AlBedaa Primary and Secondary School

UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

PHYSICS
Grade 11 – Advanced [101 C]. AY: 2024 – 2025 . . . T 1.
End of Term 1 Final Summative Assessment Preparation.

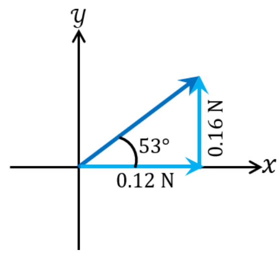
Contents.

You may use the following equations.	
$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ $y = y_0 + \bar{v}_yt$	$v_y = v_{y0} - gt$ $\bar{v}_y = \frac{1}{2}(v_y + v_{y0})$ $v_y^2 = v_{y0}^2 - 2g(y - y_0)$
$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$ $x = x_0 + \bar{v}_xt$	$v_y = v_{x0} + a_xt$ $\bar{v}_x = \frac{1}{2}(v_x + v_{x0})$ $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$
$a_x = \frac{d}{dt}v_x = \frac{d}{dt}\left(\frac{d}{dt}x\right) = \frac{d^2}{dt^2}x$	$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\, dt'$
$\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t')\, dt'$	$\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$
$H = y_0 + \frac{v_{y0}^2}{2g}$	$R = \frac{v_0^2}{g}\sin 2\theta_0$
$\vec{F}_{\text{net}} = m\vec{a}$	$f_{\text{k}} = \mu_{\text{k}}N$
$A_x = A\cos\theta$ $A_y = A\sin\theta$	$A = \sqrt{A_x^2 + A_y^2}$
$\vec{C} = \vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y + B_y, A_z + B_z)$	

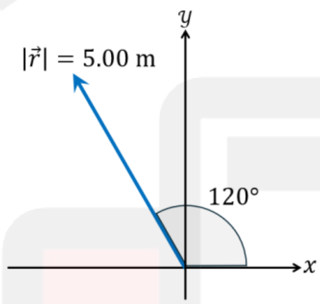
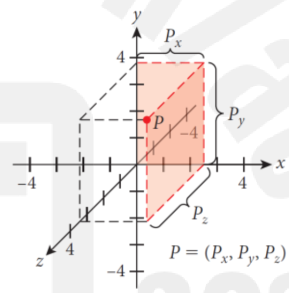
MULTIPLE CHOICE QUESTIONS.

LO – 1: Figure 1.18 Page 18, Figure 1.25 Page 22.
[1] Represent a point in one-, two- and three-dimensional space in terms of its Cartesian coordinates.
[2] Represent a vector in terms of its components in Cartesian coordinates- in two, and three-dimensional space.

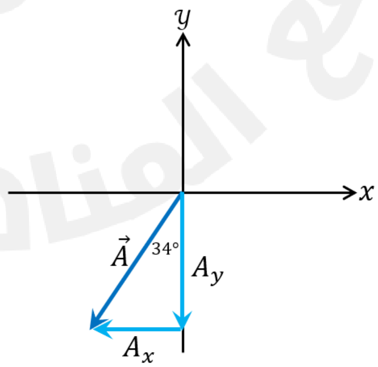
Questions – 1: A force has an x component of 0.12 N and a y component of 0.16 N. Find the magnitude and direction of this force.

Magnitude	Direction	
$\left\|\vec{F}\right\| = \sqrt{F_x^2 + F_y^2}$ $\left\|\vec{F}\right\| = \sqrt{0.12^2 + 0.16^2}$ $\left\|\vec{F}\right\| = 0.20\ \text{N}$	$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$ $\theta = \tan^{-1}\left(\frac{0.16}{0.12}\right)$ $\theta = 53°$	

Question – 2: A displacement vector lying in the xy plane has a magnitude of 50.0 m and is directed at an angle of 120° to the positive x axis. What are the rectangular components of this vector?

$r_x = \|\vec{r}\| \cos\theta$ $r_x = 50.0 \cos 120°$ $r_x = -25.0\ \text{m}$ $r_y = \|\vec{r}\| \sin\theta$ $r_y = 50.0 \sin 120°$ $r_y = 43.3\ \text{m}$ $\vec{r} = (-25.0\ \text{m})\,\hat{x} + (43.3\ \text{m})\,\hat{y}$		**FIGURE 1.15** Representation of a point *P* in a three-dimensional space in terms of its Cartesian coordinates.

Question – 3: Vector $\vec{A}$ is in the direction 34.0° clockwise from the negative y-axis. The x-component of $\vec{A}$ is $A_x = -16.0$ m. (a) What is the y-component of $\vec{A}$? (b) What is the magnitude of $\vec{A}$?

$\tan 34° = \frac{\|A_x\|}{\|A_y\|}$ $\left\|A_y\right\| = \frac{\|A_x\|}{\tan 34°}$ $\left\|A_y\right\| = \frac{16.0}{\tan 34°}$ $\left\|A_y\right\| = 23.6\ \text{m}$ $A_y = -23.6\ \text{m}$	$\left\|\vec{A}\right\| = \sqrt{(A_x)^2 + (A_y)^2}$ $\left\|\vec{A}\right\| = \sqrt{(-16.0)^2 + \left(\frac{16.0}{\tan 34°}\right)^2}$ $\left\|\vec{A}\right\| = 28.6$	

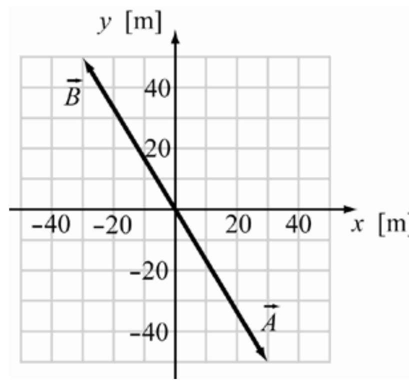
LO – 2: Page 21, Questions: 1.99, 1.100, 102, 104 Page: 30.
Find the length and direction of a two-dimensional vector from its Cartesian components.

Q1.99 sketch the vectors with the components $\vec{A} = (A_x\,, A_y) = (30.0\text{ m}, -50.0\text{ m})$ and $\vec{B} = (B_x\,, B_y) = (-30.0\text{ m}, 50.0\text{ m})$, and find the magnitudes of these vectors.

$$|\vec{A}| = \sqrt{30.0^2 + (-50.0)^2} = 58.3\text{ m}$$

$$|\vec{B}| = \sqrt{(-30.0)^2 + (50.0)^2} = 58.3\text{ m}$$

$\vec{A}$ and $\vec{B}$ are equal in magnitudes and opposite in directions.

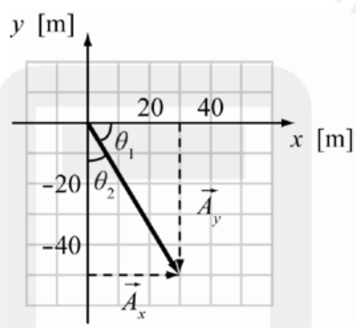


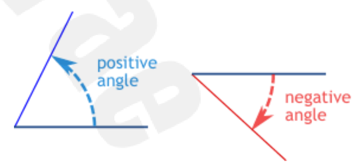
Q1.100 What angle does $\vec{A} = (A_x\,, A_y) = (30.0\text{ m}, -50.0\text{ m})$ make with the positive x − axis? What angle does it make with the negative y − axis?

$$\theta_1 = \tan^{-1}\left(\frac{-50.0}{30.0}\right) = -59.0°$$

The minus sign signifies a clockwise direction.

$$\theta_2 = 90.0° - 59.0° = 31.0°$$

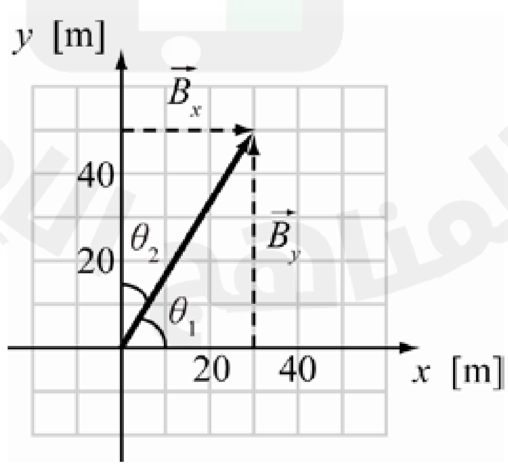




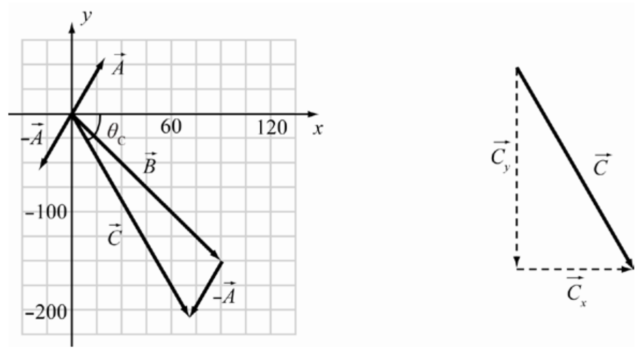
Q1.102 What angle does $\vec{B} = (B_x\,, B_y) = (30.0\text{ m}, 50.0\text{ m})$ make with the positive x − axis? What angle does it make with the positive y − axis?

$$\theta_1 = \tan^{-1}\left(\frac{50.0}{30.0}\right) = 59.0°$$

$$\theta_2 = 90.0° - 59.0° = 31.0°$$

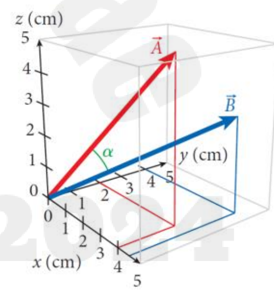


Q 1.104 Find the magnitude and direction of $-\vec{A}+\vec{B}$, Where $\vec{A}=(23.0, 59.0), \vec{B}=(90.0, -150.0)$.

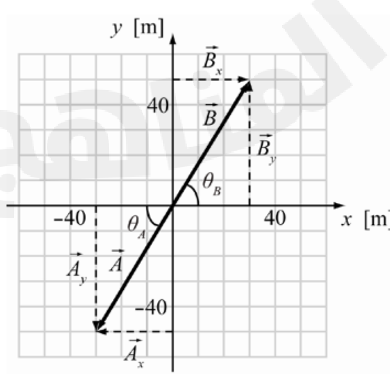
$-(23.0, 59.0)+(90.0, -150.0)=(67.0, -209)$ $\lvert -\vec{A}+\vec{B}\rvert=\sqrt{(67.0)^2+(-209)^2}=219$ $\theta = \tan^{-1}\left(\frac{-209}{67.0}\right)=-72.2° \quad \text{OR} \quad 288°$	

LO – 3: Pages 21,23 Example 1.5, Q 1.80, Q 1.103
Find the angle between two position vectors in the cartesian coordinates.

Example1.5 Page 22.
What is the angle α between the two position vectors shown in the Figure 1.25,
$\vec{A}=(4.00, 2.00, 5.00)$ cm and $\vec{B}=(4.50, 4.00, 3.00)$ cm?

Solution	Diagram
$\alpha = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{\lvert\vec{A}\rvert\,\lvert\vec{B}\rvert}\right)$ $\alpha = \cos^{-1}\left(\frac{A_xB_x+A_yB_y+A_zB_z}{\left(\sqrt{A_x^2+A_y^2+A_z^2}\right)\left(\sqrt{B_x^2+B_y^2+B_z^2}\right)}\right)$ $\alpha = \cos^{-1}\left(\frac{(4.00)(4.50)+(2.00)(4.00)+(5.00)(3.00)}{\left(\sqrt{4.00^2+2.00^2+5.00^2}\right)\left(\sqrt{4.50^2+4.00^2+3.00^2}\right)}\right)$ $\alpha = 24.7°$	 **FIGURE 1.25** Calculating the angle between two position vectors.

Q1.80 Express the vectors $\vec{A}=(A_x, A_y)=(-30.0\text{ m}, -50.0\text{ m})$ and $\vec{B}=(B_x, B_y)=(30.0\text{ m}, 50.0\text{ m})$ by giving their magnitude and direction as measured from positive $x-$axis.

$\lvert\vec{A}\rvert=\sqrt{(-30.0)^2+(-50.0)^2}=58.3$ $\lvert\vec{B}\rvert=\sqrt{(30.0)^2+(50.0)^2}=58.3$ $\theta_A=180°+\tan^{-1}\left(\frac{-50.0}{-30.0}\right)=239.0°$ $\theta_B=\tan^{-1}\left(\frac{50.0}{30.0}\right)=59.0°$	

Q1.103 Find the magnitude and direction of each of the following vectors, which are given in terms of their $x-$ and $y-$components: $\vec{A} = (23.0\,, 59.0)$ and $\vec{B} = (90.0\,, -150.0)$.

$\lvert\vec{A}\rvert = \sqrt{23.0^2 + (59.0)^2} = 63.3$	$\theta_A = \tan^{-1}\left(\frac{59.0}{23.0}\right) = 68.7°$
$\lvert\vec{B}\rvert = \sqrt{(90.0)^2 + (-150.0)^2} = 175$	$\theta_B = \tan^{-1}\left(\frac{-150.0}{90.0}\right) = -59.0°$ OR $301.0°$

LO – 4: Page 20. Questions 1.76, Q 1.79, Q 1.105, Q 106.
[1] Multiply a vector with a scalar. [2] Add or subtract vectors using Cartesian components.

Q1.76 Find the vector $\vec{C}$ that satisfies the equation $3\hat{x} + 6\hat{y} - 10\hat{z} + \vec{C} = -7\hat{x} + 14\hat{y}$.

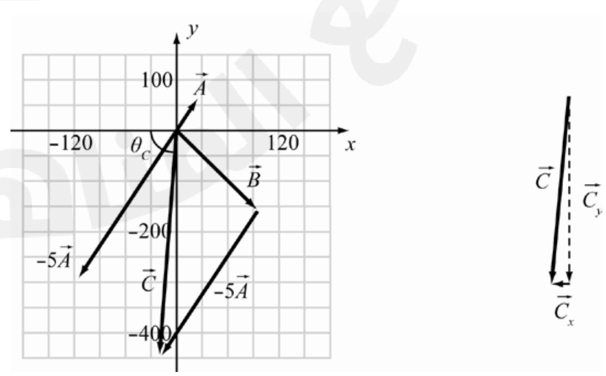
$\vec{C} = (-7\hat{x} - 3\hat{x}) + (14\hat{y} - 6\hat{y}) + (10\hat{z})$

$\vec{C} = -10\hat{x} + 8\hat{y} + 10\hat{z}$

Q1.79 Find the magnitude and direction of (a) $9\vec{B} - 3\vec{A}$ and (b) $-5\vec{A} + 8\vec{B}$,
where $\vec{A} = (23.0\,, 59.0)$, $\vec{B} = (90.0\,, -150.0)$

[a]	[b]
$9\vec{B} - 3\vec{A} = 9(90.0, -150.0) - 3(23.0\,, 59.0)$ $9\vec{B} - 3\vec{A} = (810, -1350) - (69.0\,, 177)$ $9\vec{B} - 3\vec{A} = (741\,, -1527)$ Magnitude: $\sqrt{(741)^2 + (-1527)^2} = 1697.3$ Direction: $\theta = \tan^{-1}\left(\frac{-1527}{741}\right) = -64.1°$ OR $295.9°$	$-5\vec{A} + 8\vec{B} = -5(23.0, 59.0) + 8(90.0\,, -150)$ $-5\vec{A} + 8\vec{B} = (-115\,, -295) + (720\,, -1200)$ $-5\vec{A} + 8\vec{B} = (605\,, -1495)$ Magnitude: $\sqrt{(605)^2 + (-1495)^2} = 1612.8$ Direction: $\theta = \tan^{-1}\left(\frac{-1495}{605}\right) = -68.0$ OR $292°$

Q1.105 Find the magnitude and direction of $-5\vec{A} + \vec{B}$, where $\vec{A} = (23.0\,, 59.0)$, $\vec{B} = (90.0\,, -150.0)$.

$-5\vec{A} + \vec{B} = -5(23.0, 59.0) + (90.0\,, -150)$ $-5\vec{A} + \vec{B} = (-115\,, -295) + (90\,, -150)$ $-5\vec{A} + \vec{B} = (-25\,, -445)$ Magnitude: $\sqrt{(-25)^2 + (-445)^2} = 445.7$ Direction: $\theta = 180° + \tan^{-1}\left(\frac{-445}{-25}\right) = 266.7°$	

Q1.106 Find the magnitude and direction of $-7\vec{B}+3\vec{A}$, where $\vec{A} = (23.0\,,59.0)$, $\vec{B} = (90.0\,,-150.0)$.

$-7\vec{B}+3\vec{A} = -7(90.0,-150)+3(23.0\,,59.0)$

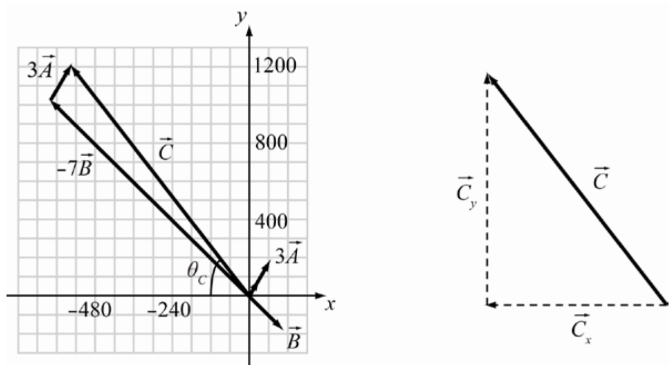
$-7\vec{B}+3\vec{A} = (-630\,,1050)+(69.0\,,177)$

$-7\vec{B}+3\vec{A} = (-561\,,1227)$

Magnitude: $\sqrt{(-561)^2+(1227)^2} = 1349.1$

Direction:

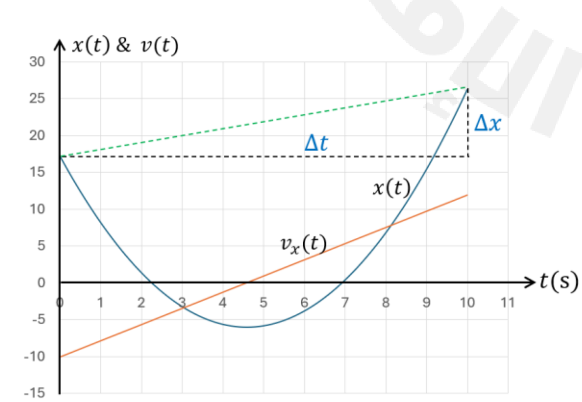
$\theta = \tan^{-1}\left(\frac{1227}{-561}\right) = -65.4°$ OR $114.6°$



LO – 5: Pages 36 – 39. Examples 2.1 & 2.2 Plus Questions 2.31, 2.32, 2.33
[1] Calculate the speed as the magnitude of instantaneous velocity.
[2] Calculate the average speed & average velocity.
[3] Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.

Example 2.1 During the time interval from 0.0 to 10.0 s, the position vector of a car on a road is given by $x(t) = a + bt + ct^2$ with $a = 17.2$ m, $b = -10.1$ m/s and $c = 1.10$ m/s^2.
What is the car's velocity as a function of time? What is the car's average velocity during this interval?

Instantaneous velocity function	Average velocity
$v(t) = \frac{d}{dt}x(t)$ $v(t) = \frac{d}{dt}(17.2 - 10.1\,t + 1.10\,t^2)$ $v(t) = -10.1 + 2.20\,t$	$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ $\bar{v} = \frac{x(10.0) - x(0.0)}{10.0 - 0.0}$ $\bar{v} = \frac{(17.2 - 101 + 110) - (17.2 - 0 + 0)}{10.0 - 0.0}$ $\bar{v} = 0.90$ m/s



Important Note:
In this particular example, the average velocity can be calculated using an alternative approach:

$\bar{v} = \frac{v(0) + v(10)}{2}$

$\bar{v} = \frac{-10.1 + 11.9}{2}$

$\bar{v} = 0.90$ m/s

This method is applicable **only if** the acceleration remains constant—in other words, if the position vector is represented by a second-degree polynomial.

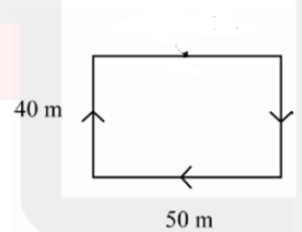
Example 2.2 Suppose a swimmer competes the first 50 m of the 100-m freestyle in 38.2 s. Once she reaches the far side of the 50-m-long pool, she turns around and swims back to the start in 42.5 s.
What are the swimmer's average velocity and average speed for

[a] the leg from the start to the far side of the pool.
[b] the return leg.
[c] the total trip.

Solution. ⊖⟵ ⟶⊕

	[a]	[b]	[c]
Average velocity	$\bar{v} = \frac{\Delta x}{\Delta t}$ $\bar{v} = \frac{50}{38.2}$ $\bar{v} = 1.31\ \mathrm{m/s}$	$\bar{v} = \frac{\Delta x}{\Delta t}$ $\bar{v} = \frac{-50}{42.5}$ $\bar{v} = -1.18\ \mathrm{m/s}$	$\bar{v} = \frac{\Sigma \Delta x}{\Sigma \Delta t}$ $\bar{v} = \frac{50 + (-50)}{38.2 + 42.5}$ $\bar{v} = 0.00\ \mathrm{m/s}$
Average speed	$\bar{s} = \lvert\bar{v}\rvert$ $\bar{s} = \lvert 1.31\rvert$ $\bar{s} = 1.31\ \mathrm{m/s}$	$\bar{s} = \lvert\bar{v}\rvert$ $\bar{s} = \lvert -1.18\rvert$ $\bar{s} = 1.18\ \mathrm{m/s}$	$\bar{s} = \frac{\ell}{\Delta t}$ $\bar{s} = \frac{100}{38.2 + 42.5}$ $\bar{s} = 1.24\ \mathrm{m/s}$

Q 2.31 Running on a 50-m by 40-m rectangular track, you complete one lap in 100 s. What is your average velocity for the lap?

$\bar{v} = \frac{\Sigma \Delta x}{\Sigma \Delta t}$ $\bar{v} = \frac{40\hat{y} + 50\hat{x} + 40(-\hat{y}) + 50(-\hat{x})}{100}$ $\bar{v} = 0.00\ \mathrm{m/s}$	

Q 2.32 An electron moves in the positive *x*-direction a distance of 2.42 m in 2.91×10^{-8}s, bounces off a moving proton, and then moves in the opposite direction of 1.69 m in 3.43×10^{-8}s.
[a] What is the average velocity of the electron over the entire time interval?
[b] What is the average speed of the electron over the entire time interval?

[a]	[b]
$\bar{v} = \frac{\Sigma \Delta x}{\Sigma \Delta t}$ $\bar{v} = \frac{2.42\,\hat{x} + 1.69(-\hat{x})}{(2.91 \times 10^{-8}) + (3.43 \times 10^{-8})}$ $\bar{v} = (1.15 \times 10^{7}\ \mathrm{m/s})\,\hat{x}$	$\bar{s} = \frac{\ell}{\Delta t}$ $\bar{s} = \frac{2.42 + 1.69}{(2.91 \times 10^{-8}) + (3.43 \times 10^{-8})}$ $\bar{s} = 6.48 \times 10^{7}\ \mathrm{m/s}$

Q 2.33 The graph describes the position of a particle in one dimension as a function of time.
[a] In which time interval does the particle have its maximum speed? What is that speed?
[b] What is the average velocity in the time interval between – 5 s and + 5 s?
[c] What is the average speed in the time interval between – 5 s and + 5 s?
[d] What is the ratio of the velocity in the interval between 2 s and 3 s to the velocity in the interval between 3 s and 4 s?
[e] At what time(s) is the particle's velocity zero?

In a position-time graph, negative time values are often used to represent moments that occur before a chosen reference point or starting time (usually $t = 0$). These negative time values allow us to see the object's position at earlier times, giving a complete view of its motion over time. For instance, in theoretical or experimental setups, the motion might have started earlier, and tracking back helps analyze past positions leading up to $t = 0$. In practical terms, negative time values can help us understand the object's trajectory or motion pattern from a prior point, aiding in understanding trends or calculating certain parameters more accurately.

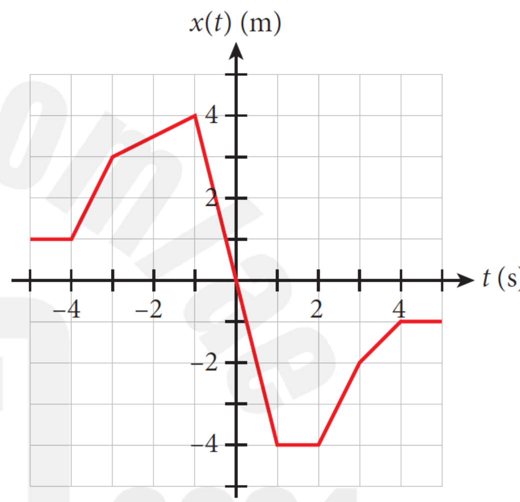
[a] the maximum speed occurs at the steepest slope occurring between –1 s and +1 s.

$$s_{\max} = \frac{|4 - -4|}{1 - -1} = 4 \text{ m/s}$$

[b] $\bar{v} = \dfrac{\Delta x}{\Delta t}$

$$\bar{v} = \frac{-1-1}{5 - -5}$$

$$\bar{v} = -0.2 \text{ m/s}$$



[c] $\bar{s} = \dfrac{\ell}{\Delta t}$

$$\bar{s} = \frac{|1-1| + |3-1| + |4-3| + |-4-4| + |-4 - -4| + |-2 - -4| + |-1 - -2| + |-1 - -1|}{5 - -5}$$

$$\bar{s} = \frac{14}{10} = 1.4 \text{ m/s}$$

[d] $v_{2\to3} = \dfrac{v(3) - v(2)}{3-2} \quad \Longrightarrow \quad v_{2\to3} = \dfrac{-2 - -4}{1} = 2 \text{ m/s}$

$$v_{3\to4} = \frac{v(4) - v(3)}{4-3} \quad \Longrightarrow \quad v_{3\to4} = \frac{-1 - -2}{1} = 1 \text{ m/s}$$

$$v_{2\to3} : v_{3\to4} = 2 : 1$$

[e] the velocity is zero during the intervals: $[-5 \text{ s}, -4 \text{ s}]$, $[1,2]$, $[4,5]$

LO – 6: Figure 2.7 & 2.16. Questions 2.12, 2.13, 2.26, 2.33, 2.42, 2.51.
[1] Interpret motion of an object from its position-time graph.
[2] Interpret the motion of an object from a velocity-time graph.

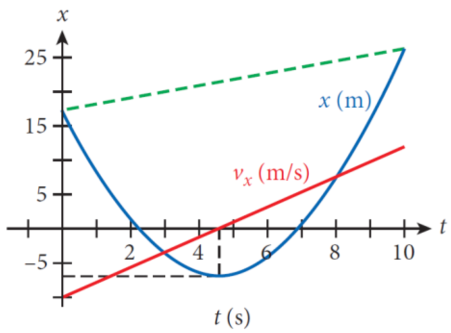


FIGURE 2.7 Graph of the position x and velocity v_x as a function of the time t. The slope of the dashed line represents the average velocity for the time interval from 0 to 10 s.

Summary of Example 2.1: Time Depnedency of Velocity.

$$x(t) = 17.2 - 10.1\, t + 1.10\, t^2$$

$$v_x(t) = 2.20\, t - 10.1$$

$$a_x(t) = 2.20 \text{ m/s}^2 \text{ … Constant Acceleration.}$$

While the average velocity $\bar{v}$ is

$$\bar{v} = \frac{x(10.0) - x(0.0)}{10.0 - 0.0}$$

$$\bar{v} = 0.90 \text{ m/s}$$

Solved Problem 2.2. PROBLEM

Assuming a constant acceleration of $a_x = 4.3 \text{ m/s}^2$ starting from rest, what is the airplane's takeoff velocity after 18.4 s? How far down the runway has the plane moved by the time it takes off?

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$x = 0.0 + 0.0 + \frac{1}{2}(4.3)t^2$$

$$x(t) = 2.15\, t^2$$

$$v_x(t) = 4.3\, t$$

$$v_x(18.4) = 4.3\,(18.4)$$

$$v_x(18.4) = 79 \text{ m/s}$$

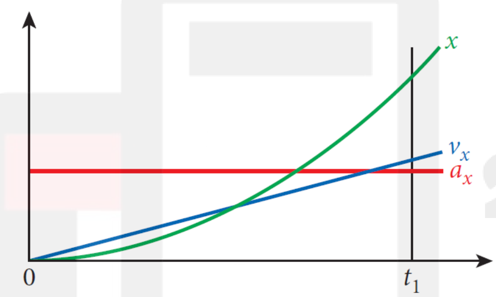
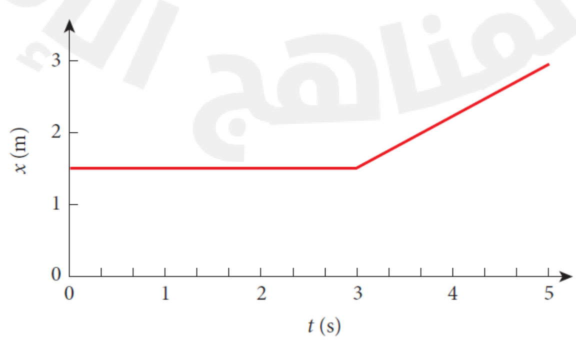


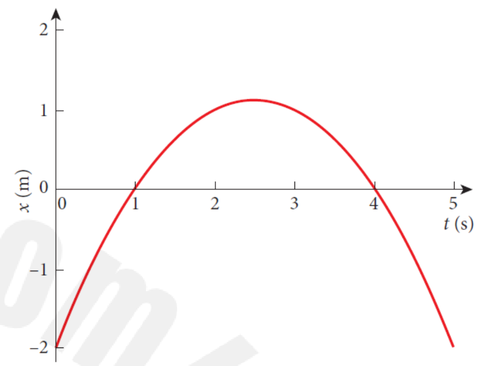
FIGURE 2.16 The acceleration, velocity, and displacement of the plane before takeoff.

Q 2.12 The figure describes the position of an object as a function of time. Which one of the following statements is true?



[a] The position of the object is constant.
[b] The velocity of the object is constant.
[c] The object moves in the positive *x*-direction until $t = 3$ s, and then the object is at rest.
[d] The object's position is constant until $t = 3$ s, and then the object begins to move in the positive *x*-direction.
[e] The object moves in the positive *x*-direction from $t = 0$ to $t = 3$ s and then moves in the negative *x*-direction from $t = 3$ s to $t = 5$ s.

The figure describes the position of an object as a function of time. Refer to it to answer questions 2.13 – 2.16.

Analyzing.	Graph.
The position – time graph is a quadratic function of time. $x(t) = at^2 + bt + c$, where a, b and c are constants. $x(0) = 0 + 0 + c = -2 \quad \Rightarrow c = -2.$ $x(1) = a + b + (-2) = 0 \quad \Rightarrow a + b = 2$ ….. (1) $x(4) = 16a + 4b + (-2) = 0 \quad \Rightarrow 16a + 4b = 2$ …. (2) Solving (1) and (2): $a = -0.5$ & $b = 2.5$ $x(t) = -0.5t^2 + 2.5t - 2$ $v(t) = -t + 2.5 \Rightarrow v(1) = -1.0 + 2.5 = 1.5$ m/s $a(t) = -1.0$ m/s^2	

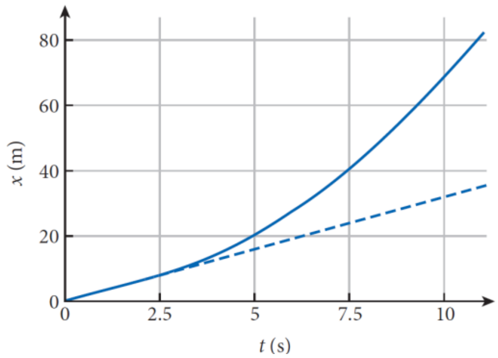
Q 2.13 Which one of the following statements is true at $t = 1.0\ s$?
[a] The *x*-component of the velocity of the object is zero.
[b] The *x*-component of the acceleration of the object is zero.
[c] The *x*-component of the velocity of the object is positive.
[d] The *x*-component of the velocity of the object is negative.

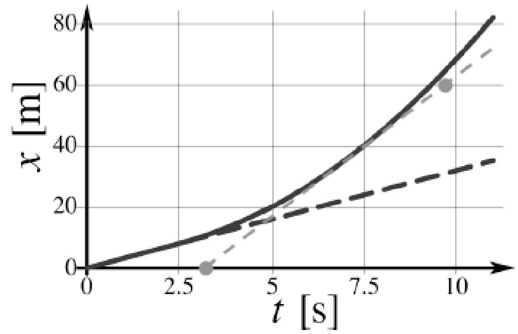
Q 2.26 A car moves along a road with a constant velocity. Starting at time $t = 2.5\ s$, the driver accelerates with a constant acceleration. The resulting position of the car as a function of time is shown by the blue curve in the figure.

[a] What is the value of the constant velocity of the car before 2.5 s?
Hint: The dashed blue line is the path the car would take in the absence of the acceleration.)

[b] What is the velocity of the car at $t = 7.5$ s? Use a graphical technique (i.e., draw a slope).

[c] What is the value of the constant acceleration?





[a] The constant velocity is equal to the slope of the $x - t$ graph during the interval [0.0 s, 2.5 s].

$$v = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \implies v = \frac{x(2.5) - x(0.0)}{2.5 - 0.0} \implies v = \frac{8.0 - 0.0}{2.5 - 0.0} = 3.2 \text{ m/s}$$

[b] The velocity at $t = 7.5\ s$ is an instantaneous velocity, slope of the tangent.

$$v = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \implies v = \frac{x(9.8) - x(3.4)}{9.8 - 3.4} \implies v = \frac{60.0 - 0.0}{9.8 - 3.4} = 9.4 \text{ m/s}$$

[c] The acceleration is the change in velocity during the interval [2.5 s, 7.5 s] $(2.5, 3.2)$ and $(7.5, 9.4)$

$$a = \frac{v(t_2) - v(t_1)}{t_2 - t_1} \implies a = \frac{v(7.5) - v(2.5)}{7.5 - 2.5} \implies a = \frac{9.4 - 3.2}{7.5 - 2.5} = 1.24 \text{ m/s}^2$$

ANALYTICAL APPROACH.

The position – time graph is a quadratic function of time.

$x(t) = at^2 + bt + c$, where a, b and c are constants.

$x(0) = 0 + 0 + c = 0 \implies c = 0$

$x(5) = 25a + 5b = 20$

$5a + b = 4 \ldots\ldots(1)$

$x(7.5) = 56.25\,a + 7.5\,b = 40$

$7.5\,a + b = \frac{16}{3} \ldots\ldots(2)$

$a = \frac{8}{15}$ & $b = \frac{4}{3}$

$x(t) = \frac{8}{15}t^2 + \frac{4}{3}t$

$v(t) = \frac{16}{15}t + \frac{4}{3}$

$v(7.5) = \frac{16 \times 7.5}{15} + \frac{4}{3} = 9.3 \text{ m/s}$

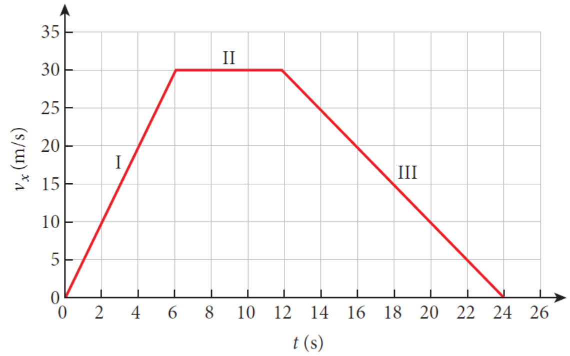
$a(t) = \frac{16}{15} = 1.07 \text{ m/s}^2$

Q 2.33: Solved in LO – 5.

Q 2.42 A fellow student found in the performance data for his new car the velocity-versus-time graph shown in the figure.

[a] Find the average acceleration of the car during each of the segments I, II, and III.

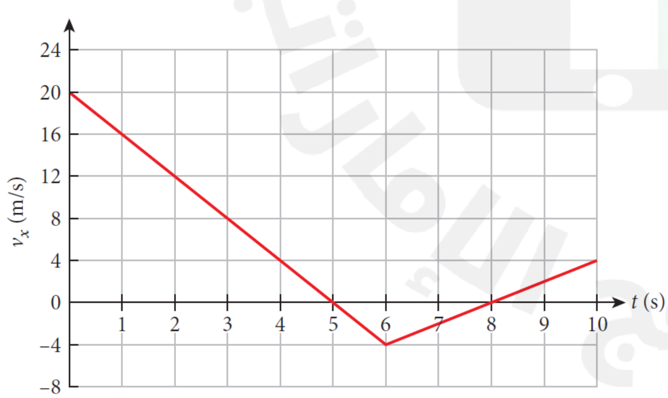
[b] What is the total distance traveled by the car from $t = 0$ s to $t = 24$ s?



[a]	[b]
$a = \dfrac{\Delta v}{\Delta t}$ $a_I = \dfrac{30 - 0}{6 - 0} = 5.0\ \text{m/s}^2$ $a_{II} = \dfrac{30 - 30}{12 - 4} = 0.0\ \text{m/s}^2$ $a_{III} = \dfrac{0 - 30}{24 - 12} = -2.5\ \text{m/s}^2$	Displacement = Bounded Area. $\Delta x = \frac{1}{2}(\text{base}_1 + \text{base}_2)(\text{Altitude})$ $\Delta x = \frac{1}{2}(24 + 6)(30)$ $\Delta x = 450\ \text{m}$

Q 2.51 A car is moving along the x-axis and its velocity, v_x varies with time as shown in the figure. If $x_0 = 2.0$ m at $t_0 = 2.0\ s$, what is the position of the car at $t = 10.0$ s?

$$\Delta x = \frac{1}{2}(\text{base})(\text{Height}) + \frac{1}{2}(\text{base})(\text{Height}) + \frac{1}{2}(\text{base})(\text{Height})$$

	Displacement = Bounded Area. $\Delta x =$ Bounded Area. $x - x_0 = \frac{1}{2}(3)(12) + \frac{1}{2}(3)(-4) + \frac{1}{2}(2)(4)$ $x - 2.0 = 16$ $x = 18\ \text{m}$

LO – 7: Figure 2.27 & 2.28. Questions 2.66, 2.67, 2.69.
[1] Interpret motion graphs for objects under free fall.
[2] Apply the constant-acceleration equations to free-fall motion

SOLVED PROBLEM 2.5 Melon Drop.
Suppose you decide to drop a melon from rest from the first observation platform of the Eiffel Tower. The initial height h from which the melon is released is 58.3 m above the head of your French friend Pierre, who is standing on the ground right below you. At the same instant you release the melon, Pierre shoots an arrow straight up with an initial velocity of 25.1 m/s. (Of course, Pierre makes sure the area around him is cleared and gets out of the way quickly after he shoots his arrow.)

PROBLEM
(a) How long after you drop the melon will the arrow hit it? (b) At what height above Pierre's head does this collision occur?

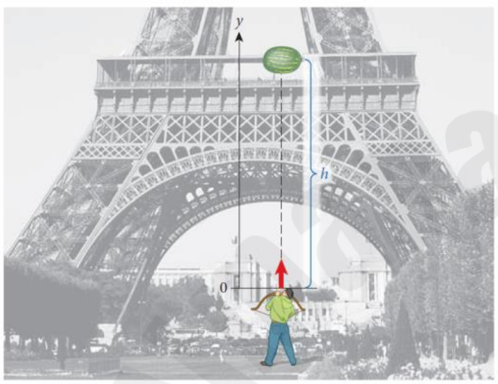


FIGURE 2.26 The melon drop (melon and person are not drawn to scale!).

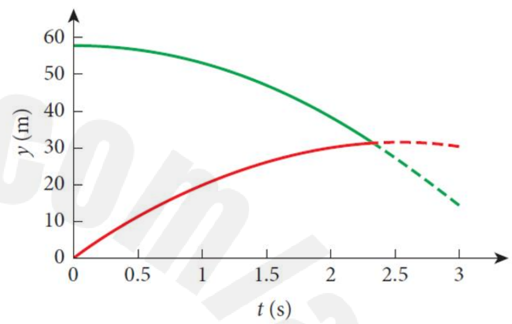
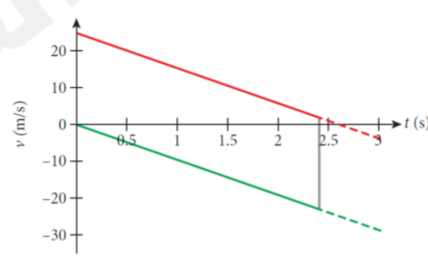


FIGURE 2.27 Position as a function of time for the arrow (red curve) and the melon (green curve).

The melon	The Arrow	
$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$	The arrow hits the melon when $y_m(t) = y_a(t)$
$y_m(t) = h - \frac{1}{2}gt^2$	$y_a(t) = v_{a0}t - \frac{1}{2}gt^2$	$58.3 - 4.905\, t_c^2 = 25.1\, t_c - 4.905\, t_c^2$
$y_m(t) = 58.3 - 4.905\, t^2$	$y_a(t) = 25.1\, t - 4.905\, t^2$	$t_c = 2.32$ s
		c: collision.

ADDITIONAL QUESTION What are the velocities of melon and arrow at the moment of the collision?

The melon	The Arrow	
$v_y = v_{y0} - gt$	$v_y = v_{y0} - gt$	
$v_m(t) = -9.81\, t$	$v_a(t) = 25.1 - 9.81\, t$	
$v_m(2.32) = (-9.81)\,(2.32)$	$v_a(2.32) = 25.1 + (-9.81)\,(2.32)$	
$v_m(2.32) = -22.8$ m/s	$v_a(2.32) = 2.34$ m/s	FIGURE 2.28 Velocities of the arrow (red curve) and melon (green curve) as a function of time.

Q 2.66 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?

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$v_y = v_{y0} - gt$

$0 = 26.4 - (9.81 \times t_{max})$

$t_{max} = 2.69\ s$

$t_{total} = 2 \times 2.69\ \mathrm{s}$

$t_{total} = 5.39\ \mathrm{s}$

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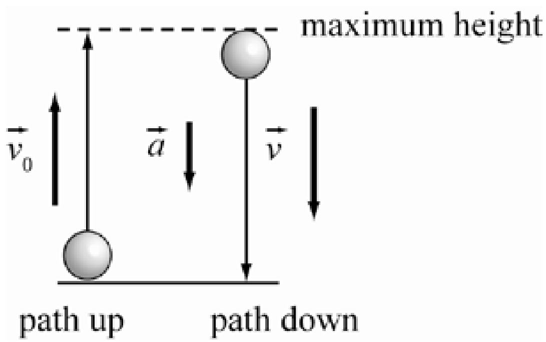

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Q 2.67 A stone is thrown upward, form ground level, with an initial velocity of 10.0 m/s.
[a] What is the velocity of the stone after 0.50 s?
[b] How high above ground level is the stone after 0.50 s?

[a]	[b]
$v_y = v_{y0} - gt$ $v_y = 10.0 + (-9.81)(0.50)$ $v_y = 5.1\ \mathrm{m/s}$	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ $y = 0.0 + (10.0 \times 0.50) - \frac{1}{2}(9.81)(0.50)^2$ $y = 3.8\ \mathrm{m}$

Q 2.69 A ball is thrown directly downward, with an initial speed of 10.0 m/s, from a height of 50.0 m. After what time interval does the ball strike the ground?

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$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

$-50.0 = 0.0 + (-10.0\ t) - \frac{1}{2}(9.81)t^2$

Shift + Solve:

$t = 2.33\ \mathrm{s}$ ☑

$t = -4.37\ \mathrm{s}$ ⊗ time cant be negative.

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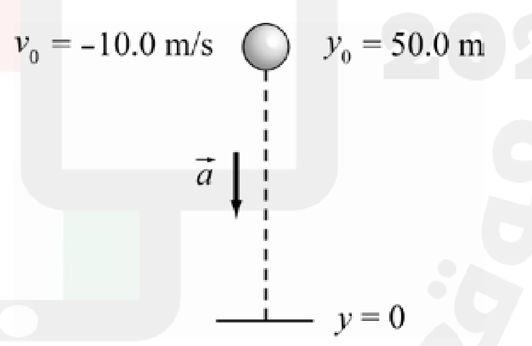

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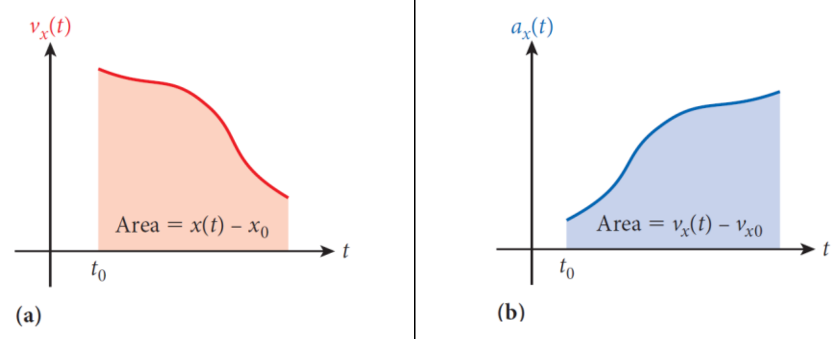
LO – 8: Figure 2.13. Questions 2.48, 2.53

Determine an object's change in velocity by the area under the curve in an acceleration versus time graph.

FIGURE 2.13 Geometrical interpretation of the integrals of
(a) velocity and
(b) acceleration with respect to time.



In calculus, you probably learned that the geometrical interpretation of the definite integral is an area under a curve. This is true for equations

$$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt' \qquad \text{and} \qquad v_x(t) = v_{x0} + \int_{t_0}^{t} a_x(t')\,dt'$$

We can interpret the area under the curve of $v_x(t)$ between (t_0) and (t) as the difference in the position between these two times, as shown in Figure 2.13a.

Figure 2.13b shows that the area under the curve of $a_x(t)$ in the time interval between (t_0) and (t) is the velocity difference between these two times.

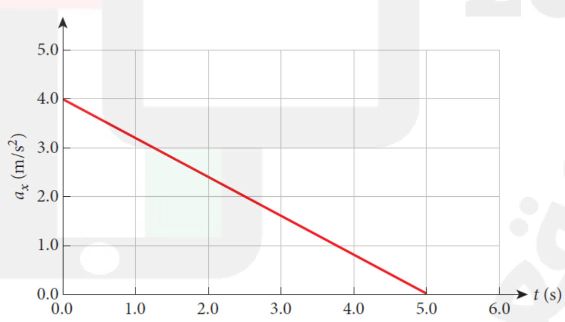
Q 2.48 A car moving in the *x*-direction has an acceleration a_x that varies with time as shown in the figure. At the moment $t =$ 0.0 s, the car is located at $x =$ 12 m and has a velocity of 6.0 m/s in the positive *x*-direction. What is the velocity of the car at $t =$ 5.0 s?

$\Delta v = \text{Bounded Area}$

$$v(5) - v(0) = \frac{1}{2}(\text{base})(\text{Height})$$

$$v(5) - 6 = \frac{1}{2}(5.0)(4.0)$$

$$v(5) = 16.0\ \text{m/s}$$



Analytical Approach:

$$a_x(t) - a_0 = \text{slope}\,(t - t_0)$$
$$a_x(t) - 4.0 = -0.8\,(t - 0)$$
$$a_x(t) = -0.8\,t + 4.0$$

$$v_x(t) = -0.4\,t^2 + 4.0\,t + 6.0$$
$$v_x(5) = -0.4\,(25) + 4.0\,(5) + 6.0$$
$$v_x(5) = 16.0\ \text{m/s}$$

$$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt'$$

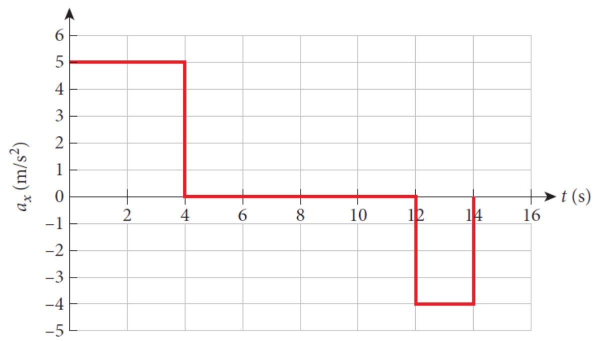
$$x(t) = 12 + \int_{0}^{t} (-0.4\,t^2 + 4.0\,t + 6.0)\,dt$$

$$x(t) = 12 - \frac{0.4}{3}t^3 + 2t^2 + 6t$$

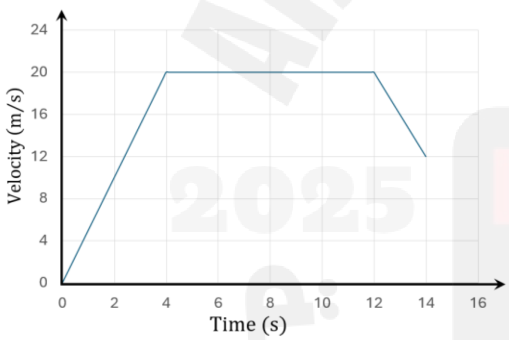
Q 2.53 A motorcycle starts from rest and accelerates as shown in the figure. Determine (a) the motorcycle's speed at $t = 4.00$ s and at $t = 14.0$ s and (b) the distance traveled in the first 14.0 s.

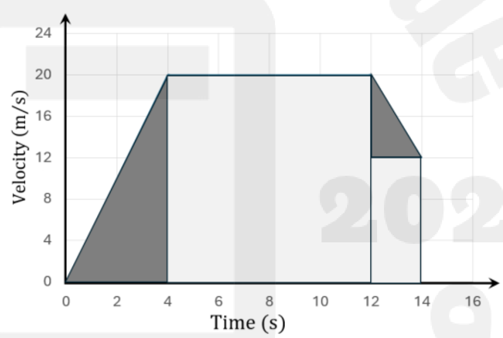
$$a = \begin{cases} 5.0\ \text{m/s}^2 & [0,4] \\ 0.0\ \text{m/s}^2 & [4,12] \\ -4.0\ \text{m/s}^2 & [12,14] \end{cases}$$

$$v(t) = \begin{cases} 5t & [0,4] \\ 20.0 & [4,12] \\ -4t + 20 & [12,14] \end{cases}$$



Δv = Bounded Area	Δv = Bounded Area
$v(4) - v(0) = (\text{base})(\text{Height})$	$v(14) - v(12) = (\text{base})(\text{Height})$
$v(4) - 0 = (4.0)(5.0)$	$v(14) - 20 = (2.0)(-4.0)$
$v(4) = 20.0\ \text{m/s}$	$v(14) = 12.0\ \text{m/s}$





$$\Delta x = \frac{1}{2}(\text{base})(\text{Height}) + (\text{base})(\text{Height}) + \frac{1}{2}(\text{base})(\text{Height}) + (\text{base})(\text{Height})$$

$$x - x_0 = \frac{1}{2}(4)(20) + (8)(20) + \frac{1}{2}(2)(8) + (2)(12)$$

$$x = 232\ \text{m}$$

LO – 9: Figure 2.13. Questions 2.49, 2.50
[1] Calculate a particle's change in velocity by integrating its acceleration function with respect to time.
[2] Calculate a particle's change in position by integrating its velocity function with respect to time.

Q 2.49 The velocity as a function of time for a car on an amusement park ride is given as $v = At^2 + Bt$ with constants $A = 2.0\ \mathrm{m/s^3}$ and $B = 1.0\ \mathrm{m/s^2}$. If the car starts at the origin, what is its position at $t =\ 3.0$ s?

$v(t) = 2t^2 + t$ $x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt'$ $x(t) = 0 + \int_{0}^{t} (2t^2 + t)\,dt$	$x(t) = \frac{2t^3}{3} + \frac{t^2}{2}$ $x(3) = \frac{2(3)^3}{3} + \frac{(3)^2}{2}$ $x(3) = 22.5\ \mathrm{m}$ $x(3) = 23\ \mathrm{m}$

Q 2.50 An object starts from rest and has an acceleration given by $a = Bt^2 - \frac{1}{2}Ct$, where $B = 2.0\ \mathrm{m/s^4}$ and $C = -4.0\ \mathrm{m/s^3}$ B = 2.0.
a) What is the object's velocity after 5.0 s?
b) How far has the object moved after $t =\ 5.0$ s?

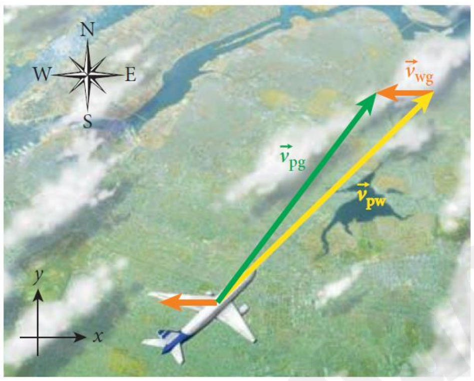
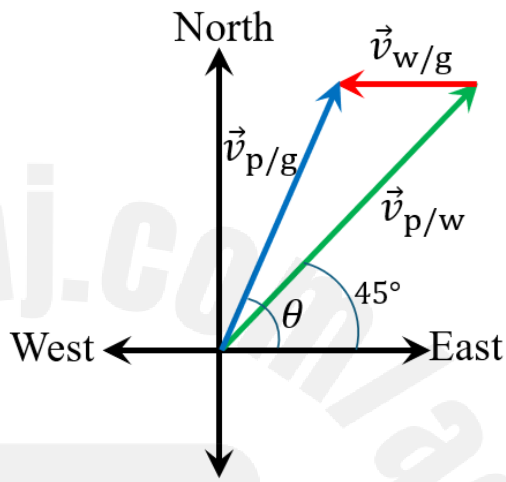
$a(t) = 2t^2 + 2t$ $v_x(t) = v_{x0} + \int_{t_0}^{t} a_x(t')\,dt'$ $v_x(t) = 0 + \int_{0}^{t} (2t^2 + 2t)\,dt$ $v_x(t) = \frac{2t^3}{3} + t^2$ $v_x(5) = \frac{2(5)^3}{3} + (5)^2$ $v_x(5) =\ 108.3\ \mathrm{m/s}$ $v_x(5) =\ 110\ \mathrm{m/s}$	$x(t) = x_0 + \int_{t_0}^{t} v_x(t')\,dt'$ $x(t) = 0 + \int_{0}^{t} \left(\frac{2t^3}{3} + t^2\right) dt$ $x(t) = \frac{2}{12}t^4 + \frac{t^3}{3}$ $x(5) = \frac{2}{12}(5)^4 + \frac{(5)^3}{3}$ $x(5) = 145.8\ \mathrm{m}$ $x(5) = 150\ \mathrm{m}$

LO – 10: Example 3.3. Example 3.4 Questions 3.63

[1] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and along a single axis.

[2] Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at constant velocity and in two dimensions

Example 3.3 Airplanes move relative to the air that surrounds them. Suppose a pilot points his plane in the northeast direction. The airplane moves with a speed of 160. m/s relative to the wind, and the wind is blowing at 32.0 m/s in a direction from east to west (measured by an instrument at a fixed point on the ground).

 **FIGURE 3.19** Velocity of an airplane with respect to the wind (yellow), the velocity of the wind with respect to the ground (orange), and the resultant velocity of the airplane with respect to the ground (green).	

PROBLEM What is the velocity vector-speed and direction-of the airplane relative to the ground? How far off course does the wind blow this plane in 2.0 h?

$\vec{v}_{p/g} = \vec{v}_{p/w} + \vec{v}_{w/g}$ $\vec{v}_{p/g} = 160\cos 45°\,\hat{x} + 160\sin 45°\,\hat{y} + (-32\,\hat{x})$ $\vec{v}_{p/g} = \left(\frac{160}{\sqrt{2}} - 32\right)\hat{x} + \left(\frac{160}{\sqrt{2}}\right)\hat{y}$ $\lvert\vec{v}_{p/g}\rvert = \sqrt{\left(\frac{160}{\sqrt{2}} - 32\right)^2 + \left(\frac{160}{\sqrt{2}}\right)^2}$ $\lvert\vec{v}_{p/g}\rvert = 139\ \text{m/s}$	$\theta = \tan^{-1}\left(\frac{v_{(p/g)y}}{v_{(p/g)x}}\right)$ $\theta = \tan^{-1}\left(\frac{\frac{160}{\sqrt{2}}}{\frac{160}{\sqrt{2}} - 32}\right)$ $\theta = 54.4°$	$\lvert\vec{r}_T\rvert = \lvert\vec{v}_{w/g}\rvert t$ $\lvert\vec{r}_T\rvert = 32 \times 2 \times 3600$ $\lvert\vec{r}_T\rvert = 230.4\ \text{km}$

EXAMPLE 3.4 Let's suppose rain is falling straight down on a car, as indicated by the white lines in Figure 3.20. A stationary observer outside the car would be able to measure the velocities of the rain (blue arrow) and of the moving car (red arrow).

However, if you are sitting inside the moving car, the outside world of the stationary observer (including the street, as well as the rain) moves with a relative velocity of $\vec{v} = -\vec{v}_{car}$. The velocity of this relative motion has to be added to all outside events as observed from inside the moving car. This motion results in a velocity vector $\vec{v}'_{rain}$ for the rain as observed from inside the moving car (Figure 3.21); mathematically, this vector is a sum, $\vec{v}'_{rain} = \vec{v}_{rain} - \vec{v}_{car}$, where $\vec{v}_{rain}$ and $\vec{v}_{car}$ are the velocity vectors of the rain and the car as observed by the stationary observer.

FIGURE 3.20 The velocity vectors of a moving car and of rain falling straight down on the car, as viewed by a stationary observer.

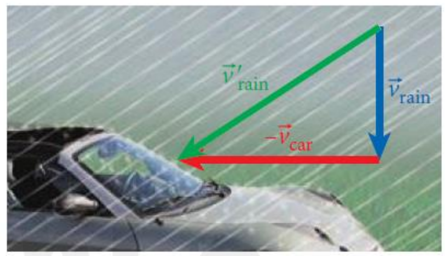


FIGURE 3.21 The velocity vector $\vec{v}'_{rain}$ of rain, as observed from inside the moving car.

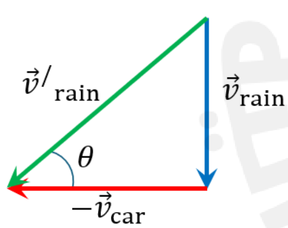
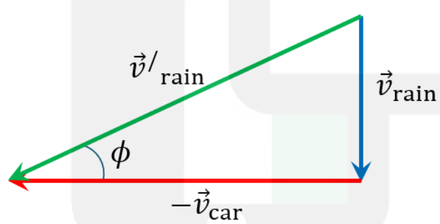
CONCEPT CHECK 3.8 It is raining, and there is practically no wind. While driving through the rain, you speed up. What happens to the angle of the rain relative to the horizontal that you observe from inside the car?

a) It increases.

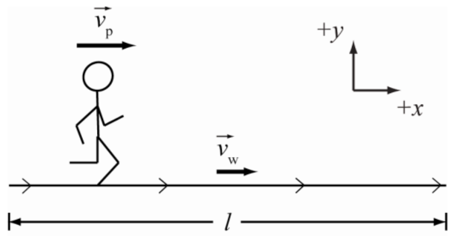
b) It decreases.

c) It stays the same.

d) It can increase or decrease, depending on the direction in which you are driving.

Before	After	
 $\theta = \tan^{-1}\left(\frac{\lvert\vec{v}_{rain}\rvert}{\lvert\vec{v}_{car}\rvert}\right)$	 $\phi = \tan^{-1}\left(\frac{\lvert\vec{v}_{rain}\rvert}{\lvert\vec{v}_{car}\rvert}\right)$	$\phi < \theta$

Q 3.63 You are walking on a moving walkway in an airport. The length of the walkway is 59.1 m. If your velocity relative to the walkway is 2.35 m/s and the walkway moves with a velocity of 1.77 m/s, how long will it take you to reach the other end of the walkway?

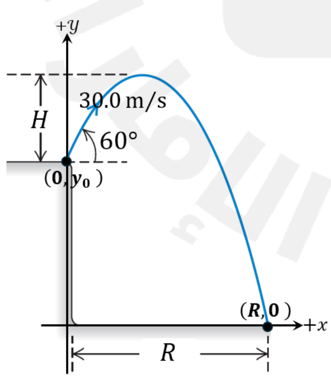
p: passenger, w: walkway, g: ground. $\vec{v}_{pw} = 2.35\,\hat{x}$ m/s $\vec{v}_{wg} = 1.77\,\hat{x}$ m/s	$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$ $\vec{v}_{pg} = 2.35\,\hat{x} + 1.77\,\hat{x}$ $\vec{v}_{pg} = 4.12\,\hat{x}$ m/s $t = \dfrac{\ell}{\lvert\vec{v}_{pg}\rvert}$ $t = \dfrac{59.1}{4.12} = 14.3$ s	

LO – 11: MCQ 3.1, 3.2, 3.4, 3.6, 3.10, 3.11
Calculate the particle's position, displacement, and velocity at a given instant during the flight given the launch velocity.

MCQ 3.1 An arrow is shot horizontally with a speed of 20. m/s from the top of a tower 60. m high. The time to reach the ground will be

(a) 8.9 s (b) 7.1 s (c) 3.5 s (d) 2.6 s (e) 1.0 s	$y = y_0 + v_{y0}t - \frac{1}{2}\,g\,t^2$ $0 = 60 + 0 - \frac{1}{2}(9.81)\,t^2$ $t = 3.5$ s	Quick Way: $t = \sqrt{\dfrac{2h}{9.81}}$ $t = \sqrt{\dfrac{2 \times 60}{9.81}}$ $t = 3.5$ s

MCQ 3.2 A projectile is launched from the top of a building with an initial velocity of 30.0 m/s at an angle of 60.0° above the horizontal. The magnitude of its velocity at t = 5.00 s after the launch is

(a) − 23.1 m/s (b) 7.3 m/s (c) 15.0 m/s (d) 27.5 m/s (e) 50.4 m/s		$t_{max} = \dfrac{v_0 \sin\theta_0}{g}$ $t_{max} = \dfrac{(30)\sin 60°}{9.81}$ $t_{max} = 2.65$ s $v_y = -gt$ $v_y = -(9.81)(5 - 2.65)$ $v_y = -23.1$ m/s	$v_x = v_0 \cos\theta_0$ $v_x = (30)(\cos 60°)$ $v_x = 15.0$ m/s $\lvert\vec{v}\rvert = \sqrt{(v_x)^2 + (v_y)^2}$ $\lvert\vec{v}\rvert = \sqrt{(15.0)^2 + (-23.1)^2}$ $\lvert\vec{v}\rvert = 27.5$ m/s

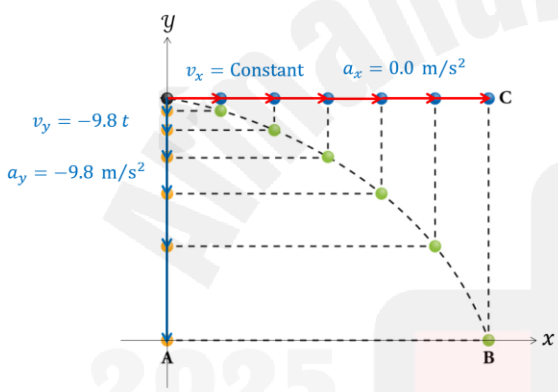
MCQ 3.4 During practice two baseball outfielders throw a ball to the shortstop. In both cases the distance is 40.0 m. Outfielder 1 throws the ball with an initial speed of 20.0 m/s, outfielder 2 throws the ball with an initial speed of 30.0 m/s. In both cases the balls are thrown and caught at the same height above ground.

a) Ball 1 is in the air for a shorter time than ball 2.
b) Ball 2 is in the air for a shorter time than ball 1.
c) Both balls are in the air for the same duration.
d) The answer cannot be decided from the information given.

Because we don't have the launch angles, we **cannot definitively determine** which ball is in the air for a shorter time.

In case the two balls were thrown horizontally then the answer will be (b).

The faster ball travels horizontally faster than the slower one, so the faster ball is in the air a shorter time, and thus gains a smaller vertical velocity.

horizontally launched projectile. 	$\theta_0 = 0.0°$ $y = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$ $y = y_0 - \frac{1}{2}gt^2$ $x - x_0 = (v_0 \cos\theta_0)\, t$ $x = v_0\, t$ $t = \frac{x}{v_0}$

MCQ 3.6 For a given initial speed of an ideal projectile, there is (are)launch angle(s) for which the range of the projectile is the same.

a) only one b) two different c) more than two but a finite number of d) only one if the angle is 45° but otherwise two different e) an infinite number of	$R = \frac{v_0^2}{g}\sin 2\theta_0$ $R = \frac{v_0^2}{g} 2\sin\theta_0\cos\theta_0$ $\sin\theta_0\cos\theta_0 = \frac{Rg}{2v_0^2}$ For $\theta_0 = 45°, \sin 45° = \cos 45°$

MCQ 3.10 A baseball is launched from the bat at an angle $\theta_0 = 30.0°$ with respect to the positive *x*-axis and with an initial speed of 40.0 m/s, and it is caught at the same height from which it was hit. Assuming ideal projectile motion (positive *y*-axis upward), the velocity of the ball when it is caught is

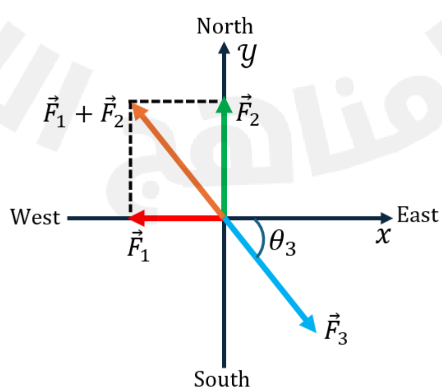
a) $(20.00\,\hat{x} + 34.64\,\hat{y})$ b) $(-20.00\,\hat{x} + 34.64\,\hat{y})$ c) $(34.64\,\hat{x} - 20.00\,\hat{y})$ d) $(34.64\,\hat{x} + 20.00\,\hat{y})$	$\vec{v} = v_x\,\hat{x} + v_y\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x} + (v_0\sin\theta_0 - g t_{\text{hang}})\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x} + \left(v_0\sin\theta_0 - g\dfrac{2v_0\sin\theta_0}{g}\right)\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x} - v_0\sin\theta_0\,\hat{y}$ $\vec{v} = (40)\cos 30°\,\hat{x} - 40\sin 30°\,\hat{y}$ $\vec{v} = 34.64\,\hat{x} - 20.00\,\hat{y}$

MCQ 3.11 In ideal projectile motion, the velocity and acceleration of the projectile at its maximum height are, respectively,

a) horizontal, vertical downward. b) horizontal, zero. c) zero, zero. d) zero, vertical downward. e) zero, horizontal.	$\vec{v} = v_x\,\hat{x} + v_y\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x} + (v_0\sin\theta_0 - g t_{\max})\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x} + \left(v_0\sin\theta_0 - g\dfrac{v_0\sin\theta_0}{g}\right)\hat{y}$ $\vec{v} = v_0\cos\theta_0\,\hat{x}$	$\vec{a} = (0, -\text{g}) = -\text{g}\hat{y}$

LO – 12: Example 4.1 & Questions 4.34, 4.81
[1] Describe an object in static equilibrium and dynamic equilibrium.
[2] State the conditions for an object to be in equilibrium.
[3] Calculate a force of unknown magnitude acting on an object in equilibrium.

Problem 4.1 Now let's consider the situation where three ropes are tied together at one point, with a team pulling on each rope. Suppose team 1 is pulling due west with a force of 2750N, and team 2 is pulling due north with a force of 3630 N. Can a third team pull in such a way that the three rug of war ends at a standstill, that is, no team is able to move the rope? If yes, what is the magnitude and direction of the force needed to accomplish this?

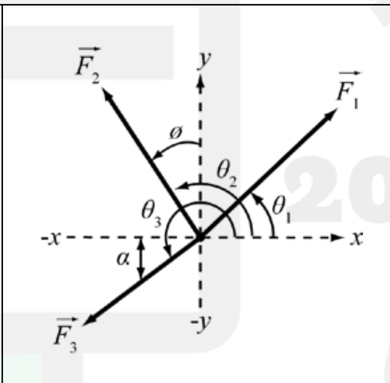


A third force $\vec{F}_3$ CAN pull in such a way that the three rug of war ends at a standstill. (At Equilibrium).

$\vec{F}_1 = -2750\ \hat{x}$ $\vec{F}_2 = +3630\ \hat{y}$ $\vec{F}_1 + \vec{F}_2 = -2750\ \hat{x} + 3630\ \hat{y}$ $\vec{F}_{\text{net}} = 0 \quad \Longrightarrow \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$	$\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$ $\vec{F}_3 = 2750\ \hat{x} - 3630\ \hat{y}$ $\lvert\vec{F}_3\rvert = \sqrt{(2750)^2 + (-3630)^2} = 4554\ \text{N}$ $\theta_3 = \tan^{-1}\left(\frac{-3630}{2750}\right) = -52.9°$

Q 4.34 In a physics laboratory class, three massless ropes are tied together at a point. A pulling force is applied along each rope: $F_1 = 150$ N at $60.0°$, $F_2 = 200.$ at $100°$, $F_3 = 100.$ N, at $190°$. What is the magnitude of a fourth force and the angle at which it acts to keep the point at the center of the system stationary? (All angles are measured from the positive *x*-axis.)

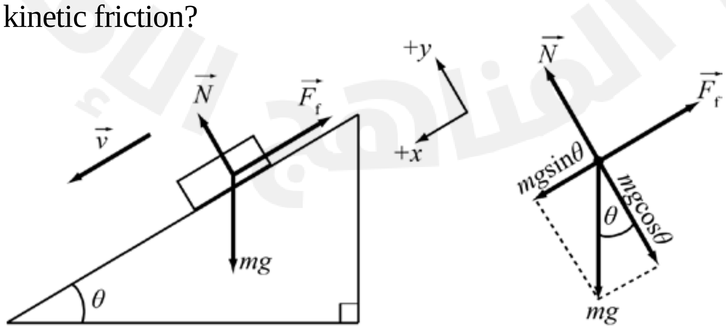
Force	Magnitude	Direction	x – component.	y – component.	$\vec{F}_{\text{net}} = -58.2\ \hat{x} + 310\ \hat{y}$
F_1	150.0	60.0°	$(150)\ (\cos 60°)$	$(150)\ (\sin 60°)$	F_4 that would balance $\vec{F}_{\text{net}}$ is:
F_2	200.	100°	$(200)\ (\cos 100°)$	$(200)\ (\sin 100°)$	
F_3	100.	190°	$(100)\ (\cos 190°)$	$(100)\ (\sin 190°)$	$\vec{F}_4 = 58.2\ \hat{x} - 310\ \hat{y}$

$\lvert\vec{F}_4\rvert = \sqrt{F_x^2 + F_y^2}$ $\lvert\vec{F}_4\rvert = \sqrt{58.2^2 + (-310)^2}$ $\lvert\vec{F}_4\rvert = 315\ \text{N}$	$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$ $\theta = \tan^{-1}\left(\frac{-310}{58.2}\right)$ $\theta = -79.4°\ \text{OR}\ 281°$	

Q 4.81 A block of mass 5.00 kg is sliding at a constant velocity down an inclined plane that makes an angle of 37° with respect to the horizontal.

[a] What is the friction force?

[b] What is the coefficient of kinetic friction?



Along $x-$axis: $a = 0.0\ \text{m/s}^2$, the velocity is constant along the plane.	Along $y-$axis: $a = 0.0\ \text{m/s}^2$, No motion along $y-$axis.	$F_{\text{friction}} = \mu_k F_N$
$\Sigma F_{\text{net}} = ma$ $mg\sin\theta - F_{\text{friction}} = m\,a_x$ $mg\sin\theta - F_{\text{f}} = m \times 0.0$ $F_{\text{friction}} = mg\sin\theta$ $F_{\text{friction}} = (5.00)(9.81)\sin 37°$ $F_{\text{friction}} = 29.5\ \text{N}$	$\Sigma F_{\text{net}} = ma$ $mg\cos\theta - F_{\text{Normal}} = m\,a_y$ $mg\cos\theta - F_{\text{Normal}} = m \times 0.0$ $F_{\text{Normal}} = mg\cos\theta$ $F_{\text{Normal}} = (5.00)(9.81)\cos 37°$ $F_{\text{Normal}} = 39.2\ \text{N}$	$\mu_k = \frac{F_{\text{friction}}}{F_{\text{Normal}}}$ $\mu_k = \frac{mg\sin\theta}{mg\cos\theta}$ $\mu_k = \tan\theta$ $\mu_k = \tan 37°$ $\mu_k = 0.75$

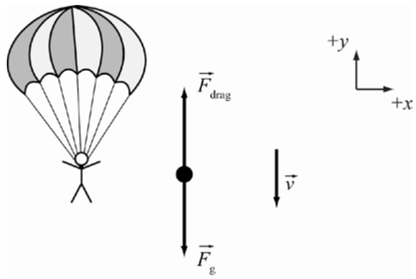
LO – 13: Example 4.7 & Questions 4.55
[1] Apply the relationship between the drag force on an object moving through air and the speed of the object.
[2] Determine the terminal speed of an object falling through air.

EXAMPLE 4.7 Sky Diving.
An 80.0-kg skydiver falls through the air with a density of $1.15\ \text{kg/m}^3$. Assume that his drag coefficient is $c_{\text{d}} = 0.570$. When he falls in the spread-eagle position, as shown in Figure 4.21a, his body presents an area $A_1 = 0.940\ \text{m}^2$ to the wind, whereas when he dives headfirst, with arms close to the body and legs together, as shown in Figure 4.21b, his area is reduced to $A_2 = 0.210\ \text{m}^2$.

PROBLEM
What are the terminal speeds in both cases?

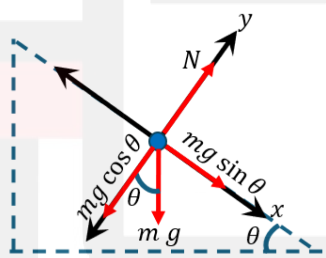
 (a)	 (b)
$v = \sqrt{\frac{mg}{K}} = \sqrt{\frac{mg}{\frac{1}{2}c_{\text{d}}A\rho}}$ $v = \sqrt{\frac{(80.0)(9.81)}{(0.5)(0.570)(0.940)(1.15)}}$ $v = 50.5\ \text{m/s}$	$v = \sqrt{\frac{mg}{K}} = \sqrt{\frac{mg}{\frac{1}{2}c_{\text{d}}A\rho}}$ $v = \sqrt{\frac{(80.0)(9.81)}{(0.5)(0.570)(0.210)(1.15)}}$ $v = 107\ \text{m/s}$

Q 4.55 A skydiver of mass 82.3 kg (including outfit and equipment) floats downward suspended from her parachute, having reached terminal speed. The drag coefficient is 0.533, and the area of her parachute is 20.11 m^2. The density of air is 1.14 kg/m^3. What is the air's drag force on her?

$F_{darg} = F_g$ $F_{darg} = (82.3)(9.81)$ $F_{darg} = 807 \text{ N}$	

LO – 14: Solved Problem (4.1), Example (4.8), Solved Problem (4.4), Example (4.9)
[1] Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle
[2] Draw free-body diagrams and apply Newton's second law for objects on horizontal, vertical, or inclined planes in situations involving friction.

Solved Problem 4.1
A snowboarder (mass 72.9 kg, height 1.79 m) glides down a slope with an angle of 22° with respect to the horizontal (Figure 4.14a). If we can neglect friction, what is his acceleration?

Diagram	Free – Body Diagram
 (a)	

Apply Newton's 2nd law along y – axis:	Apply Newton's 2nd law along x – axis:
No motion along y – axis. $\Sigma F_y = m\, a_y$ $N - mg \cos\theta = m(0.0)$ $N = mg \cos\theta \ldots\ldots (1)$	$\Sigma F_x = m\, a_x$ $mg \sin\theta = m\, a_x \ \ldots\ldots\ldots (2)$ $a_x = g \sin\theta$ $a_x = 9.8 \times \sin 22°$ $a_x = 3.7 \text{ m/s}^2$

EXAMPLE 4.8 Two Blocks Connected by a Rope-with Friction
We solved this problem in Solved Problem 4.2, with the assumptions that block 1 slides without friction across the horizontal support surface and that the rope slides without friction across the pulley. Here we will allow for friction between block 1 and the surface it slides across. For now, we will still assume that the rope slides without friction across the pulley. (Chapter 10 will present techniques that let us deal with the pulley being set into rotational motion by the rope moving across it.)

PROBLEM 1
Let the coefficient of static friction between block 1 (mass $m_1 = 2.3$ kg) and its support surface have a value of 0.73 and the coefficient of kinetic friction have a value of 0.60. (Refer back to Figure 4.17.) If block 2 has mass $m_2 = 1.9$ kg , will block 1 accelerate from rest?

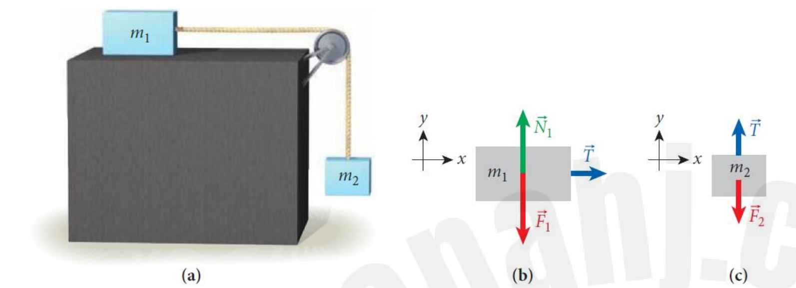


FIGURE 4.17 (a) Block 2 hanging vertically from a rope that runs over a pulley and is connected to block 1, resting on a horizontal frictionless surface. (b) Free-body diagram for block 1. (c) Free-body diagram for block 2.

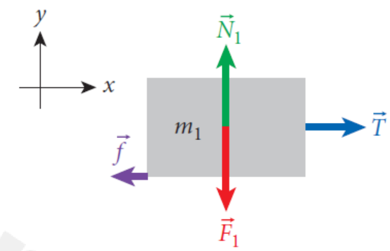


FIGURE 4.23 Free-body diagram for block 1, including the force of friction.

mass ($m_1 = 2.3$ kg)		mass ($m_2 = 1.9$ kg)	
$\vec{F}_{\text{net}} = m_1\vec{a}$ $N - m_1 g = 0$ $N = m_1 g$	$f_{\text{s,max}} = \mu_s N$ $f_{\text{s,max}} = \mu_s m_1 g$ $f_{\text{s,max}} = (0.73)(2.3)(9.81)$ $f_{\text{s,max}} = 16.5$ N	$\vec{F}_{\text{net}} = m_2\vec{a}$ $T - m_2 g = 0$ $T = m_2 g$ $T = (1.9)(9.81)$ $T = 18.6$ N	$T > f_{\text{s,max}}$ So, the mass (m_1) would accelerates from rest.

PROBLEM 2 What is the value of the acceleration?

mass ($m_1 = 2.3$ kg)	mass ($m_2 = 1.9$ kg)	② in ①	
$\vec{F}_{\text{net}} = m_1\vec{a}$ $T - f = m_1 a \rightarrow$ ① $N - m_1 g = 0$ $N = m_1 g$	$\vec{F}_{\text{net}} = m_2\vec{a}$ $T - m_2 g = -(m_2 a)$ $T = m_2 g - m_2 a \rightarrow$ ②	$m_2 g - m_2 a - f = m_1 a$ $m_2 g - f = (m_1 + m_2)a$ $m_2 g - \mu_k N = (m_1 + m_2)a$ $a = \dfrac{m_2 g - \mu_k m_1 g}{m_1 + m_2}$	$a = \left(\dfrac{m_2 - \mu_k m_1}{m_1 + m_2}\right) g$ $a = \left(\dfrac{1.9 - 0.6 \times 2.3}{2.3 + 1.9}\right)(9.81)$ $a = 1.21\ \text{m/s}^2$

Self-Test Opportunity 4.4

(a) What is the maximum mass (m_2) for which the system of these two blocks does not move? (b) What is the value of the friction force if (m_2) is smaller than this value?

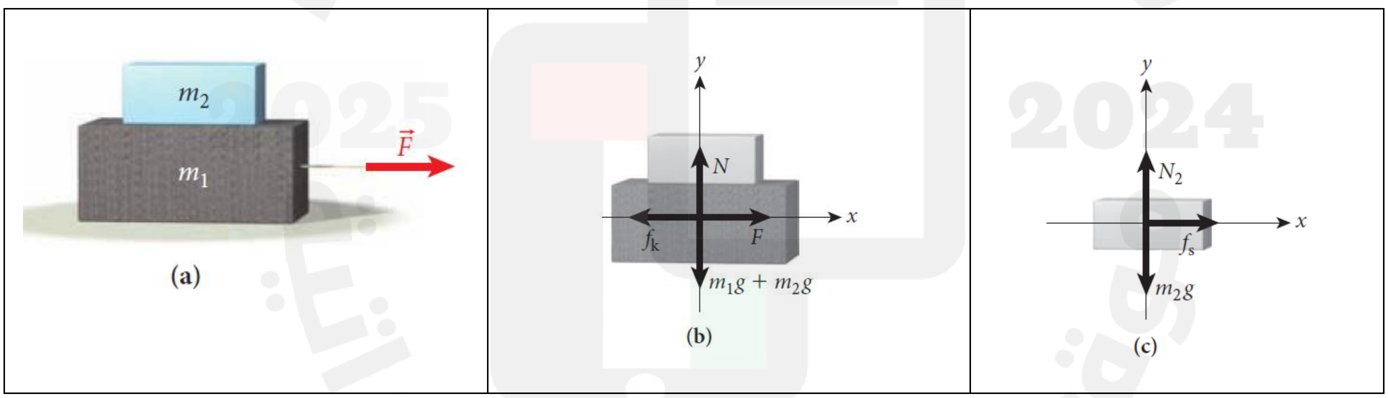
maximum (m_2)	Magnitude of friction.
$a = \left(\frac{m_2 - \mu_s m_1}{m_1 + m_2}\right) g$ $a = 0.0\ \text{m/s}^2$ $m_2 - \mu_s m_1 = 0$ $m_2 = (0.73)(2.3)$ $m_2 = 1.679\ \text{kg}$	For $m_2 < \mu_s m_1$, nothing moves; so, the acceleration has to be zero, which means that the numerator of the expression for the acceleration must be zero. Therefore, in this case, $f = m_2 g$.

SOLVED PROBLEM 4.4 Two Blocks

Two rectangular blocks are stacked on a table as shown in Figure 4.25a. The upper block has a mass of 3.40 kg, and the lower block has a mass of 38.6 kg. The coefficient of kinetic friction between the lower block and the table is 0.260. The coefficient of static friction between the blocks is 0.551. A string is attached to the lower block, and an external force $\vec{F}$ is applied horizontally, pulling on the string as shown.

PROBLEM

What is the maximum force that can be applied to the string without having the upper block slide off?



Lower Block $(m_1 = 38.6\ \text{kg})$

Along the x – axis.	Along y – axis	
$F_{\text{net}} = (m_1 + m_2)a_x$ $F - f_k = (m_1 + m_2)a$	$F_{\text{net}} = (m_1 + m_2)a_y$ $N - (m_1 + m_2)g = 0$ $N = (m_1 + m_2)g$	$F - (m_1 + m_2)\,\mu_k g = (m_1 + m_2)a$ $F = (m_1 + m_2)(\mu_k g + a)$

Upper Block ($m_2 = 3.40$ kg)

Along the x – axis.	Along y – axis		
$F_{net} = m_2 a_x$ $f_s = m_2 a$	$F_{net} = m_2 a_y$ $N_2 - m_2 g = 0$ $N_2 = m_2 g$	$N_2 \mu_s = m_2 a$ $m_2 g \mu_s = m_2 a$ $a = \mu_s g$	$F = (m_1 + m_2)(\mu_k g + a)$ $F_{max} = (m_1 + m_2)(\mu_k g + \mu_s g)$ $F_{max} = (m_1 + m_2)(\mu_k + \mu_s) g$ $F_{max} = (38.6 + 3.4)(0.26 + 0.551)(9.81)$ $F_{max} = 334$ N

EXAMPLE 4.9 Pulling a Sled

Suppose you are pulling a sled across a level snow-covered surface by exerting constant force on a rope, at an angle θ relative to the ground.

PROBLEM 1

If the sled, including its load, has a mass of 15.3 kg, the coefficients of friction between the sled and the snow are $\mu_s = 0.076$ and $\mu_k = 0.070$, and you pull with a force of 25.3 N on the rope at an angle of 24.5° relative to the horizontal ground, what is the sled's acceleration?

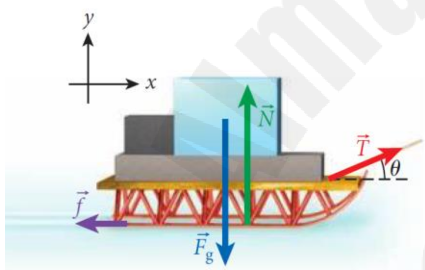
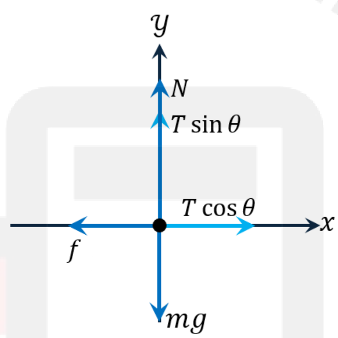


FIGURE 4.26 Free-body diagram of the sled and its load.



Vertically.	Horizontally.	
$N + T\sin\theta - mg = 0$ $N = mg - T\sin\theta$	$T\cos\theta - f = m\,a$ $T\cos\theta - \mu\,N = m\,a$ $T\cos\theta - \mu\,(mg - T\sin\theta) = m\,a$ $a = \dfrac{\mu\,T\sin\theta + T\cos\theta - \mu mg}{m}$	$a = \dfrac{T}{m}(\cos\theta + \mu\sin\theta) - \mu g$

Substitute $\mu_s = 0.076$ First.

$$a = \frac{25.3}{15.3}(\cos 24.5° + 0.076 \sin 24.5°) - (0.076 \times 9.81)$$

$$a = 0.81\ \mathrm{m/s^2}$$

Since the acceleration is positive, the sled will move.

Now Substitute $\mu_k = 0.070$

$$a = \frac{25.3}{15.3}(\cos 24.5° + 0.070 \sin 24.5°) - (0.070 \times 9.81)$$

$$a = 0.87 \text{ m/s}^2$$

PROBLEM 2
What angle of the rope with the horizontal will produce the maximum acceleration of the sled for the given value of the magnitude of the pulling force, T? What is that maximum value of a?

Derive the function of the acceleration with respect to the angle θ, then find the roots of the first derivative.

$a = \frac{T}{m}(\cos\theta + \mu \sin\theta) - \mu g$	$\mu = \tan\theta$
$\frac{d}{d\theta}\left(\frac{T}{m}(\cos\theta + \mu \sin\theta) - \mu g\right) = \frac{T}{m}(-\sin\theta + \mu\cos\theta)$	$\theta = \tan^{-1}\mu$
$\frac{T}{m}(-\sin\theta + \mu\cos\theta) = 0$	$\theta = \tan^{-1} 0.07$
$\mu\cos\theta = \sin\theta$	$\theta = 4.0°$
	$a = \frac{25.3}{15.3}(\cos 4.0° + 0.070 \sin 4.0°) - (0.070 \times 9.81)$
	$a = 0.97 \text{ m/s}^2$

LO – 15: Solved Problem (4.2), Example (4.4), Questions 4.35, 4.48, 4.96
[1] Identify that the direction of the force due to the pull on the rope acts exactly in the direction along the rope.
[2] Describe how the force with which we pull on the massless rope is transmitted through the entire rope unchanged, even if the rope passes over a pulley.

SOLVED PROBLEM 4.2 Two Blocks Connected by a Rope
In this classic problem, a hanging mass causes the acceleration of a second mass that is resting on a horizontal surface (Figure 4.17a). Block 1, of mass $m_1 = 3.00$ kg, rests on a horizontal frictionless surface and is connected via a massless rope (for simplicity, oriented in the horizontal direction) running over a massless pulley to block 2 of mass $m_2 = 1.30$ kg.

PROBLEM
What is the acceleration of block 1 and of block 2?

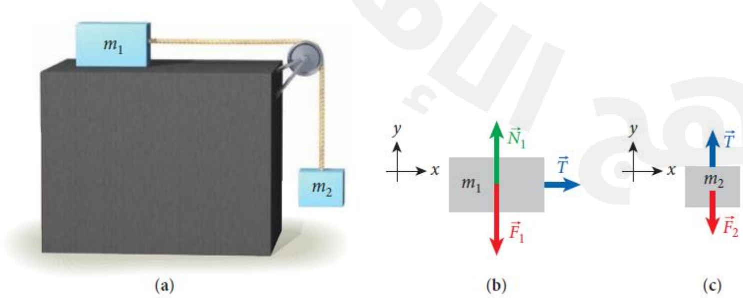


FIGURE 4.17 (a) Block 2 hanging vertically from a rope that runs over a pulley and is connected to block 1, resting on a horizontal frictionless surface. (b) Free-body diagram for block 1. (c) Free-body diagram for block 2.

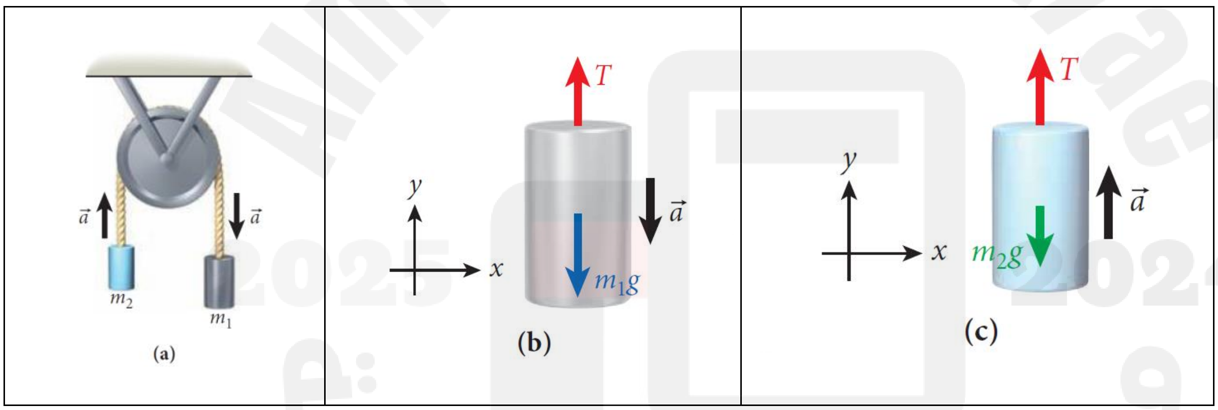
mass ($m_1 = 3.00$ kg)		mass ($m_2 = 1.30$ kg)	$m_1\, a = m_2 g - m_2\, a$
Vertically.	Horizontally.	Vertically.	$m_1\, a + m_2\, a = m_2 g$
$\vec{F}_{\text{net}} = m_1\vec{a}$	$\vec{F}_{\text{net}} = m_1\vec{a}$	$\vec{F}_{\text{net}} = m_2\vec{a}$	$a = \left(\frac{m_2}{m_1 + m_2}\right) g$
$N_1 - m_1 g = 0$	$T = m_1\, a$	$T - m_2 g = m_2\,(-a)$	$a = \left(\frac{1.30}{3.00 + 1.30}\right)(9.81)$
$N_1 = m_1 g$		$T = m_2 g - m_2\, a$	$a = 2.97\ \text{m/s}^2$

EXAMPLE 4.4 Atwood Machine

The Atwood machine consists of two hanging weights (with masses m_1 and m_2) connected via a rope running over a pulley. For now, we consider a friction-free case, where the pulley does not move, and the rope glides over it.

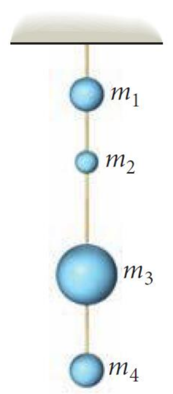
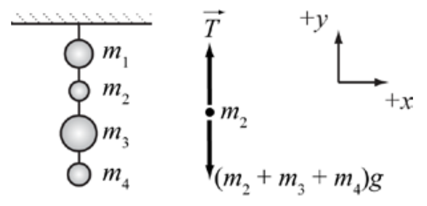
(In Chapter 10 on rotation, we will return to this problem and solve it with friction present, which causes the pulley to rotate.)

We also assume that $m_1 > m_2$. In this case, the acceleration is as shown in Figure 4.18a. (The formula derived in the following is correct for any case. If $m_1 < m_2$, then the value of the acceleration, a, will have a negative sign, which will mean that the acceleration direction is opposite to what we have assumed in working the problem.)

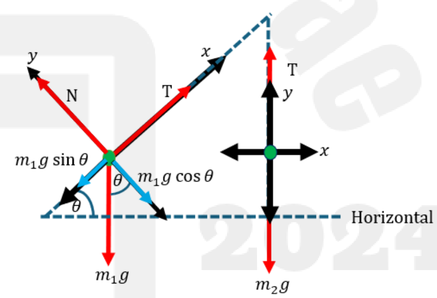


(m_1)	(m_2)	Acceleration
Vertically.	Vertically.	$m_1 g - m_1 a = m_2 g + m_2\, a$
$\vec{F}_{\text{net}} = m_1\vec{a}$	$\vec{F}_{\text{net}} = m_2\vec{a}$	$(m_1 + m_2)a = (m_1 - m_2)g$
$T - m_1 g = (m_1)(-a)$	$T - m_2 g = m_2\,(a)$	$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g)$
$T = m_1 g - m_1 a$	$T = m_2 g + m_2\, a$	

Q 4.35 Four weights of masses $m_1 = 6.50$ kg , $m_2 = 3.80$ kg , $m_3 = 10.70$ kg , $m_4 = 4.20$ kg, are hanging from a ceiling as shown in the figure. They are connected with ropes. What is the tension in the rope connecting masses m_1 and m_2?

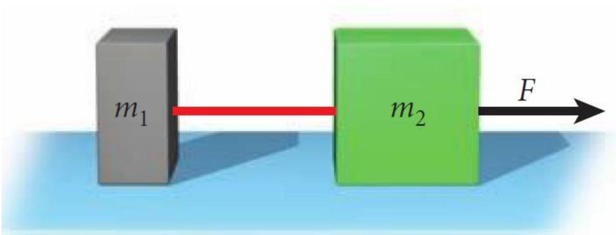
		Net forces on m_2 $\vec{F}_{\text{net}} = m_1\vec{a}$ $T - (m_2 + m_3 + m_4)(g) = (m_2 + m_3 + m_4)(0.0)$ $T = (m_2 + m_3 + m_4)(g)$ $T = (3.80 + 10.70 + 4.20)(9.81)$ $T = 183 \text{ N}$

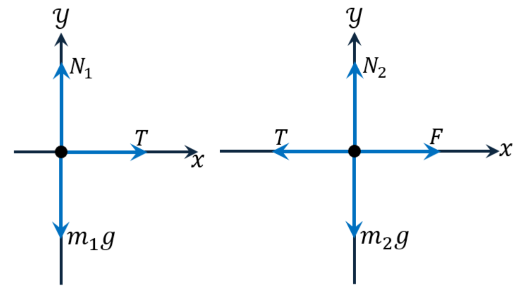
Q 4.48 A mass $m_1 = 20.0$ kg, on a frictionless ramp is attached to a light string. The string passes over a frictionless pully and is attached to a hanging mass, m_2. The ramp is at an angle of $\theta = 30.0°$ above the horizontal. The mass m_1 moves up the ramp uniformly (at constant speed). Find the value of m_2.

Diagram.	Free – Body Diagram.
	

The mass m_1 Along the x – axis. $T - m_1 g \sin\theta = m_1 a$ $T = m_1 g \sin\theta + m_1 a$ Along the y – axis. $N - m_1 g \cos\theta = m_1(0.0)$ $N = m_1 g \cos\theta$	The mass m_2 Along the y – axis. $T - m_2 g = m_2(-a)$ $T = m_2 g - m_2 a$	$m_1 g \sin\theta + m_1 a = m_2 g - m_2 a$ $(m_1 + m_2)a = (m_2 - m_1 \sin\theta)(g)$ $a = \left(\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right)(g)$ For constant speed $(a = 0.0 \text{ m/s}^2)$ $m_2 - m_1 \sin\theta = 0$ $m_2 - 20.0 \sin 30.0° = 0$ $m_2 = 10.0 \text{ kg}$

Q 4.96 Two blocks are connected by a massless rope, as shown in the figure. Block 1 has mass $m_1 = 1.267$ kg, and block 2 has mass $m_2 = 3.557$ kg. The two blocks move on a frictionless, horizontal tabletop. A horizontal external force, $F =\ 12.61$ N, acts on block 2. What is the tension in the rope connecting the two blocks?





The mass m_1	The mass m_2	
Along the $x-$axis. $T\ = m_1 a$	Along the $x-$axis. $F - T =\ m_2 a$ $F - m_1 a\ \ = m_2 a$ $F = (m_1 + m_2)a$ $12.61 = (1.267 + 3.557)a$ $a = \dfrac{F}{m_1 + m_2}$	$T\ = \left(\dfrac{m_1}{m_1 + m_2}\right)(F)$ $T\ = \left(\dfrac{1.267}{1.267 + 3.557}\right)(12.61)$ $T\ = 3.312$ N

FREE RESPONSE QUESTIONS.

LO – 16: Figures 1.18, 1.21, 1.28. Solved Problem 1.3 & Questions 1.65, 1.67, 1.97

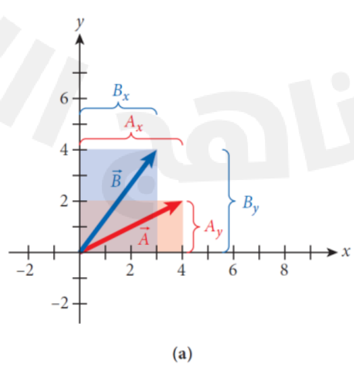
[1] Calculate the Cartesian components of a two-dimensional vector from the length and angle with respect to the x-axis.

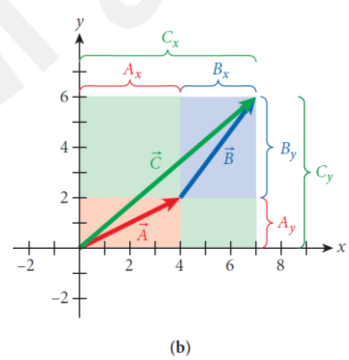
[2] Add or subtract vectors using Cartesian components.

[3] Add and subtract vectors graphically to find the resultant vectors.

[4] Identify cartesian unit vectors in two and three dimensions.

FIGURE 1.21 Vector addition by components. (a) Components of vectors $\vec{A}$ and $\vec{B}$; (b) the components of the resultant vector are the sums of the components of the individual vectors.





SOLVED PROBLEM 1.3 Hiking

You are hiking in the Florida Everglades heading southwest from your base camp, for 1.72 km. You reach a river that is too deep to cross; so, you make a 90° right turn and hike another 3.12 km to a bridge. How far away are you from your base camp?

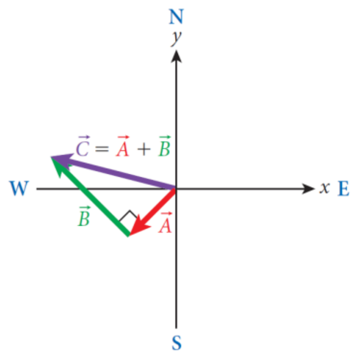


FIGURE 1.28 Hike with a 90° turn.

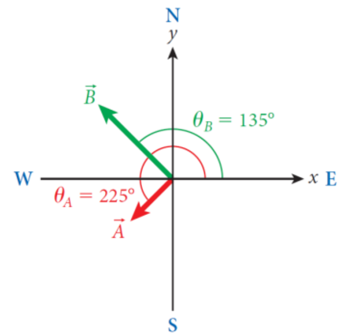


FIGURE 1.29 Angles of the two hike segments.

Apply the triangle rule:

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} = A\cos 225°\,\hat{x} + A\sin 225°\,\hat{y}$$

$$\vec{A} = \left(\frac{-1.72}{\sqrt{2}}\right)\hat{x} + \left(\frac{-1.72}{\sqrt{2}}\right)\hat{y}$$

$$\vec{B} = B\cos 135°\,\hat{x} + B\sin 135°\,\hat{y}$$

$$\vec{B} = \left(\frac{-3.12}{\sqrt{2}}\right)\hat{x} + \left(\frac{3.12}{\sqrt{2}}\right)\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = \left(\frac{-4.84}{\sqrt{2}}\right)\hat{x} + \left(\frac{1.40}{\sqrt{2}}\right)\hat{y}$$

$$|\vec{C}| = \sqrt{(C_x)^2 + (C_y)^2}$$

$$|\vec{C}| = \sqrt{\left(\frac{-4.84}{\sqrt{2}}\right)^2 + \left(\frac{1.40}{\sqrt{2}}\right)^2}$$

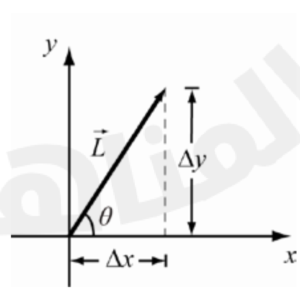
$$|\vec{C}| = 3.56 \text{ km}$$

Q 1.65 A position vector has a length of 40.0 m and is at an angle of 57.0° above the x-axis. Find the vector's components.

$$\vec{L} = (L\cos\theta)\hat{x} + (L\sin\theta)\hat{y}$$

$$\vec{L} = (40.0\cos 57.0°)\hat{x} + (40.0\sin 57.0°)\hat{y}$$

$$\vec{L} = (21.8 \text{ m })\hat{x} + (33.5 \text{ m })\hat{y}$$



Q 1.67 Find the components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$, if their lengths are given by $A = 75.0$, $B = 60.0$, $C = 25.0$, $D = 90.0$ and their directions angles are as shown in the figure. Write the vectors in terms of unit vectors.

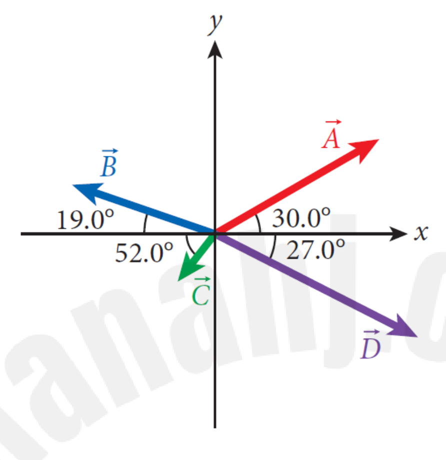
Vector	Magnitude	Direction	x – component.	y – component.
$\vec{A}$	75.0	30.0°	$(75.0)\,(\cos 30\,^\circ) = 65.0$	$(75.0)\,(\sin 30^\circ) = 37.5$
$\vec{B}$	60.0	161°	$(60.0)\,(\cos 161^\circ) = -56.7$	$(60.0)\,(\sin 161^\circ) = 19.5$
$\vec{C}$	25.0	232°	$(25.0)\,(\cos 232^\circ) = -15.4$	$(25.0)\,(\sin 232^\circ) = -19.7$
$\vec{D}$	90.0	333°	$(90.0)\,(\cos 333^\circ) = 80.2$	$(90.0)\,(\sin 333^\circ) = -40.9$

$\vec{A} = 65.0\,\hat{x} + 37.5\,\hat{y}$

$\vec{B} = -56.7\,\hat{x} + 19.5\,\hat{y}$

$\vec{C} = -15.4\,\hat{x} - 19.7\,\hat{y}$

$\vec{D} = 80.2\,\hat{x} - 40.9\,\hat{y}$



Q 1.97 Add the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ using the component method, and find their sum vector $\vec{D}$.

$\vec{A} = (-1.5 - -4)\hat{x} + (3.5 - 2)\,\hat{y}$

$\vec{A} = (2.5)\hat{x} + (1.5)\,\hat{y}$

$\vec{B} = (4 - -1.5)\hat{x} + (1 - 2.5)\,\hat{y}$

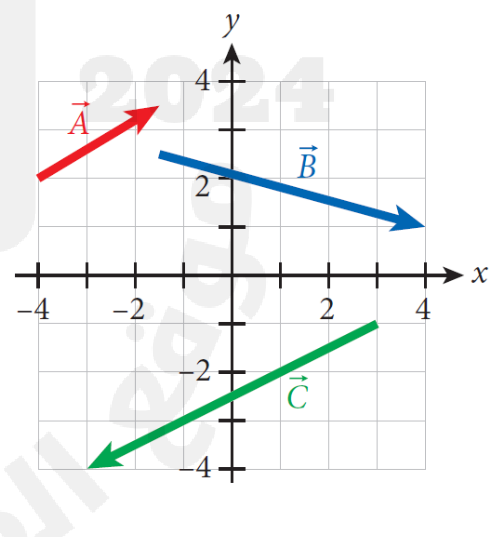
$\vec{B} = (5.5)\hat{x} + (-1.5)\,\hat{y}$

$\vec{C} = (-3 - 3)\hat{x} + (-4 - -1)\,\hat{y}$

$\vec{C} = (-6)\hat{x} + (-3)\,\hat{y}$

$\vec{D} = \vec{A} + \vec{B} + \vec{C}$

$\vec{D} = (2.0)\hat{x} + (-3.0)\,\hat{y}$



LO – 17: Example 2.1 & Questions 2.34, 2.35, 2.85, 2.66, 2.67, 2.70

[1] Solve problems related to position and displacement.

[2] Calculate the instantaneous velocity at a specific time as the rate of change of the position function, which is the slope of the position function in the specific time.

[3] Describe the motion of an object in a straight line with constant acceleration.

[4] Apply, in the direction of motion, the constant-acceleration equations to relate acceleration, velocity, position, and time for an object moving with constant acceleration.

Example 2.1 Solved in LO – 5

Q 2.34 The position of a particle moving along the x-axis is given by $x = (11 + 14t - 2.0t^2)$, where (t) is in seconds and (x) is in meters. What is the average velocity during the time interval from $t = 1.0$ s to $t = 4.0$ s?

$$\bar{v} = \frac{x(t_f) - x(t_i)}{t_f - t_i}$$

$$\bar{v} = \frac{x(4.0) - x(1.0)}{4.0 - 1.0}$$

$$\bar{v} = \frac{[11 + (14 \times 4.0) - (2.0 \times 4.0^2)] - [11 + (14 \times 1.0) - (2.0 \times 1.0^2)]}{3.0}$$

$$\bar{v} = \frac{35 - 23}{3.0} = 4 \text{ m/s}$$

Q 2.35 The position of a particle moving along the x-axis is given by $x = 3.0t^2 - 2.0t^3$, where (x) is in meters and (t) is in seconds. What is the position of the particle when it achieves its maximum speed in the positive x – direction?

Position	Velocity	Acceleration
$x(t) = 3.0t^2 - 2.0t^3$	$v(t) = \frac{d}{dt}\,x(t)$ $v(t) = \frac{d}{dt}\,(3.0t^2 - 2.0t^3)$ $v(t) = 6.0\,t - 6.0t^2$	$a(t) = \frac{d}{dt}\,v(t)$ $a(t) = \frac{d}{dt}\,(6.0\,t - 6.0t^2)$ $a(t) = 6.0 - 12\,t$

Speed will be maximum at the moment when the acceleration is zero.

$6.0 - 12\,t = 0 \quad \Longrightarrow t = 0.5 \text{ s}$

$x(0.5) = (3.0)(0.5)^2 - (2.0)(0.5)^3 = 0.5 \text{ m}$

Q 2.85 A train traveling at 40.0 m/s is headed straight toward another train, which is at rest on the same track. The moving train decelerates at 6.0 m/s, and the stationary train is 100.0 m away. How far from the stationary train will the moving train be when it comes to a stop?

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$0.0 = (40)^2 + 2(-6.0)(x - 0)$

$x = 133.3 \text{ m}$

The trains collide.

Q 2.66 & 2.67 Solved in LO – 7

Q 2.70 An object is thrown vertically upward and has a speed of 20.0 m/s when it reaches two thirds of its maximum height above the launch point. Determine its maximum height.

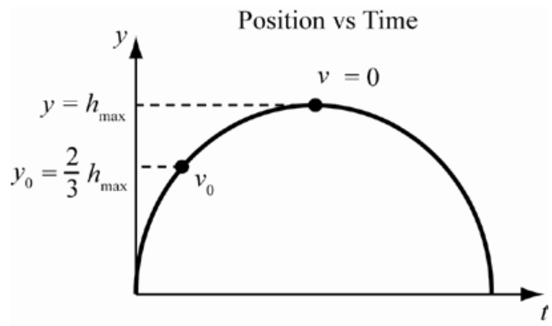
$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

$$0.0 = (20)^2 + 2(-9.81)\left(\frac{2}{3}y_{\max} - y_{\max}\right)$$

$$\frac{1}{3}y_{\max} = \frac{400}{2 \times 9.81}$$

$$y_{\max} = \frac{3 \times 400}{2 \times 9.81}\ \text{m}$$

$$y_{\max} = 61.2\ \text{m}$$



LO – 18: Questions 3.27, 3.39, 3.43, 3.47

[1] Calculate the components of a velocity vector (v_x, v_y, v_z) by the time derivative of the position vector.

[2] Define maximum height, range of a projectile and time of flight.

[3] Calculate the maximum height, range of a projectile and the time of flight for a projectile.

Q 3.27 An object moves in the xy − plane. The *x*- and *y*-coordinates of the object as a function of time are given by the following equations: $x(t) = 4.9\,t^2 + 2t + 1$ and $y(t) = 3t + 2$. What is the velocity vector of the object as a function of time? What is its acceleration vector at the time $t = 2$ s?

The velocity	The Acceleration	
$\vec{v} = v_x\hat{x} + v_y\hat{y}$ $\vec{v} = \frac{dx(t)}{dt}\hat{x} + \frac{dy(t)}{dt}\hat{y}$ $\vec{v} = (9.8\,t + 2)\hat{x} + 3\hat{y}$	$\vec{a} = a_x\hat{x} + a_y\hat{y}$ $\vec{a} = \frac{dv_x(t)}{dt}\hat{x} + \frac{dv_y(t)}{dt}\hat{y}$ $\vec{a} = 9.8\,\hat{x}$	$\vec{a}(t = 2) = 9.8\ \text{m/s}^2\ \hat{x}$

Q 3.39 A rabbit runs in a garden such that the x- and y-components of its displacement as functions of time are given by $x(t) = -0.45\,t^2 - 6.5t + 25$ and $y(t) = 0.35\,t^2 + 8.3t + 34$. (Both *x* and *y* are in meters and *t* is in seconds.)

a) Calculate the rabbit's position (magnitude and direction) at $t = 10.0$ s.

b) Calculate the rabbit's velocity at $t = 10.0$ s.

c) Determine the acceleration vector at $t = 10.0$ s.

$$\vec{r}(t) = x(t)\,\hat{x} + y(t)\,\hat{y}$$

$$\vec{r}(t) = (-0.45\,t^2 - 6.5t + 25)\,\hat{x} + (0.35\,t^2 + 8.3t + 34)\,\hat{y}$$

$\vec{r}(10.0) = (-45\ - 65 + 25)\,\hat{x} + (35\ + 83 + 34)\,\hat{y}$

$\vec{r}(10.0) = (-85)\,\hat{x} + (152)\,\hat{y}$

$|\vec{r}(10.0)| = 174\text{ m}$

$\theta = \tan^{-1}\left(\frac{152}{-85}\right)$

$\theta = -60.8°$

$\vec{v}(t) = \frac{d}{dt}x(t)\,\hat{x} + \frac{d}{dt}y(t)\,\hat{y}$

$\vec{v}(t) = \frac{d}{dt}(-0.45\,t^2 - 6.5t + 25)\,\hat{x} + \frac{d}{dt}(0.35\,t^2 + 8.3t + 34)\,\hat{y}$

$\vec{v}(t) = (-0.9\,t - 6.5)\,\hat{x} + (0.70\,t + 8.3)\,\hat{y}$

$\vec{v}(10) = -15.5\,\hat{x} + 15.3\,\hat{y}$

$|\vec{v}(10.0)| = 21.8\ \text{m/s}$

$\phi = \tan^{-1}\left(\frac{15.3}{-15.5}\right)$

$\phi = -44.6°$

$\vec{a}(t) = \frac{d}{dt}v_x(t)\,\hat{x} + \frac{d}{dt}v_y(t)\,\hat{y}$

$\vec{a}(t) = \frac{d}{dt}(-0.9\,t - 6.5)\,\hat{x} + \frac{d}{dt}(0.70\,t + 8.3)\,\hat{y}$

$\vec{a}(t) = (-0.9)\,\hat{x} + (0.70)\,\hat{y}$

$\vec{a}(10) = (-0.9)\,\hat{x} + (0.70)\,\hat{y}$

$|\vec{a}(10.0)| = 1.14\ \text{m/s}^2$

$\psi = \tan^{-1}\left(\frac{0.70}{-0.90}\right)$

$\psi = -37.9°$

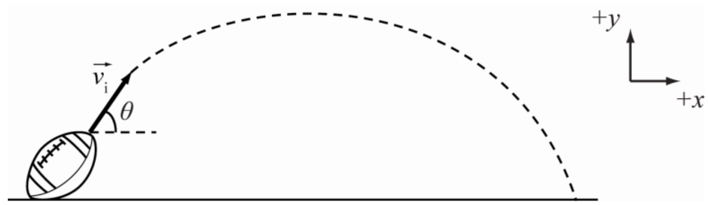
Q 3.43 A football is kicked with an initial speed of 27.5 m/s and a launch angle of 56.7°. What is its hang time (the time until it hits the ground again)?

Given: $v_0 = 27.5\ \text{m/s} \quad \theta = 56.7°$

$$t_{\text{hang}} = \frac{2\, v_0 \sin\theta_0}{g}$$

$$t_{\text{hang}} = \frac{2 \times 27.5 \times \sin 56.7°}{9.81}$$

$$t_{\text{hang}} = 4.69\ s$$



Q 3.47 A football player kicks a ball with a speed of 22.4 m/s at an angle of 49.0° above the horizontal from a distance of 39.0 m from the goalpost.
[a] By how much does the ball clear or fall short of clearing the crossbar of the goalpost if that bar is 3.05 m heigh?
[b] What is the vertical velocity of the ball at the time it reaches the goalpost?

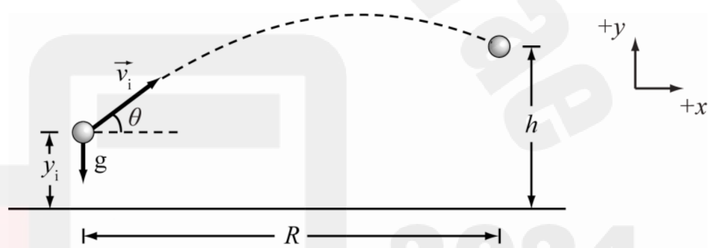
$$x = v_x\, t$$

$$x = (v_0 \cos\theta_0)\, t$$

$$39.0 = (22.4 \cos 49°)\, t$$

$$t = \frac{39.0}{22.4 \cos 49°}\ \text{s}$$

$v_0 = 22.4\ \text{m/s}\ , \theta_0 = 49°\ ,\ x = 39.0\ \text{m},\ y_0 = 0.0\ \text{m}$



[a]

$$y = y_0 + (\tan\theta_0)\, x - \frac{g}{2v_0^2(\cos\theta_0)^2} x^2$$

$$y = 0.0 + (\tan 49°)(39.0) - \frac{(9.81)(39.0)^2}{(2)(22.4)^2(\cos 49°)^2}$$

$$y = 10.3\ \text{m}$$

The ball will clear the goalpost by:
$(10.3 - 3.05 = 7.27\ \text{m})$

[b]

$$v_y = -g\, t + v_0 \sin\theta_0$$

$$v_y = \left(-9.81 \times \frac{39.0}{22.4 \cos 49°}\right) + (22.4 \times \sin 49°)$$

$$v_y = -9.13\ \text{m/s}$$

LO – 19: Example 4.2, Solved Problem 4.2 & Questions 4.26, 4.75, 4.79, 4.81
[1] Solve problems related to objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law.
[2] Solve problems related to multiple connected masses moving in a system and involving friction (e.g., Atwood machines) connected by light strings with tensions (and pulleys).

EXAMPLE 4.2 A gymnast of mass 55 kg hangs vertically from a pair of parallel rings (Figure 4.11a).

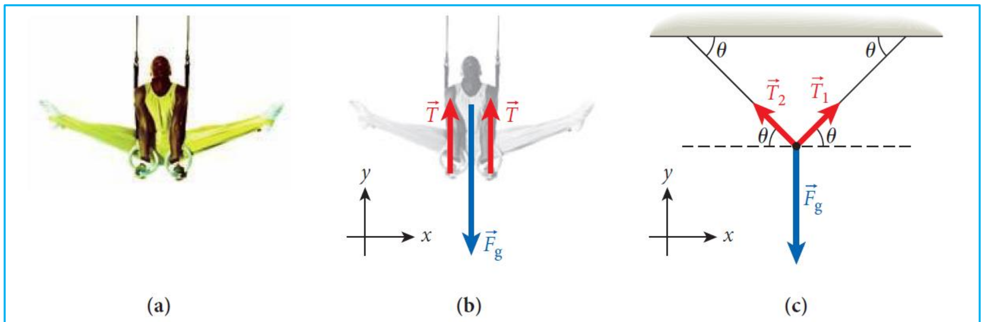


FIGURE 4.11 (a) Still rings in men's gymnastics. (b) Free-body diagram for problem 1. (c) Free-body diagram for problem 2.

PROBLEM 1
If the ropes supporting the rings are vertical and attached to the ceiling directly above, what is the tension in each rope?

$F_{net} = ma$ $2T - mg = m(0.0)$ $T = \frac{mg}{2}$	$T = \frac{(55.0)(9.81)}{2}$ $T = 270.\text{ N}$

PROBLEM 2
If the ropes are attached so that they make an angle $\theta = 45°$ with the ceiling (Figure 4.11c), what is the tension in each rope?

Horizontally, the tension's components would balance each other out.

Vertically, the gymnast is at equilibrium.

$F_{net} = ma$ $(T\sin\theta + T\sin\theta) - mg = m(0.0)$ $T = \frac{mg}{2\sin\theta}$	$T = \frac{(55.0)(9.81)}{2 \times \sin 45°}$ $T = 382\text{ N}$

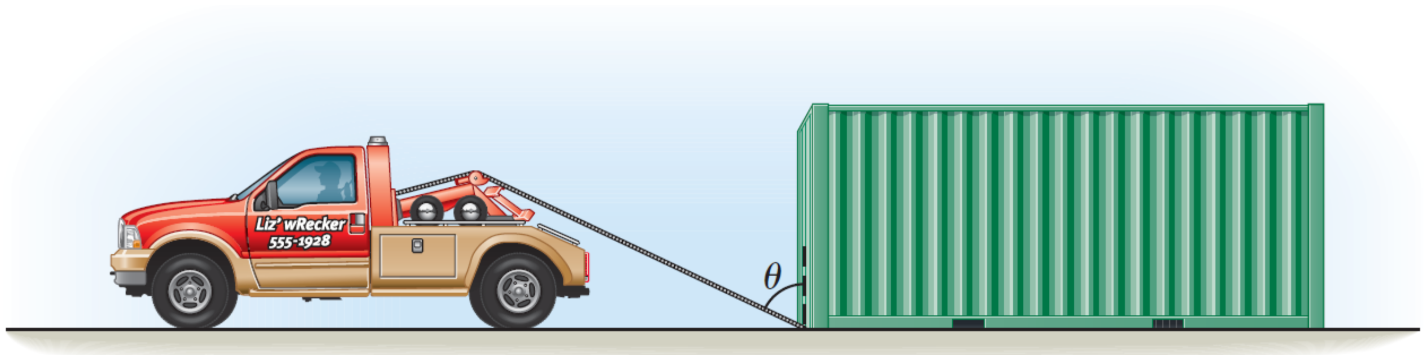
PROBLEM 3
How does the tension in the ropes change as the angle θ between the ceiling and the ropes becomes smaller and smaller?
The tension in the ropes,

$$T = \frac{mg}{2\sin\theta}$$

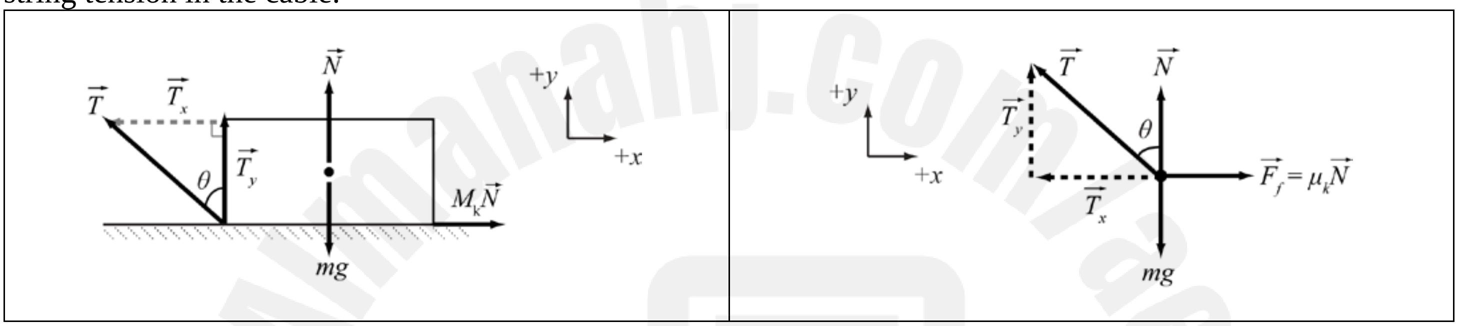
, gets larger. As θ approaches zero, the tension becomes infinitely large. In reality, of course, the gymnast has only finite strength and cannot hold his position with small angles.

Solved Problem 4.2 Solved in LO – 15

Q 4.26 A tow truck of mass M is using a cable to pull a shipping container of mass m across a horizontal surface as shown in the figure. The cable is attached to the container at the front bottom corner and makes an angle θ with the vertical as shown. The coefficient of kinetic friction between the surface and the crate is μ.

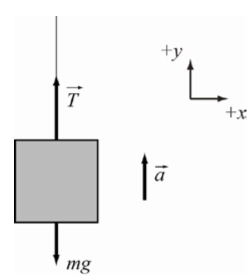


a) Draw a free-body diagram for the container.
b) Assuming that the truck pulls the container at a constant speed, write an equation for the magnitude T of the string tension in the cable.

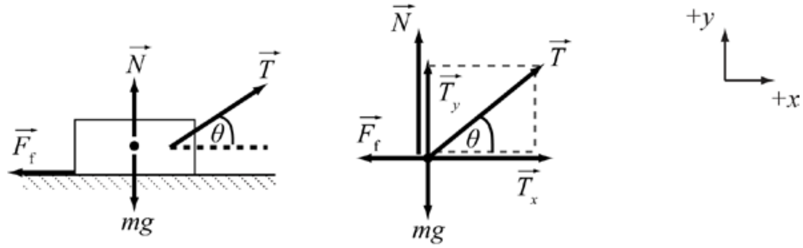


Horizontally	Vertically	
$F_{\text{net}} = ma$ $\mu_k N - T\sin\theta = m(0.0)$ $N = \dfrac{T\sin\theta}{\mu_k}$	$F_{\text{net}} = ma$ $(N + T\cos\theta) - mg = m(0.0)$ $\dfrac{T\sin\theta}{\mu_k} + T\cos\theta - mg = 0$	$T = \dfrac{mg}{\left(\dfrac{\sin\theta}{\mu_k} + \cos\theta\right)}$ $T = \dfrac{\mu_k mg}{\sin\theta + \mu_k\cos\theta}$

Q 4.75 A block of mass 20.0 kg supported by a vertical massless cable is initially at rest. The block is then pulled upward with a constant acceleration of $2.32\ \text{m/s}^2$.
a) What is the tension in the cable?
b) What is the net force acting on the mass?
c) What is the speed of the block after it has traveled 2.00 m?

[a] Tension	[b] Force	[c] Speed	
$F_{\text{net}} = ma$ $T - mg = m(a)$ $T = m(a + g)$ $T = (20.0)(2.32 + 9.81)$ $T = 243\ \text{N}$	$F_{\text{net}} = ma$ $F_{\text{net}} = (20.0)(2.32)$ $F_{\text{net}} = 46.4\ \text{N}$	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ $v_y^2 = 0 + 2(2.32)(2 - 0)$ $v_y = 3.05\ \text{m/s}$	

Q 4.79 A tractor pulls a sled of mass $M = 1000.\,kg$ across level ground. The coefficient of kinetic friction between the sled and the ground is $\mu_k = 0.600$. The tractor pulls the sled by a rope that connects to the sled at an angle of $\theta = 30.0°$ above the horizontal. What magnitude of tension in the rope is necessary to move the sled horizontally with an acceleration $a = 2.00\ \text{m/s}^2$?

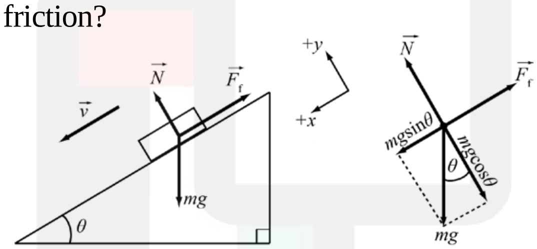


Horizontally $F_{net} = ma$ $T\cos\theta - \mu_k N = ma$ $N = \dfrac{T\cos\theta - ma}{\mu_k}$	Vertically $F_{net} = ma$ $(T\sin\theta + N) - mg = m(0.0)$ $T\sin\theta + \dfrac{T\cos\theta - ma}{\mu_k} - mg = 0$ $T = \dfrac{mg + \dfrac{ma}{\mu_k}}{\sin\theta + \dfrac{\cos\theta}{\mu_k}}$	$T = \dfrac{\mu_k mg + ma}{\cos\theta + \mu_k\sin\theta}$ $T = \dfrac{(0.6 \times 1000 \times 9.81) + (1000 \times 2)}{\cos 30° + 0.6\sin 30°}$ $T = 6760\ \text{N}$

Q 4.81 A block of mass 5.00 kg is sliding at a constant velocity down an inclined plane that makes an angle of 37° with respect to the horizontal.

[a] What is the friction force?

[b] What is the coefficient of kinetic friction?



Along x – axis: $a = 0.0\ \text{m/s}^2$, the velocity is constant along the plane.	Along y – axis: $a = 0.0\ \text{m/s}^2$, No motion along y – axis.	$F_{friction} = \mu_k F_N$ $\mu_k = \dfrac{F_{friction}}{F_{Normal}}$ $\mu_k = \dfrac{mg\sin\theta}{mg\cos\theta}$ $\mu_k = \tan\theta$ $\mu_k = \tan 37°$ $\mu_k = 0.75$
$\Sigma F_{net} = ma$ $mg\sin\theta - F_{friction} = m\,a_x$ $mg\sin\theta - F_f = m \times 0.0$ $F_{friction} = mg\sin\theta$ $F_{friction} = (5.00)(9.81)\sin 37°$ $F_{friction} = 29.5\ \text{N}$	$\Sigma F_{net} = ma$ $mg\cos\theta - F_{Normal} = m\,a_y$ $mg\cos\theta - F_{Normal} = m \times 0.0$ $F_{Normal} = mg\cos\theta$ $F_{Normal} = (5.00)(9.81)\cos 37°$ $F_{Normal} = 39.2\ \text{N}$	

the end