الملف كتاب الطالب الوحدة الأولى

موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ فيزياء ⇦ الفصل الأول

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1 Overview

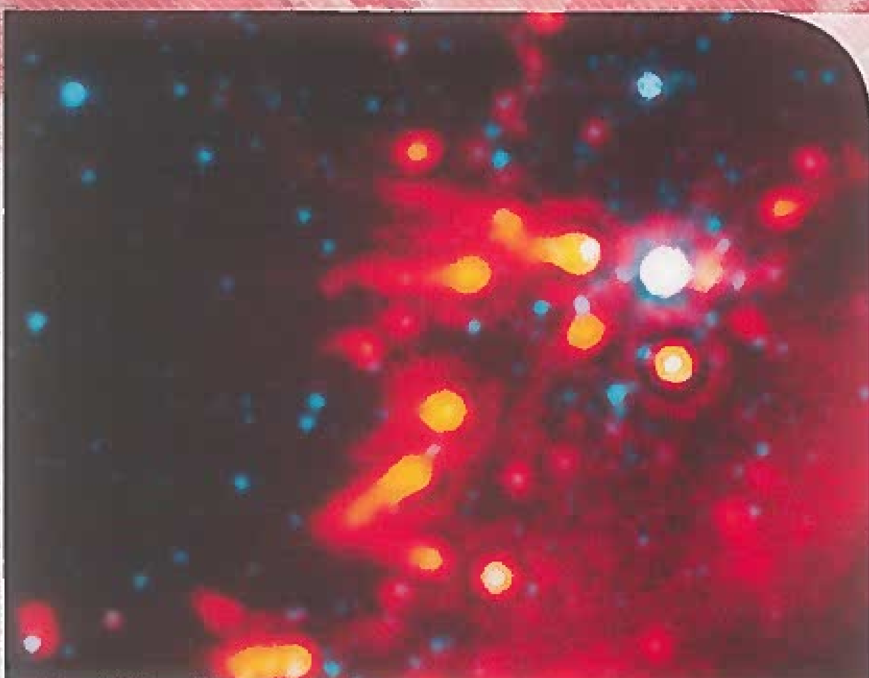


FIGURE 1.1 An image of the W5 star-forming region.

The dramatic image in Figure 1.1 could be showing any of several things: a colored liquid spreading out in a glass of water, or perhaps biological activity in some organism, or maybe even an artist's idea of mountains on some unknown planet. If we said the view was 70 wide, would that help you decide what the picture shows? Probably not–you need to know if we mean, for example, 70 meters or 70 millionths of a centimeter or 70 thousand kilometers.

In fact, this infrared image taken by the Spitzer Space Telescope shows huge clouds of gas and dust about 70 light-years across. (A light-year is the distance traveled by light in 1 year, about 10 quadrillion meters.) These clouds are about 6500 light-years away from Earth and contain newly formed stars embedded in the glowing regions. The technology that enables us to see images such as this one is at the forefront of contemporary astronomy, but it depends in a real way on the basic ideas of numbers, units, and vectors presented in this chapter.

The ideas described in this chapter are not necessarily principles of physics, but they help us to formulate and communicate physical ideas and observations. We will use the concepts of units, scientific notation, significant figures, and vector quantities throughout the course. Once you have understood these concepts, we can go on to discuss physical descriptions of motion and its causes.

WHAT WE WILL LEARN

- The use of scientific notation and the appropriate number of significant figures is important in physics.
- We will become familiar with the international unit system and the definitions of the base units as well as methods of converting among other unit systems.
- We will use available length, mass, and time scales to establish reference points for grasping the vast diversity of systems in physics.
- We will apply a problem-solving strategy that will be useful in analyzing and understanding problems throughout this course and in science and engineering applications.
- We will work with vectors: vector addition and subtraction, multiplication of vectors, unit vectors, and length and direction of vectors.

1.1 Why Study Physics?

Perhaps your reason for studying physics can be quickly summed up as "Because it is required for my major!" While this motivation is certainly compelling, the study of science, and particularly physics, offers a few additional benefits.

Physics is the science on which all other natural and engineering sciences are built. All modern technological advances–from laser surgery to television, from computers to refrigerators, from cars to airplanes–trace back directly to basic physics. A good grasp of essential physics concepts gives you a solid foundation on which to construct advanced knowledge in all sciences. For example, the conservation laws and symmetry principles of physics also hold true for all scientific phenomena and many aspects of everyday life.

The study of physics will help you grasp the scales of distance, mass, and time, from the smallest constituents inside the nuclei of atoms to the galaxies that make up our universe. All natural systems follow the same basic laws of physics, providing a unifying concept for understanding how we fit into the overall scheme of the universe.

Physics is intimately connected with mathematics because it brings to life the abstract concepts used in trigonometry, algebra, and calculus. The analytical thinking and general techniques for problem solving that you learn here will remain useful for the rest of your life.

Science, especially physics, helps remove irrationality from our explanations of the world around us. Prescientific thinking resorted to mythology to explain natural phenomena. If you read the daily news, you will find that some misconceptions from prescientific thinking persist even today. You may not find the answer to the meaning of life in this course, but at the very least you will come away with some of the intellectual tools that enable you to weed out inconsistent, logically flawed theories and misconceptions that contradict experimentally verifiable facts. Scientific progress over the last millennium has provided a rational explanation for most of what occurs in the natural world surrounding us.

Through consistent theories and well-designed experiments, physics has helped us obtain a deeper understanding of our surroundings and has given us greater ability to control them. In a time when the consequences of air and water pollution, limited energy resources, and global warming threaten the continued existence of huge portions of life on Earth, the need to understand the results of our interactions with the environment has never been greater. Much of environmental science is based on fundamental physics, and physics drives much of the technology essential to progress in chemistry and the life sciences. You may well be called upon to help decide public policy in these areas, whether as a scientist, an engineer, or simply as a citizen. Having an objective understanding of basic scientific issues is of vital importance in making such decisions. Thus, you need to acquire scientific literacy, an essential tool for every citizen in our technology-driven society.

You cannot become scientifically literate without command of the necessary elementary tools, just as it is impossible to make music without the ability to play an instrument. This is the main purpose of this text: to properly equip you to make sound contributions to the important discussions and decisions of our time. You will emerge from reading and working with this text with a deeper appreciation for the fundamental laws that govern our universe and for the tools that humanity has developed to uncover them, tools that transcend cultures and historic eras.

1.2 Working with Numbers

Scientists have established logical rules to govern how they communicate quantitative information to one another. If you want to report the result of a measurement–for example, the distance between two cities, your own weight, or the length of a lecture–you have to specify this result in multiples of a standard unit. Thus, a measurement is the combination of a number and a unit.

At first thought, writing down numbers doesn't seem very difficult. However, in physics, we need to deal with two complications: how to deal with very big or very small numbers, and how to specify precision.

Scientific Notation

If you want to report a really big number, it becomes tedious to write it out. For example, the human body contains approximately 7,000,000,000,000,000,000,000,000,000 atoms. If you used this number often, you would surely like to have a more compact notation for it. This is exactly what **scientific notation** is. It represents a number as the product of a number greater than or equal to 1 and less than 10 (called the *mantissa*) and a power (or exponent) of 10:

$$\text{number} = \text{mantissa} \times 10^{\text{exponent}}. \tag{1.1}$$

The number of atoms in the human body can thus be written compactly as 7×10^{27}, where 7 is the mantissa and 27 is the exponent.

Another advantage of scientific notation is that it makes it easy to multiply and divide large numbers. To multiply two numbers in scientific notation, we multiply their mantissas and then add their exponents. If we wanted to estimate, for example, how many atoms are contained in the bodies of all the people on Earth, we could do this calculation rather easily. Earth hosts approximately 7 billion ($=7 \times 10^9$) humans. All we have to do to find our answer is to multiply 7×10^{27} by 7×10^9. We do this by multiplying the two mantissas and adding the exponents:

$$(7 \times 10^{27})\ (7 \times 10^{9}) = (7 \times 7) \times 10^{27+9} = 49 \times 10^{36} = 4.9 \times 10^{37}. \tag{1.2}$$

In the last step, we follow the common convention of keeping only one digit in front of the decimal point of the mantissa and adjusting the exponent accordingly. (But be advised that we will have to further adjust this answer–read on!)

Division with scientific notation is equally straightforward: If we want to calculate A/B, we divide the mantissa of A by the mantissa of B and subtract the exponent of B from the exponent of A.

Concept Check 1.1

The total surface area of Earth is $A = 4\pi R^2 = 4\pi(6370\text{ km})^2 = 5.099 \times 10^{14}\text{ m}^2$. Assuming there are 7.0 billion humans on the planet, what is the available surface area per person?

a) $7.3 \times 10^4\text{ m}^2$
b) $7.3 \times 10^{24}\text{ m}^2$
c) $3.6 \times 10^{24}\text{ m}^2$
d) $3.6 \times 10^4\text{ m}^2$

Significant Figures

When we specified the number of atoms in the average human body as 7×10^{27}, we meant to indicate that we know it is at least 6.5×10^{27} but smaller than 7.5×10^{27}. However, if we had written 7.0×10^{27}, we would have implied that we know the true number is somewhere between 6.95×10^{27} and 7.05×10^{27}. This statement is more precise than the previous statement.

As a general rule, the number of digits you write in the mantissa specifies how precisely you claim to know it. The more digits specified, the more precision is implied (see Figure 1.2). We call the number of digits in the mantissa the number of **significant figures.** Here are some rules about using significant figures followed in each case by an example:

- The number of significant figures is the number of reliably known digits. For example, 1.62 has three significant figures; 1.6 has two significant figures.

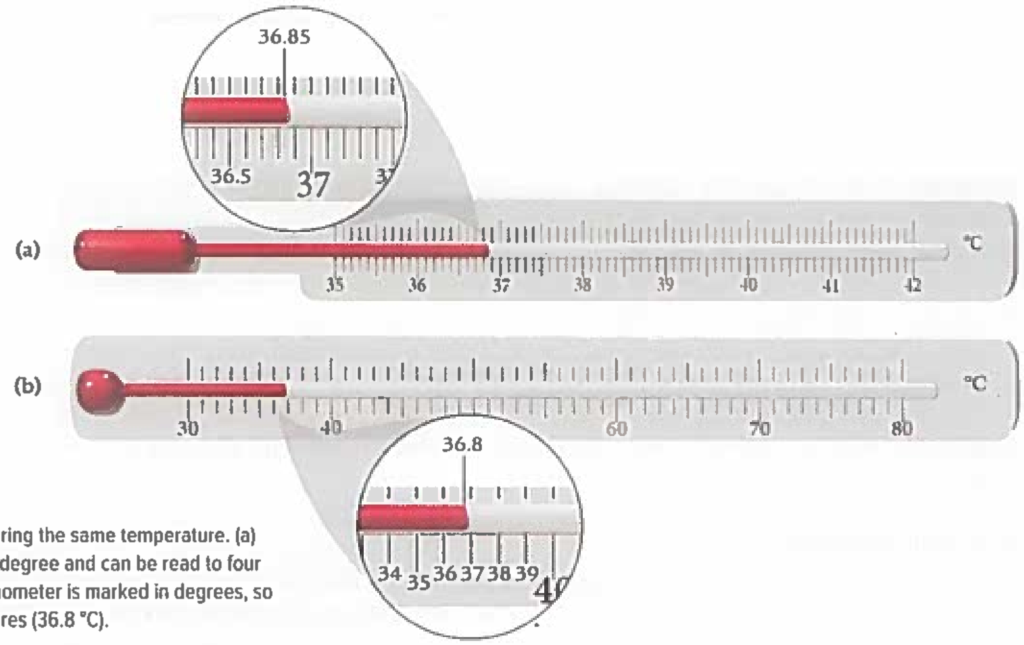


FIGURE 1.2 Two thermometers measuring the same temperature. (a) The thermometer is marked in tenths of a degree and can be read to four significant figures (36.85 °C); (b) the thermometer is marked in degrees, so it can be read to only three significant figures (36.8 °C).

- If you give a number as an integer, you specify it with infinite precision. For example, if someone says that he or she has 3 children, this means exactly 3, no less and no more.
- Leading zeros do not count as significant digits. The number 1.62 has the same number of significant digits as 0.00162. There are three significant figures in both numbers. We start counting significant digits from the left at the first nonzero digit.
- Trailing zeros, on the other hand, do count as significant digits. The number 1.620 has four significant figures. Writing a trailing zero implies greater precision!
- Numbers in scientific notation have as many significant figures as their mantissa. For example, the number 9.11×10^{-31} has three significant figures because that's what the mantissa (9.11) has. The size of the exponent has no influence.
- You can never have more significant figures in a result than you start with in any of the factors of a multiplication or division. For example, 1.23/3.4461 is not equal to 0.3569252. Your calculator may give you that answer, but calculators do not automatically display the correct number of significant figures. Correctly, $1.23/3.4461 = 0.357$. You must round a calculator result to the proper number of significant figures—in this case, three, which is the number of significant figures in the numerator.
- You can only add or subtract when there are significant figures for that place in every number. For example, $1.23 + 3.4461 = 4.68$, and not 4.6761 as you may think. This rule, in particular, requires some getting used to.

Concept Check 1.2

How many significant figures are in each of the following numbers?

a) 2.150
b) 0.000215
c) 215.00
d) 0.215000
e) 0.215 + 0.21

Concept Check 1.3

For the two numbers $x = 0.43$ and $y = 3.53$, which of the following has the greatest number of significant figures?

a) the sum, $x + y$
b) the product, $x\,y$
c) the difference, $x - y$
d) the number x
e) the number y

To finish this discussion of significant figures, let's reconsider the total number of atoms contained in the bodies of all people on Earth. We started with two quantities that were given to only one significant figure. Therefore, the result of our multiplication needs to be properly rounded to one significant digit. The combined number of atoms in all human bodies is thus correctly stated as 5×10^{37}.

1.3 SI Unit System

In high school, you may have been introduced to the international system of units and compared it to the British system of units in common use in the United States. You may have driven on a freeway on which the distances are posted both in miles and in kilometers or purchased food where the price was quoted per pound and per kilogram.

Table 1.1	Unit Names and Abbreviations for the Base Units of the SI System of Units	
Unit	**Abbreviation**	**Base Unit for**
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	current
kelvin	K	temperature
mole	mol	amount of a substance
candela	cd	luminous intensity

The international system of units is often abbreviated as SI (for Système International). Sometimes the units in this system are called *metric units*. The **SI unit system** is the standard used for scientific work around the world. The seven base units for the SI system are given in Table 1.1.

The first letters of the first four base units provide another commonly used name for the SI system: the MKSA system. We will only use the first three units (meter, kilogram, and second) in the entire first part of this book and in all of mechanics. The current definitions of these base units are as follows:

- 1 meter (m) is the distance that a light beam in vacuum travels in 1/299,792,458 of a second. Originally, the meter was related to the size of the Earth (Figure 1.3).
- 1 kilogram (kg) is defined as the mass of the international prototype of the kilogram. This prototype is kept just outside Paris, France, under carefully controlled environmental conditions.
- 1 second (s) is the time interval during which 9,192,631,770 oscillations of the electromagnetic wave (see Chapter 31) that corresponds to the transition between two specific states of the cesium-133 atom. Until 1967, the standard for the second was 1/86,400 of a mean solar day. However, the atomic definition is more precise and more reliably reproducible.

FIGURE 1.3 Originally, the meter was defined as 1 ten-millionth of the length of the meridian through Paris from the North Pole to the Equator.

Notation Convention: It is common practice to use roman letters for unit abbreviations and italic letters for physical quantities. We follow this convention in this book. For example, m stands for the unit meter, while m is used for the physical quantity mass. Thus, the expression $m = 17.2$ kg specifies that the mass of an object is 17.2 kilograms.

Units for all other physical quantities can be derived from the seven base units of Table 1.1. The unit for area, for example, is m^2. The units for volume and mass density are m^3 and kg/m^3, respectively. The units for velocity and acceleration are m/s and m/s^2, respectively. Some derived units were used so often that it became convenient to give them their own names and symbols. Often the name is that of a famous physicist. Table 1.2 lists the 20 derived SI units with special names. In the two rightmost columns of the table, the named unit is listed in terms of other named units and then in terms of SI base units. Also included in this table are the radian and steradian, the dimensionless units of angle and solid angle, respectively.

FIGURE 1.4 Prototype of the kilogram, stored near Paris, France.

You can obtain SI-recognized multiples of the base units and derived units by multiplying them by various factors of 10. These factors have universally accepted letter abbreviations that are used as prefixes, shown in Table 1.3. The use of standard prefixes (factors of 10) makes it easy to determine, for example, how many centimeters (cm) are in a kilometer (km):

$$1 \text{ km} = 10^3 \text{ m} = 10^3 \text{ m} \cdot (10^2 \text{ cm/m}) = 10^5 \text{ cm}. \qquad (1.3)$$

In comparison, note how tedious it is to figure out how many inches are in a mile:

$$1 \text{ mile} = (5{,}280 \text{ feet/mile}) \cdot (12 \text{ inches/foot}) = 63{,}360 \text{ inches}. \qquad (1.4)$$

As you can see, not only do you have to memorize particular conversion factors in the British system, but calculations also become more complicated. For calculations in the SI system, you only have to know the standard prefixes shown in Table 1.3 and how to add or subtract integers in the powers of 10.

Table 1.2 Common SI Derived Units

Derived or Dimensionless Unit	Name	Symbol	Equivalent	Expressions
Absorbed dose	gray	Gy	J/kg	$m^2\ s^{-2}$
Angle	radian	rad	–	–
Capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
Catalytic activity	katal	kat	–	s^{-1} mol
Dose equivalent	sievert	Sv	J/kg	$m^2\ s^{-2}$
Electric charge	coulomb	C	–	s A
Electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
Electric potential	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
Electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
Energy	joule	J	N m	$m^2\ kg\ s^{-2}$
Force	newton	N	–	$m\ kg\ s^{-2}$
Frequency	hertz	Hz	–	s^{-1}
Illuminance	lux	lx	lm/m^2	m^{-2} cd
Inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Luminous flux	lumen	lm	cd sr	cd
Magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}\ A^{-1}$
Magnetic field	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
Power	watt	W	J/s	$m^2\ kg\ s^{-3}$
Pressure	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
Radioactivity	becquerel	Bq	—	s^{-1}
Solid angle	steradian	sr	—	—
Temperature	degree Celsius	°C	—	K

Table 1.3 SI Standard Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta-	Y	10^{-24}	yocto-	y
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{18}	exa-	E	10^{-18}	atto-	a
10^{15}	peta-	P	10^{-15}	femto-	f
10^{12}	tera-	T	10^{-12}	pico-	p
10^{9}	giga-	G	10^{-9}	nano-	n
10^{6}	mega-	M	10^{-6}	micro-	μ
10^{3}	kilo-	k	10^{-3}	milli-	m
10^{2}	hecto-	h	10^{-2}	centi-	c
10^{1}	deka-	da	10^{-1}	deci-	d

FIGURE 1.5 The *Mars Climate Orbiter*, a victim of faulty unit conversion.

The international system of units was adopted by France in 1799 and is now used daily in almost all countries of the world, the one notable exception being the United States. Since we use British units in our daily lives, this book will indicate British units where appropriate, to establish connections with everyday experiences.

The use of British units can be costly. The cost can range from a small expense, such as that incurred by car mechanics who need to purchase two sets of wrench socket sets, one metric and one British, to the very expensive loss of the *Mars Climate Orbiter* spacecraft (Figure 1.5) in September 1999. The crash of this spacecraft has been blamed on the fact that one of the engineering teams used British units and the other one SI units. The two teams relied on each other's numbers, without realizing that the units were not the same.

The use of powers of 10 is not completely consistent even within the SI system itself. The notable exception is in the time units, which are not factors of 10 times the base unit (second):

- 365 days form a year,
- a day has 24 hours,
- an hour contains 60 minutes, and
- a minute consists of 60 seconds.

Early metric pioneers tried to establish a completely consistent set of metric time units, but these attempts failed. The not-exactly-metric nature of time units extends to some derived units. For example, a European sedan's speedometer does not show speeds in meters per second, but in kilometers per hour.

EXAMPLE 1.1 Units of Land Area

The unit of land area used in countries that use the SI system is the hectare, defined as 10,000 m^2. In some countries , land area is given in acres; an acre is defined as 43,560 ft^2.

PROBLEM

You just bought a plot of land with dimensions 2.00 km by 4.00 km. What is the area of your new purchase in hectares and acres?

SOLUTION

The area A is given by

$$A = \text{length}\times\text{width} = (2.00\text{ km})(4.00\text{ km}) = (2.00\times10^3\text{ m})(4.00\times10^3\text{ m})$$
$$A = 8.00\text{ km}^2 = 8.00\times10^6\text{ m}^2.$$

The area of this plot of land in hectares is then

$$A = 8.00\times10^6\text{ m}^2\,\frac{1\text{ hectare}}{10{,}000\text{ m}^2} = 8.00\times10^2\text{ hectares} = 800.\text{ hectares}.$$

To find the area of the land in acres, we need the length and width in British units:

$$\text{length} = 2.00\text{ km}\,\frac{1\text{ mi}}{1.609\text{ km}} = 1.24\text{ mi}\,\frac{5{,}280\text{ ft}}{1\text{ mi}} = 6{,}563\text{ ft}$$
$$\text{width} = 4.00\text{ km}\,\frac{1\text{ mi}}{1.609\text{ km}} = 2.49\text{ mi}\,\frac{5{,}280\text{ ft}}{1\text{ mi}} = 13{,}130\text{ ft}.$$

The area is then

$$A = \text{length}\times\text{width} = (1.24\text{ mi})(2.49\text{ mi}) = (6{,}563\text{ ft})(13{,}130\text{ ft})$$
$$A = 3.09\text{ mi}^2 = 8.61\times10^7\text{ ft}^2.$$

In acres, this is

$$A = 8.61\times10^7\text{ ft}^2\,\frac{1\text{ acre}}{43{,}560\text{ ft}^2} = 1980\text{ acres}.$$

Metrology: Research on Measures and Standards

The work of defining the standards for the base units of the SI system is by no means complete. A great deal of research is devoted to refining measurement technologies and pushing them to greater precision. This field of research is called **metrology.** In the United States, the laboratory that has the primary responsibility for this work is the National Institute of Standards and Technology (NIST). NIST works in collaboration with similar institutes in other countries to refine accepted standards for the SI base units.

One current research project is to find a definition of the kilogram based on reproducible quantities in nature. This definition would replace the current definition of the kilogram, which is based on the mass of a standard object kept in Sèvres on the outskirts of Paris. The most promising effort in this direction seems to be Project Avogadro, which attempts to define the kilogram using highly purified silicon crystals.

Research on keeping time ever more precisely is one of the major tasks of NIST and similar institutions.

Greater precision in timekeeping is needed for many applications in our information-based society, where signals can travel around the world in less than 0.2 second. The Global Positioning System (GPS) is one example of technology that would be impossible to realize without the precision of atomic clocks and the physics research that enters into their construction. The GPS system also relies on Einstein's theory of relativity, which we will study in Chapter 13.

1.4 The Scales of Our World

The most amazing fact about physics is that its laws govern every object, from the smallest to the largest. The scales of the systems for which physics holds predictive power span many orders of magnitude (powers of 10), as we'll see in this section.

Nomenclature: In the following, you will read "on the order of" several times. This phrase means "within a factor of 2 or 3."

Length Scales

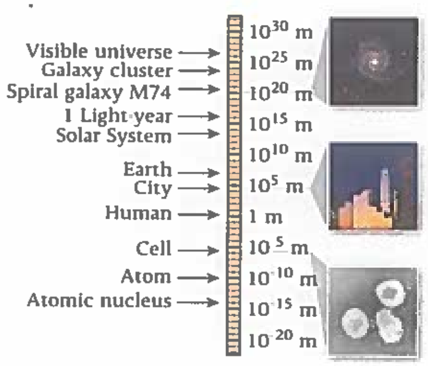


FIGURE 1.7 Range of length scales for physical systems. The pictures top to bottom are the spiral galaxy M74, the Dallas skyline, and the SARS virus.

Length is defined as the distance measurement between two points in space. Figure 1.7 shows some length scales for common objects and systems that span over 40 orders of magnitude.

Let's start with ourselves. On average, in the United States, a woman is 1.62 m (5 ft 4 in) tall, and a man measures 1.75 m (5 ft 9 in). Thus, human height is on the order of a meter. If you reduce the length scale for a human body by a factor of a million, you arrive at a micrometer. This is the typical diameter of a cell in your body or a bacterium.

If you reduce the length of your measuring stick by another factor of 10,000, you are at a scale of 10^{-10} m, the typical diameter of an individual atom. This is the smallest size we can resolve with the aid of the most advanced microscopes.

Inside the atom is its nucleus, with a diameter about 1/10,000 that of the atom, on the order of 10^{-14} m. The individual protons and neutrons that make up the atomic nucleus have a diameter of approximately 10^{-15} m = 1 fm (a femtometer).

Considering objects larger than ourselves, we can look at the scale of a typical city, on the order of kilometers. The diameter of Earth is just a little bigger than 10,000 km (12,760 km, to be more precise). As discussed earlier, the definition of the meter is now stated in terms of the speed of light. However, the meter was originally defined as 1 ten-millionth of the length of the meridian through Paris from the North Pole to the Equator. If a quarter circle has an arc length of 10 million meters (= 10,000 km), then the circumference of the entire circle would be exactly 40,000 km. Using the modern definition of the meter, the equatorial circumference of the Earth is 40,075 km, and the circumference along the meridian is 40,008 km.

The distance from the Earth to the Moon is 384,000 km, and the distance from the Earth to the Sun is greater by a factor of approximately 400, or about 150 million km. This distance is called an *astronomical unit* and has the symbol AU. Astronomers used this unit before the distance from Earth to Sun became known with accuracy, but it is still convenient today. In SI units, an astronomical unit is

$$1 \text{ AU} = 1.495\,98 \times 10^{11} \text{ m}. \tag{1.5}$$

The diameter of our Solar System is conventionally stated as approximately 10^{13} m, or 60 AU.

We have already remarked that light travels in vacuum at a speed of approximately 300,000 km/s. Therefore, the distance between Earth and Moon is covered by light in just over 1 second, and light from the Sun takes approximately 8 minutes to reach Earth. In order to cover distance scales outside our Solar System, astronomers have introduced the (non-SI, but handy) unit of the light-year, the distance that light travels in 1 year in vacuum:

$$1 \text{ light-year} = 9.46 \times 10^{15} \text{ m}. \tag{1.6}$$

The nearest star to our Sun is just over 4 light-years away. The Andromeda Galaxy, the sister galaxy of our Milky Way, is about 2.5 million light-years $= 2 \times 10^{22}$ m away.

Finally, the radius of the visible universe is approximately 14 billion light-years $= 1.5 \times 10^{26}$ m. Thus, about 41 orders of magnitude span between the size of an individual proton and that of the entire visible universe.

Mass Scales

Mass is the amount of matter in an object. When you consider the range of masses of physical objects, you obtain an even more awesome span of orders of magnitude (Figure 1.8) than for lengths.

Atoms and their parts have incredibly small masses. The mass of an electron is only 9.11×10^{-31} kg. A proton's mass is 1.67×10^{-27} kg, roughly a factor of 2000 more than the mass of an electron. An individual atom of lead has a mass of 3.46×10^{-25} kg.

The mass of a single cell in the human body is on the order of 10^{-13} kg to 10^{-12} kg. Even a fly has more than 100 million times the mass of a cell, at approximately 10^{-4} kg.

A car's mass is on the order of 10^3 kg, and that of a passenger plane is on the order of 10^5 kg.

A mountain typically has a mass of 10^{12} kg to 10^{14} kg, and the combined mass of all the water in all of the Earth's oceans is estimated to be on the order of 10^{21} kg.

The mass of the entire Earth can be specified fairly precisely at 6.0×10^{24} kg. The Sun has a mass of 2.0×10^{30} kg, or about 300,000 times the mass of the Earth. Our entire galaxy, the Milky Way, is estimated to have 200 billion stars in it and thus has a mass around 3×10^{41} kg. Finally, the entire universe contains billions of galaxies. Depending on the assumptions about dark matter, a currently active research topic (see Chapter 12), the mass of the universe as a whole is roughly 10^{51} kg. However, you should recognize that this number is an estimate and may be off by a factor of up to 100.

Interestingly, some objects have no mass. For example, photons, the "particles" that light is made of, have zero mass.

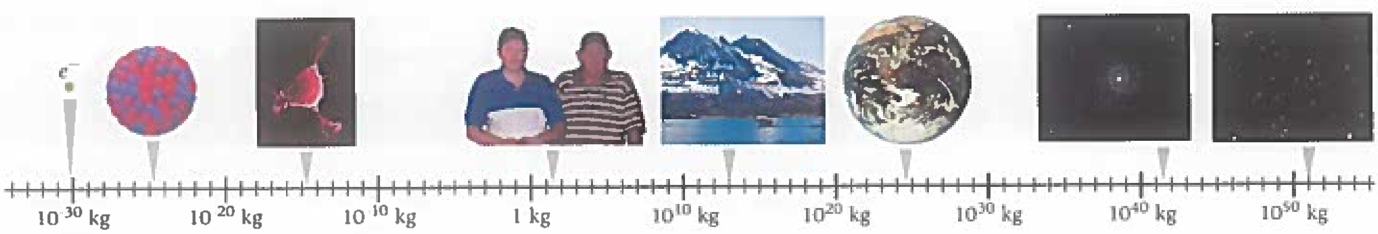


FIGURE 1.8 Range of mass scales for physical systems.

Time Scales

Time is the duration between two events. Human time scales lie in the range from a second (the typical duration of a human heartbeat) to a century (about the life expectancy of a person born now). Incidentally, human life expectancy is increasing at an ever faster rate. During the Roman Empire, 2000 years ago, a person could expect to live only 25 years. In 1850, actuary tables listed the mean lifetime of a human as 39 years. Now that number is 80 years. Thus, it took almost 2000 years to add 50% to human life expectancy, but in the last 150 years, life expectancy has doubled again. This is perhaps the most direct evidence that science has basic benefits for all of us. Physics contributes to this progress by aiding the development of more sophisticated medical imaging and treatment equipment, and today's fundamental research will enter clinical practice tomorrow. Laser surgery, cancer radiation therapy, magnetic resonance imaging, and positron emission tomography are just a few examples of technological advances that have helped increase life expectancy.

In their research, the authors of this book study ultrarelativistic heavy-ion collisions. These collisions occur during time intervals on the order of 10^{-22} s, more than a million times shorter than the time intervals we can measure directly. During this course, you will learn that the time scale for the oscillation of visible light is 10^{-15} s, and that of audible sound is 10^{-3} s.

The longest time span we can measure *indirectly* or *infer* is the age of the universe. Current research puts this number at 13.7 billion years, but with an uncertainty of up to 0.2 billion years.

We cannot leave this topic without mentioning one interesting fact to ponder during your next class lecture. Lectures typically last 50 minutes at most universities. A century, by comparison, has $100 \times 365 \times 24 \times 60 \approx 50{,}000{,}000$ minutes. So a lecture lasts about 1 millionth of a century, leading to a handy (non-SI) time unit, the microcentury = duration of one lecture.

1.5 General Problem-Solving Strategy

Physics involves more than solving problems–but problem solving is a big part of it. At times, while you are laboring over your homework assignments, it may seem that's all you do. However, repetition and practice are important parts of learning.

A basketball player spends hours practicing the fundamentals of free throw shooting. Many repetitions of the same action enable a player to become very reliable at this task. You need to develop the same philosophy toward solving mathematics and physics problems: You have to practice good problem-solving techniques. This work will pay huge dividends, not just during the remainder of this physics course, not just during exams, not even just in your other science classes, but also throughout your entire career.

What constitutes a good problem-solving strategy? Everybody develops his or her own routines, procedures, and shortcuts. However, here is a general blueprint that should help get you started:

1. **THINK** Read the problem carefully. Ask yourself what quantities are known, what quantities might be useful but are unknown, and what quantities are asked for in the solution. Write down these quantities and represent them with their commonly used symbols. Convert into SI units, if necessary.

2. **SKETCH** Make a sketch of the physical situation to help you visualize the problem. For many learning styles, a visual or graphical representation is essential, and it is often essential for defining the variables.

3. **RESEARCH** Write down the physical principles or laws that apply to the problem. Use equations representing these principles to connect the known and unknown quantities to each other. In some cases, you will immediately see an equation that has only the quantities that you know and the one unknown that you are supposed to calculate, and nothing else. More often you may have to do a bit of deriving, combining two or more known equations into the one that you need. This requires some experience, more than any of the other steps listed here. To the beginner, the task of deriving a new equation may look daunting, but you will get better at it the more you practice.

4. **SIMPLIFY** Do not plug numbers into your equation yet! Instead, simplify your result algebraically as much as possible. For example, if your result is expressed as a ratio, cancel out common factors in the numerator and the denominator. This step is particularly helpful if you need to calculate more than one quantity.

5. **CALCULATE** Put the numbers with units into the equation and get to work with a calculator. Typically, you will obtain a number and a physical unit as your answer.

6. **ROUND** Determine the number of significant figures that you want to have in your result. As a rule of thumb, a result obtained by multiplying or dividing should be rounded to the same number of significant figures as in the input quantity that is given with the least number of significant figures. You should not round in intermediate steps, as rounding too early might give you a wrong solution.

7. **DOUBLE-CHECK** Step back and look at the result. Judge for yourself if the answer (both the number and the units) seems realistic. You can often avoid handing in a wrong solution by making this final check. Sometimes the units of your answer are simply wrong, and you know you must have made an error. Or sometimes the order of magnitude comes out totally wrong. For example, if your task is to calculate the mass of the Sun (we will do this later in this book), and your answer comes out near 10^6 kg (only a few thousand tons), you know you must have made a mistake somewhere.

Let's put this strategy to work in the following example.

SOLVED PROBLEM 1.1 Volume of a Cylinder

PROBLEM

Nuclear waste material in a physics laboratory is stored in a cylinder of height $4\frac{3}{16}$ inches and circumference $8\frac{3}{16}$ inches. What is the volume of this cylinder, measured in metric units?

SOLUTION

In order to practice problem-solving skills, we will go through each of the steps of the strategy outlined above.

THINK From the question, we know that the height of the cylinder, converted to centimeters, is

$$\begin{aligned} h &= 4\tfrac{13}{16} \text{ in} = 4.8125 \text{ in} \\ &= (4.8125 \text{ in})\cdot(2.54 \text{ cm/in}) \\ &= 12.22375 \text{ cm.} \end{aligned}$$

Also, the circumference of the cylinder is specified as

$$\begin{aligned} c &= 8\tfrac{3}{16} \text{ in} = 8.1875 \text{ in} \\ &= (8.1875 \text{ in})\cdot(2.54 \text{ cm/in}) \\ &= 20.79625 \text{ cm.} \end{aligned}$$

Even though it is clear that the number of significant digits in our SI-converted values for h and c are clearly too high, we keep them for our intermediate calculations, and we only round our final answer to the proper number of significant digits.

SKETCH Next, we produce a sketch, something like Figure 1.9. Note that the given quantities are shown with their symbolic representations, not with their numerical values. The circumference is represented by the thicker circle (oval, actually, in this projection).

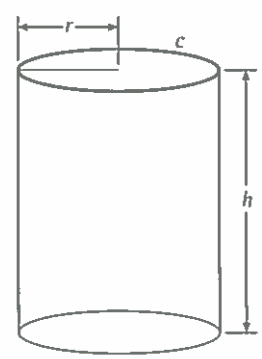


FIGURE 1.9 Sketch of right cylinder.

RESEARCH Now we have to find the volume of the cylinder in terms of its height and its circumference. This relationship is not commonly listed in collections of geometric formulas. Instead, the volume of a cylinder is given as the product of base area and height:

$$V = \pi r^2 h.$$

Once we find a way to connect the radius and the circumference, we'll have the formula we need. The top and bottom areas of a cylinder are circles, and for a circle we know that

$$c = 2\pi r.$$

SIMPLIFY Remember: We do not plug in numbers yet! To simplify our numerical task, we can solve the second equation for r and insert this result into the first equation:

$$c = 2\pi r \Rightarrow r = \frac{c}{2\pi}$$

$$V = \pi r^2 h = \pi\left(\frac{c}{2\pi}\right)^2 h = \frac{c^2 h}{4\pi}.$$

– *Continued*

CALCULATE Now it's time to get out the calculator and put in the numbers:

$$V = \frac{c^2 h}{4\pi}$$
$$= \frac{(20.79625\text{ cm})^2 \cdot (12.22375\text{ cm})}{4\pi}$$
$$= 420.69239\text{ cm}^3.$$

ROUND The output of the calculator has again made our result look much more precise than we can claim realistically. We need to round. Since the input quantities are given to only three significant figures, our result needs to be rounded to three significant figures. Our final answer is $V = 421.\text{ cm}^3$.

DOUBLE-CHECK Our last step is to check that the answer is reasonable. First, we look at the unit we got for our result. Cubic centimeters are a unit for volume, so our result passes its first test. Now let's look at the magnitude of our result. You might recognize that the height and circumference given for the cylinder are close to the corresponding dimensions of a soda can. If you look on a can of your favorite soft drink, it will list the contents as 12 fluid ounces and also give you the information that this is 355 mL. Because 1 mL = 1 cm^3, our answer is reasonably close to the volume of the soda can. Note that this does *not* tell us that our calculation is correct, but it shows that we are not way off.

Suppose the researchers decide that a soda can is not large enough for holding the waste in the lab and replace it with a larger cylindrical container with a height of 44.6 cm and a circumference of 62.5 cm. If we want to calculate the volume of this replacement cylinder, we don't need to do all of Solved Problem 1.1 over again. Instead, we can go directly to the algebraic formula we derived in the Simplify step and substitute our new data into that, ending up with a volume of 13,900 cm^3 when rounded to three significant figures. This example illustrates the value of waiting to substitute in numbers until algebraic simplification has been completed.

In Solved Problem 1.1, you can see that we followed the seven steps outlined in our general strategy. It is tremendously helpful to train your brain to follow a certain procedure in attacking all kinds of problems. This is not unlike following the same routine whenever you are shooting free throws in basketball, where frequent repetition helps you build the muscle memory essential for consistent success, even when the game is on the line.

Perhaps more than anything, an introductory physics class should enable you to develop methods to come up with your own solutions to a variety of problems, eliminating the need to accept "authoritative" answers uncritically. The method we used in Solved Problem 1.1 is extremely useful, and we will practice it again and again in this book. However, to make a simple point, one that does not require the full set of steps used for a solved problem, we'll sometimes use an illustrative example.

EXAMPLE 1.2 Volume of a Barrel of Oil

PROBLEM

The volume of a barrel of oil is 159 L. We need to design a cylindrical container that will hold this volume of oil. The container needs to have a height of 1.00 m to fit in a transportation container. What is the required circumference of the cylindrical container?

SOLUTION

Starting with the equation we derived in the Simplify step in Solved Problem 1.1, we can relate the circumference, c, and the height, h, of the container to the volume, V, of the container:

$$V = \frac{c^2 h}{4\pi}.$$

Solving for the circumference, we get

$$c = \sqrt{\frac{4\pi V}{h}}.$$

The volume in SI units is

$$V = 159\ \text{L}\frac{1000\ \text{mL}}{\text{L}}\frac{1\ \text{cm}^3}{1\ \text{mL}}\frac{1\ \text{m}^3}{10^6\ \text{cm}^3} = 0.159\ \text{m}^3.$$

The required circumference is then

$$c = \sqrt{\frac{4\pi V}{h}} = \sqrt{\frac{4\pi\,(0.159\ \text{m}^3)}{1.00\ \text{m}}} = 1.41\ \text{m}.$$

As you may have already realized from the preceding problem and example, a good command of algebra is essential for success in an introductory physics class. For engineers and scientists, most universities and colleges also require calculus, but at many schools an introductory physics class and a calculus class can be taken concurrently. This first chapter does not contain any calculus, and subsequent chapters will review the relevant calculus concepts as you need them. However, there is another field of mathematics that is used extensively in introductory physics: trigonometry. Virtually every chapter of this book uses right triangles in some way. Therefore, it is a good idea to review the formulas for sine, cosine, and the like, as well as the indispensable Pythagorean Theorem. Let's look at another solved problem, which makes use of trigonometric concepts.

SOLVED PROBLEM 1.2 View from the Willis Tower

PROBLEM

It goes without saying that one can see farther from a tower than from ground level; the higher the tower, the farther one can see. The Willis Tower (formerly named Sears Tower) in Chicago has an observation deck, which is 412 m above ground. How far can one see out over Lake Michigan from this observation deck under perfect weather conditions? (Assume eye level is at 413 m above the level of the lake.)

SOLUTION

THINK As we have stressed before, this is the most important step in the problem-solving process. A little preparation at this stage can save a lot of work at a later stage. Perfect weather conditions are specified, so fog or haze is not a limiting factor. What else could determine how far one can see? If the air is clear, one can see mountains that are quite far away. Why mountains? Because they are very tall. But the landscape around Chicago is flat. What then could limit the viewing range? Nothing, really; one can see all the way to the horizon. And what is the deciding factor for where the horizon is? It is the curvature of the Earth. Let's make a sketch to make this a little clearer.

SKETCH Our sketch does not have to be elaborate, but it needs to show a simple version of the Willis Tower on the surface of the Earth. It is not important that the sketch be to scale, and we elect to greatly exaggerate the height of the tower relative to the size of the Earth. See Figure 1.10.

It seems obvious from this sketch that the farthest point (point C) that one can see from the top of the Willis Tower (point B) is where the line of sight just touches the surface of the Earth tangentially. Any point on Earth's surface farther away from the Willis Tower is hidden from view (below the dashed line segment). The viewing range is then given by the distance r between that surface point C and the observation deck (point B) on top of the tower, at height h. Included in the sketch is also a line from the center of Earth (point A) to the foot of the Willis Tower. It has length R, which is the radius of Earth. Another line of the same length, R, is drawn to the point where the line of sight touches the Earth's surface tangentially.

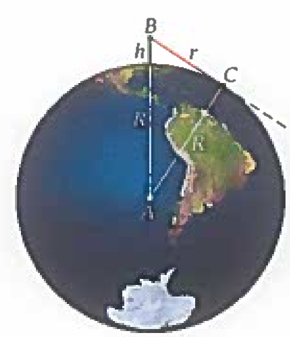


FIGURE 1.10 Distance from the top of the Willis Tower (B) to the horizon (C).

– Continued

RESEARCH As you can see from the sketch, a line drawn from the center of the Earth to the point where the line of sight touches the surface (A to C) will form a right angle with that line of sight (B to C); that is, the three points A, B, and C form the corners of a right triangle. This is the key insight, which enables us to use trigonometry and the Pythagorean Theorem to attack the solution of this problem. Examining the sketch in Figure 1.9, we find

$$r^2 + R^2 = (R+h)^2.$$

SIMPLIFY Remember, we want to find the distance to the horizon, for which we used the symbol r in the previous equation. Isolating that variable on one side of our equation gives

$$r^2 = (R+h)^2 - R^2.$$

Now we can simplify the square and obtain

$$r^2 = R^2 + 2hR + h^2 - R^2 = 2hR + h^2.$$

Finally, we take the square root and obtain our final algebraic answer:

$$r = \sqrt{2hR + h^2}.$$

CALCULATE Now we are ready to insert numbers. The accepted value for the radius of Earth is $R = 6.37 \times 10^6$ m, and $h = 413$ m $= 4.13 \times 10^2$ m was given in the problem. This leads to

$$r = \sqrt{2(4.13{\times}10^2\,\text{m})(6.37{\times}10^6\,\text{m}) + (4.13{\times}10^2\,\text{m})^2} = 7.25382{\times}10^4\,\text{m}.$$

ROUND The Earth's radius was given to three-digit precision, as was the elevation of the eye level of the observer. So we round to three digits and give our final result as

$$r = 7.25{\times}10^4\,\text{m} = 72.5\,\text{km}.$$

DOUBLE-CHECK Always check the units first. Since the problem asked "how far," the answer needs to be a distance, which has the dimension of length and thus the base unit meter. Our answer passes this first check. What about the magnitude of our answer? Since the Willis Tower is almost 0.5 kilometer high, we expect the viewing range to be at least several kilometers; so a multikilometer range for the answer seems reasonable. Lake Michigan is slightly more than 80 km wide, if you look toward the east from Chicago. Our answer then implies that you cannot see the Michigan shore of Lake Michigan if you stand on top of the Willis Tower. Experience shows that this is correct, which gives us additional confidence in our answer.

Concept Check 1.4

Two sailors are at the tops of their ships' masts in the open ocean. The mast of ship A is twice as high as that of ship B. How much farther can sailor A see than sailor B?

a) no farther

b) a little more than 40% farther

c) twice as far

d) almost three times as far

e) four times as far

Problem-Solving Guidelines: Limits

In Solved Problem 1.2, we found a handy formula, $r = \sqrt{2hR + h^2}$, for how far one can see on the surface of Earth from an elevation h, where R is the radius of Earth. There is another test that we can perform to check the validity of this formula. We did not include it in the Double-Check step, because it deserves seperate consideration. This general problem-solving technique is examining the limits of an equation.

What does "examining the limits" mean? In terms of Solved Problem 1.2, it means that instead of just inserting the given number for h into our formula and computing the solution, we can also step back and think about what should happen to the distance r one can see if h becomes very large or very small. Obviously, the smallest that h can become is zero. In this case, r will also approach zero. This is expected, of course; if your eye level is at ground level, you cannot see very far. On the other hand, we can ponder what happens if h becomes large compared to the radius of Earth (see Figure 1.11). (Yes, it is impossible to build a tower that tall but h could also stand for the altitude of a satellite above ground.) In that case, we expect that the viewing range will eventually simply be the height h. Our formula also bears out this expectation, because as h becomes large compared to R, the first term in the square root can be neglected, and we find $\lim\limits_{h\to\infty} \sqrt{2hR + h^2} = h$.

What we have illustrated by this example is a general guideline: If you derive a formula, you can check its validity by substituting extreme values of the variables in the formula and checking if these limits agree with common sense. Often the formula simplifies drastically

FIGURE 1.11 Viewing range in the limit of very large h.

at a limit. If your formula has a limiting behavior that is correct, it does not necessarily mean that the formula itself is correct, but it gives you additional confidence in its validity.

Problem-Solving Guidelines: Ratios

Another very common class of physics problems asks what happens to a quantity that depends on a certain parameter if that parameter changes by a given factor. These problems provide excellent insight into physical concepts and take almost no time to do. This is true, in general, *if* two conditions are met: First, you have to know what formula to use; and second, you have to know how to solve this general class of problems. But that is a big *if*. Studying will equip your memory with the correct formulas, but you need to acquire the skill of solving problems of this general type.

Here is the trick: Write down the formula that connects the dependent quantity to the parameter that changes. Write it twice, once with the dependent quantity and the parameters indexed (or labeled) with 1 and once with them indexed with 2. Then form ratios of the indexed quantities by dividing the right-hand sides and the left-hand sides of the two equations. Next, insert the factor of change for the parameter (expressed as a ratio) and do the calculation to find the factor of change for the dependent quantity (also expressed as a ratio).

Here is an example that demonstrates this method.

EXAMPLE 1.3 Change in Volume

PROBLEM

If the radius of a cylinder increases by a factor of 2.73, by what factor does the volume change? Assume that the height of the cylinder stays the same.

SOLUTION

The formula that connects the volume of a cylinder, V, and its radius, r, is

$$V = \pi r^2 h.$$

The way the problem is phrased, V is the dependent quantity and r is the parameter it depends on. The height of the cylinder, h, also appears in the equation but remains constant, according to the problem statement.

Following the general problem-solving guideline, we write the equation twice, once with 1 as indexes and once with 2:

$$V_1 = \pi r_1^2 h$$

$$V_2 = \pi r_2^2 h.$$

Now we divide the second equation by the first, obtaining

$$\frac{V_2}{V_1} = \frac{\pi r_2^2 h}{\pi r_1^2 h} = \left(\frac{r_2}{r_1}\right)^2.$$

As you can see, h did not receive an index because it stayed constant in this problem; it canceled out in the division.

The problem states that the change in radius is given by:

$$r_2 = 2.73 r_1.$$

We substitute for r_2 in our ratio:

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2.73 r_1}{r_1}\right)^2 = 2.73^2 = 7.4529,$$

or

$$V_2 = 7.45 V_1,$$

where we have rounded the solution to the three significant digits that the quantity given in the problem had. Thus, the answer is that the volume of the cylinder increases by a factor of 7.45 when you increase its radius by a factor of 2.73.

Problem-Solving Guidelines: Estimation

Sometimes you don't need to solve a physics problem exactly. When an estimate is all that is asked for, knowing the order of magnitude of some quantity is enough. For example, an answer of 1.24×10^{20} km is for most purposes not much different from 1×10^{20} km. In such cases, you can round off all the numbers in a problem to the nearest power of 10 and carry out the necessary arithmetic. For example, the calculation in Solved Problem 1.1 reduces to

$$\frac{(20.8\text{ cm})^2 \times (12.2\text{ cm})}{4\pi} \approx \frac{(2\times10^1\text{ cm})^2 \times (10\text{ cm})}{10} = \frac{4\times10^3\text{ cm}^3}{10} = 400\text{ cm}^3,$$

which is pretty close to our answer of 420. cm^3. Even an answer of 100 cm^3 (rounding 20.8 cm to 10 cm) has the correct order of magnitude for the volume. Notice that you can often round the number π to 3 or round π^2 to 10. With practice, you can find more tricks of approximation like these that can make estimations simpler and faster.

The technique of gaining useful results through careful estimation was made famous by the 20th-century physicist Enrico Fermi (1901–1954), who estimated the energy released by the Trinity nuclear explosion on July 16, 1945, near Socorro, New Mexico, by observing how far a piece of paper was blown by the wind from the blast. There is a class of estimation problems called *Fermi problems* that can yield interesting results when reasonable assumptions are made about quantities that are not known exactly.

Estimates are useful to gain insight into a problem before turning to more complicated methods of calculating a precise answer. For example, one could estimate how many tacos people eat and how many taco stands there are in town before investing in a complete business plan to construct a taco stand. In order to practice estimation skills, let's estimate the amount of carbon dioxide that is added to Earth's atmosphere annually by humans breathing.

EXAMPLE 1.4 Greenhouse Gas Production

PROBLEM

The concentration of greenhouse gases, including carbon dioxide (CO_2), in the Earth's atmosphere is increasing. Estimate how much CO_2 is added to the atmosphere each year by humans breathing.

SOLUTION

Since we are asked to estimate, we have to come up with the order of magnitude of the amount. The precise number does not matter so much. Let's start with the amount of CO_2 in one breath, estimate how many breaths each of us takes per year, and then multiply by the number of humans on the planet.

When we breathe, we take in air that is a mixture of 21% oxygen and 78% nitrogen (plus traces of other gases). We exhale air that has approximately 16% oxygen and 5% CO_2. Even though our lung capacity is approximately 3 to 5 L, we only use about 10% of that capacity in normal breathing. So, let's say that one breath of air is approximately 0.4 L. Then, 5% of 0.4 L is 2×10^{-2} L. You may remember from high school science that 22.4 L of gas comprise 1 mole, and that 1 mole of CO_2 has a mass of $2 \times 16\text{ g} + 12\text{ g} = 44\text{ g}$. This means that in one breath we produce

$$m_1 = \frac{(2\times10^{-2}\text{ L})(44\text{ g})}{22.4\text{ L}} \approx 4\times10^{-2}\text{ g}$$

of CO_2.

We take a breath of air about once every 4 seconds. (You can use a stopwatch to convince yourself that this is true, or you can count the number of breaths you take in a minute.) This means that we breathe about 1000 times per hour, and since a year has about 10,000 hours, we take $N = 10^7$ breaths per year.

Now, we can put all this together and get our estimate. Humanity (~7 billion, or 7×10^9, humans) produces about

$$M = Nm_1N_{\text{humans}} = 10^7(4 \times 10^{-2}\text{ g})(7 \times 10^9) \approx 3 \times 10^{15}\text{ g} = 3 \times 10^{12}\text{ kg of } CO_2$$

Concept Check 1.5

Estimate the number of liters of gasoline consumed each day in the United States by commuters driving to work.

a) 37,854 liters
b) 378,541 liters
c) 3,785,411,784 liters
d) 37,854,118 liters
e) 378,541,178 liters

In other words, our estimate indicates that humans add approximately 3 billion metric tons of CO_2 to Earth's atmosphere each year just by breathing. Remember, this is just an estimate; we cannot be sure of the mantissa, but we can be reasonably confident of the exponent. Our answer certainly is billions of tons, but we cannot be sure if it is 1 or 3 billion.

For comparison, measurements indicate that the amount of CO_2 in the atmosphere increases by approximately 15 billion metric tons per year. This amount is clearly much higher than our estimate, but is mainly due to burning of fossil fuels.

1.6 Vectors

Vectors are mathematical descriptions of quantities that have both magnitude and direction. The magnitude of a vector is a nonnegative number, often combined with a physical unit. Many vector quantities are important in physics and, indeed, in all of science. Therefore, before we start this study of physics, you need to be familiar with vectors and some basic vector operations.

Vectors have a starting point and an ending point. For example, consider a flight from Seattle to New York. To represent the change of the plane's position, we can draw an arrow from the plane's departure point to its destination (Figure 1.11). (Real flight paths are not exactly straight lines due to the fact that the Earth is a sphere and due to airspace restrictions and air traffic regulations, but a straight line is a reasonable approximation for our purpose.) This arrow represents a *displacement vector*, which always goes from somewhere to somewhere else. Any vector quantity has a magnitude and a direction. If the vector represents a physical quantity, such as displacement, it will also have a physical unit. A quantity that can be represented without giving a direction is called a **scalar.** A scalar quantity has just a magnitude and possibly a physical unit. Examples of scalar quantities are time and temperature.

This book denotes a vector quantity by a letter with a small horizontal arrow pointing to the right above it. For example, in the drawing of the trip from Seattle to New York (Figure 1.12), the displacement vector has the symbol $\vec{C}$. In the rest of this section, you will learn how to work with vectors: how to add and subtract them and how to multiply them. In order to perform these operations, it is very useful to introduce a coordinate system in which to represent vectors.

Cartesian Coordinate System

A **Cartesian coordinate system** is defined as a set of two or more axes with angles of 90° between each pair. These axes are said to be orthogonal to each other. In a two-dimensional space, the coordinate axes are typically labeled x and y. We can then uniquely specify any point P in the two-dimensional space by giving its coordinates P_x and P_y along the two coordinate axes, as shown in Figure 1.13. We will use the notation (P_x,P_y) to specify a

FIGURE 1.12 Flight path from Seattle to New York as an example of a vector.

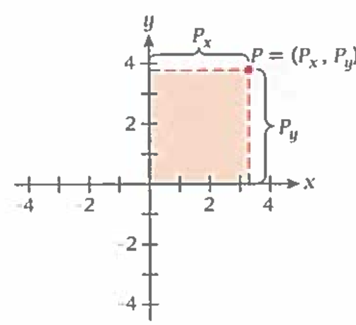


FIGURE 1.13 Representation of a point P in two-dimensional space in terms of its Cartesian coordinates.

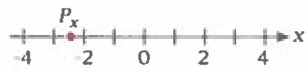


FIGURE 1.14 Representation of a point P in a one-dimensional Cartesian coordinate system.

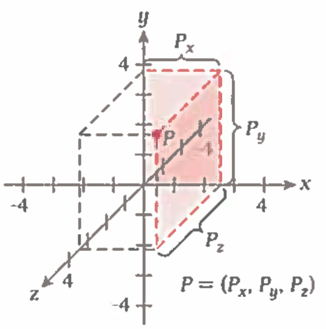


FIGURE 1.15 Representation of a point P in a three-dimensional space in terms of its Cartesian coordinates.

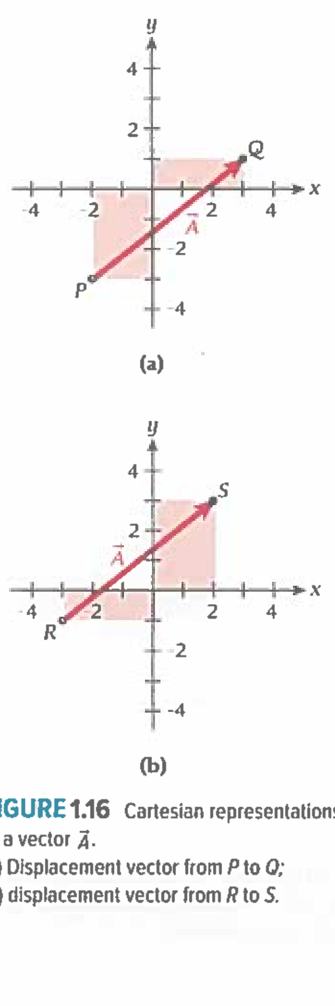


FIGURE 1.16 Cartesian representations of a vector $\vec{A}$.
(a) Displacement vector from P to Q;
(b) displacement vector from R to S.

point in terms of its coordinates. In Figure 1.13, for example, the point P has the position (3.3,3.8), because its x-coordinate has a value of 3.3 and its y-coordinate has a value of 3.8. Note that each coordinate is a number and can have a positive or negative value or be zero.

We can also define a one-dimensional coordinate system, for which any point is located on a single straight line, conventionally called the x-axis. Any point in this one-dimensional space is then uniquely defined by specifying one number, the value of the x-coordinate, which again can be negative, zero, or positive (Figure 1.14). The point P in Figure 1.14 has the x-coordinate $P_x = -2.5$.

Clearly, one- and two-dimensional coordinate systems are easy to draw, because the surface of paper has two dimensions. In a three-dimensional coordinate system, the third coordinate axis is perpendicular to the other two; thus, to be represented accurately, it would have to stick straight out of the plane of the page. In order to draw a three-dimensional coordinate system, we have to rely on conventions that make use of the techniques for perspective drawings. We represent the third axis by a line that is at a 45° angle with the other two (Figure 1.15).

In a three-dimensional space, we have to specify three numbers to uniquely determine the coordinates of a point. We use the notation $P=(P_x,P_y,P_z)$ to accomplish this. It is possible to construct Cartesian coordinate systems with more than three orthogonal axes, although they are almost impossible to visualize. Modern string theories for example, are usually constructed in 10-dimensional spaces. However, for the purposes of this book and for almost all of physics, three dimensions are sufficient. As a matter of fact, for most applications, the essential mathematical and physical understanding can be obtained from two-dimensional representations.

Cartesian Representation of Vectors

The example of the flight from Seattle to New York established that vectors are characterized by two points: start and finish, represented by the tail and head of an arrow, respectively. Using the Cartesian representation of points, we can define the Cartesian representation of a displacement vector as the difference in the coordinates of the end point and the starting point. Since the difference between the two points for a vector is all that matters, we can shift the vector around in space as much as we like. As long as we do not change the length or direction of the arrow, the vector remains mathematically the same. Consider the two vectors in Figure 1.16.

Figure 1.16a shows the displacement vector $\vec{A}$ that points from point $P=(-2, -3)$ to point $Q=(3, 1)$. With the notation just introduced, the **components** of $\vec{A}$ are the coordinates of point Q minus those of point P, $\vec{A} = (3-(-2), 1-(-3)) = (5, 4)$. Figure 1.16b shows another vector from point $R=(-3, -1)$ to point $S=(2, 3)$. The difference between these coordinates is $(2-(-3), 3-(-1)) = (5, 4)$, which is the same as the vector $\vec{A}$ pointing from P to Q.

For simplicity, we can shift the beginning of a vector to the origin of the coordinate system, and the components of the vector will be the same as the coordinates of its end point (Figure 1.17). As a result, we see that we can represent a vector in Cartesian coordinates as

$$\vec{A} = (A_x, A_y) \text{ in two-dimensional space} \tag{1.7}$$

$$\vec{A} = (A_x, A_y, A_z) \text{ in three-dimensional space} \tag{1.8}$$

where A_x, A_y, and A_z are numbers. Note that the notation for a point in Cartesian coordinates is similar to the notation for a vector in Cartesian coordinates. Whether the notation specifies a point or a vector will be clear from the context of the reference.

Graphical Vector Addition and Subtraction

Suppose that the direct flight from Seattle to New York shown in Figure 1.12 was not available, and you had to make a connection through Dallas (Figure 1.18). Then the displacement vector $\vec{C}$ for the flight from Seattle to New York is the sum of a displacement vector $\vec{A}$ from Seattle to Dallas and a displacement vector $\vec{B}$ from Dallas to New York:

$$\vec{C} = \vec{A} + \vec{B}. \tag{1.9}$$

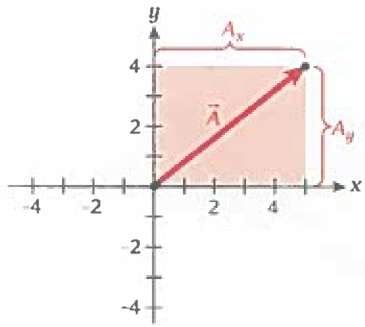


FIGURE 1.17 Cartesian components of vector $\vec{A}$ in two dimensions.

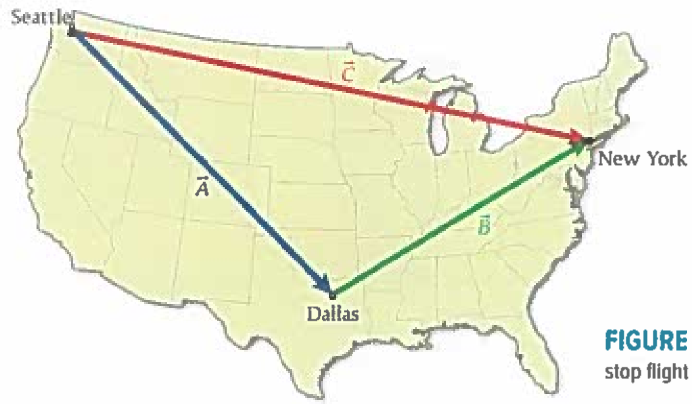


FIGURE 1.18 Direct flight versus one-stop flight as an example of vector addition.

This example shows the general procedure for vector addition in a graphical way: Move the tail of vector $\vec{B}$ to the head of vector $\vec{A}$; then the vector from the tail of vector $\vec{A}$ to the head of vector $\vec{B}$ is the sum vector, or **resultant,** of the two.

If you add two real numbers, the order does not matter: $3+5=5+3$. This property is called the *commutative property of addition*. Vector addition is also commutative:

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}. \tag{1.10}$$

Figure 1.19 demonstrates this commutative property of vector addition graphically. It shows the same vectors as in Figure 1.18, but also shows the beginning of vector $\vec{A}$ moved to the tip of vector $\vec{B}$ (dashed arrows)–note that the resultant vector is the same as before.

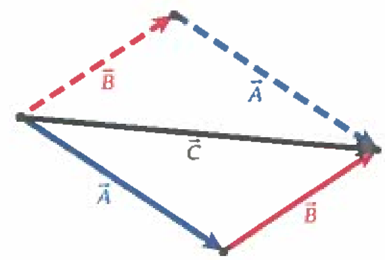


FIGURE 1.19 Commutative property of vector addition.

Next, the inverse (or reverse or negative) vector, $-\vec{C}$, of the vector $\vec{C}$ is a vector with the same length as $\vec{C}$ but pointing in the opposite direction (Figure 1.20). For the vector representing the flight from Seattle to New York, for example, the inverse vector is the return trip. Clearly, if you add $\vec{C}$ and its inverse vector, $-\vec{C}$, you end up at the point you started from. Thus, we find

$$\vec{C}+(-\vec{C})=\vec{C}-\vec{C}=(0,0,0), \tag{1.11}$$

and the magnitude is zero, $\left|\vec{C}-\vec{C}\right|=0$. This seemingly simple identity shows that we can treat vector subtraction as vector addition, by simply adding the inverse vector. For example, the vector $\vec{B}$ in Figure 1.19 can be obtained as $\vec{B}=\vec{C}-\vec{A}$. Therefore, vector addition and subtraction follow exactly the same rules as the addition and subtraction of real numbers.

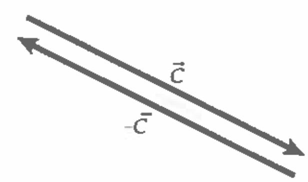


FIGURE 1.20 Inverse vector $-\vec{C}$ of a vector $\vec{C}$.

Vector Addition Using Components

Graphical vector addition illustrates the concepts very well, but for practical purposes the component method of vector addition is much more useful. (This is because calculators are easier to use and much more precise than rulers and graph paper.) Let's consider the component method for addition of three-dimensional vectors. The equations for two-dimensional vectors are special cases that arise by neglecting the z-components. Similarly, the one-dimensional equation can be obtained by neglecting all y- and z-components.

If you add two three-dimensional vectors, $\vec{A}=(A_x, A_y, A_z)$ and $\vec{B}=(B_x, B_y, B_z)$, the resulting vector is

$$\vec{C}=\vec{A}+\vec{B}=(A_x, A_y, A_z)+(B_x, B_y, B_z)=(A_x+B_x, A_y+B_y, A_z+B_z). \tag{1.12}$$

In other words, the components of the sum vector are the sums of the components of the individual vectors:

$$\begin{aligned} C_x &= A_x + B_x \\ C_y &= A_y + B_y \\ C_z &= A_z + B_z. \end{aligned} \tag{1.13}$$

The relationship between graphical and component methods is illustrated in Figure 1.21. Figure 1.21a shows two vectors $\vec{A}=(4, 2)$ and $\vec{B}=(3, 4)$ in two-dimensional space, and

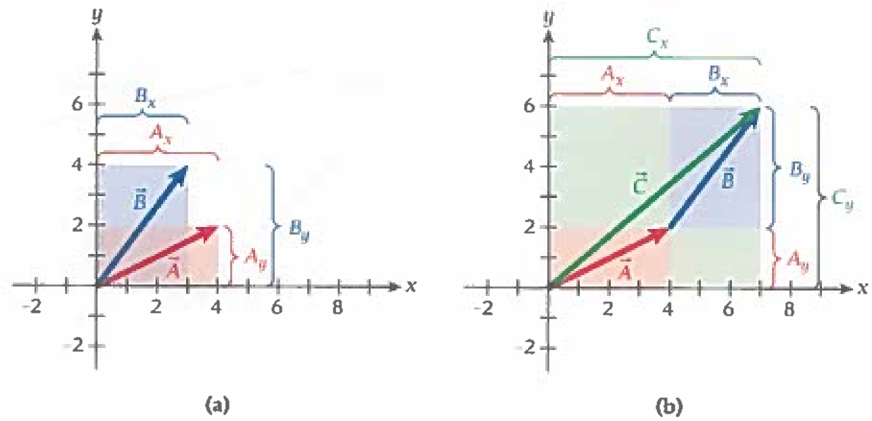


FIGURE 1.21 Vector addition by components. (a) Components of vectors $\vec{A}$ and $\vec{B}$; (b) the components of the resultant vector are the sums of the components of the individual vectors.

Figure 1.21b displays their sum vector $\vec{C}=(4+3, 2+4)=(7,6)$. Figure 1.21b clearly shows that $C_x=A_x + B_x$, because the whole is equal to the sum of its parts.

In the same way, we can take the difference $\vec{D}=\vec{A}-\vec{B}$, and the Cartesian components of the difference vector are given by

$$\begin{aligned} D_x &= A_x - B_x \\ D_y &= A_y - B_y \\ D_z &= A_z - B_z. \end{aligned} \qquad (1.14)$$

Multiplication of a Vector with a Scalar

What is $\vec{A}+\vec{A}+\vec{A}$? If your answer to this question is $3\vec{A}$, you already understand multiplying a vector with a scalar. The vector that results from multiplying the vector $\vec{A}$ with the scalar 3 is a vector that points in the same direction as the original vector $\vec{A}$ but is 3 times as long.

Multiplication of a vector with an arbitrary positive scalar–that is, a positive number–results in another vector that points in the same direction but has a magnitude that is the product of the magnitude of the original vector and the value of the scalar. Multiplication of a vector by a negative scalar results in a vector pointing in the opposite direction to the original with a magnitude that is the product of the magnitude of the original vector and the magnitude of the scalar.

Again, the component notation is useful. For the multiplication of a vector $\vec{A}$ with a scalar s, we obtain:

$$\vec{E}=s\vec{A}=s(A_x, A_y, A_z)=(sA_x, sA_y, sA_z). \qquad (1.15)$$

In other words, each component of the vector $\vec{A}$ is multiplied by the scalar in order to arrive at the components of the product vector:

$$\begin{aligned} E_x &= sA_x \\ E_y &= sA_y \\ E_z &= sA_z. \end{aligned} \qquad (1.16)$$

Unit Vectors

There is a set of special vectors that make much of the math associated with vectors easier. Called **unit vectors,** they are vectors of magnitude 1 directed along the main coordinate axes of the coordinate system. In two dimensions, these vectors point in the positive x-direction and the positive y-direction. In three dimensions, a third unit vector points in the positive z-direction. In order to distinguish these as unit vectors, we give them the symbols $\hat{x}$, $\hat{y}$, and $\hat{z}$. Their component representation is

$$\begin{aligned} \hat{x} &= (1,0,0) \\ \hat{y} &= (0,1,0) \\ \hat{z} &= (0,0,1). \end{aligned} \qquad (1.17)$$

Figure 1.22a shows the unit vectors in two dimensions, and Figure 1.22b shows the unit vectors in three dimensions.

What is the advantage of unit vectors? We can write any vector as a sum of these unit vectors instead of using the component notation; each unit vector is multiplied by the corresponding Cartesian component of the vector:

$$\begin{aligned}\vec{A} &= (A_x, A_y, A_z) \\ &= (A_x, 0, 0) + (0, A_y, 0) + (0, 0, A_z) \\ &= A_x(1,0,0) + A_y(0,1,0) + A_z(0,0,1) \\ &= A_x\hat{x} + A_y\hat{y} + A_z\hat{z}.\end{aligned} \tag{1.18}$$

In two dimensions, we have

$$\vec{A} = A_x\hat{x} + A_y\hat{y}. \tag{1.19}$$

This unit vector representation of a general vector will be particularly useful for multiplying two vectors.

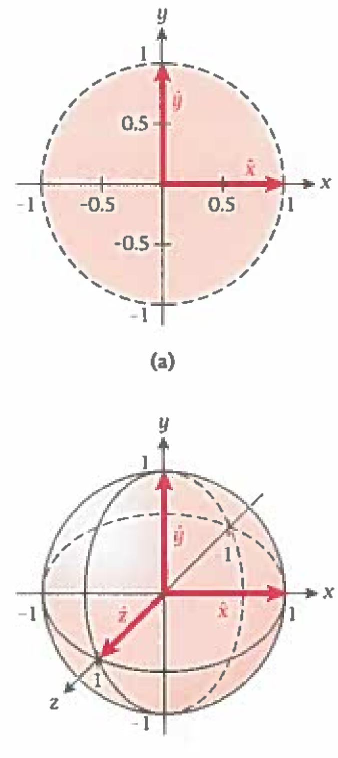


FIGURE 1.22 Cartesian unit vectors in (a) two and (b) three dimensions.

Vector Length and Direction

If we know the component representation of a vector, how can we find its length (magnitude) and the direction it is pointing in? Let's look at the most important case: a vector in two dimensions. In two dimensions, a vector $\vec{A}$ can be specified uniquely by giving the two Cartesian components, A_x and A_y. We can also specify the same vector by giving two other numbers: its length A and its angle θ with respect to the positive x-axis.

Let's take a look at Figure 1.23 to see how we can determine A and θ from A_x and A_y. Figure 1.23a shows the graphical representation of equation 1.19. The vector $\vec{A}$ is the sum of the vectors $A_x\hat{x}$ and $A_y\hat{y}$. Since the unit vectors $\hat{x}$ and $\hat{y}$ are by definition orthogonal to each other, these vectors form a 90° angle. Thus, the three vectors $\vec{A}$, $A_x\hat{x}$, and $A_y\hat{y}$ form a right triangle with side lengths A, A_x, and A_y, as shown in Figure 1.23b.

Now we can employ basic trigonometry to find θ and A. Using the Pythagorean Theorem results in

$$A = \sqrt{A_x^2 + A_y^2}. \tag{1.20}$$

We can find the angle θ from the definition of the tangent function

$$\theta = \tan^{-1}\frac{A_y}{A_x}. \tag{1.21}$$

In using equation 1.21, you must be careful that θ is in the correct quadrant. We can also invert equations 1.20 and 1.21 to obtain the Cartesian components of a vector of given length and direction:

$$A_x = A\cos\theta \tag{1.22}$$

$$A_y = A\sin\theta. \tag{1.23}$$

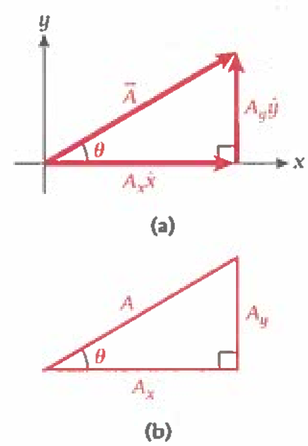


FIGURE 1.23 Length and direction of a vector. (a) Cartesian components A_x and A_y; (b) length A and angle θ.

You will encounter these trigonometric relations again and again throughout introductory physics. If you need to refamiliarize yourself with trigonometry, consult the mathematics primer provided in Appendix A.

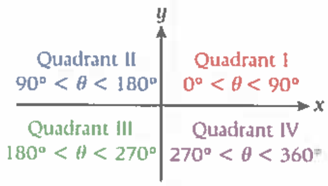
Scalar Product of Vectors

Above we saw how to multiply a vector with a scalar. Now we will define one way of multiplying a vector with a vector and obtain the **scalar product.** The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\alpha, \tag{1.24}$$

Concept Check 1.6

Into which quadrant do each of the following vectors point?



a) $A = (A_x, A_y)$ with $A_x = 1.5$ cm, $A_y = -1.0$ cm

b) a vector with length 2.3 cm and direction angle 131°

c) the inverse vector of $B = (0.5$ cm, 1.0 cm$)$

d) the sum of the unit vectors in the x- and y-directions

FIGURE 1.24 Two vectors $\vec{A}$ and $\vec{B}$ and the angle α between them.

where α is the angle between the vectors $\vec{A}$ and $\vec{B}$, as shown in Figure 1.24. Note the use of the larger dot (•) as the multiplication sign for the scalar product between vectors, in contrast to the smaller dot (·) that is used for the multiplication of scalars. Because of the dot, the scalar product is often referred to as the *dot product*.

If two vectors form a 90° angle, then the scalar product has the value zero. In this case, the two vectors are orthogonal to each other. The scalar product of a pair of orthogonal vectors is zero.

If $\vec{A}$ and $\vec{B}$ are given in Cartesian coordinates as $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$, then their scalar product can be shown to be equal to:

$$\vec{A} \bullet \vec{B} = (A_x, A_y, A_z) \bullet (B_x, B_y, B_z) = A_x B_x + A_y B_y + A_z B_z. \tag{1.25}$$

From equation 1.25, we can see that the scalar product has the commutative property:

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}. \tag{1.26}$$

This result is not surprising, since the commutative property also holds for the multiplication of two scalars.

For the scalar product of any vector with itself, we have, in component notation, $\vec{A} \bullet \vec{A} = A_x^2 + A_y^2 + A_z^2$. Then, from equation 1.24, we find $\vec{A} \bullet \vec{A} = |\vec{A}|\,|\vec{A}|\cos\alpha = |\vec{A}|\,|\vec{A}| = |\vec{A}|^2$ (because the angle between the vector $\vec{A}$ and itself is zero, and the cosine of that angle has the value 1). Combining these two equations, we obtain the expression for the length of a vector that was introduced in the previous subsection:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}. \tag{1.27}$$

We can also use the definition of the scalar product to compute the angle between two arbitrary vectors in three-dimensional space:

$$\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}|\cos\alpha \Rightarrow \cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|} \Rightarrow \alpha = \cos^{-1}\left(\frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|}\right). \tag{1.28}$$

For the scalar product, the same distributive property that is valid for the conventional multiplication of numbers holds:

$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}. \tag{1.29}$$

The following example puts the scalar product to use.

EXAMPLE 1.5 Angle Between Two Position Vectors

PROBLEM

What is the angle α between the two position vectors shown in Figure 1.25, $\vec{A} = (4.00, 2.00, 5.00)$ cm and $\vec{B} = (4.50, 4.00, 3.00)$ cm?

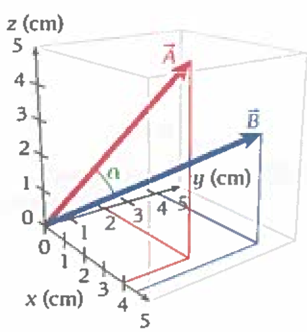


FIGURE 1.25 Calculating the angle between two position vectors.

SOLUTION

To solve this problem, we have to put the numbers for the components of each of the two vectors into equation 1.27 and equation 1.25 then use equation 1.28:

$$|A| = \sqrt{4.00^2 + 2.00^2 + 5.00^2}\ \text{cm} = 6.71\ \text{cm}$$

$$|B| = \sqrt{4.50^2 + 4.00^2 + 3.00^2}\ \text{cm} = 6.73\ \text{cm}$$

$$A \bullet B = A_x B_x + A_y B_y + A_z B_z = (4.00{\times}4.50 + 2.00{\times}4.00 + 5.00{\times}3.00)\ \text{cm}^2 = 41.0\ \text{cm}^2$$

$$\alpha = \cos^{-1}\frac{41.0\ \text{cm}^2}{6.71\ \text{cm}{\times}6.73\ \text{cm}} = 24.7°.$$

Scalar Product for Unit Vectors. On page 26 we introduced unit vectors in the three-dimensional Cartesian coordinate system: $\hat{x} = (1,0,0)$, $\hat{y} = (0,1,0)$, and $\hat{z} = (0,0,1)$. With our definition (1.25) of the scalar product, we find

$$\hat{x} \bullet \hat{x} = \hat{y} \bullet \hat{y} = \hat{z} \bullet \hat{z} = 1 \tag{1.30}$$

and

$$\begin{gathered} \hat{x} \bullet \hat{y} = \hat{x} \bullet \hat{z} = \hat{y} \bullet \hat{z} = 0 \\ \hat{y} \bullet \hat{x} = \hat{z} \bullet \hat{x} = \hat{z} \bullet \hat{y} = 0. \end{gathered} \tag{1.31}$$

Now we see why the unit vectors are called that: Their scalar products with themselves have the value 1. Thus, the unit vectors have length 1, or unit length, according to equation 1.27. In addition, any pair of different unit vectors has a scalar product that is zero, meaning that these vectors are orthogonal to each other. Equations 1.30 and 1.31 thus state that the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ form an orthonormal set of vectors, which makes them extremely useful for the description of physical systems.

Self-Test Opportunity 1.1

Show that equations 1.30 and 1.31 are correct by using equation 1.25 and the definitions of the unit vectors.

Geometrical Interpretation of the Scalar Product. In the definition of the scalar product $\vec{A} \bullet \vec{B} = |\vec{A}|\,|\vec{B}| \cos\alpha$ (equation 1.24), we can interpret $|\vec{A}|\cos\alpha$ as the projection of the vector $\vec{A}$ onto the vector $\vec{B}$ (Figure 1.26a). In this drawing, the line $|\vec{A}|\cos\alpha$ is rotated by 90° to show the geometrical interpretation of the scalar product as the area of a rectangle with sides $|\vec{A}|\cos\alpha$ and $|\vec{B}|$. In the same way, we can interpret $|\vec{B}|\cos\alpha$ as the projection of the vector $\vec{B}$ onto the vector $\vec{A}$ and construct a rectangle with side lengths $|\vec{B}|\cos\alpha$ and $|\vec{A}|$ (Figure 1.26b). The areas of the two yellow rectangles in Figure 1.25 are identical and are equal to the scalar product of the two vectors $\vec{A}$ and $\vec{B}$.

Finally, if we substitute from equation 1.28 for the cosine of the angle between the two vectors, the projection $|\vec{A}|\cos\alpha$ of the vector $\vec{A}$ onto the vector $\vec{B}$ can be written as

$$|\vec{A}|\cos\alpha = |\vec{A}|\frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{\vec{A} \bullet \vec{B}}{|\vec{B}|},$$

and the projection $|\vec{B}|\cos\alpha$ of the vector $\vec{B}$ onto the vector $\vec{A}$ can be expressed as

$$|\vec{B}|\cos\alpha = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}|}.$$

Vector Product

The **vector product** (or cross product) between two vectors $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ is defined as

$$\begin{aligned} \vec{C} &= \vec{A} \times \vec{B} \\ C_x &= A_y B_z - A_z B_y \\ C_y &= A_z B_x - A_x B_z \\ C_z &= A_x B_y - A_y B_x. \end{aligned} \tag{1.32}$$

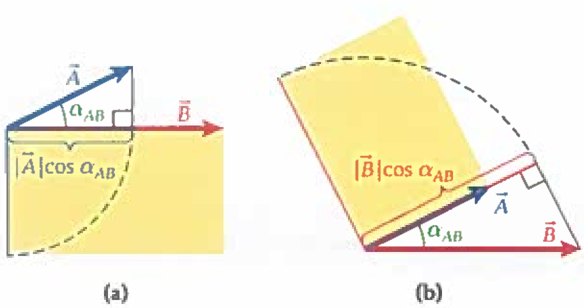


FIGURE 1.26 Geometrical interpretation of the scalar product as an area. (a) The projection of $\vec{A}$ onto $\vec{B}$. (b) The projection of $\vec{B}$ onto $\vec{A}$.

In particular, for the vector products of the Cartesian unit vectors, this definition implies

$$\begin{aligned}\hat{x}\times\hat{y}&=\hat{z}\\ \hat{y}\times\hat{z}&=\hat{x}\\ \hat{z}\times\hat{x}&=\hat{y}.\end{aligned}\tag{1.33}$$

The absolute magnitude of the vector $\vec{C}$ is given by

$$\left|\vec{C}\right|=\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta.\tag{1.34}$$

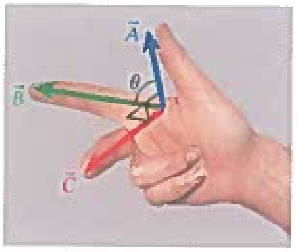


FIGURE 1.27 Vector product.

Here θ is the angle between $\vec{A}$ and $\vec{B}$, as shown in Figure 1.27. This result implies that the magnitude of the vector product of two vectors is at its maximum when $\vec{A}\perp\vec{B}$ and is zero when $\vec{A}\parallel\vec{B}$. We can also interpret the right-hand side of this equation as either the product of the magnitude of vector $\vec{A}$ times the component of $\vec{B}$ perpendicular to $\vec{A}$ or the product of the magnitude of $\vec{B}$ times the component of $\vec{A}$ perpendicular to $\vec{B}$: $\left|\vec{C}\right|=\left|\vec{A}\right|B_{\perp A}=\left|\vec{B}\right|A_{\perp B}$. Either interpretation is valid.

The direction of the vector $\vec{C}$ can be found using the right-hand rule: If vector $\vec{A}$ points along the direction of the thumb and vector $\vec{B}$ points along the direction of the index finger, then the vector product is perpendicular to *both* vectors and points along the direction of the middle finger, as shown in Figure 1.27.

It is important to realize that for the vector product, the order of the factors matters:

$$\vec{B}\times\vec{A}=-\vec{A}\times\vec{B}.\tag{1.35}$$

Thus, the vector product differs from both regular multiplication of scalars and multiplication of vectors to form a scalar product.

We'll see immediately from the definition of the vector product that for any vector $\vec{A}$, the vector product with itself is always zero:

$$\vec{A}\times\vec{A}=0.\tag{1.36}$$

Finally, there is a handy rule for a double vector product involving three vectors: The vector product of the vector $\vec{A}$ with the vector product of the vectors $\vec{B}$ and $\vec{C}$ is the sum of two vectors, one pointing in the direction of the vector $\vec{B}$ and multiplied by the scalar product $\vec{A}\bullet\vec{C}$ and another one pointing in the direction of the vector $\vec{C}$ and multiplied by $-\vec{A}\bullet\vec{B}$:

$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\bullet\vec{C})-\vec{C}(\vec{A}\bullet\vec{B}).\tag{1.37}$$

This *BAC-CAB rule* is straightforward to prove using the Cartesian components in the definitions of the vector product and the scalar product, but the proof is cumbersome and thus omitted here. However, the rule occasionally comes in handy, in particular when dealing with torque and angular momentum. And the mnemonic BAC-CAB makes it fairly easy to remember.

SOLVED PROBLEM 1.3 Hiking

PROBLEM

You are hiking in the Florida Everglades heading southwest from your base camp, for 1.72 km. You reach a river that is too deep to cross; so you make a 90° right turn and hike another 3.12 km to a bridge. How far away are you from your base camp?

SOLUTION

THINK If you are hiking, you are moving in a two-dimensional plane: the surface of Earth (because the Everglades are flat). Thus, we can use two-dimensional vectors to characterize the various segments of the hike. Making one straight-line hike, then performing a turn, followed by another straight-line hike amounts to a problem of vector addition that is asking for the length of the resultant vector.

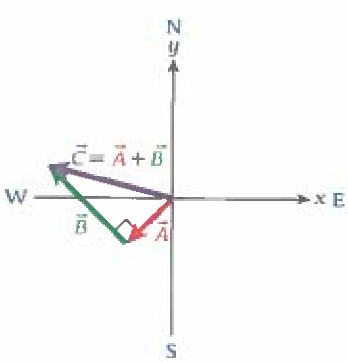


FIGURE 1.28 Hike with a 90° turn.

SKETCH Figure 1.28 presents a coordinate system in which the y-axis points north and the x-axis points east, as is conventional. The first portion of the hike, in the southwestern direction, is indicated by the vector $\vec{A}$, and the second portion by the vector $\vec{B}$. The figure also shows the resultant vector, $\vec{C}=\vec{A}+\vec{B}$, for which we want to determine the length.

RESEARCH If you have drawn the sketch with sufficient accuracy, making the lengths of the vectors in your drawing to be proportional to the lengths of the segments of the hike (as was done in Figure 1.28), then you can measure the length of the vector $\vec{C}$ to determine the distance from your base-camp at the end of the second segment of the hike. However, the given distances are specified to three significant digits, so the answer should also have three significant digits. Thus, we cannot rely on the graphical method but must use the component method of vector addition.

In order to calculate the components of the vectors, we need to know their angles relative to the positive x-axis. For the vector $\vec{A}$, which points southwest, this angle is θ_A=225°, as shown in Figure 1.29. The vector $\vec{B}$ has an angle of 90° relative to $\vec{A}$, and thus θ_B=135° relative to the positive x-axis. To make this point clearer, the starting point of $\vec{B}$ has been moved to the origin of the coordinate system in Figure 1.29. (Remember: We can move vectors around at will. As long as we leave the direction and length of a vector the same, the vector remains unchanged.)

Now we have everything in place to start our calculation. We have the lengths and directions of both vectors, allowing us to calculate their Cartesian components. Then, we will add their components to calculate the components of the vector $\vec{C}$, from which we can calculate the length of this vector.

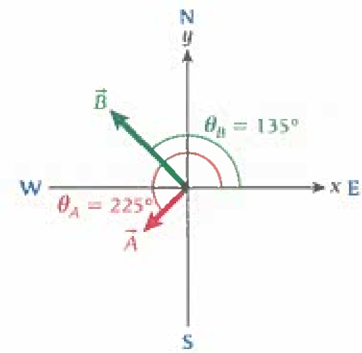


FIGURE 1.29 Angles of the two hike segments.

SIMPLIFY The components of the vector $\vec{C}$ are:

$$C_x = A_x + B_x = A\cos\theta_A + B\cos\theta_B$$
$$C_y = A_y + B_y = A\sin\theta_A + B\sin\theta_B.$$

Thus, the length of the vector $\vec{C}$ is (compare with equation 1.20)

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{(A\cos\theta_A + B\cos\theta_B)^2 + (A\sin\theta_A + B\sin\theta_B)^2}.$$

CALCULATE Now all that is left is to put in the numbers to obtain the vector length:

$$C = \sqrt{((1.72\text{ km})\cos 225° + (3.12\text{ km})\cos 135°)^2 + ((1.72\text{ km})\sin 225° + (3.12\text{ km})\sin 135°)^2}$$
$$= \sqrt{\left(1.72\times(-\sqrt{1/2}) + 3.12\times(-\sqrt{1/2})\right)^2 + \left(1.72\times(-\sqrt{1/2}) + 3.12\times\sqrt{1/2}\right)^2}\text{ km.}$$

Entering these numbers into a calculator, we obtain:

$$C = 3.562695609\text{ km.}$$

ROUND Because the initial distances were given to three significant figures, our final answer should also have (at most) the same precision. Rounding to three significant figures yields our final answer:

$$C = 3.56\text{ km.}$$

DOUBLE-CHECK This problem was intended to provide practice with vector concepts. However, if you forget for a moment that the displacements are vectors and note that they form a right triangle, you can immediately calculate the length of side C from the Pythagorean Theorem as follows:

$$C = \sqrt{A^2 + B^2} = \sqrt{1.72^2 + 3.12^2}\text{ km} = 3.56\text{ km.}$$

Here we also rounded our result to three significant figures, and we see that it agrees with the answer obtained using the longer procedure of vector addition.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- Large and small numbers can be represented using scientific notation, consisting of a mantissa and a power of ten.
- Physical systems are described by the SI system of units. These units are based on reproducible standards and provide convenient methods of scaling and calculation. The base units of the SI system include meter (m), kilogram (kg), second (s), and ampere (A).
- Physical systems have widely varying sizes, masses, and time scales, but the same physical laws govern all of them.
- A number (with a specific number of significant figures) or a set of numbers (such as components of a vector) must be combined with a unit or units to describe physical quantities.
- Vectors in three dimensions can be specified by their three Cartesian components, $\vec{A}=(A_x, A_y, A_z)$. Each of these Cartesian components is a number.
- Vectors can be added or subtracted. In Cartesian components, $\vec{C}=\vec{A}+\vec{B}=(A_x, A_y, A_z)+(B_x, B_y, B_z)$ $=(A_x+B_x, A_y+B_y, A_z+B_z)$.
- Multiplication of a vector with a scalar results in another vector in the same or opposite direction but of different magnitude, $\vec{E}=s\vec{A}=s(A_x, A_y, A_z)=(sA_x, sA_y, sA_z)$.
- Unit vectors are vectors of length 1. The unit vectors in Cartesian coordinate systems are denoted by $\hat{x}$, $\hat{y}$, and $\hat{z}$.
- The length and direction of a two-dimensional vector can be determined from its Cartesian components: $A=\sqrt{A_x^2+A_y^2}$ and $\theta=\tan^{-1}(A_y/A_x)$.
- The Cartesian components of a two-dimensional vector can be calculated from the vector's length and angle with respect to the x-axis: $A_x=A\cos\theta$ and $A_y=A\sin\theta$.
- The scalar product, or dot product, of two vectors yields a scalar quantity and is defined as $\vec{A}\bullet\vec{B}=A_xB_x+A_yB_y+A_zB_z$.
- The vector product, or cross product, of two vectors yields another vector and is defined as $\vec{A}\times\vec{B}=\vec{C}=(A_yB_z-A_zB_y, A_zB_x-A_xB_z, A_xB_y-A_yB_x)$.

ANSWERS TO SELF-TEST OPPORTUNITIES

1.1 Equation 1.30

$$\hat{x}\bullet\hat{x}=(1,0,0)\bullet(1,0,0)=1\cdot1+0\cdot0+0\cdot0=1$$
$$\hat{y}\bullet\hat{y}=(0,1,0)\bullet(0,1,0)=0\cdot0+1\cdot1+0\cdot0=1$$
$$\hat{z}\bullet\hat{z}=(0,0,1)\bullet(0,0,1)=0\cdot0+0\cdot0+1\cdot1=1$$

Equation 1.31

$$\hat{x}\bullet\hat{y}=(1,0,0)\bullet(0,1,0)=1\cdot0+0\cdot1+0\cdot0=0$$
$$\hat{x}\bullet\hat{z}=(1,0,0)\bullet(0,0,1)=1\cdot0+0\cdot0+0\cdot1=0$$
$$\hat{y}\bullet\hat{z}=(0,1,0)\bullet(0,0,1)=0\cdot0+1\cdot0+0\cdot1=0$$
$$\hat{y}\bullet\hat{x}=(0,1,0)\bullet(1,0,0)=0\cdot1+1\cdot0+0\cdot0=0$$
$$\hat{z}\bullet\hat{x}=(0,0,1)\bullet(1,0,0)=0\cdot1+0\cdot0+1\cdot0=0$$
$$\hat{z}\bullet\hat{y}=(0,0,1)\bullet(0,1,0)=0\cdot0+0\cdot1+1\cdot0=0$$

PROBLEM-SOLVING GUIDELINES: NUMBERS, UNITS, AND VECTORS

1. Try to use our seven-step strategy for problem solving, even if you have no idea how to arrive at the final solution. Sometimes the process of making a sketch can give you a hint about what to do next.

2. In general, you should try to convert all given units to SI units before you start working with numbers. Working with quantities in SI units makes computations easier.

3. In most situations, the number of significant figures your final solution should be rounded to is the number of significant figures in the least precisely given quantity.

4. Estimating a solution for a problem can be very useful for obtaining an idea of the order of magnitude of the solution. Often, estimating can be used as a double-check.

5. In working with vectors, you should generally use the Cartesian coordinate system. Make use of your knowledge of trigonometry when solving vector problems!

6. The graphical method for vector addition and subtraction is useful for making sketches. But the component method is more precise, and thus preferred if you have to arrive at a numerical answer.

MULTIPLE-CHOICE QUESTIONS

1.1 Which of the following is the frequency of the musical note C5?

a) 376 g b) 483 m/s c) 523 Hz d) 26.5 J

1.2 If $\vec{A}$ and $\vec{B}$ are vectors and $\vec{B} = -\vec{A}$, which of the following is true?

a) The magnitude of $\vec{B}$ is equal to the negative of the magnitude of $\vec{A}$.

b) $\vec{A}$ and $\vec{B}$ are perpendicular.

c) The direction angle of $\vec{B}$ is equal to the direction angle of $\vec{A}$ plus 180°.

d) $\vec{A} + \vec{B} = 2\vec{A}$.

1.3 Compare three SI units: millimeter, kilogram, and microsecond. Which is the largest?

a) millimeter c) microsecond

b) kilogram d) The units are not comparable.

1.4 What is(are) the difference(s) between 3.0 and 3.0000?

a) 3.0000 could be the result from an intermediate step in a calculation; 3.0 has to result from a final step.

b) 3.0000 represents a quantity that is known more precisely than 3.0.

c) There is no difference.

d) They convey the same information, but 3.0 is preferred for ease of writing.

1.5 A speed of 7 mm/μs is equal to ___.

a) 7000 m/s b) 70 m/s c) 7 m/s d) 0.07 m/s

1.6 A round object, whose diameter is approximately 3 centimeters, is to be used to determine the value of π to three significant figures by carefully measuring its diameter and its circumference. For this calculation to be done properly, the measurements must be made to the nearest ___.

a) hundredth of a mm c) mm e) in

b) tenth of a mm d) cm

1.7 What is the sum of $5.786 \cdot 10^3$ m and $3.19 \cdot 10^4$ m?

a) 6.02×10^{23} m c) 8.976×10^3 m

b) 3.77×10^4 m d) 8.98×10^3 m

1.8 What is the number of carbon atoms in 0.5 nanomoles of carbon? One mole contains 6.02×10^{23} atoms.

a) 3.2×10^{14} atoms e) 3.19×10^{17} atoms

b) 3.19×10^{14} atoms f) $3. \times 10^{17}$ atoms

c) $3. \times 10^{14}$ atoms

d) 3.2×10^{17} atoms

1.9 The resultant of the two-dimensional vectors (1.5 m, 0.7 m), (−3.2 m, 1.7 m), and (1.2 m, −3.3 m) lies in quadrant ___.

a) I b) II c) III d) IV

1.10 By how much does the volume of a cylinder change if the radius is halved and the height is doubled?

a) The volume is quartered. d) The volume doubles.

b) The volume is cut in half. e) The volume quadruples.

c) There is no change in the volume.

1.11 How is the number 0.009834 expressed in scientific notation?

a) 9.834×10^4 c) 9.834×10^3

b) 9.834×10^{-4} d) 9.834×10^{-3}

1.12 How many significant figures does the number 0.4560 have?

a) five c) three e) one

b) four d) two

1.13 How many watts are in 1 gigawatt (GW)?

a) 10^3 c) 10^9 e) 10^{15}

b) 10^6 d) 10^{12}

1.14 What is the limit of $\gamma = 1/\sqrt{1-(v/c)^2}$, where c is a constant and $v \rightarrow 0$?

a) $\gamma = 1$ c) $\gamma = 2$ e) $\gamma = v/2$

b) $\gamma = 0$ d) $\gamma = v$

1.15 For the two vectors $\vec{A} = (2, 1, 0)$ and $\vec{B} = (0, 1, 2)$, what is their scalar product, $\vec{A} \bullet \vec{B}$?

a) 3 b) 6 c) 2 d) 0 e) 1

1.16 For the two vectors $\vec{A} = (2, 1, 0)$ and $\vec{B} = (0, 1, 2)$, what is their vector product, $\vec{A} \times \vec{B}$?

a) (2, −4, 2) c) (2, 0, 2) e) (0, 0, 0)

b) (1, 0, 1) d) (3, −2, 1)

CONCEPTUAL QUESTIONS

1.17 In Europe, cars' gas consumption is measured in liters per 100 kilometers. In the United States, the unit used is miles per gallon.

a) How are these units related?

b) How many miles per gallon does your car get if it consumes 12.2 liters per 100 kilometers?

c) What is your car's gas consumption in liters per 100 kilometers if it gets 27.4 miles per gallon?

d) Can you draw a curve plotting miles per gallon versus liters per 100 kilometers? If yes, draw the curve.

1.18 If you draw a vector on a sheet of paper, how many components are required to describe it? How many components does a vector in real space have? How many components would a vector have in a four-dimensional world?

1.19 Since vectors in general have more than one component and thus more than one number is used to describe them, they are obviously more difficult to add and subtract than single numbers. Why then work with vectors at all?

1.20 If $\vec{A}$ and $\vec{B}$ are vectors specified in magnitude-direction form, and $\vec{C} = \vec{A} + \vec{B}$ is to be found and to be expressed in magnitude-direction form, how is this done? That is, what is the procedure for adding vectors that are given in magnitude-direction form?

1.21 Suppose you solve a problem and your calculator's display reads 0.0000000036. Why not just write this down? Is there any advantage to using the scientific notation?

1.22 Since the British system of units is more familiar to most people in the United States, why is the international (SI) system of units used for scientific work in the United States?

1.23 Is it possible to add three equal-length vectors and obtain a vector sum of zero? If so, sketch the arrangement of the three vectors. If not, explain why not.

1.24 Is mass a vector quantity? Why or why not?

1.25 Two flies sit exactly opposite each other on the surface of a spherical balloon. If the balloon's volume doubles, by what factor does the distance between the flies change?

1.26 What is the ratio of the volume of a cube of side r to that of a sphere of radius r? Does your answer depend on the particular value of r?

1.27 Consider a sphere of radius r. What is the length of a side of a cube that has the same surface area as the sphere?

1.28 The mass of the Sun is 2×10^{30} kg, and the Sun contains more than 99% of all the mass in the Solar System. Astronomers estimate there are approximately 100 billion stars in the Milky Way and approximately 100 billion galaxies in the universe. The Sun and other stars are predominantly composed of hydrogen; a hydrogen atom has a mass of approximately 2×10^{-27} kg.

a) Assuming that the Sun is an average star and the Milky Way is an average galaxy, what is the total mass of the universe?

b) Since the universe consists mainly of hydrogen, can you estimate the total number of atoms in the universe?

1.29 A futile task is proverbially said to be "like trying to empty the ocean with a teaspoon." Just how futile is such a task? Estimate the number of teaspoonfuls of water in the Earth's oceans.

1.30 The world's population passed 6.5 billion in 2006. Estimate the amount of land area required if each person were to stand in such a way as to be unable to touch another person. Compare this area to the land area of your home state (or country).

1.31 Advances in the field of nanotechnology have made it possible to construct chains of single metal atoms linked one to the next. Physicists are particularly interested in the ability of such chains to conduct electricity with little resistance. Estimate how many gold atoms would be required to make such a chain long enough to wear as a necklace. How many would be required to make a chain that encircled the Earth? If 1 mole of a substance is equivalent to roughly 6.022×10^{23} atoms, how many moles of gold are required for each necklace?

1.32 One of the standard clichés in physics courses is to talk about approximating a cow as a sphere. How large a sphere makes the best approximation to an average dairy cow? That is, estimate the radius of a sphere that has the same mass and density as a dairy cow.

1.33 Estimate the mass of your head. Assume that its density is that of water, 1000 kg/m^3.

1.34 Estimate the number of hairs on your head.

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 1.2

1.35 How many significant figures are in each of the following numbers?

a) 4.01
b) 4.010
c) 4
d) 2.00001
e) 0.00001
f) 2.1 − 1.10042
g) 7.01×3.1415

1.36 Two different forces, acting on the same object, are measured. One force is 2.0031 N and the other force, in the same direction, is 3.12 N. These are the only forces acting on the object. Find the total force on the object *to the correct number of significant figures.*

1.37 Three quantities, the results of measurements, are to be added. They are 2.0600, 3.163, and 1.12. What is their sum *to the correct number of significant figures*?

1.38 Given the equation $w = xyz$, and $x = 1.1 \times 10^3$, $y = 2.48 \times 10^{-2}$, and $z = 6.000$, what is w, in scientific notation and with the correct number of significant figures?

1.39 Write this quantity in scientific notation: one ten-millionth of a centimeter.

1.40 Write this number in scientific notation: one hundred fifty-three million.

Section 1.3

1.41 How many centimeters are in 30.7484 kilometers?

1.42 What metric prefixes correspond to the following powers of 10?

a) 10^3
b) 10^{-2}
c) 10^{-3}

1.43 How many millimeters in a kilometer?

1.44 A hectare is a hundred ares, and an are is a hundred square meters. How many hectares are there in a square kilometer?

1.45 The unit of pressure in the SI system is the pascal. What is the SI name for 1 one-thousandth of a pascal?

1.46 The masses of four sugar cubes are measured to be 25.3 g, 24.7 g, 26.0 g, and 25.8 g. Express the answers to the following questions in scientific notation, with standard SI units and an appropriate number of significant figures.

a) If the four sugar cubes were crushed and all the sugar collected, what would be the total mass, in kilograms, of the sugar?

b) What is the average mass, in kilograms, of these four sugar cubes?

•1.47 What is the surface area of a right cylinder of height 20.5 cm and radius 11.9 cm?

Section 1.4

1.48 You step on your brand-new digital bathroom scale, and it reads 125.4 pounds. What is your mass in kilograms?

1.49 The distance from the center of the Moon to the center of the Earth ranges from approximately 356,000 km to 407,000 km.

a) What is the minimum distance to the Moon in meters?

b) What is the maximum distance to the Moon in meters?

1.50 In Major League baseball, the pitcher delivers his pitches from a distance of 60 feet, 6 inches from home plate. What is the distance in meters?

1.51 A flea hops in a straight path along a meter stick, starting at 0.7 cm and making successive jumps, which are measured to be 3.2 cm, 6.5 cm, 8.3 cm, 10.0 cm, 11.5 cm, and 15.5 cm. Express the answers to the following questions in scientific notation, with units of meters and an appropriate number of significant figures. What is the total distance covered by the flea in these six hops? What is the average distance covered by the flea in a single hop?

•1.52 One cubic centimeter of water has a mass of 1 gram. A milliliter is equal to a cubic centimeter. What is the mass, in kilograms, of a liter of water? A metric ton is a thousand kilograms. How many cubic centimeters of water are in a metric ton of water? If a metric ton of water were held in a thin-walled cubical tank, how long (in meters) would each side of the tank be?

•1.53 The speed limit on a particular stretch of road is 72.4 kilometers per hour. Express this speed limit in millifurlongs per microfortnight. A furlong is $\frac{1}{5}$ kilometer, and a fortnight is a period of 2 weeks.

•1.54 Calculate the weight of a pint of water in grams, assuming that the density of water is 1000. kg/m^3 and that the weight of 1.00 kg of a substance is 2.51 pounds. The volume of 1.00 fluid ounce is 29.6 mL. A pint is 16 fluid ounces.

Section 1.5

1.55 If the radius of a planet is larger than that of Earth by a factor of 8.7, how much bigger is the surface area of the planet than Earth's?

1.56 If the radius of a planet is larger than that of Earth by a factor of 5.8, how much bigger is the volume of the planet than Earth's?

1.57 What is the maximum distance from which a sailor on top of the mast of ship 1, at 34 m above the ocean's surface, can see another sailor on top of the mast of ship 2, at 26 m above the ocean's surface?

1.58 You are flying in a jetliner at an altitude of 10,668 m. How far away is the horizon?

1.59 How many cubic centimeters are in 1.56 barrels of oil?

1.60 A car's gasoline tank has the shape of a right rectangular box with a square base whose sides measure 62 cm. Its capacity is 52 L. If the tank has only 1.5 L remaining, how deep is the gasoline in the tank, assuming the car is parked on level ground?

•1.61 The volume of a sphere is given by the formula $\frac{4}{3}\pi r^3$, where r is the radius of the sphere. The average density of an object is simply the ratio of its mass to its volume. Using the numerical data found in Table 12.1, express the answers to the following questions in scientific notation, with SI units and an appropriate number of significant figures.

a) What is the volume of the Sun?

b) What is the volume of the Earth?

c) What is the average density of the Sun?

d) What is the average density of the Earth?

•1.62 A tank is in the shape of an inverted cone, having height $h = 2.5$ m and base radius $r = 0.75$ m. If water is poured into the tank at a rate of 15 L/s, how long will it take to fill the tank?

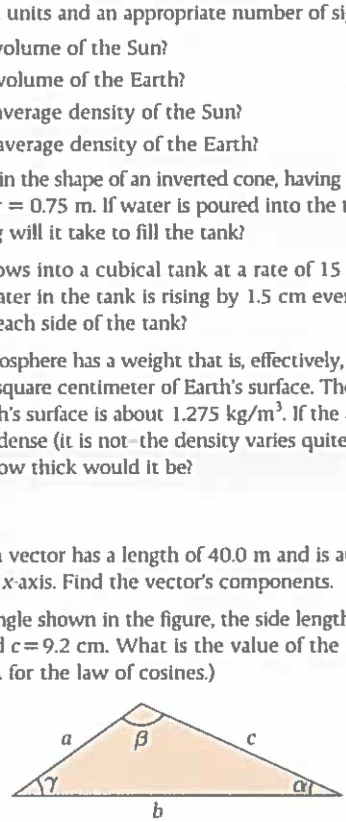
•1.63 Water flows into a cubical tank at a rate of 15 L/s. If the top surface of the water in the tank is rising by 1.5 cm every second, what is the length of each side of the tank?

••1.64 The atmosphere has a weight that is, effectively, about 6.8 kilograms for every square centimeter of Earth's surface. The average density of air at the Earth's surface is about 1.275 kg/m^3. If the atmosphere were uniformly dense (it is not–the density varies quite significantly with altitude), how thick would it be?

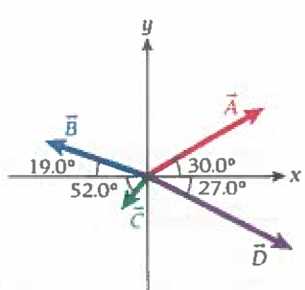
Section 1.6

1.65 A position vector has a length of 40.0 m and is at an angle of 57.0° above the x-axis. Find the vector's components.

1.66 In the triangle shown in the figure, the side lengths are $a = 6.6$ cm, $b = 13.7$ cm, and $c = 9.2$ cm. What is the value of the angle γ? (*Hint:* See Appendix A for the law of cosines.)



1.67 Find the components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, if their lengths are given by $A = 75.0$, $B = 60.0$, $C = 25.0$, $D = 90.0$ and their direction angles are as shown in the figure. Write the vectors in terms of unit vectors.



•1.68 Use the components of the vectors from Problem 1.67 to find

a) the sum $\vec{A} + \vec{B} + \vec{C} + \vec{D}$ in terms of its components

b) the magnitude and direction of the sum $\vec{A} - \vec{B} + \vec{D}$

•1.69 The Bonneville Salt Flats, located in Utah near the border with Nevada, not far from Interstate I80, cover an area of over 30,000 acres. A race car driver on the Flats first heads north for 4.47 km, then makes a sharp turn and heads southwest for 2.49 km, then makes another turn and heads east for 3.59 km. How far is he from where he started?

•1.70 A map in a ship's log gives directions to the location of a buried treasure. The starting location is an old oak tree. According to the map, the treasure's location is found by proceeding 20 paces north from the oak tree and then 30 paces northwest. At this location, an iron pin is sunk in the ground. From the iron pin, walk 10 paces south and dig. How far (in paces) from the oak tree is the spot at which digging occurs?

••1.71 The next page of the ship's log contains a set of directions that differ from those on the map in Problem 1.70. These say the treasure's location is found by proceeding 20 paces north from the old oak tree and then 30 paces northwest. After finding the iron pin, one should "walk 12 paces nor'ward and dig downward 3 paces to the treasure box." What is the vector that points from the base of the old oak tree to the treasure box? What is the length of this vector?

••1.72 The Earth's orbit has a radius of 1.5×10^{11} m, and that of Venus has a radius of 1.1×10^{11} m. Consider these two orbits to be perfect circles (though in reality they are ellipses with slight eccentricity). Write the direction and length of a vector from Earth to Venus (take the direction from Earth to Sun to be 0°) when Venus is at the maximum angular separation in the sky relative to the Sun.

••1.73 A friend walks away from you a distance of 550 m, and then turns an unknown angle, and walks an additional 178 m in the new direction. You use a laser range-finder to find out that his final distance from you is 432 m. What is the angle between his initial departure direction and the direction to his final location? Through what angle did he turn? (There are two possibilities.)

Additional Exercises

1.74 The radius of Earth is 6378. km. What is its circumference to three significant figures?

1.75 Estimate the product of 4,308,229 and 44 to one significant figure (show your work and do not use a calculator), and express the result in standard scientific notation.

1.76 Find the vector $\vec{C}$ that satisfies the equation $3\hat{x} + 6\hat{y} - 10\hat{z} + \vec{C} = -7\hat{x} + 14\hat{y}$.

1.77 A position vector has components $x = 34.6$ m and $y = -53.5$ m. Find the vector's length and angle with the x-axis.

1.78 For the planet Mars, calculate the distance around the Equator, the surface area, and the volume. The radius of Mars is $3.39 \cdot 10^6$ m.

•1.79 Find the magnitude and direction of (a) $9\vec{B} - 3\vec{A}$ and (b) $-5\vec{A} + 8\vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

•1.80 Express the vectors $\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$ by giving their magnitude and direction as measured from the positive x-axis.

•1.81 The force F that a spring exerts on you is directly proportional to the distance x that you stretch it beyond its resting length. Suppose that when you stretch a spring 8.00 cm, it exerts a force of 200. N on you. How much force will it exert on you if you stretch it 40.0 cm?

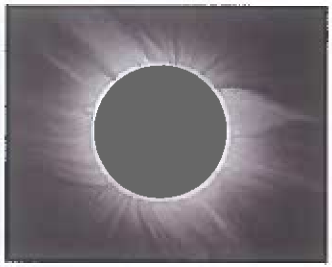
•1.82 The distance a freely falling object drops, starting from rest, is proportional to the square of the time it has been falling. By what factor will the distance fallen change if the time of falling is three times as long?

•1.83 A pilot decides to take his small plane for a Sunday afternoon excursion. He first flies north for 155.3 kilometers, then makes a 90° turn to his right and flies on a straight line for 62.5 kilometers, then makes another 90° turn to his right and flies 47.5 kilometers on a straight line.

a) How far away from his home airport is he at this point?

b) In which direction does he need to fly from this point on to make it home in a straight line?

c) What was the farthest distance from the home airport that he reached during the trip?



Total solar eclipse.

•1.84 As the photo shows, during a total eclipse, the Sun and the Moon appear to the observer to be almost exactly the same size. The radii of the Sun and Moon are $r_S = 6.96 \times 10^8$ m and $r_M = 1.74 \times 10^6$ m, respectively. The distance between the Earth and the Moon is $d_{EM} = 3.84 \times 10^8$ m.

a) Determine the distance from the Earth to the Sun at the moment of the eclipse.

b) In part (a), the implicit assumption is that the distance from the observer to the Moon's center is equal to the distance between the centers of the Earth and the Moon. By how much is this assumption incorrect, if the observer of the eclipse is on the Equator at noon? [*Hint:* Express this quantitatively, by calculating the relative error as a ratio: (assumed observer-to-Moon distance − actual observer-to-Moon distance)/(actual observer-to-Moon distance).]

c) Use the corrected observer-to-Moon distance to determine a corrected distance from Earth to the Sun.

•1.85 A hiker travels 1.50 km north and turns to a heading of 20.0° north of west, traveling another 1.50 km along that heading. Subsequently, he then turns north again and travels another 1.50 km. How far is he from his original point of departure, and what is the heading relative to that initial point?

•1.86 Assuming that 1 mole (6.02×10^{23} molecules) of an ideal gas has a volume of 22.4 L at standard temperature and pressure (STP) and that nitrogen, which makes up 78% of the air we breathe, is an ideal gas, how many nitrogen molecules are there in an average 0.5L breath at STP?

•1.87 On August 27, 2003, Mars approached as close to Earth as it will for over 50,000 years. If its angular size (the planet's diameter, measured by the angle the radius subtends) on that day was measured by an astronomer to be 24.9 seconds of arc, and its diameter is known to be 6784 km, how close was the approach distance? Be sure to use an appropriate number of significant figures in your answer.

•1.88 A football field's length is exactly 100 meters, and its width is $53\frac{1}{3}$ meters. A player stands at the exact center of the field and kicks the ball to a teammate standing at one corner of the field. Let the origin of coordinates be at the center of the football field and the x-axis point along the longer side of the field, with the y-direction parallel to the shorter side of the field.

a) Write the direction and length of a vector pointing from the player to the receiver.

b) Consider the other three possibilities for the location of the teammate at corners of the field. Repeat part (a) for each.

•1.89 The circumference of the Cornell Electron Storage Ring is 768.4 m. Express the diameter in centimeters, to the proper number of significant figures.

••1.90 Roughly 4% to 5% of what you exhale is carbon dioxide. Assume that 22.4 L is the volume of 1 mole (6.02×10^{23} molecules) of carbon dioxide and that you exhale 0.5 L per breath.

a) Estimate how many carbon dioxide molecules you breathe out each day.

b) If each mole of carbon dioxide has a mass of 44.0 g, how many kilograms of carbon dioxide do you exhale in a year?

••1.91 The Earth's orbit has a radius of 1.5×10^{11} m, and that of Mercury has a radius of 4.6×10^{10} m. Consider these orbits to be perfect circles (though in reality they are ellipses with slight eccentricity). Write down the direction and length of a vector from Earth to Mercury (take the direction from Earth to Sun to be 0°) when Mercury is at the maximum angular separation in the sky relative to the Sun.

••1.92 The star (other than the Sun) that is closest to Earth is Proxima Centauri. Its distance from Earth can be measured using parallax. *Parallax* is half the apparent angular shift of the star when observed from Earth at points on opposite sides of the Sun. The parallax of Proxima Centauri is 769 milliarcseconds. How far away is Proxima Centauri? (Give your answer in light-years with the correct number of significant digits.)

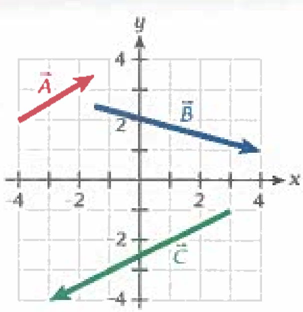


FIGURE FOR PROBLEMS 1.93 THROUGH 1.98

1.93 Write the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Cartesian coordinates.

1.94 Calculate the length and direction of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.

1.95 Add the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ graphically.

1.96 Determine the difference vector $\vec{E} = \vec{B} - \vec{A}$ graphically.

1.97 Add the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ using the component method, and find their sum vector $\vec{D}$.

1.98 Use the component method to determine the length of the vector $\vec{F} = \vec{C} - \vec{A} - \vec{B}$.

1.99 Sketch the vectors with the components $\vec{A} = (A_x, A_y) = (30.0$ m, -50.0 m) and $\vec{B} = (B_x, B_y) = (-30.0$ m, 50.0 m), and find the magnitudes of these vectors.

1.100 What angle does $\vec{A} = (A_x, A_y) = (30.0$ m, -50.0 m) make with the positive x-axis? What angle does it make with the negative y-axis?

1.101 Sketch the vectors with the components $\vec{A} = (A_x, A_y) = (-30.0$ m, -50.0 m) and $\vec{B} = (B_x, B_y) = (30.0$ m, 50.0 m), and find the magnitudes of these vectors.

1.102 What angle does $\vec{B} = (B_x, B_y) = (30.0$ m, 50.0 m) make with the positive x-axis? What angle does it make with the positive y-axis?

1.103 Find the magnitude and direction of each of the following vectors, which are given in terms of their x- and y-components: $\vec{A} = (23.0, 59.0)$, and $\vec{B} = (90.0, -150.0)$.

1.104 Find the magnitude and direction of $-\vec{A} + \vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

1.105 Find the magnitude and direction of $-5\vec{A} + \vec{B}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

1.106 Find the magnitude and direction of $-7\vec{B} + 3\vec{A}$, where $\vec{A} = (23.0, 59.0)$, $\vec{B} = (90.0, -150.0)$.

1.107 Which of the six cases shown in the figure has the largest absolute value of the scalar product of the vectors $\vec{A}$ and $\vec{B}$?

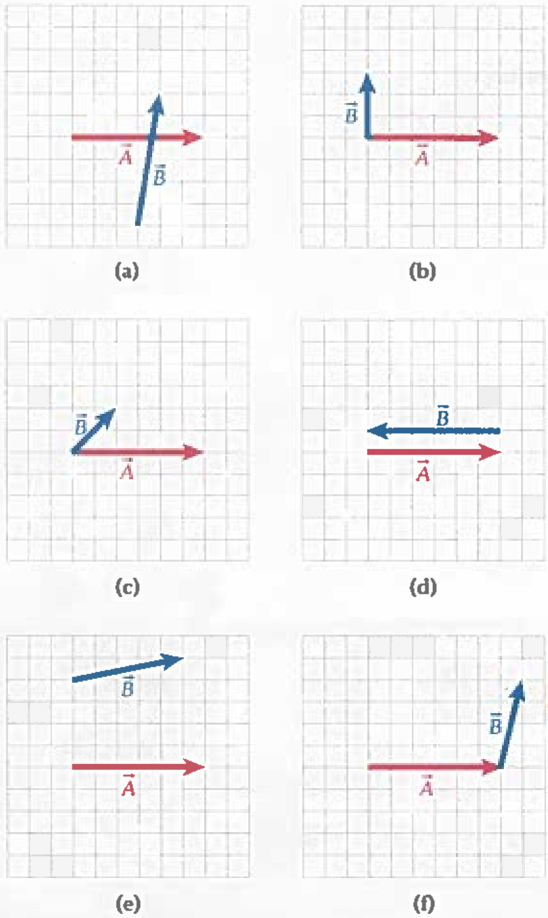


Figure for Problems 1.107 to 1.112

1.108 Which of the six cases shown in the figure has the smallest absolute value of the scalar product of the vectors $\vec{A}$ and $\vec{B}$?

1.109 Which of the six cases shown in the figure has the largest absolute value of the vector product of the vectors $\vec{A}$ and $\vec{B}$?

1.110 Which of the six cases shown in the figure has the smallest absolute value of the vector product of the vectors $\vec{A}$ and $\vec{B}$?

•**1.111** Rank order the six cases shown in the figure from the smallest absolute value to the largest absolute value of the scalar product of the vectors $\vec{A}$ and $\vec{B}$.

•**1.112** Rank order the six cases shown in the figure from the smallest absolute value to the largest absolute value of the vector product of the vectors $\vec{A}$ and $\vec{B}$.

1.113 Toward the end of their lives many stars become much bigger. Assume that they remain spherical in shape and that their masses do not change in this process. If the radius of a star increases by a factor of 11.4, by what factors do the following change:

a) its surface area,

b) its circumference,

c) its volume?

1.114 Toward the end of their lives many stars become much bigger. Assume that they remain spherical in shape and that their masses do not change in this process. If the circumference of a star increases by a factor of 12.5, by what factors do the following change:

a) its surface area,

b) its radius,

c) its volume?

1.115 Toward the end of their lives many stars become much bigger. Assume that they remain spherical in shape and that their masses do not change in this process. If the volume of a star increases by a factor of 872, by what factors do the following change:

a) its surface area,

b) its circumference,

c) its diameter?

•**1.116** Toward the end of their lives many stars become much bigger. Assume that they remain spherical in shape and that their masses do not change in this process. If the surface area of a star increases by a factor of 274, by what factors do the following change:

a) its radius,

b) its volume,

c) its density?