الملف أوراق عمل الوحدة الخامسة Matrices and Equations of Systems
موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ رياضيات ⇦ الفصل الأول

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول	
1	مراجعة لامتحان منتصف الفصل الأول
2	حساب المثلثات القائمة الزاوية
3	مراجعة في وحدة القوى
4	نموذج الاجابة لامتحان الوزارة
5	التوزيع الزمني للفصل الاول

CHAPTER 5 Systems of Equations and Matrices

LESSON 1 Multivariable Linear Systems and Row Operations

NewVocabulary multivariable linear system, row-echelon form, Gaussian elimination, augmented matrix, coefficient matrix, reduced row-echelon form, Gauss-Jordan elimination,

Gaussian Elimination

KeyConcept Operations that Produce Equivalent Systems

Each of the following operations produces an equivalent system of linear equations.

- Interchange any two equations.
- Multiply one of the equations by a nonzero real number.
- Add a multiple of one equation to another equation.

Example 1 Gaussian Elimination with a System

Write the system of equations in triangular form using Gaussian elimination. Then solve the system.

$$
\begin{aligned}
5x - 5y - 5z &= 35 \\
-x + 2y - 3z &= -12 \\
3x - 2y + 7z &= 30
\end{aligned}
$$

KeyConcept Row-Echelon Form

A matrix is in row-echelon form if the following conditions are met.

- Rows consisting entirely of zeros (if any) appear at the bottom of the matrix.
- The first nonzero entry in a row is 1, called a *leading 1*.
- For two successive rows with nonzero entries, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

$$\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Example 2 **Identify an Augmented Matrix in Row-Echelon Form**

Determine whether each matrix is in row-echelon form.

a. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 4 & 2 \end{array}\right]$

b. $\left[\begin{array}{cccc|c} 1 & 6 & 2 & -11 & 10 \\ 0 & 1 & -5 & 8 & -7 \\ 0 & 0 & 0 & 1 & 14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 1 & 5 & -6 & 10 \\ 0 & 1 & 9 & -3 \\ 0 & 1 & 0 & 14 \end{array}\right]$

Gauss-Jordan Elimination

Example 3 **Use Gauss-Jordan Elimination**

Solve the system of equations.

$x - y + z = 0$

$-x + 2y - 3z = -5$

$2x - 3y + 5z = 8$

Example 4 No Solution and Infinitely Many Solutions

Solve each system of equations.

a. $-5x - 2y + z = 2$
$4x - y - 6z = 2$
$-3x - y + z = 1$

b. $3x + 5y - 8z = -3$
$2x + 5y - 2z = -7$
$-x - y + 4z = -1$

<u>*LESSON 2*</u>

Matrix Multiplication, Inverses, and Determinants

NewVocabulary identity matrix | inverse matrix | invertible | singular matrix | determinant

Multiply Matrices

matrix A	$\cdot$	matrix B	$=$	AB
3×2		2×4		3×4

Equal

Dimensions of AB

KeyConcept Matrix Multiplication

Words If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix obtained by adding the products of the entries of a row in A to the corresponding entries of a column in B.

Symbols If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix in which

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ir}b_{rj}.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ir} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rj} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2n} \\ \vdots & \vdots & \cdots & \vdots & & \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & & \vdots & & \\ c_{m1} & c_{m2} & & c_{mj} & \cdots & c_{mn} \end{bmatrix}$$

<u>*Example 1*</u> **Multiply Matrices**

Use matrices $A = \begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 6 \\ 3 & 5 & 1 \end{bmatrix}$ to find each product, if possible.

a. AB

b. BA

KeyConcept Properties of Matrix Multiplication

For any matrices A, B, and C for which the matrix product is defined and any scalar k, the following properties are true.

Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$k(AB) = (kA)B = A(kB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

KeyConcept Identity Matrix

Words The identity matrix of order n, denoted I_n, is an $n \times n$ matrix consisting of all 1s on its main diagonal, from upper left to lower right, and 0s for all other elements.

Symbols

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Example 2 Solve a System of Linear Equations

Write the system of equations as a matrix equation, $AX = B$. Then use Gauss-Jordan elimination on the augmented matrix to solve the system.

$-x_1 + x_2 - 2x_3 = 2$

$-2x_1 + 3x_2 - 4x_3 = 5$

$3x_1 - 4x_2 + 7x_3 = -1$

Inverses and Determinants

KeyConcept Inverse of a Square Matrix

Let A be an $n \times n$ matrix. If there exists a matrix B such that $AB = BA = I_n$, then B is called the **inverse** of A and is written as A^{-1}. So, $AA^{-1} = A^{-1}A = I_n$.

Example 3 **Verify an Inverse Matrix**

Determine whether $A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ are inverse matrices.

Example 4 **Inverse of a Matrix**

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*.

a. $A = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

KeyConcept Inverse and Determinant of a 2 × 2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. A is invertible if and only if $ad - cb \neq 0$.

If A is invertible, then $A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

The number $ad - cb$ is called the **determinant** of the 2 × 2 matrix and is denoted

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

Example 5 **Determinant and Inverse of a 2 × 2 Matrix**

Find the determinant of each matrix. Then find the inverse of the matrix, if it exists.

a. $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

KeyConcept Determinant of a 3 × 3 Matrix

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Then $\det(A) = |A| = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$.

Example 6 **Determinant and Inverse of a 3 × 3 Matrix**

Find the determinant of $C = \begin{bmatrix} -3 & 2 & 4 \\ 1 & -1 & 2 \\ -1 & 4 & 0 \end{bmatrix}$. **Then find** C^{-1}, **if it exists.**

LESSON 3 Solving Linear Systems using Inverses and Cramer's Rule

NewVocabulary square system Cramer's Rule

KeyConcept Invertible Square Linear Systems

Let A be the coefficient matrix of a system of n linear equations in n variables given by $AX = B$, where X is the matrix of variables and B is the matrix of constants. If A is invertible, then the system of equations has a unique solution given by $X = A^{-1}B$.

Example 1 Solve a 2 × 2 System Using an Inverse Matrix

Use an inverse matrix to solve the system of equations, if possible.

$$2x - 3y = -1$$
$$-3x + 5y = 3$$

KeyConcept Cramer's Rule

Let A be the coefficient matrix of a system of n linear equations in n variables given by $AX = B$. If $\det(A) \neq 0$, then the unique solution of the system is given by

$$x_1 = \frac{|A_1|}{|A|},\ x_2 = \frac{|A_2|}{|A|},\ x_3 = \frac{|A_3|}{|A|},\ \ldots,\ x_n = \frac{|A_n|}{|A|},$$

where A_i is obtained by replacing the ith column of A with the column of constant terms B. If $\det(A) = 0$, then $AX = B$ has either no solution or infinitely many solutions.

Example 2 Use Cramer's Rule to Solve a 2 × 2 System

Use Cramer's Rule to find the solution of the system of linear equations, if a unique solution exists.

$$3x_1 + 2x_2 = 6$$
$$-4x_1 - x_2 = -13$$

LESSON 4 Partial Fractions

NewVocabulary partial fraction | partial fraction decomposition

Example 1 **Denominator with Nonrepeated Linear Factors**

Find the partial fraction decomposition of $\frac{x+13}{x^2-x-20}$.

Example 2 **Improper Rational Expression**

Find the partial fraction decomposition of $\frac{2x^2+5x-4}{x^2-x}$.

Example 3 **Denominator with Repeated Linear Factors**

Find the partial fraction decomposition of $\dfrac{-x^2 - 3x - 8}{x^3 + 4x^2 + 4x}$.

Example 4 **Denominator with Prime Quadratic Factors**

Find the partial fraction decomposition of $\dfrac{x^3 + 2x}{(x^2 + 1)^2}$

LESSON 5

Linear Optimization

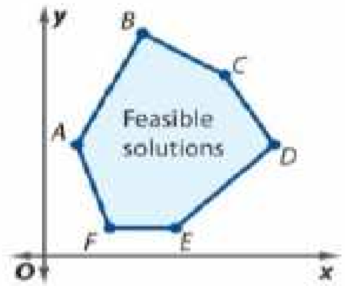
NewVocabulary: optimization | linear programming | objective function | feasible solutions | multiple optimal solutions | unbounded | constraints

Linear Programming

KeyConcept Vertex Theorem for Optimization

Words If a linear programming problem can be optimized, an optimal value will occur at one of the vertices of the region representing the set of feasible solutions.

Example The maximum or minimum value of $f(x, y) = ax + by + c$ over the set of feasible solutions graphed occurs at point A, B, C, D, E, or F.



KeyConcept Linear Programming

To solve a linear programming problem, follow these steps.

Step 1 Graph the region corresponding to the solution of the system of constraints.

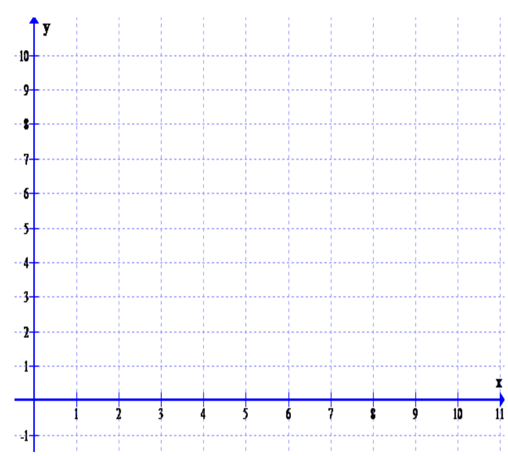
Step 2 Find the coordinates of the vertices of the region formed.

Step 3 Evaluate the objective function at each vertex to determine which x- and y-values, if any, maximize or minimize the function.

Example 1 **Maximize and Minimize an Objective Function**

Find the maximum and minimum values of the objective function $f(x, y) = x + 3y$ and for what values of x and y they occur, subject to the following constraints.

$x + y \leq 8$ $\quad$ $2x - y \leq 5$ $\quad$ $x \geq 0$ $\quad$ $y \geq 0$



Example 2 **Optimization at Multiple Points**

Find the maximum value of the objective function $f(x, y) = 4x + 2y$ and for what values of x and y it occurs, subject to the following constraints.

$y + 2x \leq 18$
$y \leq 6$
$x \leq 8$
$x \geq 0$
$y \geq 0$