شرح الدرس الرابع Normal Distributions من الوحدة الثامنة

تاريخ نشر الملف على موقع المناهج: 2023-10-31 09:11:05 || اسم المدرس: محمد زياد

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم

روابط مواد الصف الحادي عشر المتقدم على تلغرام

الرياضيات	اللغة الانجليزية	اللغة العربية	التربية الاسلامية

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول

ورقة عمل الدرس الثالث Analyzing population data من الوحدة الثامنة متبوعة بالإجابات	1
شرح الدرس الثالث Analyzing population data من الوحدة الثامنة	2
ورقة عمل الدرس الثاني Using statistical experiments من الوحدة الثامنة متبوعة بالحل	3
شرح الدرس الثاني Using statistical experiments من الوحدة الثامنة	4

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول	
[مراجعة الامتحان التكويني الثاني](#)	5

Normal Distributions

Sample space Ω:

The set of all possible outcomes of an experiment

Ex1:

For the following experiments write the sample space

    1) Rolling a die:

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

    2) tossing two coins

$$\Omega = \{HH, TT, TH, HT\}$$

Random variable X:

A random variable is a function that assigns values to each of an experiment's outcomes.

- A random variable can be either discrete (having specific values) or continuous (any value in a continuous range).

Ex2: Identify the random variable in each distribution and classify it as discrete or continuous. Explain your reasoning.

    1. the number of texts received per week

       discrete

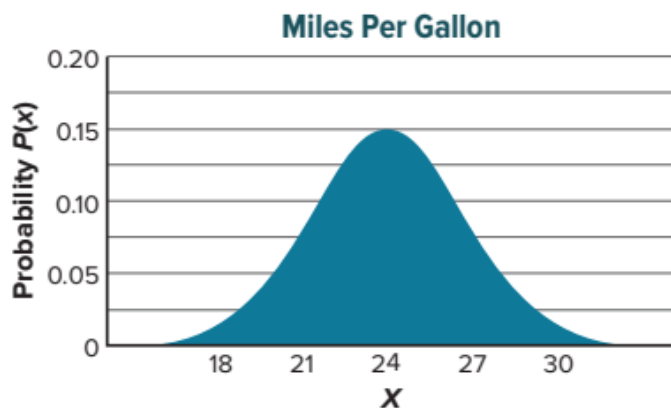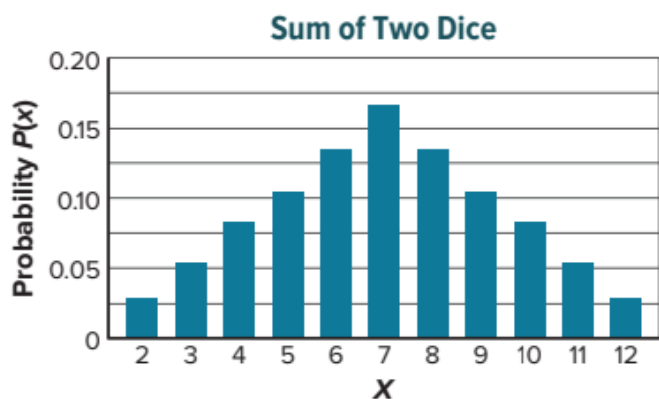   2. the number of "likes" for a Web page

       discrete

    3. the height of a plant after a specific amount of time

       Continous

Probability Distributions

A probability distribution is a mapping of each outcome of a statistical experiment to its probability of occurrence.

The probability distribution of a random variable X must satisfy the conditions:
1) The probability of each value of X must be between 0 and 1.
2) The sum of the probabilities of all values of X must equal 1

Ex3:

FUNDRAISING At a fundraising dinner, the underside of 100 plates were randomly tagged with a sticker to indicate winning a cash prize. The frequency table shows the number of winning plates for each prize. Construct a relative frequency table, and graph the probability distribution.

Prize, (X)	Frequency
$5	50
$50	40
$100	9
$1000	1

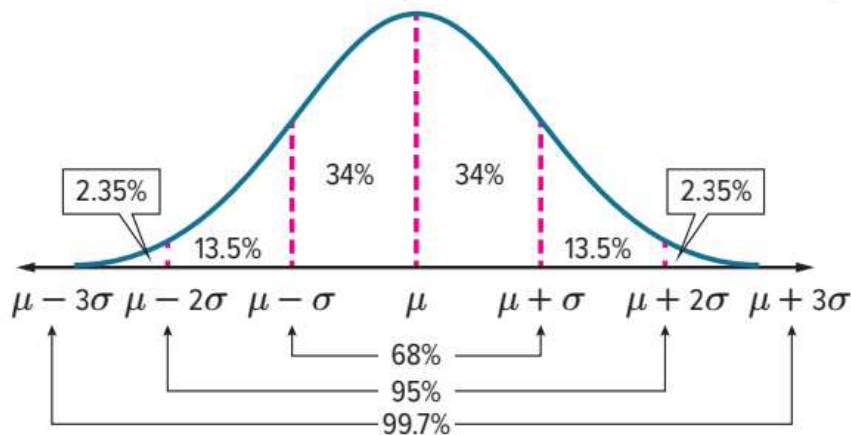| x	Frequency	Relative frequency
5	50	$\frac{50}{100} = 0.5$
50	40	$\frac{40}{100} = 0.4$
100	9	$\frac{9}{100} = 0.09$
1000	1	$\frac{1}{100} = 0.01$
Sum	100	1
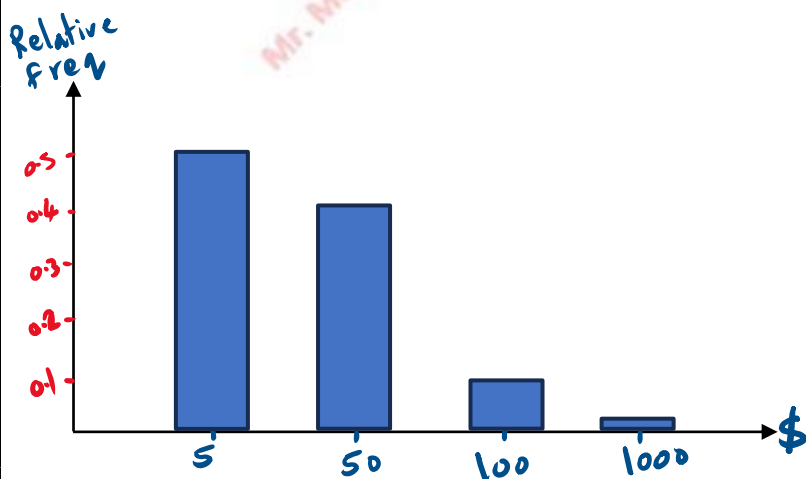
Normal Distribution
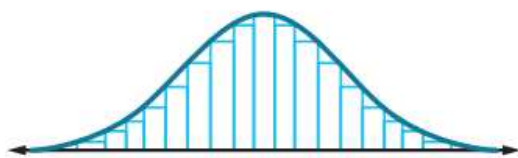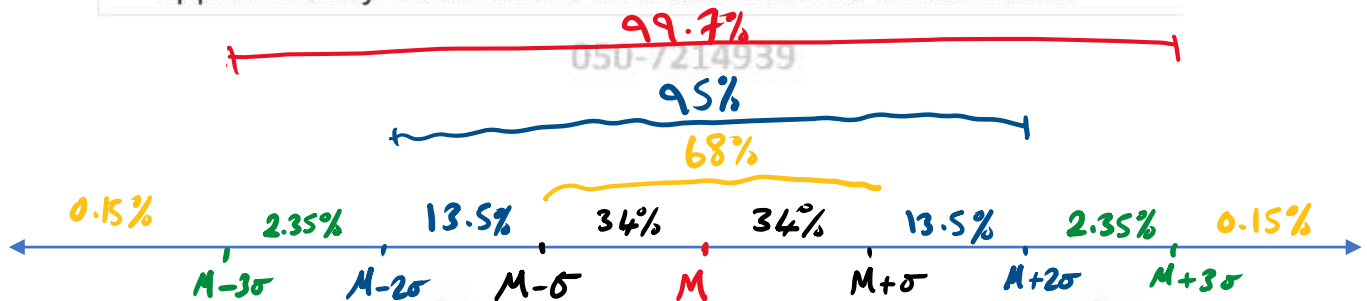
Key Concept • The Normal Distribution

- The graph of a normal distribution is continuous, bell-shaped, and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve approaches, but never touches, the x-axis.
- The total area under the curve is equal to 1, or 100%.

## Key Concept • The Empirical Rule

In a normal distribution with mean μ and standard deviation σ,

2.35% 34% 34% 2.35%
13.5% 13.5%

$\mu - 3\sigma \quad \mu - 2\sigma \quad \mu - \sigma \quad \mu \quad \mu + \sigma \quad \mu + 2\sigma \quad \mu + 3\sigma$

68%
95%
99.7%

- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.

99.7%
95%
68%

0.15%   2.35%   13.5%   34%   34%   13.5%   2.35%   0.15%

$M-3\sigma \quad M-2\sigma \quad M-\sigma \quad M \quad M+\sigma \quad M+2\sigma \quad M+3\sigma$

The data are normally distributed, symmetric about the mean, and bell-shaped.
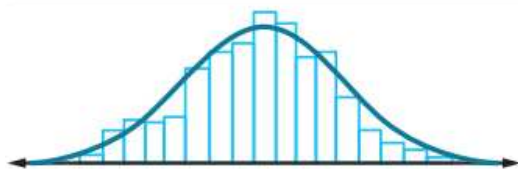
The data are approximately normally distributed. The data can be modeled by the normal distribution.

The data are skewed to the left. A normal curve would not be the best curve to model the distribution.

Ex4: **A normal distribution has a mean of 186.4 and a standard deviation of 48.9.**

a. What range of values represents the middle 99.7% of the data?

$$186.4 - 3(48.9) = 39.7$$
$$186.4 + 3(48.9) = 333.1$$

$M-3\sigma \longleftarrow \longrightarrow M+3\sigma$

$$39.7 < x < 333.1$$

b. What percent of data will be greater than 235.3?

$$186.4 + 48.9 = 235.3 \quad \text{above the mean by } 1\sigma$$

$$13.5\% + 2.35\% + 0.15\% = 16\%$$

$13.5 + 2.35 + 0.15$

M   M+σ

c. What range of values represents the upper 2.5% of the data?

above M+2σ

$$186.4 + 2(48.9) = 284.2$$

all values greater than 284.2

2.5%

2.35 0.15

M   +1σ +2σ +3σ

Ex5: **A normal distribution has a mean of 80 and a standard deviation of 5**

a. What range of values represents the middle 95% of the data?
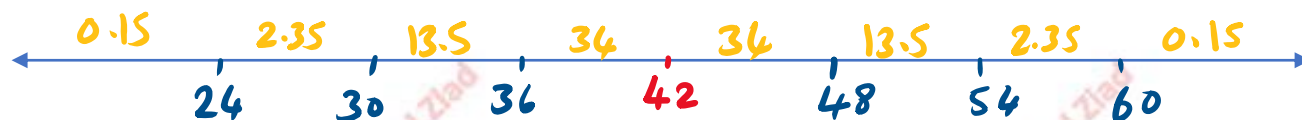
$$80 - 2(5) = 70$$
$$80 + 2(5) = 90$$

$-2\sigma \longleftarrow \longrightarrow +2\sigma$

$$70 < x < 90$$

b. What percent of data will be less than 85?

0.15	2.35	13.5	34	34	13.5	2.35	0.15
65	70	75	80 M		85	90	95

$$0.15 + 2.35 + 13.5 + 34 + 34 = 84\%$$

Ex6: A normal distribution has a mean of 42 and a standard deviation of 6.

   a. Find the range of values that represents the middle 95% of the distribution.

| 0.15 | 2.35 | 13.5 | 34 | 34 | 13.5 | 2.35 | 0.15 |

$$24 \quad 30 \quad 36 \quad 42 \quad 48 \quad 54 \quad 60$$

95% between $M - 2\sigma$ & $M + 2\sigma$

$$42 - 2(6) = 30 \qquad 30 < X < 54$$
$$42 + 2(6) = 54$$

   b. Find the range of values that upper 16% of the distribution.

above $M + \sigma$

above 48

   c. What percent of the data will be greater than 48?

16%

   d. What percent of the data will be less than 30?

2.5 %

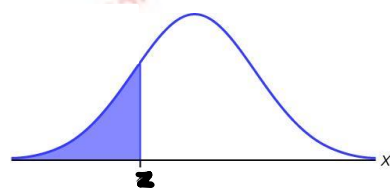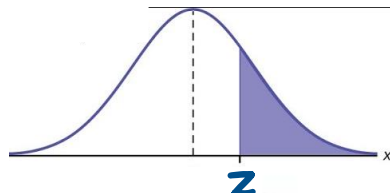## The Standard Normal Distribution

The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1

$$z = \frac{X - \mu}{\sigma}$$

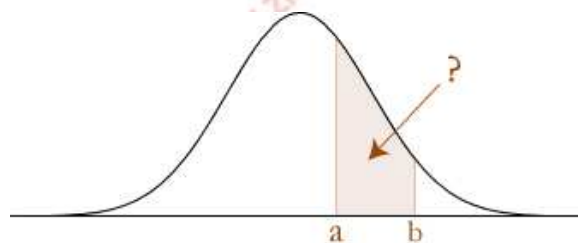**Ex7:** Find the z-value for each standard normal distribution. σ = 9.8, X = 55.4, and μ = 68.34

$$Z = \frac{55.4 - 68.34}{9.8}$$

$$= -1.32$$

Area under the curve of the Standard Normal Distribution using calculator

Area	Graph		
$P(z < k)$		Mode 3 1 [AC] shift 1 5 1 →1:P(2:Q(3:R(4:▸t	menu Distrib 2 lower −10 upper k
$P(z > k)$		Mode 3 1 [AC] shift 1 5 3 1:P(2:Q(→3:R(4:▸t	menu Distrib 2 lower k upper 10

Note:

To find $P(a < z < b) = P(z < b) - P(z < a)$

050-7214939

Ex8: find the area under the normal curve for each interval.

a) z > 0.58

0.28096

b) z < -1.56

0.05938

c) -2.29 < z < 2.76

$= P(z < 2.76) - P(z < -2.29)$

$= 0.99711 - 0.011011$

$= 0.986099$

Ex9: The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

a. What percent of the scores are between 70 and 80?

$$Z_1 = \frac{x_1 - M}{\sigma} = \frac{70 - 100}{12.3} = -2.44 \quad x_1 \quad x_2$$

$$Z_2 = \frac{x_2 - M}{\sigma} = \frac{80 - 100}{12.3} = -1.63$$

$$P(-2.44 < z < -1.63) = P(z < -1.63) - P(z < -2.44)$$

$$= 0.05155 - 0.00734$$

$$= 0.044207 = 4.42\%$$

b. What percent of the scores are over 115?

$$Z = \frac{115 - 100}{12.3} = 1.22 \quad x$$

$$P(z > 1.22) = 0.11123 = 11.123\%$$

c. If 160 people take the test, how many would you expect to score lower than 75?

$$Z = \frac{75 - 100}{12.3} = -2.03 \quad x$$

$$P(z < -2.03) = 0.021178$$

Expected number = Area under normal distribution × total

$$= 0.021178 \times 160$$

$$= 3.38$$

$$\approx 3 \text{ students}$$