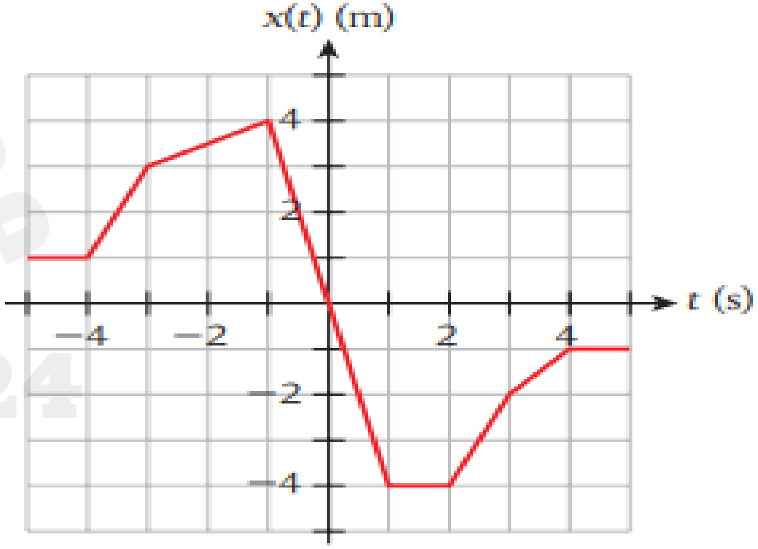
a) In which time interval does the particle have its maximum speed? What is that speed?

b) What is the average velocity in the time interval between -5 s and +5 s?

c) What is the average speed in the time interval between -5 s and +5 s?

d) What is the ratio of the velocity in the interval between 2 s and 3 s to the velocity in the interval between 3 s and 4 s?

e) At what time(s) is the particle's velocity zero?



6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

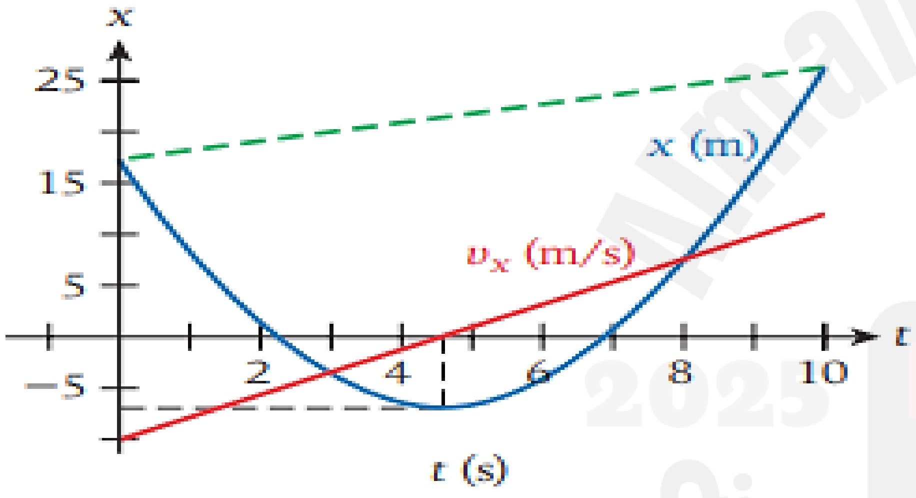
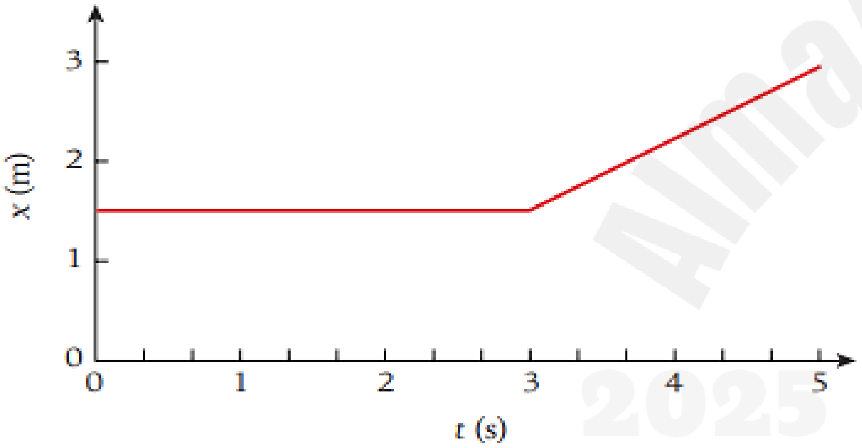


FIGURE 2.7 Graph of the position x and velocity v_x as a function of the time t. The slope of the dashed line represents the average velocity for the time interval from 0 to 10 s.

FIGURE 2.16 The acceleration, velocity, and displacement of the plane before takeoff.

6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

2.12 The figure describes the position of an object as a function of time. Which one of the following statements is true?



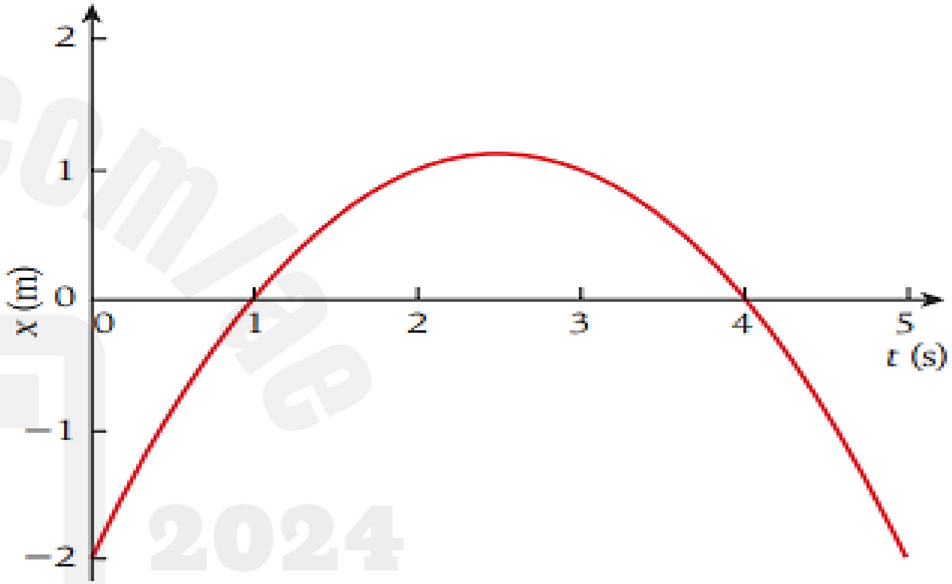
a) The position of the object is constant.

b) The velocity of the object is constant.

c) The object moves in the positive x-direction until $t = 3$ s, and then the object is at rest.

d) The object's position is constant until $t = 3$ s, and then the object begins to move in the positive x-direction.

e) The object moves in the positive x-direction from $t = 0$ to $t = 3$ s and then moves in the negative x-direction from $t = 3$ s to $t = 5$ s.



This figure describes the position of an object as a function of time. Refer to it to answer Questions 2.13–2.16.

2.13 Which one of the following statements is true at $t = 1$ s?

a) The x-component of the velocity of the object is zero.

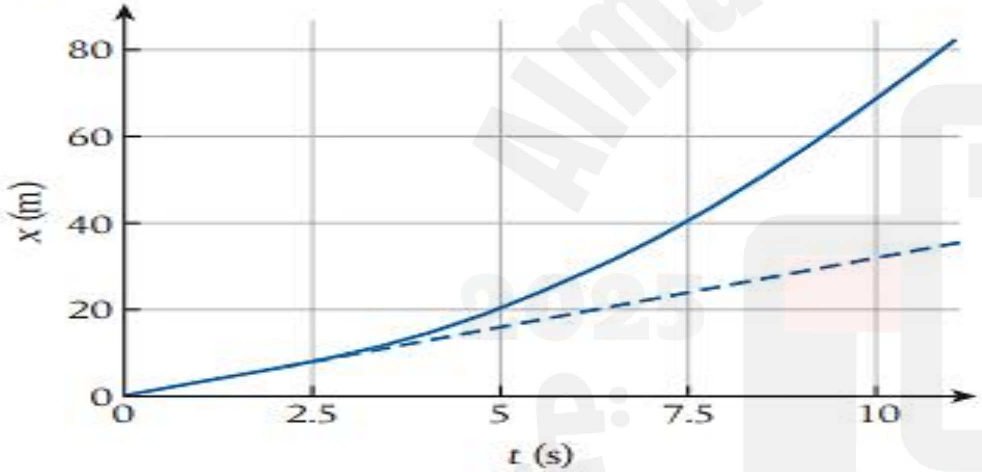
b) The x-component of the acceleration of the object is zero.

c) The x-component of the velocity of the object is positive.

d) The x-component of the velocity of the object is negative.

6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

2.26 A car moves along a road with a constant velocity. Starting at time $t = 2.5$ s, the driver accelerates with constant acceleration. The resulting position of the car as a function of time is shown by the blue curve in the figure.



a) What is the value of the constant velocity of the car before 2.5 s? (*Hint:* The dashed blue line is the path the car would take in the absence of the acceleration.)

b) What is the velocity of the car at $t = 7.5$ s? Use a graphical technique (i.e., draw a slope).

c) What is the value of the constant acceleration?

6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

2.33 The graph describes the position of a particle in one dimension as a function of time.

a) In which time interval does the particle have its maximum speed? What is that speed?

b) What is the average velocity in the time interval between -5 s and +5 s?

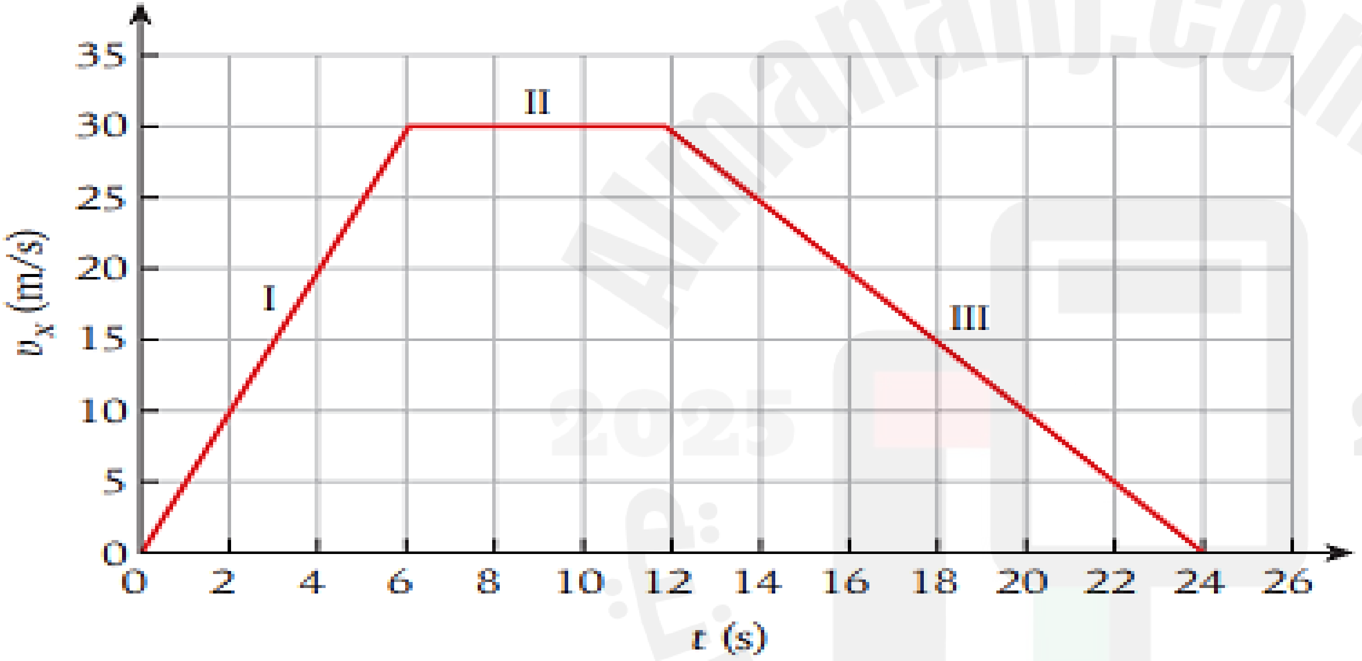
c) What is the average speed in the time interval between -5 s and +5 s?

d) What is the ratio of the velocity in the interval between 2 s and 3 s to the velocity in the interval between 3 s and 4 s?

e) At what time(s) is the particle's velocity zero?

6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

2.42 A fellow student found in the performance data for his new car the velocity-versus-time graph shown in the figure.

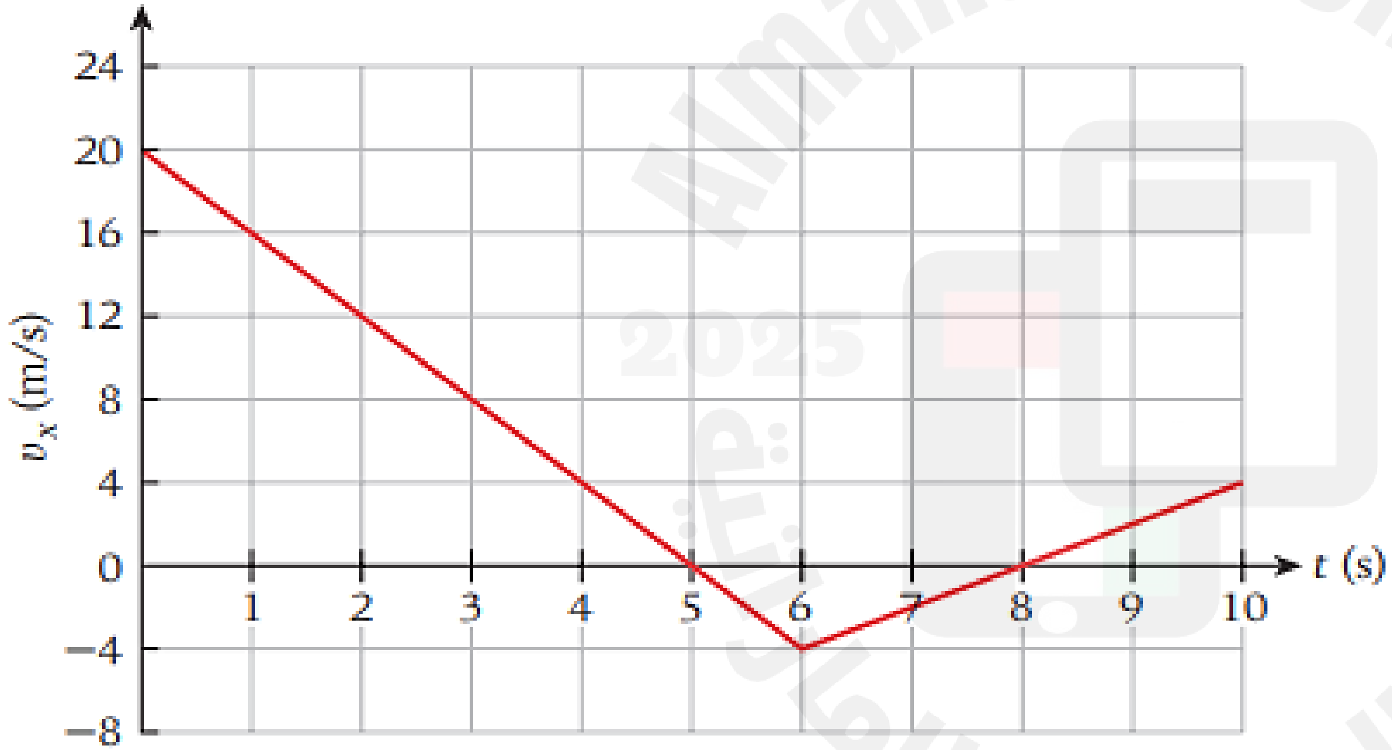


a) Find the average acceleration of the car during each of the segments I, II, and III.

b) What is the total distance traveled by the car from $t = 0$ s to $t = 24$ s?

6	[1] Interpret motion of an object from its position-time graph. [2] Interpret the motion of an object from a velocity-time graph.	FIGURE (2.7 , 2.16)	(38 , 45)
		Q. [2.12/2.13/2.26/2.33/ 2.42/2.51]	59,60, 61,62

•**2.51** A car is moving along the x-axis and its velocity, v_x, varies with time as shown in the figure. If $x_0 = 2.0$ m at $t_0 = 2.0$ s, what is the position of the car at $t = 10.0$ s?



7	[1] Interpret motion graphs for objects under free fall. [2] Apply the constant-acceleration equations to free-fall motion	STUDENT TEXTBOOK	50-54
		FIGURE (2.27 , 2.28) Q. [2.66/2.67/2.69]	54 63

2.8 Free Fall

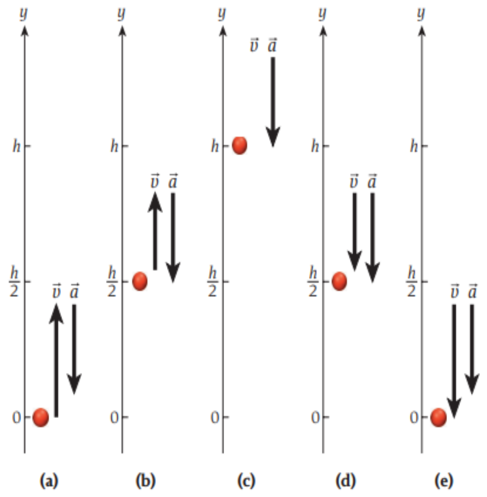


FIGURE 2.24 The velocity vector and acceleration vector of a ball thrown straight up in the air. (a) The ball is initially thrown upward at $y = 0$. (b) The ball going upward at a height of $y = h/2$. (c) The ball at its maximum height of $y = h$. (d) The ball coming down at $y = h/2$. (e) The ball back at $y = 0$ going downward.

Concept Check 2.9

If the reaction time of person B determined with the meter stick method is twice as long as that of person A, then the displacement h_B measured for person B in terms of the displacement h_A for person A is

a) $h_B = 2h_A$.

b) $h_B = \frac{1}{2} h_A$.

c) $h_B = \sqrt{2} h_A$.

d) $h_B = 4h_A$.

e) $h_B = \sqrt{\frac{1}{2}} h_A$.

Concept Check 2.8

A ball is thrown upward with a speed v_1, as shown in Figure 2.24. The ball reaches a maximum height of $y = h$. What is the ratio of the speed of the ball, v_2, at $y = h/2$ in Figure 2.24b, to the initial upward speed of the ball, v_1, at $y = 0$ in Figure 2.24a?

a) $v_2/v_1 = 0$

b) $v_2/v_1 = 0.50$

c) $v_2/v_1 = 0.71$

d) $v_2/v_1 = 0.75$

e) $v_2/v_1 = 0.90$

Concept Check 2.7

Throwing a ball straight up into the air provides an example of free-fall motion. At the instant the ball reaches its maximum height, which of the following statements is true?

a) The ball's acceleration vector points down, and its velocity vector points up.

b) The ball's acceleration is zero, and its velocity vector points up.

c) The ball's acceleration vector points up, and its velocity vector points up.

d) The ball's acceleration vector points down, and its velocity is zero.

e) The ball's acceleration vector points up, and its velocity is zero.

f) The ball's acceleration is zero, and its velocity vector points down.

Reaction Time

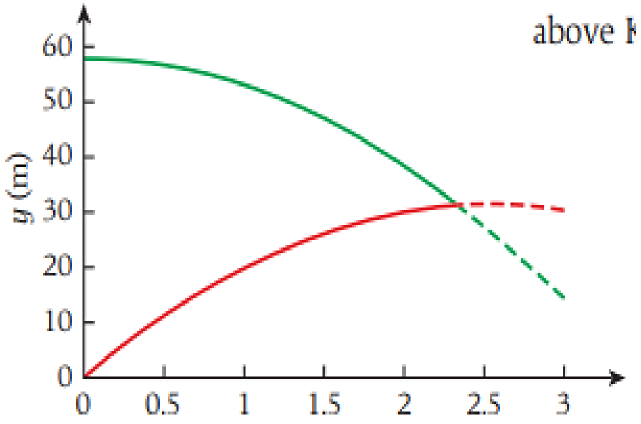
PROBLEM

If the meter stick falls 0.20 m before you catch it, what is your reaction time?

7	[1] Interpret motion graphs for objects under free fall. [2] Apply the constant-acceleration equations to free-fall motion	STUDENT TEXTBOOK	50-54
		FIGURE (2.27 , 2.28) Q. [2.66/2.67/2.69]	54 63

2.8 Free Fall

above h



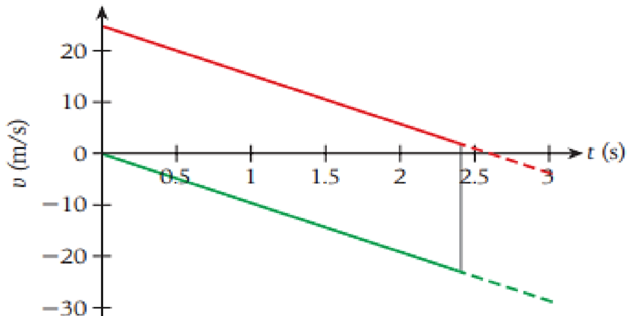


FIGURE 2.28 Velocities of the arrow (red curve) and melon (green curve) as a function of time.

SOLVED PROBLEM 2.5 Melon Drop

Suppose you decide to drop a melon from rest from the first observation platform of the Eiffel Tower. The initial height h from which the melon is released is 58.3 m above the head of your French friend Khaled, who is standing on the ground right below you. At the same instant you release the melon, Khaled shoots an arrow straight up with an initial velocity of 25.1 m/s. (Of course, Khaled makes sure the area around him is cleared and gets out of the way quickly after he shoots his arrow.)

PROBLEM

(a) How long after you drop the melon will the arrow hit it? (b) At what height above Khaled's head does this collision occur?

$$y_m(t) = h - \tfrac{1}{2}gt^2$$

$$y_a(t) = v_{a0}t - \tfrac{1}{2}gt^2.$$

$$t_c = \frac{58.3\text{ m}}{25.1\text{ m/s}} = 2.32271\text{ s}$$

The key insight is that at t_c, the moment when the melon and arrow collide, their coordinates are identical:

$$y_a(t_c) = y_m(t_c).$$

$$y_m(t_c) = 58.3\text{ m} - \tfrac{1}{2}(9.81\text{ m/s}^2)(2.32271\text{ s})^2 = 31.8376\text{ m}.$$

SIMPLIFY Inserting t_c into the two equations of motion and setting them equal results in

$$h - \tfrac{1}{2}gt_c^2 = v_{a0}t_c - \tfrac{1}{2}gt_c^2 \Rightarrow$$

$$h = v_{a0}t_c \Rightarrow$$

$$t_c = \frac{h}{v_{a0}}.$$

(i) $y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$

(ii) $y = y_0 + \bar{v}_y t$

(iii) $v_y = v_{y0} - gt$

(iv) $\bar{v}_y = \tfrac{1}{2}(v_y + v_{y0})$

(v) $v_y^2 = v_{y0}^2 - 2g(y - y_0)$

$$v_m(t_c) = -(9.81\text{ m/s}^2)(2.32\text{ s}) = -22.8\text{ m/s}$$

$$v_a(t_c) = (25.1\text{ m/s}) - (9.81\text{ m/s}^2)(2.32\text{ s}) = 2.34\text{ m/s}.$$

7	[1] Interpret motion graphs for objects under free fall. [2] Apply the constant-acceleration equations to free-fall motion	2.8 Free Fall	STUDENT TEXTBOOK	50-54
			FIGURE (2.27 , 2.28) Q. [2.66/2.67/2.69]	54 63

2.66 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?

2.67 A stone is thrown upward, from ground level, with an initial velocity of 10.0 m/s.

a) What is the velocity of the stone after 0.50 s?

b) How high above ground level is the stone after 0.50 s?

2.69 A ball is thrown directly downward, with an initial speed of 10.0 m/s, from a height of 50.0 m. After what time interval does the ball strike the ground?

8	Determine an object's change in velocity by the area under the curve in an acceleration versus time graph.	STUDENT TEXTBOOK	42-43
		FIGURE (2.13) Q. [2.48/2.53]	43 62

2.6 Finding Displacement and Velocity from Acceleration

The fact that integration is the inverse operation of differentiation is known as the *Fundamental Theorem of Calculus*. It allows us to reverse the differentiation process leading from displacement to velocity to acceleration and instead integrate the equation for velocity (2.6) to obtain displacement and the equation for acceleration (2.13) to obtain velocity. Let's start with the equation for the *x*-component of the velocity:

$$v_x(t) = \frac{dx(t)}{dt} \Rightarrow$$

$$\int_{t_0}^{t} v_x(t')\,dt' = \int_{t_0}^{t} \frac{dx(t')}{dt'}\,dt' = x(t) - x(t_0) \Rightarrow$$

$$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt'. \qquad (2.14)$$

$$a_x(t) = \frac{dv_x(t)}{dt} \Rightarrow$$

$$\int_{t_0}^{t} a_x(t')\,dt' = \int_{t_0}^{t} \frac{dv_x(t')}{dt'}\,dt' = v_x(t) - v_x(t_0) \Rightarrow$$

$$v_x(t) = v_{x0} + \int_{t_0}^{t} a_x(t')\,dt'.$$

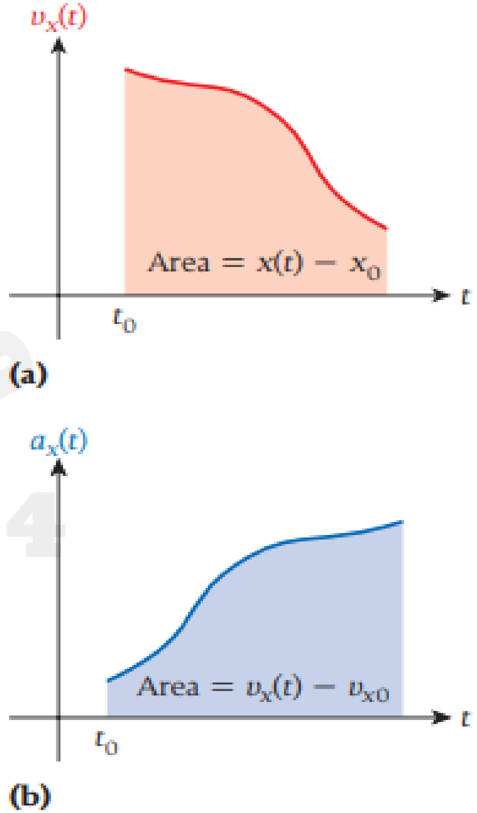
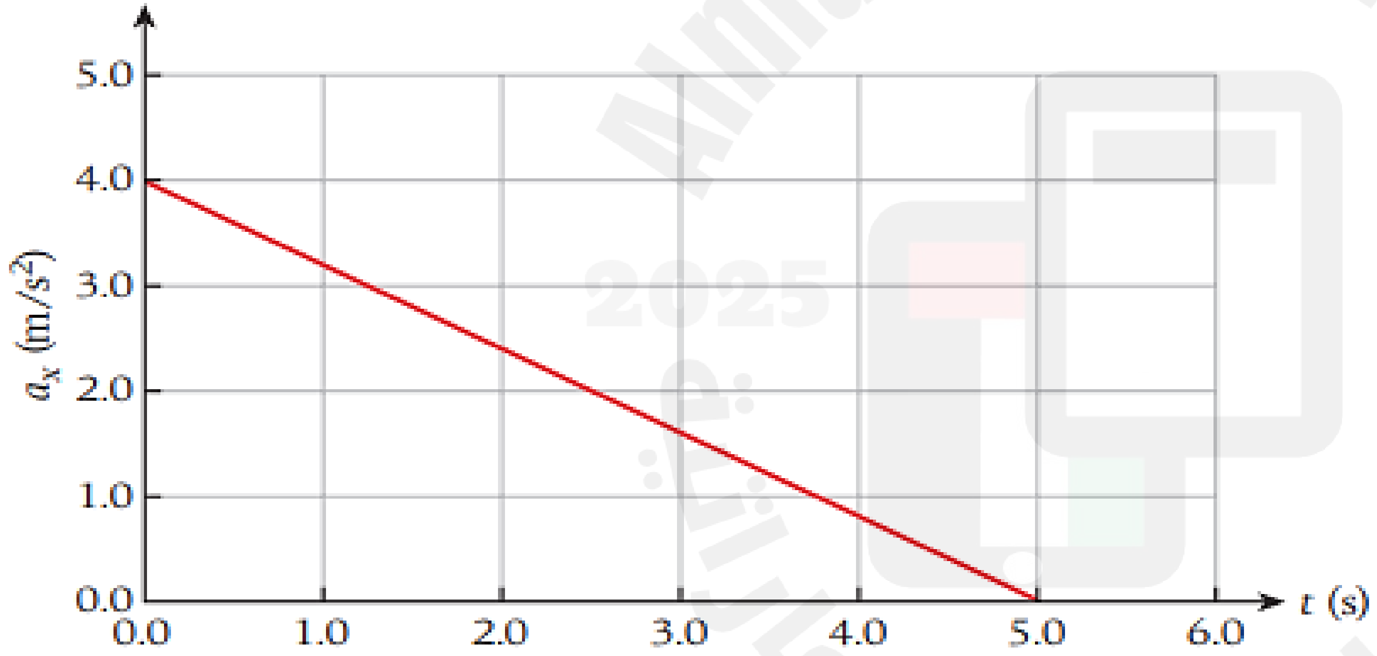


FIGURE 2.13 Geometrical interpretation of the integrals of (a) velocity and (b) acceleration with respect to time.

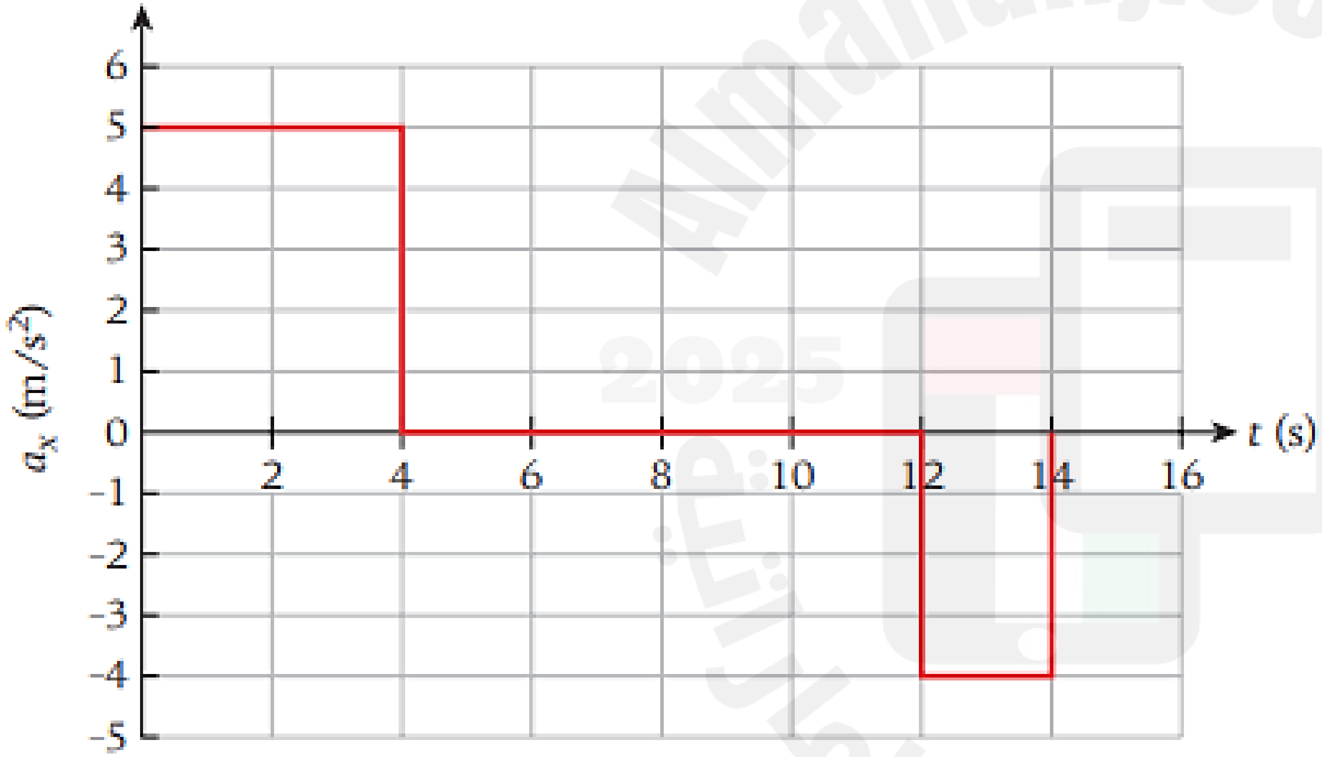
8	Determine an object's change in velocity by the area under the curve in an acceleration versus time graph.	STUDENT TEXTBOOK	42-43
		FIGURE (2.13) Q. [2.48/2.53]	43 62

2.48 A car moving in the x-direction has an acceleration a_x that varies with time as shown in the figure. At the moment $t = 0.0$ s, the car is located at $x = 12$ m and has a velocity of 6.0 m/s in the positive x-direction. What is the velocity of the car at $t = 5.0$ s?



8	Determine an object's change in velocity by the area under the curve in an acceleration versus time graph.	STUDENT TEXTBOOK	42-43
		FIGURE (2.13) Q. [2.48/2.53]	43 62

•2.53 A motorcycle starts from rest and accelerates as shown in the figure. Determine (a) the motorcycle's speed at $t = 4.00$ s and at $t = 14.0$ s and (b) the distance traveled in the first 14.0 s.



9	[1] Calculate a particle's change in velocity by integrating its acceleration function with respect to time. [2] Calculate a particle's change in position by integrating its velocity function with respect to time.	STUDENT TEXTBOOK	42
		Q. [2.49/2.50]	62

2.49 The velocity as a function of time for a car on an amusement park ride is given as $v = At^2 + Bt$ with constants $A = 2.0\ m/s^3$ and $B = 1.0\ m/s^2$

If the car starts at the origin, what is its position at $t = 3.0\ s$?

2.50 An object starts from rest and has an acceleration given by $a = Bt^2 - 12Ct$, where $B = 2.0\ m/s^4$ and $C = -4.0\ m/s^3$

a) What is the object's velocity after 5.0 s?

b) How far has the object moved after $t = 5.0\ s$?

10	[1] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and along a single axis. [2] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and in two dimensions	STUDENT TEXTBOOK	80
		EXAMPLE 3.3 EXAMPLE 3.4 Q. [3.63]	81 82 88

3.6 Relative Motion

To study motion, we have allowed ourselves to shift the origin of the coordinate system by properly choosing values for x_0 and y_0. In general, x_0 and y_0 are constants that can be chosen freely. If this choice is made intelligently, it can help make a problem more manageable. For example, when we calculated the path of the projectile, $y(x)$, we set $x_0 = 0$ to simplify our calculations. The freedom to select values for x_0 and y_0 arises from the fact that our ability to describe any kind of motion does not depend on the location of the origin of the coordinate system.

So far, we have examined physical situations where we have kept the origin of the coordinate system at a fixed location during the motion of the object we wanted to consider. However, in some physical situations, it is impractical to choose a reference system with a fixed origin. Consider, for example, a jet plane landing on an aircraft carrier that is going forward at full throttle at the same time. You want to describe the plane's motion in a coordinate system fixed to the carrier, even though the carrier is moving. The reason why this is important is that the plane needs to come to rest *relative* to the carrier at some fixed location on the deck. The reference frame from which we view motion makes a big difference in how we describe the motion, producing an effect known as **relative velocity.**

Another example of a situation for which we cannot neglect relative motion is a transatlantic flight from Detroit, Michigan, to Frankfurt, Germany, which takes 8 h and 10 min. Using the same aircraft and going in the reverse direction, from Frankfurt to Detroit, takes 9 h and 10 min, a full hour longer. The primary reason for this difference is that the prevailing wind at high altitudes, the jet stream, tends to blow from west to east at speeds as high as 67 m/s (150 mph). Even though the airplane's speed relative to the air around it is the same in both directions, that air is moving with its own speed. Thus, the relationship of the coordinate system of the air inside the jet stream to the coordinate system in which the locations of Detroit and Frankfurt remain fixed is important in understanding the difference in flight times.

For a more easily analyzed example of a moving coordinate system, let's consider motion on a moving walkway, as is typically found in airport terminals. This system is an example of one-dimensional relative motion. Suppose that the walkway surface moves with a certain velocity, v_{wt}, relative to the terminal. We use the subscripts w for walkway and t for terminal. Then a coordinate system that is fixed to the walkway surface has exactly velocity v_{wt} relative to a coordinate system attached to the terminal. The man shown in Figure 3.17 is walking with a velocity v_{mw} as measured in a coordinate system on the walkway, and he has a velocity $v_{mt} = v_{mw} + v_{wt}$ with respect to the terminal. The two velocities v_{mw} and v_{wt} add as vectors since the corresponding displacements add as vectors. (We will show this explicitly when we generalize to three dimensions.) For example, if the walkway moves with $v_{wt} = 1.5$ m/s and the man moves with $v_{mw} = 2.0$ m/s, then he will progress through the terminal with a velocity of $v_{mt} = v_{mw} + v_{wt} = 2.0 \text{ m/s} + 1.5 \text{ m/s} = 3.5$ m/s.

One can achieve a state of no motion relative to the terminal by walking in the direction opposite of the motion of the walkway with a velocity that is exactly the negative of the walkway velocity. Children often try to do this. If a child were to walk with $v_{mw} = -1.5$ m/s on this walkway, her velocity would be zero relative to the terminal.

10	[1] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and along a single axis.	STUDENT TEXTBOOK	80
	[2] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and in two dimensions	EXAMPLE 3.3 EXAMPLE 3.4 Q. [3.63]	81 82 88

EXAMPLE 3.3 Airplane in a Crosswind

Airplanes move relative to the air that surrounds them. Suppose a pilot points his plane in the northeast direction. The airplane moves with a speed of 160. m/s relative to the wind, and the wind is blowing at 32.0 m/s in a direction from east to west (measured by an instrument at a fixed point on the ground).

PROBLEM

What is the velocity vector—speed and direction—of the airplane relative to the ground? How far off course does the wind blow this plane in 2.0 h?

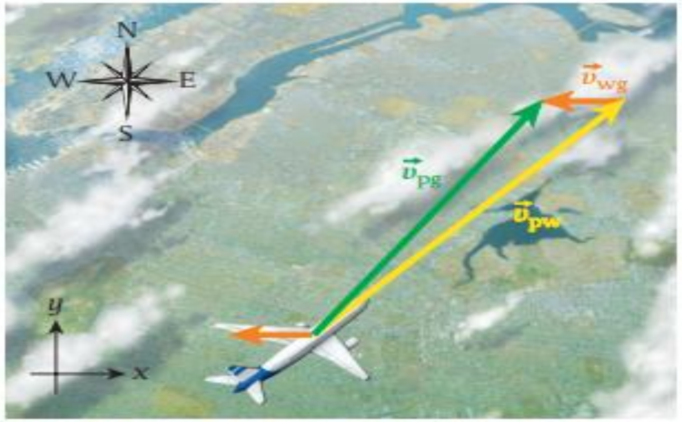


FIGURE 3.19 Velocity of an airplane with respect to the wind (yellow), the velocity of the wind with respect to the ground (orange), and the resultant velocity of the airplane with respect to the ground (green).

airplane

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}.$$

Here $\vec{v}_{pw}$ is the velocity of the plane with respect to the wind and has these components:

$$v_{pw,x} = v_{pw}\cos\theta = 160 \text{ m/s} \cdot \cos 45° = 113 \text{ m/s}$$

$$v_{pw,y} = v_{pw}\sin\theta = 160 \text{ m/s} \cdot \sin 45° = 113 \text{ m/s}.$$

$$v_{wg,x} = -32 \text{ m/s}$$

$$v_{wg,y} = 0.$$

$$v_{pg,x} = v_{pw,x} + v_{wg,x} = 113 \text{ m/s} - 32 \text{ m/s} = 81 \text{ m/s}$$

$$v_{pg,y} = v_{pw,y} + v_{pw,y} = 113 \text{ m/s}.$$

$$v_{pg} = \sqrt{v_{pg,x}^2 + v_{pg,y}^2} = 139 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{pg,y}}{v_{pg,x}}\right) = 54.4°.$$

$$\left|\vec{r}_T\right| = \left|\vec{v}_{wg}\right| t = 32.0 \text{ m/s} \times 7200 \text{ s} = 230.4 \text{ km}.$$

10	[1] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and along a single axis. [2] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and in two dimensions	STUDENT TEXTBOOK	80
		EXAMPLE 3.3 EXAMPLE 3.4 Q. [3.63]	81 82 88

EXAMPLE 3.4 Driving through Rain

Let's supppose rain is falling straight down on a car, as indicated by the white lines in Figure 3.20. A stationary observer outside the car would be able to measure the velocities of the rain (blue arrow) and of the moving car (red arrow).

However, if you are sitting inside the moving car, the outside world of the stationary observer (including the street, as well as the rain) moves with a relative velocity of $\vec{v} = -\vec{v}_{car}$. The velocity of this relative motion has to be added to all outside events as observed from inside the moving car. This motion results in a velocity vector $\vec{v}\,'_{rain}$ for the rain as observed from inside the moving car (Figure 3.21); mathematically, this vector is a sum, $\vec{v}\,'_{rain} = \vec{v}_{rain} - \vec{v}_{car}$, where $\vec{v}_{rain}$ and $\vec{v}_{car}$ are the velocity vectors of the rain and the car as observed by the stationary observer.

FIGURE 3.20 The velocity vectors of a moving car and of rain falling straight down on the car, as viewed by a stationary observer.

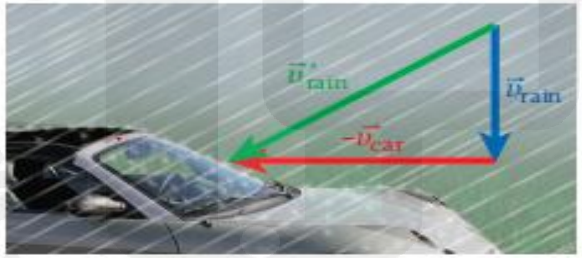


FIGURE 3.21 The velocity vector $\vec{v}\,'_{rain}$ of rain, as observed from inside the moving car.

Concept Check 3.8

It is raining, and there is practically no wind. While driving through the rain, you speed up. What happens to the angle of the rain relative to the horizontal that you observe from inside the car?

a) It *increases*.

b) It *decreases*.

c) It stays the same.

d) It can increase or decrease, depending on the direction in which you are driving.

10	[1] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and along a single axis. [2] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and in two dimensions	STUDENT TEXTBOOK	80
		EXAMPLE 3.3 EXAMPLE 3.4 Q. [3.63]	81 82 88

3.63 You are walking on a moving walkway in an airport. The length of the walkway is 59.1 m. If your velocity relative to the walkway is 2.35 m/s and the walkway moves with a velocity of 1.77 m/s, how long will it take you to reach the other end of the walkway?

11	Calculate the particle's position, displacement, and velocity at a given instant during the flight given the launch velocity	STUDENT TEXTBOOK	68-78
		MCQ. (3.1/3.2/3.4/3.6/3.10 3.11)	87

3.1 An arrow is shot horizontally with a speed of 20. m/s from the top of a tower 60. m high. The time to reach the ground will be

a) 8.9 s.
b) 7.1 s.
c) 3.5 s.
d) 2.6 s.
e) 1.0 s.

3.2 A projectile is launched from the top of a building with an initial velocity of 30.0 m/s at an angle of 60.0° above the horizontal. The magnitude of its velocity at $t = 5.00$ s after the launch is

a) −23.0 m/s.
b) 7.3 m/s.
c) 15.0 m/s.
d) 27.5 m/s.
e) 50.4 m/s.

11	Calculate the particle's position, displacement, and velocity at a given instant during the flight given the launch velocity	STUDENT TEXTBOOK	68-78
		MCQ. (3.1/3.2/3.4/3.6/3.10 3.11)	87

3.4 During practice two baseball outfielders throw a ball to the shortstop. In both cases the distance is 40.0 m. Outfielder 1 throws the ball with an initial speed of 20.0 m/s, outfielder 2 throws the ball with an initial speed of 30.0 m/s. In both cases the balls are thrown and caught at the same height above ground.

a) Ball 1 is in the air for a shorter time than ball 2.
b) Ball 2 is in the air for a shorter time than ball 1.
c) Both balls are in the air for the same duration.
d) The answer cannot be decided from the information given.

3.6 For a given initial speed of an ideal projectile, there is (are) _____ launch angle(s) for which the range of the projectile is the same.

a) only one
b) two different
c) more than two but a finite number of
d) only one if the angle is 45° but otherwise two different
e) an infinite number of

11	Calculate the particle's position, displacement, and velocity at a given instant during the flight given the launch velocity	STUDENT TEXTBOOK	68-78
		MCQ. (3.1/3.2/3.4/3.6/3.10 3.11)	87

3.10 A baseball is launched from the bat at an angle $\theta_0 = 30.0°$ with respect to the positive x-axis and with an initial speed of 40.0 m/s, and it is caught at the same height from which it was hit. Assuming ideal projectile motion (positive y-axis upward), the velocity of the ball when it is caught is

a) $(20.00\,\hat{x} + 34.64\,\hat{y})$ m/s.

b) $(-20.00\,\hat{x} + 34.64\,\hat{y})$ m/s.

c) $(34.64\,\hat{x} - 20.00\,\hat{y})$ m/s.

d) $(34.64\,\hat{x} + 20.00\,\hat{y})$ m/s.

3.11 In ideal projectile motion, the velocity and acceleration of the projectile at its maximum height are, respectively,

a) horizontal, vertical downward.

b) horizontal, zero.

c) zero, zero.

d) zero, vertical downward.

e) zero, horizontal.

12	[1] Describe an object in static equilibrium and dynamic equilibrium. [2] State the conditions for an object to be in equilibrium. [3] Calculate a force of unknown magnitude acting on an object in equilibrium.	STUDENT TEXTBOOK	97-99
		EXAMPLE 4.1 Q 4.34 / Q 4.81	100 122 / 125

4.4 Newton's Laws

Newton's First Law:

If the net force on an object is equal to zero, the object will remain at rest if it was at rest. If it was moving, it will remain in motion in a straight line with the same constant velocity.

Newton's Second Law:

If a net external force, $\vec{F}_{net}$, acts on an object with mass m, the force will cause an acceleration, $\vec{a}$, in the same direction as the force:

$$\vec{F}_{net} = m\vec{a}.$$

Newton's Third Law:

The forces that two interacting objects exert on each other are always exactly equal in magnitude and opposite in direction:

$$\vec{F}_{1\to2} = -\vec{F}_{2\to1}.$$

Newton's First Law says there are two possible states for an object with no net force on it: An object at rest is said to be in **static equilibrium.** An object moving with constant velocity is said to be in **dynamic equilibrium.**

$$\vec{F}_{net} = 0 \,;$$

12	[1] Describe an object in static equilibrium and dynamic equilibrium. [2] State the conditions for an object to be in equilibrium. [3] Calculate a force of unknown magnitude acting on an object in equilibrium.	STUDENT TEXTBOOK	97-99
		EXAMPLE 4.1 Q 4.34 / Q 4.81	100 122 / 125

EXAMPLE 4.1 Modified Tug-of-War

In a tug-of-war competition, two teams try to pull each other across a line. If neither team is moving, then the two teams exert equal and opposite forces on a rope. This is an immediate consequence of Newton's Third Law. That is, if one team pulls on the rope with a force of magnitude *F*, the other team necessarily has to pull on the rope with a force of the same magnitude but in the opposite direction.

Children playing tug-of-war.

PROBLEM

Now let's consider the situation where three ropes are tied together at one point, with a team pulling on each rope. Suppose team 1 is pulling due west with a force of 2750 N, and team 2 is pulling due north with a force of 3630 N. Can a third team pull in such a way that the three-team tug-of-war ends at a standstill, that is, no team is able to move the rope? If yes, what is the magnitude and direction of the force needed to accomplish this?

$$0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\Leftrightarrow \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$$

$$= (2750 \text{ N})\hat{x} - (3630 \text{ N})\hat{y}.$$

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = \sqrt{(2750 \text{ N})^2 + (-3630 \text{ N})^2} = 4554 \text{ N}$$

$$\theta_3 = \tan^{-1}\left(\frac{F_{3,y}}{F_{3,x}}\right) = \tan^{-1}\left(\frac{-3630 \text{ N}}{2750 \text{ N}}\right) = -52.9°.$$

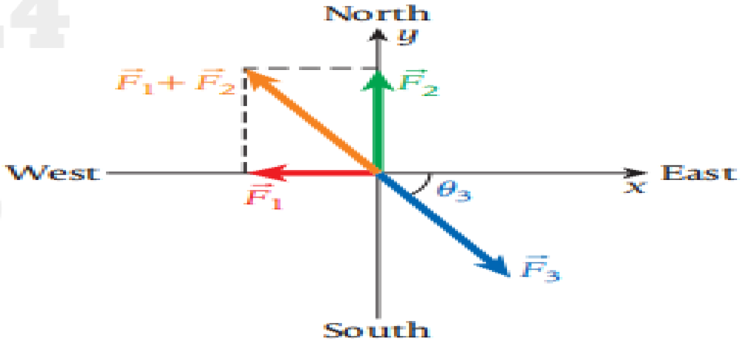


FIGURE 4.9 Addition of force vectors in the three-team tug-of-war.

12	[1] Describe an object in static equilibrium and dynamic equilibrium. [2] State the conditions for an object to be in equilibrium. [3] Calculate a force of unknown magnitude acting on an object in equilibrium.	STUDENT TEXTBOOK EXAMPLE 4.1 Q 4.34 / Q 4.81	97-99 100 122 / 125

4.34 In a physics laboratory class, three massless ropes are tied together at a point. A pulling force is applied along each rope: F1 = 150. N at 60.0°, F2 = 200. N at 100.°, F3 = 100. N at 190.°. What is the magnitude of a fourth force and the angle at which it acts to keep the point at the center of the system stationary? (All angles are measured from the positive x-axis.)

12	[1] Describe an object in static equilibrium and dynamic equilibrium. [2] State the conditions for an object to be in equilibrium. [3] Calculate a force of unknown magnitude acting on an object in equilibrium.	STUDENT TEXTBOOK	97-99
		EXAMPLE 4.1 Q 4.34 / Q 4.81	100 122 / 125

4.81 A block of mass 5.00 kg is sliding at a constant velocity down an inclined plane that makes an angle of 37.0° with respect to the horizontal.

a) What is the friction force?

b) What is the coefficient of kinetic friction?

13	[1] Apply the relationship between the drag force on an object moving through air and the speed of the object. [2] Determine the terminal speed of an object falling through air	STUDENT TEXTBOOK	111-112
		EXAMPLE 4.7 / Q 4.55	112 /124

Air Resistance

In general, the magnitude of the friction force due to air resistance, or **drag force**, can be expressed as $F_{\text{frict}} = K_0 + K_1 v + K_2 v^2 + \dots$, with the constants $K_0, K_1, K_2, \dots$ determined experimentally. For the drag force on macroscopic objects moving at relatively high speeds, we can neglect the linear term in the velocity. The magnitude of the drag force is then approximately

$$F_{\text{drag}} = Kv^2 . \quad (4.13)$$

This equation means that the force due to air resistance is proportional to the square of the speed.

When an object falls through air, the force from air resistance increases as the object accelerates until it reaches a **terminal speed**. At this point, the upward force of air resistance and the downward force due to gravity equal each other. Thus, the net force is zero, and there is no more acceleration. Because there is no more acceleration, the falling object has constant terminal speed:

$$F_g = F_{\text{drag}} \Rightarrow mg = Kv^2 .$$

Solving this for the terminal speed, we obtain

$$v = \sqrt{\frac{mg}{K}} . \quad (4.14)$$

To compute the terminal speed for a falling object, we need to know the value of the constant K. This constant depends on many variables, including the size of the cross-sectional area, A, exposed to the air stream. In general terms, the bigger the area, the bigger is the constant K. K also depends linearly on the air density, ρ. All other dependences on the shape of the object, on its inclination relative to the direction of motion, on air viscosity, and compressibility are usually collected in a drag coefficient, c_d:

$$K = \tfrac{1}{2} c_d A \rho . \quad (4.15)$$

13	[1] Apply the relationship between the drag force on an object moving through air and the speed of the object. [2] Determine the terminal speed of an object falling through air	STUDENT TEXTBOOK	111-112
		EXAMPLE 4.7 / Q 4.55	112 /124

EXAMPLE 4.7 Sky Diving

An 80.0-kg skydiver falls through air with a density of 1.15 kg/m^3. Assume that his drag coefficient is $c_d = 0.570$. When he falls in the spread-eagle position, as shown in Figure 4.20a, his body presents an area $A_1 = 0.940\ \text{m}^2$ to the wind, whereas when he dives head first, with arms close to the body and legs together, as shown in Figure 4.20b, his area is reduced to $A_2 = 0.210\ \text{m}^2$.

(a)

PROBLEM

What are the terminal speeds in both cases?

$$v = \sqrt{\frac{mg}{K}} = \sqrt{\frac{mg}{\frac{1}{2}c_d A\rho}}$$

$$v_1 = \sqrt{\frac{(80.0\ \text{kg})(9.81\ \text{m/s}^2)}{\frac{1}{2}0.570(0.940\ \text{m}^2)(1.15\ \text{kg/m}^3)}} = 50.5\ \text{m/s}$$

$$v_2 = \sqrt{\frac{(80.0\ \text{kg})(9.81\ \text{m/s}^2)}{\frac{1}{2}0.570(0.210\ \text{m}^2)(1.15\ \text{kg/m}^3)}} = 107\ \text{m/s}.$$

13	[1] Apply the relationship between the drag force on an object moving through air and the speed of the object. [2] Determine the terminal speed of an object falling through air	STUDENT TEXTBOOK	111-112
		EXAMPLE 4.7 / Q 4.55	112 /124

4.55 A skydiver of mass 82.3 kg (including outfit and equipment) floats downward suspended from his parachute, having reached terminal speed. The drag coefficient is 0.533, and the area of his parachute is 20.11 m^2 . The density of air is 1.14 kg/m^3

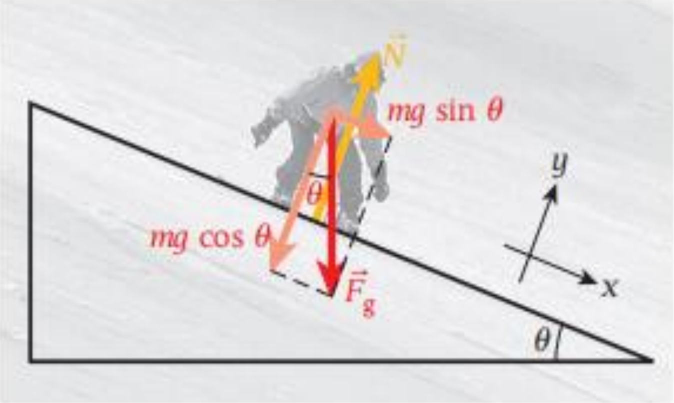
What is the air's drag force on him?

14	[1] Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle [2] Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction	SOLVED PROBLEM (4.1) EXAMPLE (4.8) SOLVED PROBLEM (4.4) EXAMPLE (4.9)	104 114 116 118

SOLVED PROBLEM 4.1 Snowboarding

PROBLEM

A snowboarder (mass 72.9 kg, height 1.79 m) glides down a slope with an angle of 22° with respect to the horizontal (Figure 4.15a). If we can neglect friction, what is his acceleration?



14	[1] Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle [2] Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction	SOLVED PROBLEM (4.1) EXAMPLE (4.8) SOLVED PROBLEM (4.4) EXAMPLE (4.9)	104 114 116 118

EXAMPLE 4.8 Two Blocks Connected by a Rope—with Friction

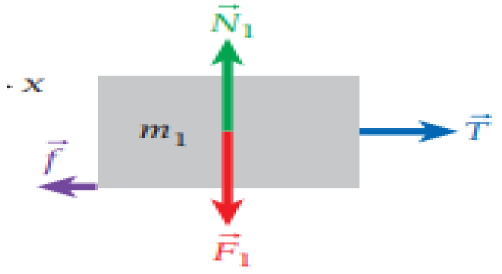
We solved this problem in Solved Problem 4.2, with the assumptions that block 1 slides without friction across the horizontal support surface and that the rope slides without friction across the pulley. Here we will allow for friction between block 1 and the surface it slides across. For now, we will still assume that the rope slides without friction across the pulley. (Chapter 10 will present techniques that let us deal with the pulley being set into rotational motion by the rope moving across it.)

PROBLEM 1

Let the coefficient of static friction between block 1 (mass $m_1 = 2.3$ kg) and its support surface have a value of 0.73 and the coefficient of kinetic friction have a value of 0.60. (Refer back to Figure 4.16.) If block 2 has mass $m_2 = 1.9$ kg, will block 1 accelerate from rest?

PROBLEM 2

What is the value of the acceleration?



14	[1] Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle [2] Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction	SOLVED PROBLEM (4.1) EXAMPLE (4.8) SOLVED PROBLEM (4.4) EXAMPLE (4.9)	104 114 116 118

SOLVED PROBLEM 4.4 Two Blocks

Two rectangular blocks are stacked on a table as shown in Figure 4.24a. The upper block has a mass of 3.40 kg, and the lower block has a mass of 38.6 kg. The coefficient of kinetic friction between the lower block and the table is 0.260. The coefficient of static friction between the blocks is 0.551. A string is attached to the lower block, and an external force $\vec{F}$ is applied horizontally, pulling on the string as shown.

PROBLEM

What is the maximum force that can be applied to the string without having the upper block slide off?

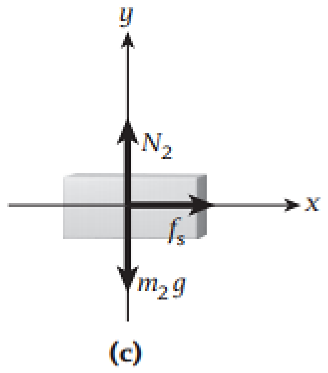


FIGURE 4.24 (a) Two stacked blocks being pulled to the right. (b) Free-body diagram for the two blocks moving together. (c) Free-body diagram for the upper block.

14	[1] Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle [2] Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction	SOLVED PROBLEM (4.1) EXAMPLE (4.8) SOLVED PROBLEM (4.4) EXAMPLE (4.9)	104 114 116 118

EXAMPLE 4.9 Pulling a Sled

Suppose you are pulling a sled across a level snow-covered surface by exerting constant force on a rope, at an angle θ relative to the ground.

PROBLEM 1

If the sled, including its load, has a mass of 15.3 kg, the coefficients of friction between the sled and the snow are $\mu_s = 0.076$ and $\mu_k = 0.070$, and you pull with a force of 25.3 N on the rope at an angle of 24.5° relative to the horizontal ground, what is the sled's acceleration?

PROBLEM 2

What angle of the rope with the horizontal will produce the maximum acceleration of the sled for the given value of the magnitude of the pulling force, T? What is that maximum value of a?

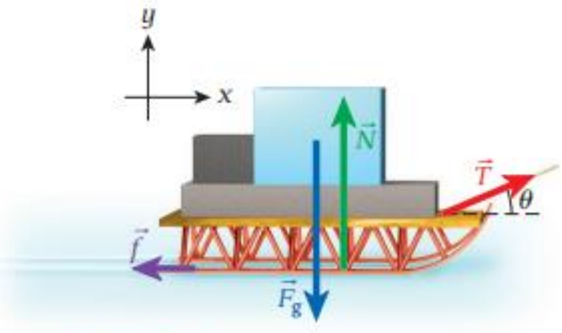
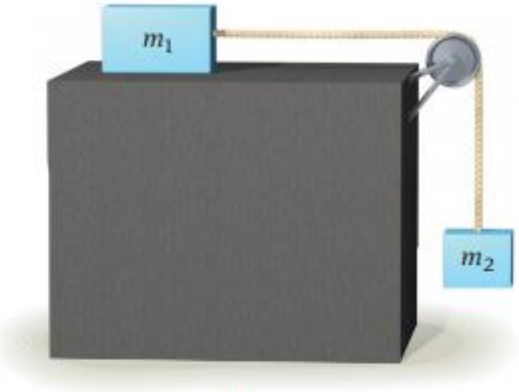


FIGURE 4.25 Free-body diagram of the sled and its load.

15	[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope. [2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley	SOLVED PROBLEM (4.2) EXAMPLE (4.4)	105 106
		Q. (4.35/4.48/4.96)	122,123,126

SOLVED PROBLEM 4.2 Two Blocks Connected by a Rope

In this classic problem, a hanging mass causes the acceleration of a second mass that is resting on a horizontal surface (Figure 4.16a). Block 1, of mass $m_1 = 3.00$ kg, rests on a horizontal frictionless surface and is connected via a massless rope (for simplicity, oriented in the horizontal direction) running over a massless pulley to block 2, of mass $m_2 = 1.30$ kg.



(a)

PROBLEM

What is the acceleration of block 1 and of block 2?

15	[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope. [2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley	SOLVED PROBLEM (4.2) EXAMPLE (4.4)	105 106
		Q. (4.35/4.48/4.96)	122,123,126

EXAMPLE 4.4 Atwood Machine

The Atwood machine consists of two hanging weights (with masses m_1 and m_2) connected via a rope running over a pulley. For now, we consider a friction-free case, where the pulley does not move, and the rope glides over it. (In Chapter 10 on rotation, we will return to this problem and solve it with friction present, which causes the pulley to rotate.) We also assume

that $m_1 > m_2$. In this case, the acceleration is as shown in Figure 4.17a. (The formula derived in the following is correct for any case. If $m_1 < m_2$, then the value of the acceleration, a, will have a negative sign, which will mean that the acceleration direction is opposite to what we have assumed in working the problem.)

(a)

$$a = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right).$$

Self-Test Opportunity 4.2

What is the acceleration of the masses on the Atwood machine at the limits where m_1 approaches infinity, m_1 approaches zero, and $m_1 = m_2$?

Self-Test Opportunity 4.3

For the Atwood machine, can you write a formula for the magnitude of the tension in the rope?

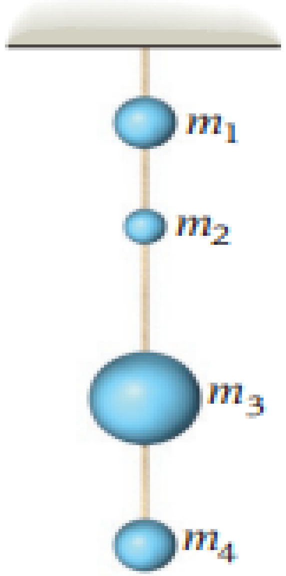
Concept Check 4.4

If you double both masses in an Atwood machine, the resulting acceleration will be

a) twice as large.
b) half as large.
c) the same.
d) one-quarter as large.
e) four times as large.

15	[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope. [2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley	SOLVED PROBLEM (4.2) EXAMPLE (4.4)	105 106
		Q. (4.35/4.48/4.96)	122,123,126

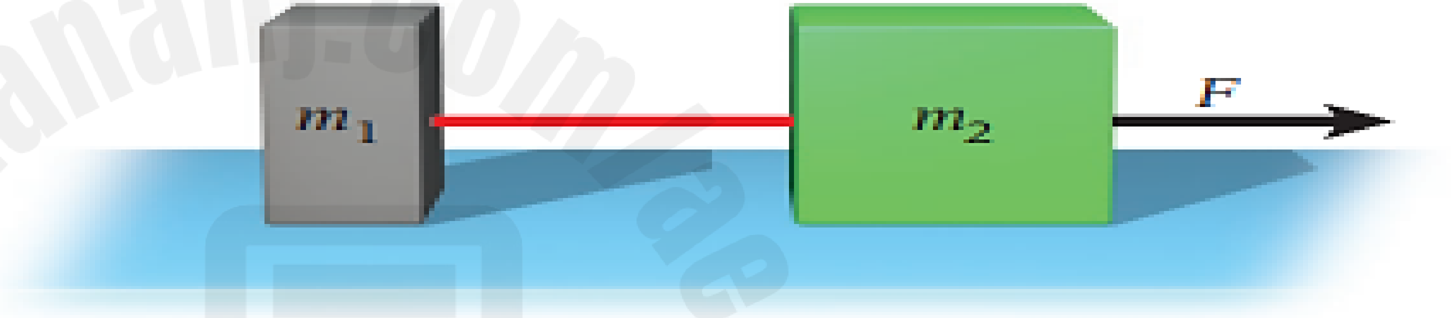
4.35 Four weights, of masses m1 = 6.50 kg, m2 = 3.80 kg, m3 = 10.70 kg, and m4 = 4.20 kg, are hanging from a ceiling as shown in the figure. They are connected with ropes. What is the tension in the rope connecting masses m1 and m2?



15	[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope. [2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley	SOLVED PROBLEM (4.2) EXAMPLE (4.4)	105 106
		Q. (4.35/4.48/4.96)	122,123,126

•**4.48** A mass, $m_1 = 20.0$ kg, on a frictionless ramp is attached to a light string. The string passes over a frictionless pulley and is attached to a hanging mass, m_2. The ramp is at an angle of $\theta = 30.0°$ above the horizontal. The mass m_1 moves up the ramp uniformly (at constant speed). Find the value of m_2.

15	[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope. [2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley	SOLVED PROBLEM (4.2) EXAMPLE (4.4)	105 106
		Q. (4.35/4.48/4.96)	122,123,126



4.96 Two blocks are connected by a massless rope, as shown in the figure. Block 1 has mass $m_1 = 1.267$ kg, and block 2 has mass $m_2 = 3.557$ kg. The two blocks move on a frictionless, horizontal tabletop. A horizontal external force, $F = 12.61$ N, acts on block 2. What is the tension in the rope connecting the two blocks?

Paper exam

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

Cartesian Representation of Vectors

$\vec{A} = (A_x, A_y)$ in two-dimensional space

$\vec{A} = (A_x, A_y, A_z)$ in three-dimensional space

Vector Addition Using Components

$$\vec{C} = \vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y + B_y, A_z + B_z).$$

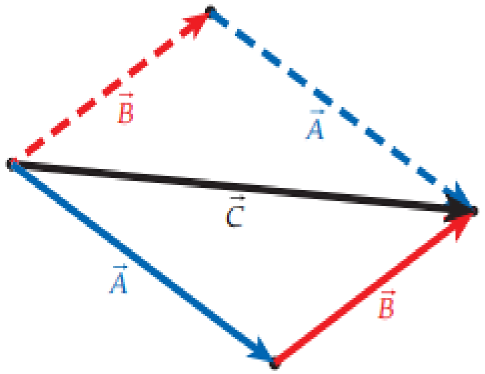


FIGURE 1.19 Commutative property of vector addition.

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z.$$

Graphical Vector Addition and Subtraction

$$\vec{C} = \vec{A} + \vec{B}.$$

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}.$$

$$\vec{C} + (-\vec{C}) = \vec{C} - \vec{C} = (0,0,0),$$

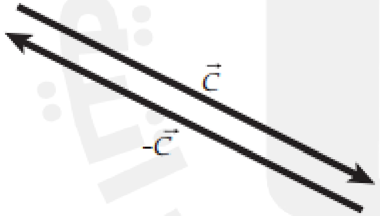
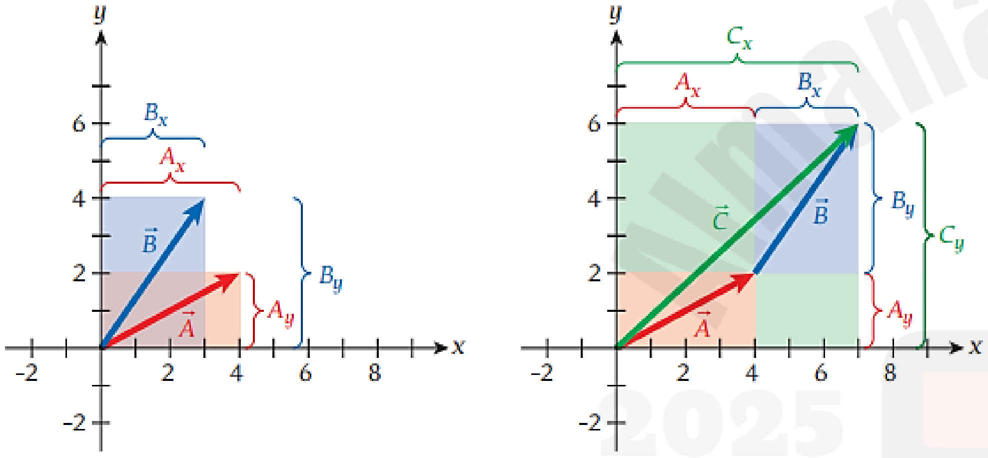


FIGURE 1.20 Inverse vector $-\vec{C}$ of a vector $\vec{C}$.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30



(a) (b)

Figure 1.21b displays their sum vector $\vec{C}=(4+3, 2+4)=(7,6)$. Figure 1.21b clearly shows that $C_x = A_x + B_x$, because the whole is equal to the sum of its parts.

In the same way, we can take the difference $\vec{D}=\vec{A}-\vec{B}$, and the Cartesian components of the difference vector are given by

$$D_x = A_x - B_x$$
$$D_y = A_y - B_y \qquad (1.14)$$
$$D_z = A_z - B_z .$$

Multiplication of a Vector with a Scalar

$$\vec{E}=s\vec{A}=s(A_x, A_y, A_z)=(sA_x, sA_y, sA_z). \qquad (1.15)$$

In other words, each component of the vector $\vec{A}$ is multiplied by the scalar in order to arrive at the components of the product vector:

$$E_x = sA_x$$
$$E_y = sA_y \qquad (1.16)$$
$$E_z = sA_z .$$

Unit Vectors

$$\hat{x}=(1,0,0)$$
$$\hat{y}=(0,1,0)$$
$$\hat{z}=(0,0,1).$$

$$\begin{aligned}\vec{A}&=(A_x, A_y, A_z)\\ &=(A_x,0,0)+(0,A_y,0)+(0,0,A_z)\\ &=A_x(1,0,0)+A_y(0,1,0)+A_z(0,0,1)\\ &=A_x\hat{x}+A_y\hat{y}+A_z\hat{z}.\end{aligned}$$

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis.	STUDENT TEXTBOOK	18-25
	Add or subtract vectors using Cartesian components.	FIGURES (1.18/1.21/1.28)	19,20,24
	Add and subtract vectors graphically to find the resultant vectors.	SOLVED PROBLEM. (1.3)	24
	Identify cartesian unit vectors in two and three dimensions.	Q. (1.65/1.67/1.97)	29,30

Vector Length and Direction

If we know the component representation of a vector, how can we find its length (magnitude) and the direction it is pointing in? Let's look at the most important case: a vector in two dimensions. In two dimensions, a vector $\vec{A}$ can be specified uniquely by giving the two Cartesian components, A_x and A_y. We can also specify the same vector by giving two other numbers: its length A and its angle θ with respect to the positive x-axis.

Let's take a look at Figure 1.23 to see how we can determine A and θ from A_x and A_y. Figure 1.23a shows the graphical representation of equation 1.19. The vector $\vec{A}$ is the sum of the vectors $A_x\hat{x}$ and $A_y\hat{y}$. Since the unit vectors $\hat{x}$ and $\hat{y}$ are by definition orthogonal to each other, these vectors form a 90° angle. Thus, the three vectors $\vec{A}$, $A_x\hat{x}$, and $A_y\hat{y}$ form a right triangle with side lengths A, A_x, and A_y, as shown in Figure 1.23b.

Now we can employ basic trigonometry to find θ and A. Using the Pythagorean Theorem results in

$$A = \sqrt{A_x^2 + A_y^2}. \tag{1.20}$$

We can find the angle θ from the definition of the tangent function

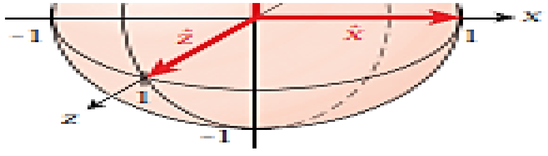
$$\theta = \tan^{-1}\frac{A_y}{A_x}. \tag{1.21}$$

In using equation 1.21, you must be careful that θ is in the correct quadrant. We can also invert equations 1.20 and 1.21 to obtain the Cartesian components of a vector of given length and direction:

$$A_x = A\cos\theta \tag{1.22}$$

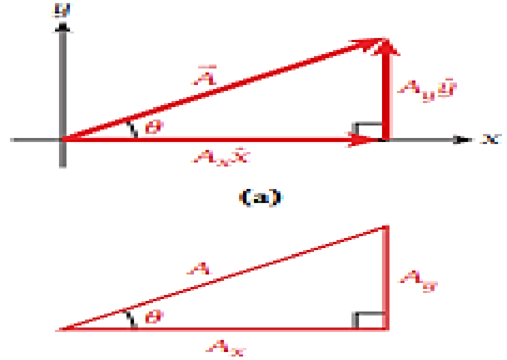
$$A_y = A\sin\theta. \tag{1.23}$$

You will encounter these trigonometric relations again and again throughout introductory physics. If you need to refamiliarize yourself with trigonometry, consult the mathematics primer provided in Appendix A.



(b)

FIGURE 1.22 Cartesian unit vectors in (a) two and (b) three dimensions.



(b)

FIGURE 1.23 Length and direction of a vector. (a) Cartesian components A_x and A_y; (b) length A and angle θ.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

Scalar Product of Vectors

Above we saw how to multiply a vector with a scalar. Now we will define one way of multiplying a vector with a vector and obtain the **scalar product.** The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}| \cos \alpha, \quad (1.24)$$

Scalar Product for Unit Vectors. On page 26 we introduced unit vectors in the three-dimensional Cartesian coordinate system: $\hat{x} = (1,0,0)$, $\hat{y} = (0,1,0)$, and $\hat{z} = (0,0,1)$. With our definition (1.25) of the scalar product, we find

$$\hat{x} \bullet \hat{x} = \hat{y} \bullet \hat{y} = \hat{z} \bullet \hat{z} = 1 \quad (1.30)$$

and

$$\begin{gathered} \hat{x} \bullet \hat{y} = \hat{x} \bullet \hat{z} = \hat{y} \bullet \hat{z} = 0 \\ \hat{y} \bullet \hat{x} = \hat{z} \bullet \hat{x} = \hat{z} \bullet \hat{y} = 0. \end{gathered} \quad (1.31)$$

Now we see why the unit vectors are called that: Their scalar products with themselves have the value 1. Thus, the unit vectors have length 1, or unit length, according to equation 1.27. In addition, any pair of different unit vectors has a scalar product that is zero, meaning that these vectors are orthogonal to each other. Equations 1.30 and 1.31 thus state that the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ form an orthonormal set of vectors, which makes them extremely useful for the description of physical systems.

Self-Test Opportunity 1.1

Show that equations 1.30 and 1.31 are correct by using equation 1.25 and the definitions of the unit vectors.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	18-25 19,20,24 24 29,30

$$\vec{A} \bullet \vec{B} = (A_x, A_y, A_z) \bullet (B_x, B_y, B_z) = A_x B_x + A_y B_y + A_z B_z.$$

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}.$$

$$\vec{A} \bullet \vec{B} = |\vec{A}| \; |\vec{B}| \cos\alpha \Rightarrow \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|} \Rightarrow \alpha = \cos^{-1}\left(\frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|}\right).$$

Geometrical Interpretation of the Scalar Product. In the definition of the scalar product $\vec{A} \bullet \vec{B} = |\vec{A}| \; |\vec{B}| \cos\alpha$ (equation 1.24), we can interpret $|\vec{A}| \cos\alpha$ as the projection of the vector $\vec{A}$ onto the vector $\vec{B}$ (Figure 1.26a). In this drawing, the line $|\vec{A}| \cos\alpha$ is rotated by 90° to show the geometrical interpretation of the scalar product as the area of a rectangle with sides $|\vec{A}| \cos\alpha$ and $|\vec{B}|$. In the same way, we can interpret $|\vec{B}| \cos\alpha$ as the projection of the vector $\vec{B}$ onto the vector $\vec{A}$ and construct a rectangle with side lengths $|\vec{B}| \cos\alpha$ and $|\vec{A}|$ (Figure 1.26b). The areas of the two yellow rectangles in Figure 1.25 are identical and are equal to the scalar product of the two vectors $\vec{A}$ and $\vec{B}$.

Finally, if we substitute from equation 1.28 for the cosine of the angle between the two vectors, the projection $|\vec{A}| \cos\alpha$ of the vector $\vec{A}$ onto the vector $\vec{B}$ can be written as

$$|\vec{A}| \cos\alpha = |\vec{A}| \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| \; |\vec{B}|} = \frac{\vec{A} \bullet \vec{B}}{|\vec{B}|},$$

and the projection $|\vec{B}| \cos\alpha$ of the vector $\vec{B}$ onto the vector $\vec{A}$ can be expressed as

$$|\vec{B}| \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|}.$$

$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}.$$

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

Vector Product

The **vector product** (or cross product) between two vectors $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ is defined as

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\begin{aligned} C_x &= A_y B_z - A_z B_y \\ C_y &= A_z B_x - A_x B_z \\ C_z &= A_x B_y - A_y B_x. \end{aligned} \quad (1.32)$$

$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}.$$

$$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta.$$

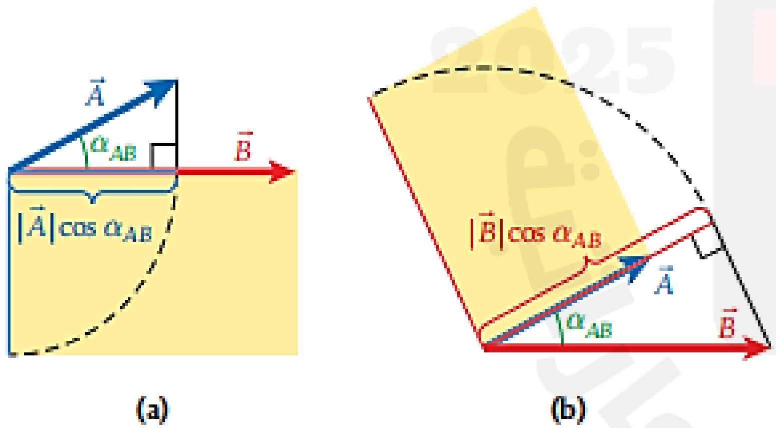


FIGURE 1.26 Geometrical interpretation of the scalar product as an area. (a) The projection of $\vec{A}$ onto $\vec{B}$. (b) The projection of $\vec{B}$ onto $\vec{A}$.

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}.$$

$$\vec{A} \times \vec{A} = 0.$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \bullet \vec{C}) - \vec{C}(\vec{A} \bullet l$$

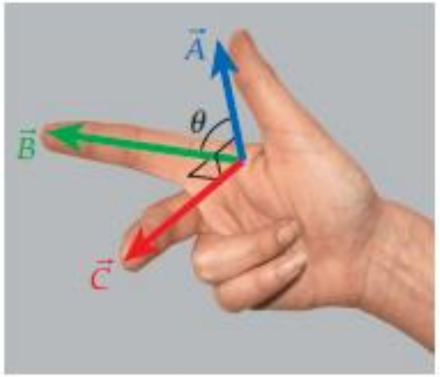


FIGURE 1.27 Vector product.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

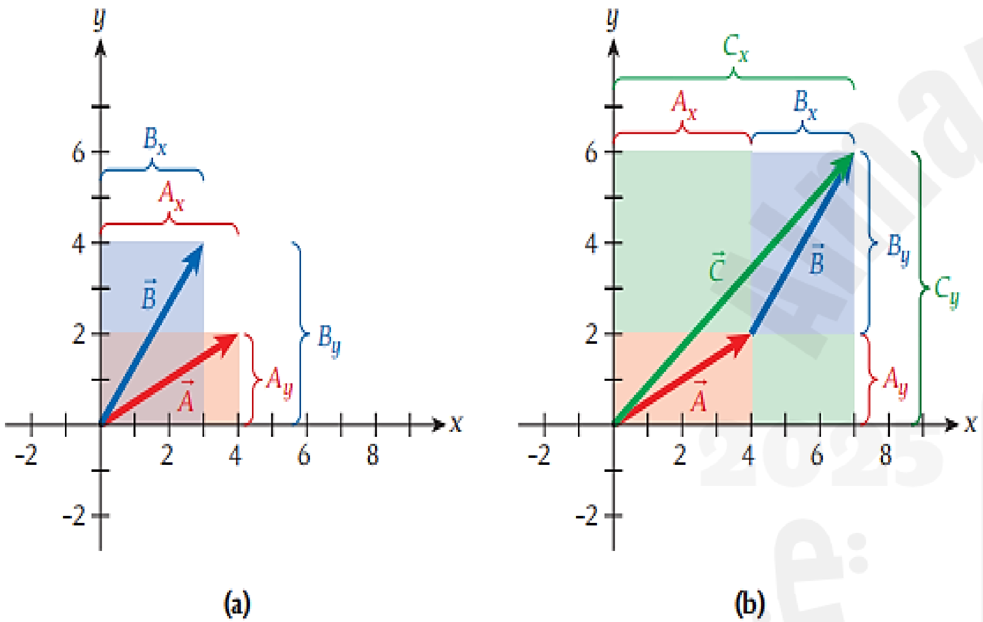


FIGURE 1.21 Vector addition by components. (a) Components of vectors $\vec{A}$ and $\vec{B}$; (b) the components of the resultant vector are the sums of the components of the individual vectors.

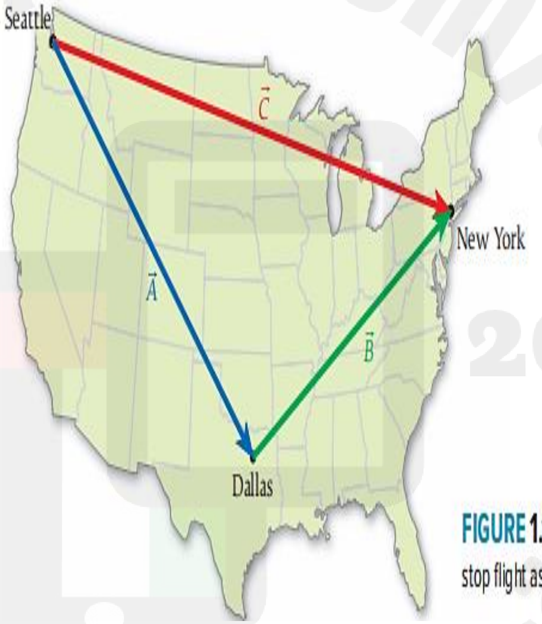


FIGURE 1.18 Direct flight versus one-stop flight as an example of vector addition.

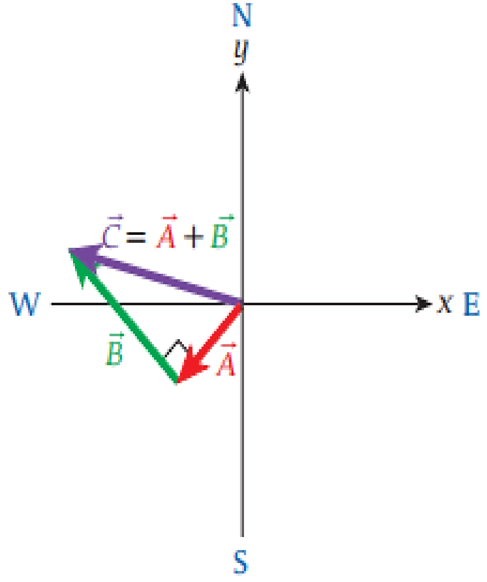


FIGURE 1.28 Hike with a 90° turn.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

SOLVED PROBLEM 1.3 Hiking

PROBLEM

You are hiking in the Florida Everglades heading southwest from your base camp, for 1.72 km. You reach a river that is too deep to cross; so you make a 90° right turn and hike another 3.12 km to a bridge. How far away are you from your base camp?

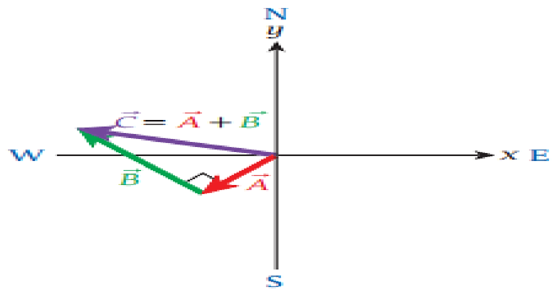


FIGURE 1.28 Hike with a 90° turn.

$$C_x = A_x + B_x = A\cos\theta_A + B\cos\theta_B$$
$$C_y = A_y + B_y = A\sin\theta_A + B\sin\theta_B.$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{(A\cos\theta_A + B\cos\theta_B)^2 + (A\sin\theta_A + B\sin\theta_B)^2}.$$

$$C = \sqrt{((1.72\text{ km})\cos 225° + (3.12\text{ km})\cos 135°)^2 + ((1.72\text{ km})\sin 225° + (3.12\text{ km})\sin 135°)^2}$$
$$= \sqrt{(1.72\times(-\sqrt{1/2}) + 3.12\times(-\sqrt{1/2}))^2 + ((1.72\times(-\sqrt{1/2}) + 3.12\times\sqrt{1/2})^2}\text{ km.}$$

$$C = 3.56\text{ km.}$$

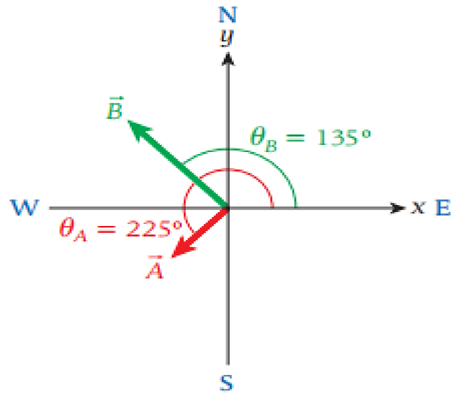
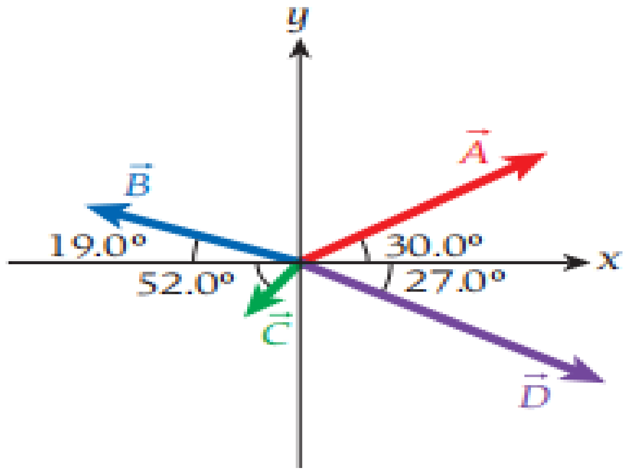


FIGURE 1.29 Angles of the two hike segments.

16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	18-25 19,20,24 24 29,30

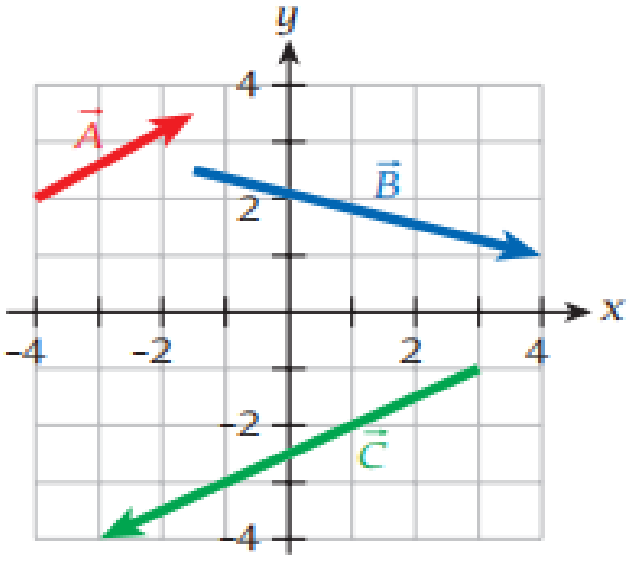
1.65 A position vector has a length of 40.0 m and is at an angle of 57.0°above the x-axis. Find the **vector's components**.

1.67 Find the components of the vectors A, B, C, and D, if their lengths are given by A=75.0, B=60.0, C=25.0, D=90.0 and their direction angles are as shown in the figure. Write the **vectors in terms of unit vectors**..



16	Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis. Add or subtract vectors using Cartesian components. Add and subtract vectors graphically to find the resultant vectors. Identify cartesian unit vectors in two and three dimensions.	STUDENT TEXTBOOK	18-25
		FIGURES (1.18/1.21/1.28) SOLVED PROBLEM. (1.3) Q. (1.65/1.67/1.97)	19,20,24 24 29,30

1.97 Add the three vectors A, B, and C using the component method, and find their sum vector D.



17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.1 Introduction to Kinematics

The study of physics is divided into several large parts, one of which is mechanics. **Mechanics,** or the study of motion and its causes, is usually subdivided. In this chapter and the next, we examine the kinematics aspect of mechanics. **Kinematics** is the study of the motion of objects. These objects may be, for example, cars, baseballs, people, plan-

displacement. **Displacement** is simply the difference between the final position vector, $\vec{r}_2 \equiv \vec{r}(t_2)$, at the end of a motion and the initial position vector, $\vec{r}_1 \equiv \vec{r}(t_1)$. We write the displacement vector as

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \qquad \Delta x = x_2 - x_1. \qquad (2.1)$$

Concept Check 2.1

The train in Figure 2.1 is

a) speeding up.

b) slowing down.

c) traveling at a constant speed.

d) moving at a rate that can't be determined from the photo.

Distance

For motion on a straight line without changing directions, the **distance,** ℓ, that a moving object travels is the absolute value of the displacement vector:

$$\ell = |\Delta\vec{r}|. \qquad (2.4)$$

$$\text{speed} \equiv v = |\vec{v}| = |v_x|. \qquad \text{average speed} \equiv \bar{v} = \frac{\ell}{\Delta t}.$$

17	➥ Solve problems related to position and displacement. ➥ Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. ➥ Describe the motion of an object in a straight line with constant acceleration. ➥ Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

We define v_x, the x-component of the velocity vector, as the change in position (i.e., the displacement component) in a given time interval divided by that time interval, $\Delta x/\Delta t$. Velocity can change from moment to moment. The velocity calculated by taking the ratio of displacement per time interval is the average of the velocity over this time interval, or the x-component of the **average velocity**, $\bar{v}_x$:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}. \tag{2.5}$$

Notation: A bar above a symbol is the notation for averaging over a finite time interval.

In calculus, a time derivative is obtained by taking a limit as the time interval approaches zero. We use the same concept here to define the **instantaneous velocity**, usually referred to as simply as the **velocity**, as the time derivative of the displacement. For the x-component of the velocity vector, this implies

$$v_x = \lim_{\Delta t \to 0} \bar{v}_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}. \tag{2.6}$$

Concept Check 2.2

Your dorm room is located 0.25 kilometers from the Dairy Store. You walk from your room to the Dairy Store and back. Which of the following statements about your trip is true?

a) The distance is 0.50 kilometer, and the displacement is 0.50 kilometer.

b) The distance is 0.50 kilometer, and the displacement is 0.00 kilometer.

c) The distance is 0.00 kilometer, and the displacement is 0.50 kilometer.

d) The distance is 0.00 kilometer, and the displacement is 0.00 kilometer.

17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

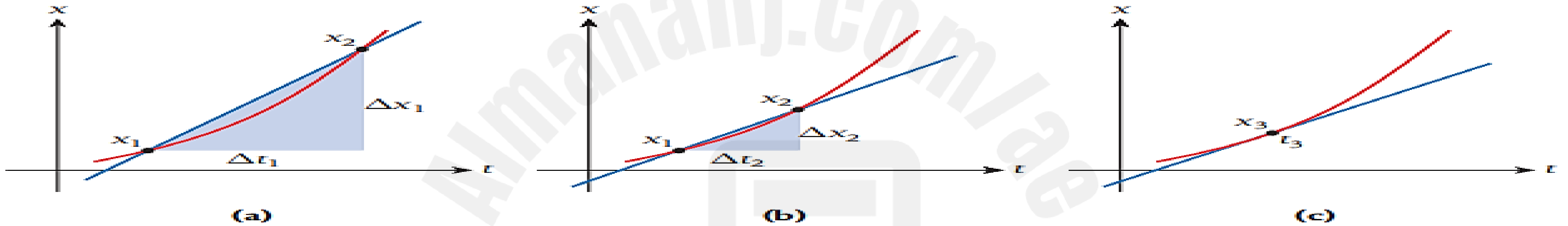
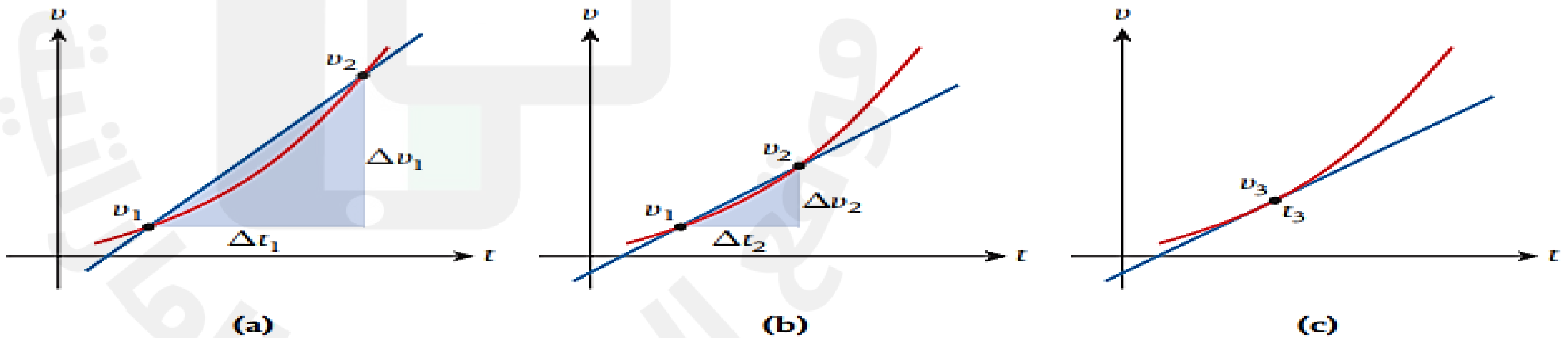


FIGURE 2.6 Instantaneous velocity as the limit of the ratio of displacement to time interval: (a) an average velocity over a large time interval; (b) an average velocity over a smaller time interval; and (c) the instantaneous velocity at a specific time, t_3.

FIGURE 2.10 Instantaneous acceleration as the limit of the ratio of velocity change to time interval: (a) average acceleration over a large time interval; (b) average acceleration over a smaller time interval; and (c) instantaneous acceleration in the limit as the time interval goes to zero.



17	• Solve problems related to position and displacement. • Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. • Describe the motion of an object in a straight line with constant acceleration. • Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

Concept Check 2.5

When you're driving a car along a straight road, you may be traveling in the positive or negative direction and you may have a positive acceleration or a negative acceleration. Match the following combinations of velocity and acceleration with the list of outcomes.

a) positive velocity, positive acceleration

b) positive velocity, negative acceleration

c) negative velocity, positive acceleration

d) negative velocity, negative acceleration

1) slowing down in positive direction

2) speeding up in negative direction

3) speeding up in positive direction

4) slowing down in negative direction

Concept Check 2.6

An example of one-dimensional motion with constant acceleration is

a) the motion of a car during a NASCAR race.

b) the Earth orbiting the Sun.

c) an object in free fall.

d) None of the above describe one-dimensional motion with constant acceleration.

Concept Check 2.4

Average acceleration is defined as the

a) displacement change per time interval.

b) position change per time interval.

c) velocity change per time interval.

d) speed change per time interval.

17	**Solve problems related to position and displacement.** **Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time.** **Describe the motion of an object in a straight line with constant acceleration.** **Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.**	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

Just as the average velocity is defined as the displacement per time interval, the x-component of **average acceleration** is defined as the velocity change per time interval:

$$\overline{a}_x = \frac{\Delta v_x}{\Delta t}. \tag{2.10}$$

Similarly, the x-component of the **instantaneous acceleration** is defined as the limit of the average acceleration as the time interval approaches 0:

$$a_x = \lim_{\Delta t \to 0} \overline{a}_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt}. \tag{2.11}$$

We can now define the acceleration vector as

$$\vec{a} = \frac{d\vec{v}}{dt}, \tag{2.12}$$

$$a_x = \frac{d}{dt} v_x = \frac{d}{dt}\left(\frac{d}{dt} x\right) = \frac{d^2}{dt^2} x.$$

17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.6 Finding Displacement and Velocity from Acceleration

$$v_x(t) = \frac{dx(t)}{dt} \Rightarrow$$

$$\int_{t_0}^{t} v_x(t')\,dt' = \int_{t_0}^{t} \frac{dx(t')}{dt'}\,dt' = x(t) - x(t_0) \Rightarrow$$

$$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt'.$$

$$a_x(t) = \frac{dv_x(t)}{dt} \Rightarrow$$

$$\int_{t_0}^{t} a_x(t')\,dt' = \int_{t_0}^{t} \frac{dv_x(t')}{dt'}\,dt' = v_x(t) - v_x(t_0) \Rightarrow$$

$$v_x(t) = v_{x0} + \int_{t_0}^{t} a_x(t')\,dt'.$$

$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{v}(t')\,dt'$$

$$\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t')\,dt'.$$

2.7 Motion with Constant Acceleration

(i) $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

(ii) $x = x_0 + \bar{v}_x t$

(iii) $v_x = v_{x0} + a_x t$

(iv) $\bar{v}_x = \frac{1}{2}(v_x + v_{x0})$

(v) $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.8 Free Fall

The acceleration due to gravity near the surface of the Earth has the value $g = 9.81\ m/s^2$. We call the vertical axis the y-axis and define the positive direction as up. Then the acceleration vector $\vec{a}$ has only a nonzero y-component, which is given by

$$a_y = -g. \tag{2.24}$$

(i) $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

(ii) $y = y_0 + \bar{v}_y t$

(iii) $v_y = v_{y0} - gt$

(iv) $\bar{v}_y = \frac{1}{2}(v_y + v_{y0})$

(v) $v_y^2 = v_{y0}^2 - 2g(y - y_0)$

Concept Check 2.8

A ball is thrown upward with a speed v_1, as shown in Figure 2.24. The ball reaches a maximum height of $y = h$. What is the ratio of the speed of the ball, v_2, at $y = h/2$ in Figure 2.24b, to the initial upward speed of the ball, v_1, at $y = 0$ in Figure 2.24a?

a) $v_2/v_1 = 0$

b) $v_2/v_1 = 0.50$

c) $v_2/v_1 = 0.71$

d) $v_2/v_1 = 0.75$

e) $v_2/v_1 = 0.90$

Concept Check 2.7

Throwing a ball straight up into the air provides an example of free-fall motion. At the instant the ball reaches its maximum height, which of the following statements is true?

a) The ball's acceleration vector points down, and its velocity vector points up.

b) The ball's acceleration is zero, and its velocity vector points up.

c) The ball's acceleration vector points up, and its velocity vector points up.

d) The ball's acceleration vector points down, and its velocity is zero.

e) The ball's acceleration vector points up, and its velocity is zero.

f) The ball's acceleration is zero, and its velocity vector points down.

17	- Solve problems related to position and displacement. - Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. - Describe the motion of an object in a straight line with constant acceleration. - Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

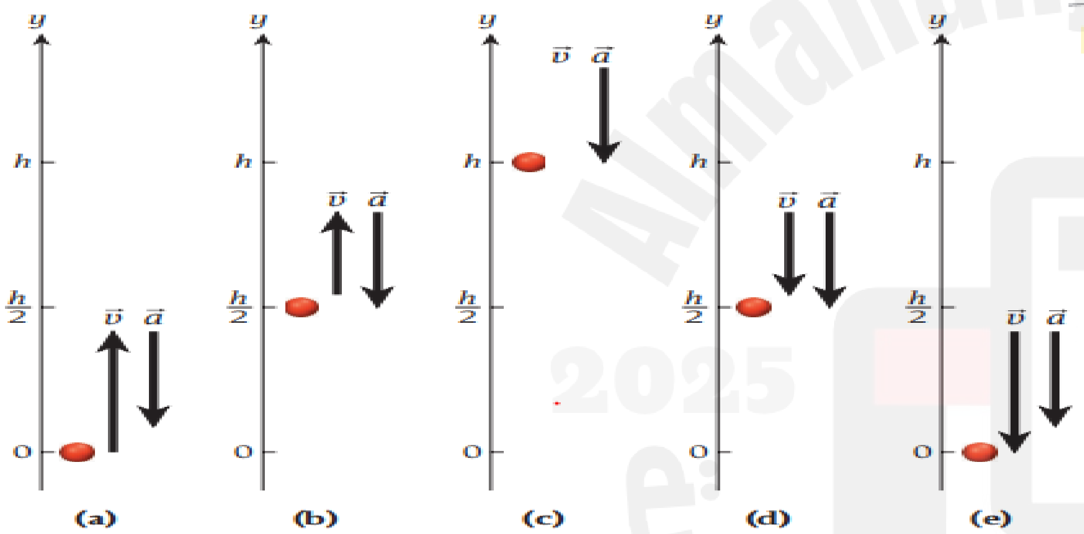


FIGURE 2.24 The velocity vector and acceleration vector of a ball thrown straight up in the air. (a) The ball is initially thrown upward at $y = 0$. (b) The ball going upward at a height of $y = h/2$. (c) The ball at its maximum height of $y = h$. (d) The ball coming down at $y = h/2$. (e) The ball back at $y = 0$ going downward.

Reaction Time $t = \sqrt{\dfrac{2h}{g}}$

Concept Check 2.9

If the reaction time of person B determined with the meter stick method is twice as long as that of person A, then the displacement h_B measured for person B in terms of the displacement h_A for person A is

a) $h_B = 2h_A$.

b) $h_B = \frac{1}{2} h_A$.

c) $h_B = \sqrt{2} h_A$.

d) $h_B = 4h_A$.

e) $h_B = \sqrt{\frac{1}{2}} h_A$.

17	- Solve problems related to position and displacement. - Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. - Describe the motion of an object in a straight line with constant acceleration. - Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

EXAMPLE 2.1 Time Dependence of Velocity

PROBLEM

During the time interval from 0.0 to 10.0 s, the position vector of a car on a road is given by $x(t) = a + bt + ct^2$, with $a = 17.2$ m, $b = -10.1$ m/s, and $c = 1.10$ m/s^2. What is the car's velocity as a function of time? What is the car's average velocity during this interval?

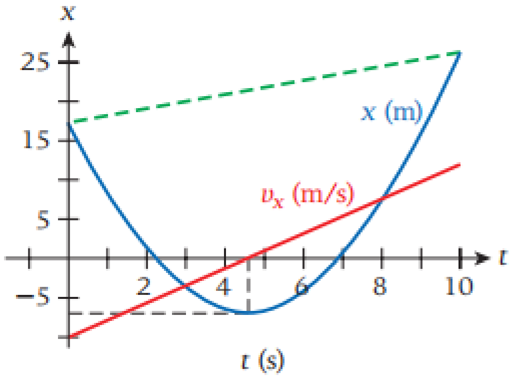


FIGURE 2.7 Graph of the position x and velocity v_x as a function of the time t. The slope of the dashed line represents the average velocity for the time interval from 0 to 10 s.

17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK	(33-40), (42-54)
		EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	38 61,64 63

2.34 The position of a particle moving along the x-axis is given by $x = (11 + 14t - 2.0t^2)$, where t is in seconds and x is in meters.

What is the **average velocity** during the time interval from t = 1.0 s to t = 4.0 s?

2.35 The position of a particle moving along the x-axis is given by $x = 3.0t^2 - 2.0t^3$, where x is in meters and t is in seconds.

What is the **position** of the particle when it achieves its **maximum speed** in the positive x-direction?

17	Solve problems related to position and displacement. Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. Describe the motion of an object in a straight line with constant acceleration. Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.85 The position of a rocket sled on a straight track is given as $x = at^3 + bt^2 + c$, where $a = 2.0\ m/s^3$, $b = 2.0\ m/s^2$, and $c = 3.0\ m$.

a) What is the sled's position between $t = 4.0\ s$ and $t = 9.0\ s$?

b) What is the average speed between $t = 4.0\ s$ and $t = 9.0\ s$?

17	- Solve problems related to position and displacement. - Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. - Describe the motion of an object in a straight line with constant acceleration. - Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.66 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?

2.67 A stone is thrown upward, from ground level, with an initial velocity of 10.0 m/s.

a) What is the velocity of the stone after 0.50 s?

b) How high above ground level is the stone after 0.50 s?

17	• Solve problems related to position and displacement. • Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time. • Describe the motion of an object in a straight line with constant acceleration. • Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.	STUDENT TEXTBOOK EXAMPLE. (2.1) Q. (2.34/2.35/2.85) Q. (2.66/2.67/2.70)	(33-40), (42-54) 38 61,64 63

2.70 An object is thrown vertically upward and has a speed of 20.0 m/s when it reaches two thirds of its maximum height above the launch point. Determine its **maximum height** ?

		STUDENT TEXTBOOK (67-72),(74-78),(80-83)	
18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	Q. (3.27/3.39) Q. (3.43/3.47)	86-87

With this set of Cartesian coordinates, a position vector can be written in component form as

$$\vec{r} = (x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}. \tag{3.1}$$

A velocity vector is

$$\vec{v} = (v_x, v_y, v_y) = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}. \tag{3.2}$$

For one-dimensional vectors, the time derivative of the position vector defines the velocity vector. This is also the case for more than one dimension:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{x} + y\hat{y} + z\hat{z}) = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} + \frac{dz}{dt}\hat{z}. \tag{3.3}$$

In the last step of this equation, we used the sum and product rules of differentiation, as well as the fact that the unit vectors are constant vectors (fixed directions along the coordinate axes and constant magnitude of 1). Comparing equations 3.2 and 3.3, we see that

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}. \tag{3.4}$$

The same procedure leads us from the velocity vector to the acceleration vector by taking the time derivative of the former:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} + \frac{dv_z}{dt}\hat{z}. \tag{3.5}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}.$$

The change in velocity of the particle is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. The average acceleration, $\vec{a}_{\text{ave}}$, for the time interval $\Delta t = t_2 - t_1$ is given by

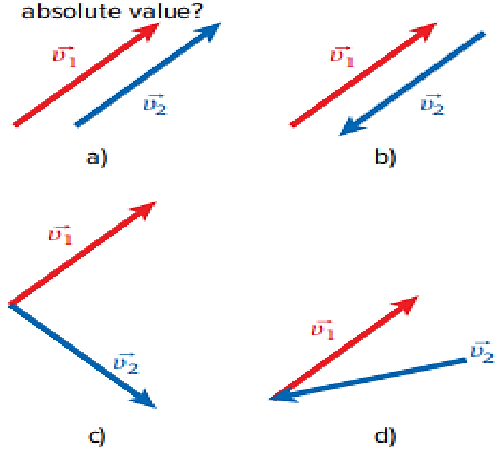
$$\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}. \tag{3.7}$$

Concept Check 3.2

In all of the cases shown in Concept Check 3.1, the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ have the same length. In which case does the acceleration $\vec{a} = \Delta\vec{v} / \Delta t$ have the *smallest* absolute value?

Concept Check 3.1

In all of the cases shown below, the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ have the same length. In which case does $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ have the *largest* absolute value?



e) All of the cases are identical.

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83) Q. (3.27/3.39) Q. (3.43/3.47)	86-87

$$\vec{a} = (0, -g) = -g\hat{y}. \tag{3.10}$$

For this special case of a constant acceleration only in the y-direction and with zero acceleration in the x-direction, we have a free-fall problem in the vertical direction and motion with constant velocity in the horizontal direction. The kinematical equations for the x-direction are those for an object moving with constant velocity:

$$x = x_0 + v_{x0}t \tag{3.11}$$

$$v_x = v_{x0}. \tag{3.12}$$

Just as in Chapter 2, we use the notation $v_{x0} \equiv v_x(t=0)$ for the initial value of the x-component of the velocity. The kinematical equations for the y-direction are those for free-fall motion in one dimension:

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$

$$y = y_0 + \bar{v}_y t$$

$$v_y = v_{y0} - gt$$

$$\bar{v}_y = \tfrac{1}{2}(v_y + v_{y0}) \tag{3.16}$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0). \tag{3.17}$$

Self-Test Opportunity 3.1

What is the dependence of $|\vec{v}|$ on the x-coordinate?

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{x0}^2 + v_{y0}^2 - 2g(y - y_0)} = \sqrt{v_0^2 - 2g(y - y_0)}. \tag{3.23}$$

Note that the initial launch angle does not appear in this equation. The absolute value of the velocity–the speed–depends only on the initial value of the speed and the difference between the y-coordinate and the initial launch height. Thus, if we release a projectile from a certain height above ground and want to know the speed with which it hits the ground, it does not matter if the projectile is shot straight up, or horizontally, or straight down. Chapter 5 will discuss the concept of kinetic energy, and then the reason for this seemingly strange fact will become more apparent.

Concept Check 3.3

At the top of the trajectory of any projectile, which of the following statement, if any, is (are) true?

a) The acceleration is zero.

b) The x-component of the acceleration is zero.

c) The y-component of the acceleration is zero.

d) The speed is zero.

e) The x-component of the velocity is zero.

f) The y-component of the velocity is zero.

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83) Q. (3.27/3.39) Q. (3.43/3.47)	86-87

Projectile Motion

Vocabulary

1- Projectile: An object shot through the air or (the motion of an object given initial velocity that then moves only under the force of gravity.)
2- Trajectory: The path of a projectile through space.
3- Flight time: The amount of time that a projectile is in the air.
4- Range: The horizontal distance traveled by a projectile.

the **range** (R), or how far the projectile will travel horizontally before returning to its original vertical position, and the **maximum height** (H) it will reach. These quantities R and H are illustrated in Figure 3.11. We find that the maximum height reached by the projectile is

$$H = y_0 + \frac{v_{y0}^2}{2g}. \quad (3.24)$$

We'll derive this equation below. We'll also derive this equation for the range:

$$R = \frac{v_0^2}{g}\sin 2\theta_0, \quad (3.25)$$

where v_0 is the absolute value of the initial velocity vector and θ_0 is the launch angle. The maximum range, for a given fixed value of v_0, is reached when $\theta_0 = 45°$.

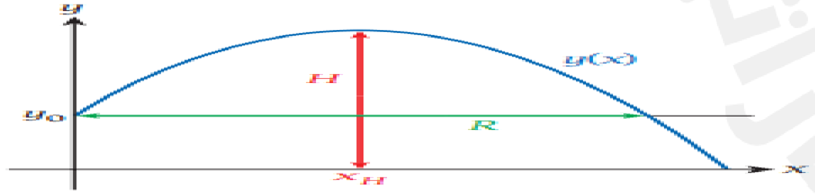


FIGURE 3.11 The maximum height (red) and range (green) of a projectile.

$$R_{max} = \frac{v_0^2}{g}.$$

$$t = \frac{R}{v_0 \cos\theta_0}$$

Concept Check 3.4

A projectile is launched from an initial height $y_0 = 0$. For a given launch angle, if the launch speed is doubled, what will happen to the range, R, and the time in the air, t_{air}?

a) R and t_{air} will both double.

b) R and t_{air} will both quadruple.

c) R will double, and t_{air} will stay the same.

d) R will quadruple, and t_{air} will double.

e) R will double, and t_{air} will quadruple.

location on the deck. The reference frame from which we view motion makes a big difference in how we describe the motion, producing an effect known as **relative velocity**.

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83) Q. (3.27/3.39) Q. (3.43/3.47)	86-87

Angled Launches

1- When a projectile is launched at an angle, the initial velocity has a vertical component as well as a horizontal component

2- If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path, and descends with increasing speed.

3- The adjoining figure shows the separate vertical-an Horizontal-motion diagrams for the trajectory of the ball

4- At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward

5- The only difference is that the directions of the two velocities are opposite

6- The adjoining figure defines two quantities associated with a trajectory

7- One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component

8- For football punts, flight time is often called hang time.

9- The forces that effect in the projectiles: 1- air resistance 2- force of gravity

The position vector for projectile motion is

$$\vec{r} = (x, y) = x\hat{x} + y\hat{y}, \qquad (3.8)$$

and the velocity vector is

$$\vec{v} = (v_x, v_y) = v_x\hat{x} + v_x\hat{y} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y}. \qquad (3.9)$$

Acceleration vector is

$$\vec{a} = (0, -g) = -g\hat{y}. \qquad (3.10)$$

- In x axis acceleration is zero, in y axis acceleration is constant (9.81) and downward.
- This is motion with constant velocity in X direction and Free Fall in Y direction (make sense ☺)

$$\vartheta_x = \vartheta_i Cos\theta \qquad \Delta X = \vartheta_i Cos\theta \times t \qquad t = \frac{\vartheta_i Sin\theta}{g} \qquad t = 2\frac{\vartheta_i Sin\theta}{g}$$

$$\Delta y = \frac{(\vartheta_i Sin\theta)^2}{2.g} \qquad \vartheta_f = \sqrt{\vartheta_x^2 + \vartheta_y^2}$$

1- Horizontal Projectile

$$v_i = \frac{\Delta x}{t} \qquad \Delta x = v_i t = v_x t \qquad (\text{ For range})$$

$$t = \sqrt{\frac{2\Delta y}{g}} \quad \text{For flight time} \qquad \Delta y = \frac{1}{2} g t^2$$

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83)	
		Q. (3.27/3.39) Q. (3.43/3.47)	86-87

3.27 An object moves in the xy-plane. The x- and y-coordinates of the object as a function of time are given by the following equations:

$x(t) = 4.9t^2 + 2t + 1$ and $y(t) = 3t + 2$.

What is the velocity vector of the object as a function of time?

What is its acceleration vector at the time $t = 2$ s ?

		STUDENT TEXTBOOK (67-72),(74-78),(80-83)	
18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	Q. (3.27/3.39) Q. (3.43/3.47)	86-87

3.39 A rabbit runs in a garden such that the x- and y-components of its displacement as functions of time are given by

$$x(t) = -0.45t^2 - 6.5t + 25 \quad \text{and} \quad y(t) = 0.35t^2 + 8.3t + 34\ .$$

(Both x and y are in meters and t is in seconds.)

a) Calculate the rabbit's position (magnitude and direction) at $t = 10.0$ s.

b) Calculate the rabbit's velocity at $t = 10.0$ s.

c) Determine the acceleration vector at $t = 10.0$ s.

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83)	
		Q. (3.27/3.39) Q. (3.43/3.47)	86-87

3.43 football is kicked with an initial speed of 27.5 m/s and a launch angle of 56.7°. What is its hang time (the time until it hits the ground again)?

18	Calculate the components of a velocity vector (vx, vy, vz) by the time derivative of the position vector. Define maximum height, range of a projectile and time of flight. Calculate the maximum height, range of a projectile and the time of flight for a projectile.	STUDENT TEXTBOOK (67-72),(74-78),(80-83)		
		Q. (3.27/3.39) Q. (3.43/3.47)	86-87	

3.47 7 A football player kicks a ball with a speed of 22.4 m/s at an angle of 49.0° above the horizontal from a distance of 39.0 m from the goal-post.

a) By how much does the ball clear or fall short of clearing the crossbar of the goalpost if that bar is 3.05 m high?

b) What is the vertical velocity of the ball at the time it reaches the goalpost?

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118) EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	 101 105 122-125

4.3 Net Force

Because forces are vectors, we must add them as vectors, using the methods developed in Chapter 1. We define the **net force** as the vector sum of all force vectors that act on an object:

$$\vec{F}_{\text{net}} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n. \quad (4.5)$$

Newton's First Law:

If the net force on an object is equal to zero, the object will remain at rest if it was at rest. If it was moving, it will remain in motion in a straight line with the same constant velocity.

Newton's Second Law:

If a net external force, $\vec{F}_{\text{net}}$, acts on an object with mass m, the force will cause an acceleration, $\vec{a}$, in the same direction as the force:

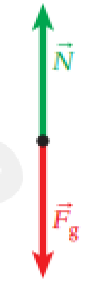
$$\vec{F}_{\text{net}} = m\vec{a}.$$

Newton's Third Law:

The forces that two interacting objects exert on each other are always exactly equal in magnitude and opposite in direction:

$$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}.$$

force vectors that act on it are drawn, is called a **free-body diagram.**



(b)

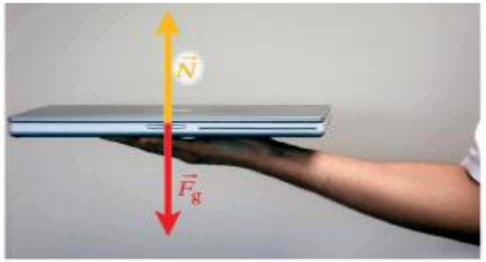
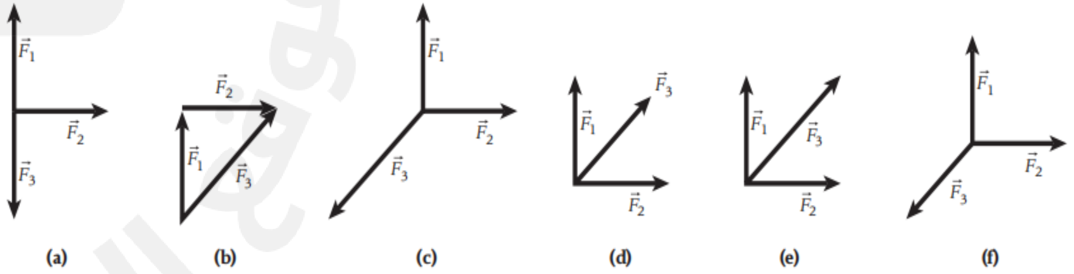


FIGURE 4.6 Force of gravity acting downward and normal force acting upward exerted by the hand holding the laptop computer.

Concept Check 4.2

Choose the set of three coplanar vectors that sum to a net force of zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$.



19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)	
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125

4.7 Friction Force

The science of friction has a name: **tribology.**

Friction always resists the motion so it is in opposite direction of the object motion

- Friction is independent of the speed of the object
- Friction is independent of the size of the contact area between object and surface
- Friction is depend on the roughness of the surface
- No friction for smooth surface
- There is a linear proportional between normal force (N) and friction force f

Identify the type of friction force acting on the couch when it begins to move.

Static friction increases up to a maximum to balance the applied force.

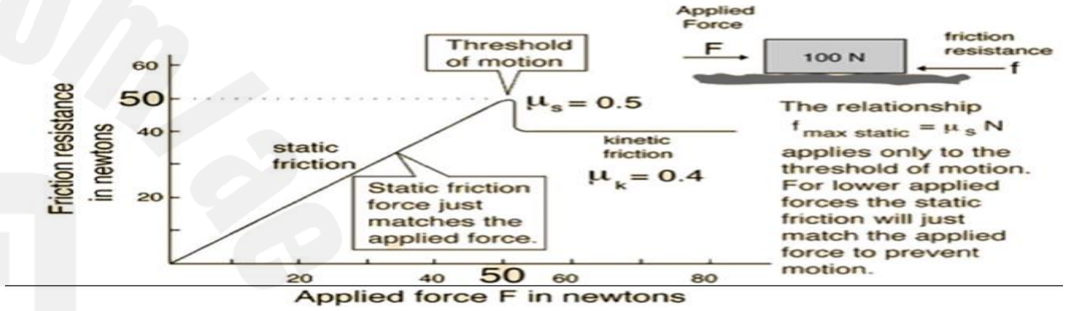
The couch accelerates when the applied force exceeds the maximum static friction force.

-* There is two kinds of friction

1- Static friction: it is an opposing force that keeps the object from moving (no move) . it is increase as the applied force increases until the maximum static friction force increase possible between the two surfaces

2- Kinetic friction: it is an opposing force on an object when it is moving. It stays constant when the object is in motion.

-* The coefficient of friction(μ)depends on the composition and qualities of the surfaces in contact , and always it is a decimal number $0 < \mu < 1$



Always the coefficient of static friction ($\mu s,max$)bigger than the coefficient of kinetic friction (μk) $\mu s,max > \mu k$ because

$f s,max > f k$

kinetic friction equation	static friction equation	
$f_k = \mu_k F_n$	$f_{s,max} = \mu_s F_n$	f_k [N] = kinetic friction $f_{s,max}$ [N] = maximum static friction F_n [N] = normal force μ_k [−] = coffiecent kinetic friction μ_s [−] = coffiecent static friction

$\mu_s > \mu_k$.

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118) EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	 101 105 122-125

Air Resistance

The drag force in this example is the air resistance

*- When the person goes down his weight (F_g)does not change

*- When the person goes down the velocity increases so the drag force increases, too

*- When the person goes down F_{net} decreases because drag force increases

$F_{net} = F_d - F_g$

*- In one point in the air(fluid) ,drag force equals person weight(F_g) and

$F_{net} = 0$ $F_d = F_g$,the person will complete his motion with the final velocity which reach it in this point(terminal velocity)

*- Terminal velocity is a constant velocity when $F_d = F_g$ and $F_{net} = 0$

*- the terminal speed equation $F_{drag} = Fg = Kv^2$

$$v = \sqrt{\frac{mg}{K}}. \qquad K = \frac{1}{2} c_d A \rho.$$

The factors effect on terminal speed

1- Object mass when K is constant , the massive one will be fast

2-Drag coefficient , there is inversely proportional between(**K**) and $\mathbf{v}_{\text{terminal}}$

$K = 1/2\, C_P\, A\, \rho$

The factors effect on the constant of drag force

1- Size of cross sectional area (A) exposed to the air stream (linearly proportional between(**K**) and **(A)**

2- Air density ρ (linearly proportional)

3- Drag constant C_P ***shows*** (the shape of the object , on its inclination relative to the direction of motion on air viscosity and compressibility

Q1) Why do you open the parachute when you are falling?

Answer: because the surface area of the parachute is big so constant drag (K) will be increase but velocity will be decrease because of the inversely proportional between (K and $\mathbf{v}_{\text{terminal}}$) so when the terminal speed decreases it protects the person when reaching the ground

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)	
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125

EXAMPLE 4.2 Still Rings

A gymnast of mass 55 kg hangs vertically from a pair of parallel rings (Figure 4.10a).

PROBLEM 1

If the ropes supporting the rings are vertical and attached to the ceiling directly above, what is the tension in each rope?

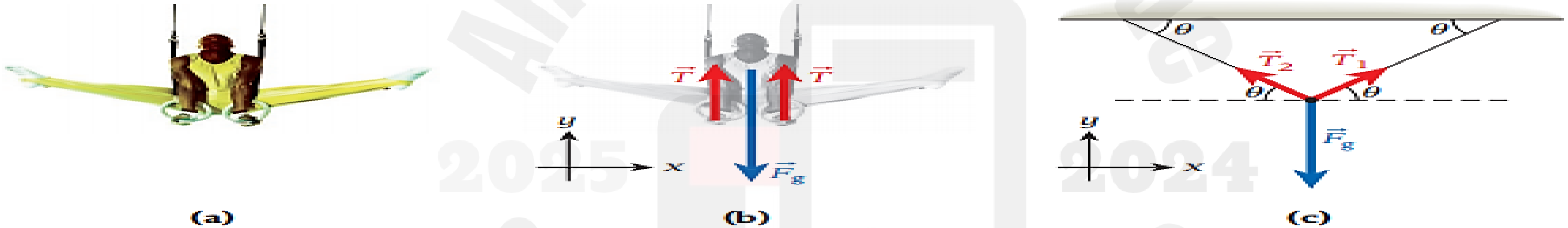
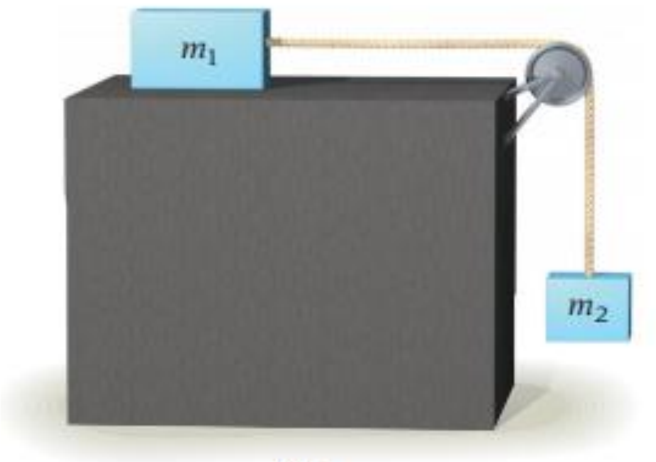


FIGURE 4.10 (a) Still rings in men's gymnastics. (b) Free-body diagram for problem 1. (c) Free-body diagram for problem 2.

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)	
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125

SOLVED PROBLEM 4.2 Two Blocks Connected by a Rope

In this classic problem, a hanging mass causes the acceleration of a second mass that is resting on a horizontal surface (Figure 4.16a). Block 1, of mass $m_1 = 3.00$ kg, rests on a horizontal frictionless surface and is connected via a massless rope (for simplicity, oriented in the horizontal direction) running over a massless pulley to block 2, of mass $m_2 = 1.30$ kg.



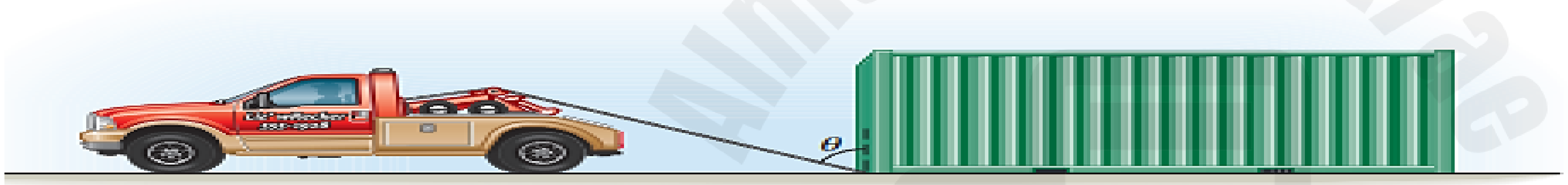
(a)

PROBLEM

What is the acceleration of block 1 and of block 2?

<table>
<tr><td rowspan="2">19</td><td rowspan="2">• Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law.
• Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).</td><td colspan="2">STUDENT TEXTBOOK (96-102), 103-112), (113-118)</td></tr>
<tr><td>EXAMPLE. (4.2)
SOLVED PROBLEM. (4.2)
Q (4.26/4.75/4.79/4.81)</td><td>101
105
122-125</td></tr>
</table>

4.26 A tow truck of mass M is using a cable to pull a shipping container of mass m across a horizontal surface as shown in the figure. The cable is attached to the container at the front bottom corner and makes an angle θ with the vertical as shown. The coefficient of kinetic friction between the surface and the crate is μ.



a) Draw a free-body diagram for the container.

b) Assuming that the truck pulls the container at a constant speed, write an equation for the magnitude T of the string tension in the cable.

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)	
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125

4.75 A block of mass 20.0 kg supported by a vertical massless cable is initially at rest. The block is then pulled upward with a constant acceleration of 2.32 m/s^2

a) What is the tension in the cable?

b) What is the net force acting on the mass?

c) What is the speed of the block after it has traveled 2.00 m?

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)	
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125

4.79 A tractor pulls a sled of mass M = 1000. kg across level ground. The coefficient of kinetic friction between the sled and the ground is $\mu k = 0.600$. The tractor pulls the sled by a rope that connects to the sled at an angle of $\theta = 30.0°$ above the horizontal. What magnitude of tension in the rope is necessary to move the sled horizontally with an acceleration $a = 2.00\ m/s^2$?

19	Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law. Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).	STUDENT TEXTBOOK (96-102), 103-112), (113-118)		
		EXAMPLE. (4.2) SOLVED PROBLEM. (4.2) Q (4.26/4.75/4.79/4.81)	101 105 122-125	

4.81 A block of mass 5.00 kg is sliding at a constant velocity down an inclined plane that makes an angle of 37.0° with respect to the horizontal.

a) What is the friction force?

b) What is the coefficient of kinetic friction?

اللهم يسر الامتحانات على أولادنا وطلابنا ، اللهم كن معهم
وقت النسيان فذكرهم، وكن معهم وقت السؤال فاجبهم،
اشرح صدورهم، ويسر أمورهم ، واحلل عقدة من ألسنتهم"