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## حل مراجعة امتحانية وفق الهيكل الوزاري الخطة B-101

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← فيزياء ← الفصل الثالث ← الملف

تاريخ إضافة الملف على موقع المناهج: 08:15:36 2024-06-01

إعداد: Jadhav Kailas

## التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



اضغط هنا للحصول على جميع روابط "الصف الحادي عشر المتقدم"

## روابط مواد الصف الحادي عشر المتقدم على تلغرام

[الرياضيات](#)

[اللغة الانجليزية](#)

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## المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة فيزياء في الفصل الثالث

[مراجعة نهائية وفق الهيكل الوزاري الخطة C](#)

1

[الهيكل الوزاري الجديد منهج بريدج الخطة M-101-A المسار المتقدم](#)

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[الهيكل الوزاري الجديد منهج بريدج الخطة M-101-B المسار المتقدم](#)

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## المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة فيزياء في الفصل الثالث

[الهيكل الوزاري الحديد منهج بريدج الخطة C-101 المسار المتقدم](#)

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[مراجعة نهائية اختبار من متعدد مع بعض الإجابات منهج انسابير](#)

5

**GRADE 11 ADVANCED**

**PHYSICS**

**2023-2024**

**TERM 3**

**REVISION PPT (EOT)**

**PREPARED BY: MR. KAILAS JADHAV**

# TERM 3 SYLLABUS GRADE 11 ADV (101-B)

## Chapter 6: Potential Energy and Energy Conservation

6.1 Potential Energy

6.2 Conservative and Nonconservative Forces

6.3 Work and Potential Energy

6.4 Potential Energy and Force (For Enrichment-إثرائي)

6.5 Conservation of Mechanical Energy

6.6 Work and Energy for the Spring Force (For Enrichment-إثرائي)

6.7 Nonconservative Forces and the Work-Energy Theorem (For Enrichment-إثرائي)

6.8 Potential Energy and Stability (For Enrichment-إثرائي)

## Chapter 7: Momentum and Collisions

7.1 Linear Momentum

7.2 Impulse

7.3 Conservation of Linear Momentum

7.4 Elastic Collisions in One Dimension

7.5 Elastic Collisions in Two or Three Dimensions (For Enrichment-إثرائي)

7.6 Totally Inelastic Collisions (For Enrichment-إثرائي)

7.7 Partially Inelastic Collisions (For Enrichment-إثرائي)

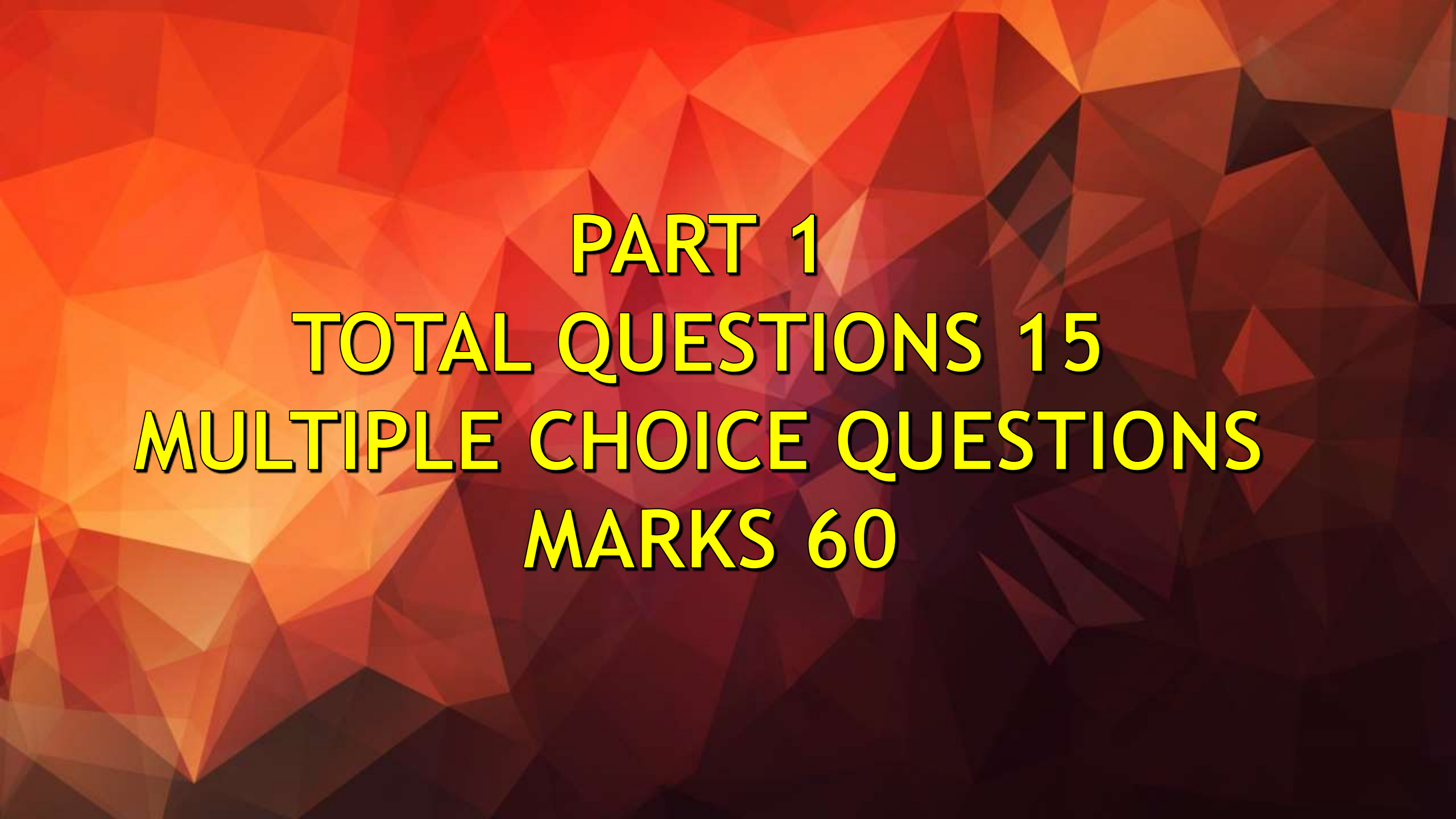
7.8 Billiards and Chaos (For Enrichment-إثرائي)



Academic Year	2024/2023
العام الدراسي	
Term	3 <sup>rd</sup>
Subject	Physics (Bridge) الفيزياء
الموضوع	
Grade	11
الصف	
Stream	Advanced/المتقدم
المسار	
Code	PHY M-101-B



Number Of MCQ	15
Markes of MCQ	4
درجة الأسئلة الموضوعية	
Number of FRQ	4
عدد الأسئلة المقالية	
Marks Per FRQ	10
الدرجات للأسئلة المقالية	
Type of All Questions	MCQ/ الأسئلة الموضوعية
نوع كافة الأسئلة	FRQ/ الأسئلة المقالية
Maximum Overall Grade	100
الدرجة القصوى الممكنة	
Exam Duration	150 min.
مدة الامتحان	



**PART 1**  
**TOTAL QUESTIONS 15**  
**MULTIPLE CHOICE QUESTIONS**  
**MARKS 60**



1

Define potential energy as the energy stored in the configuration of a system of objects that exert forces on one another.

PAGE NO. 155

- Potential energy,  $U$ , is the energy stored in the configuration of a system of objects that exert forces on one another.

## SI Potential Energy Units

From the equation  $U = mgh$  the units of gravitational potential energy must be:

$$\text{kg} \cdot (\text{m}/\text{s}^2) \cdot \text{m} = (\text{kg} \cdot \text{m}/\text{s}^2) \cdot \text{m} = \text{N} \cdot \text{m} = \text{J}$$

This shows the SI unit for potential energy is the Joule, as it is for work and all other types of energy.

## EXAMPLES OF POTENTIAL ENERGY

MECHDAILY.COM

Potential energy is stored energy. The energy comes from an object's relative position, its electric charge, internal stresses, or other factors.



Raised Object



Dynamite



Bow and Arrow



Stretched Rubber



Battery



Roller Coaster



Dam Holding Water



Sling Shot

2

- (1) Identify that the work done by a conservative force along a closed path is zero:  $W_{(A \rightarrow B)} + W_{(B \rightarrow A)} = 0$ .
- (2) Identify that for a particle moving between two points, the work done by a conservative force does not depend on the path taken by the particle:  $W_{(A \rightarrow B), \text{path} \textcircled{1}} = W_{(A \rightarrow B), \text{path} \textcircled{2}}$

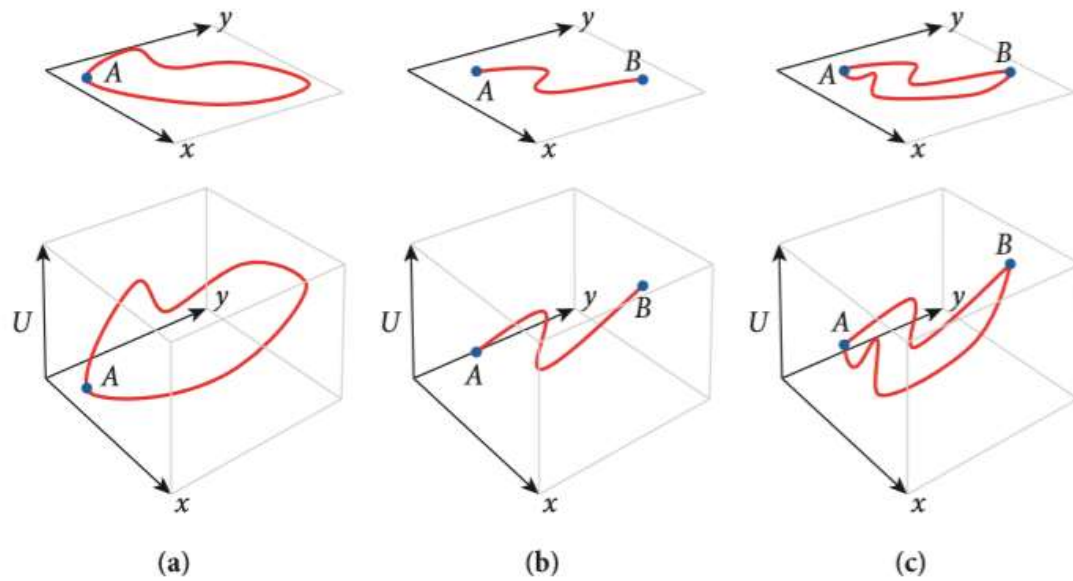
1. If we know the work,  $W_{A \rightarrow B}$ , done by a conservative force on an object as the object moves along a path from point  $A$  to point  $B$ , then we also know the work,  $W_{B \rightarrow A}$ , that the same force does on the object as it moves along the path in the reverse direction, from point  $B$  to point  $A$  (see Figure 6.5b):

$$W_{B \rightarrow A} = -W_{A \rightarrow B} \text{ (for conservative forces).} \quad (6.4)$$

The proof of this statement is obtained from the condition of zero work over a closed loop. Because the path from  $A$  to  $B$  to  $A$  forms a closed loop, the sum of the work contributions from the loop has to equal zero. In other words,

$$W_{A \rightarrow B} + W_{B \rightarrow A} = 0,$$





**FIGURE 6.5** Various paths for the potential energy related to a conservative force as a function of positions  $x$  and  $y$ , with  $U$  proportional to  $y$ . The two-dimensional plots are projections of the three-dimensional plots onto the  $xy$ -plane. (a) Closed loop. (b) A path from point  $A$  to point  $B$ . (c) Two different paths between points  $A$  and  $B$ .

2. If we know the work,  $W_{A \rightarrow B, \text{path 1}}$ , done by a conservative force on an object moving along path 1 from point  $A$  to point  $B$ , then we also know the work,  $W_{A \rightarrow B, \text{path 2}}$ , done by the same force on the object when it uses any other path (path 2) to go from point  $A$  to point  $B$  (see Figure 6.5c). The work is the same; the work done by a conservative force is independent of the path taken by the object:

$$W_{A \rightarrow B, \text{path 2}} = W_{A \rightarrow B, \text{path 1}} \quad (6.5)$$

(for arbitrary paths 1 and 2, for conservative forces).

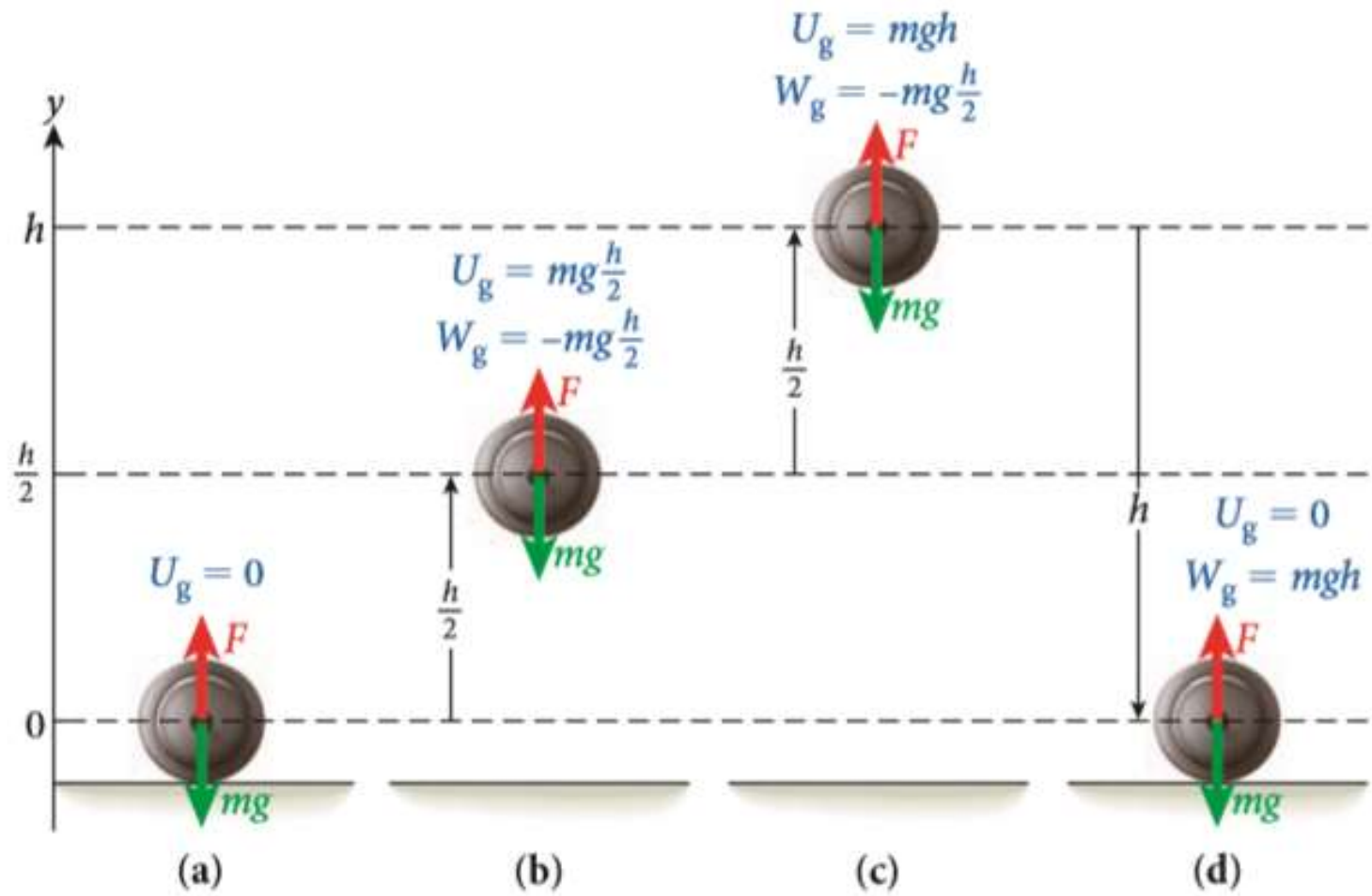
**EXAMPLE 6.1** Weightlifting**PROBLEM**

Let's consider the gravitational potential energy in a specific situation: a weightlifter lifting a barbell of mass  $m$ . What is the gravitational potential energy and the work done during the different phases of lifting the barbell?

**SOLUTION**

The weightlifter starts with the barbell on the floor, as shown in Figure 6.2a. At  $y = 0$ , the gravitational potential energy can be defined to be  $U_g = 0$ . The weightlifter then picks up the barbell, lifts it to a height of  $y = h/2$ , and holds it there, as shown in Figure 6.2b. The gravitational potential energy is now  $U_g = mgh/2$ , and the work done by gravity on the barbell is  $W_g = -mgh/2$ . The weightlifter next lifts the barbell over his head to a height of  $y = h$ , as shown in Figure 6.2c. The gravitational potential energy is now  $U_g = mgh$ , and the work done by gravity during this part of the lift is again  $W_g = -mgh/2$ . Having completed the lift, the weightlifter lets go of the barbell, and it falls to the floor, as illustrated in Figure 6.2d. The gravitational potential energy of the barbell on the floor is again  $U_g = 0$ , and the work done by gravity during the fall is  $W_g = mgh$ .





if  $\Delta U_g$  is positive, there exists the potential (hence the name *potential energy*) to allow  $\Delta U_g$  to be negative in the future, thereby extracting positive work, since  $W_g = -\Delta U_g$ .



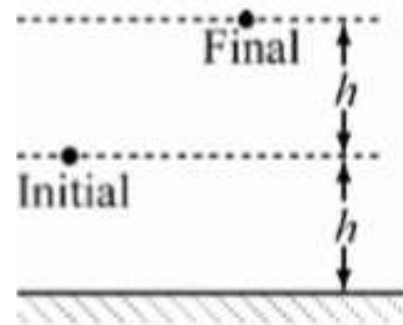
- 6.32 a) If the gravitational potential energy of a 40.0-kg rock is 500. J relative to a value of zero on the ground, how high is the rock above the ground?
- b) If the rock were lifted to twice its original height, how would the value of its gravitational potential energy change?

**THINK:** The rock's mass is  $m = 40.0$  kg and the gravitational potential energy is  $U_g = 500.$  J. Determine:

(a) the height of the rock,  $h$ , and

(b) the change,  $\Delta U_g$  if the rock is raised to twice its original height,  $2h$ .

**SKETCH:**



**RESEARCH:** Use the equation  $U_g = mgh$ . Note:  $\Delta U_g = U_g - U_{g,0}$ .

## SIMPLIFY:

$$(a) U_g = mgh \Rightarrow h = \frac{U_g}{mg}$$

$$(b) \Delta U_g = U_g - U_{g,0}$$
$$= mg(2h) - mgh$$
$$= mgh$$
$$= U_g$$

## CALCULATE:

$$(a) h = \frac{500. \text{ J}}{40.0 \text{ kg}(9.81 \text{ m/s}^2)} = 1.274 \text{ m}$$

$$(b) \Delta U_g = 500. \text{ J}$$

## ROUND:

$$(a) h = 1.27 \text{ m}$$

$$(b) \Delta U_g = 500. \text{ J} \text{ does not need to be rounded.}$$

4 Calculate the gravitational potential energy of a particle -Earth system ( $U_g = mgy$ ).

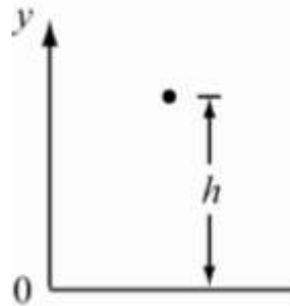
Exercises Q./6.31/6.32/6.33  
Additional Exercises Q./6.66

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**6.31** What is the gravitational potential energy of a 2.00-kg book 1.50 m above the floor?

**THINK:** The mass of the book is  $m = 2.00$  kg and its height above the floor is  $h = 1.50$  m. Determine the gravitational potential energy,  $U_g$ .

**SKETCH:**



**RESEARCH:** Taking the floor's height as  $U_g = 0$ ,  $U_g$  for the book can be determined from the formula  $U_g = mgh$ .

**SIMPLIFY:** It is not necessary to simplify.

**CALCULATE:**  $U_g = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m}) = 29.43 \text{ J}$

**ROUND:** The given initial values have three significant figures, so the result should be rounded to  $U_g = 29.4 \text{ J}$ .



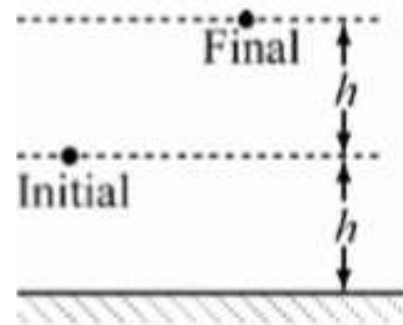
- 6.32 a) If the gravitational potential energy of a 40.0-kg rock is 500. J relative to a value of zero on the ground, how high is the rock above the ground?
- b) If the rock were lifted to twice its original height, how would the value of its gravitational potential energy change?

**THINK:** The rock's mass is  $m = 40.0$  kg and the gravitational potential energy is  $U_g = 500.$  J. Determine:

(a) the height of the rock,  $h$ , and

(b) the change,  $\Delta U_g$  if the rock is raised to twice its original height,  $2h$ .

**SKETCH:**



**RESEARCH:** Use the equation  $U_g = mgh$ . Note:  $\Delta U_g = U_g - U_{g,0}$ .

## SIMPLIFY:

$$(a) U_g = mgh \Rightarrow h = \frac{U_g}{mg}$$

$$(b) \Delta U_g = U_g - U_{g,0}$$
$$= mg(2h) - mgh$$
$$= mgh$$
$$= U_g$$

## CALCULATE:

$$(a) h = \frac{500. \text{ J}}{40.0 \text{ kg}(9.81 \text{ m/s}^2)} = 1.274 \text{ m}$$

$$(b) \Delta U_g = 500. \text{ J}$$

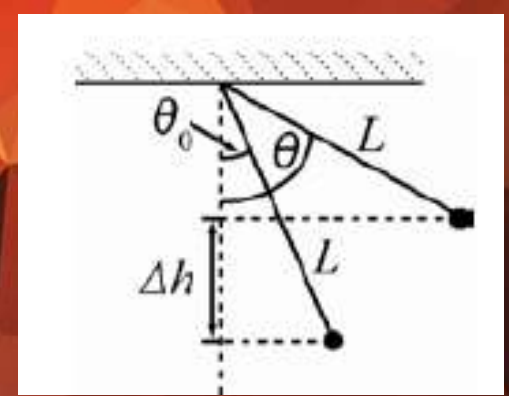
## ROUND:

$$(a) h = 1.27 \text{ m}$$

$$(b) \Delta U_g = 500. \text{ J} \text{ does not need to be rounded.}$$



6.33 A rock of mass 0.773 kg is hanging from a string of length 2.45 m on the Moon, where the gravitational acceleration is a sixth of that on Earth. What is the change in gravitational potential energy of this rock when it is moved so that the angle of the string changes from  $3.31^\circ$  to  $14.01^\circ$ ? (Both angles are measured relative to the vertical.)



**THINK:** The rock's mass is  $m = 0.773$  kg. The length of the string is  $L = 2.45$  m. The gravitational acceleration on the Moon is  $g_M = g/6$ . The initial and final angles are  $\theta_0 = 3.31^\circ$  and  $\theta = 14.01^\circ$ , respectively. Determine the rock's change in gravitational potential energy,  $\Delta U$ .

**RESEARCH:** To determine  $\Delta U$ , the change in height of the rock,  $\Delta h$ , is needed. This can be determined using trigonometry. Then  $\Delta U = mg_M \Delta h$ .

**SIMPLIFY:** To determine  $\Delta h$ :  $\Delta h = L \cos \theta_0 - L \cos \theta = L(\cos \theta_0 - \cos \theta)$ . Then

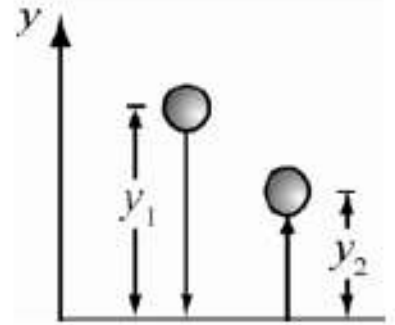
$$\Delta U = mg_M \Delta h = \frac{1}{6} mgL (\cos \theta_0 - \cos \theta).$$

**CALCULATE:**  $\Delta U = \frac{1}{6} (0.773 \text{ kg})(9.81 \text{ m/s}^2)(2.45 \text{ m})(\cos(3.31^\circ) - \cos(14.01^\circ)) = 0.08694 \text{ J}$

**ROUND:** With three significant figures in the values, the result should be rounded to  $\Delta U = 0.0869 \text{ J}$ .



**6.66** A ball of mass 1.84 kg is dropped from a height  $y_1 = 1.49$  m and then bounces back up to a height of  $y_2 = 0.87$  m. How much mechanical energy is lost in the bounce? The effect of air resistance has been experimentally found to be negligible in this case, and you can ignore it.



**THINK:** The mass of the ball is  $m = 1.84$  kg. The initial height is  $y_1 = 1.49$  m and the second height is  $y_2 = 0.87$  m. Determine the energy lost in the bounce.

**RESEARCH:** Consider the changes in the potential energy from  $y_1$  to  $y_2$ . The energy lost in the bounce is given by  $U_1 - U_2$ .

**SIMPLIFY:**  $E_{\text{lost}} = mgy_1 - mgy_2 = mg(y_1 - y_2)$

**CALCULATE:**  $E_{\text{lost}} = (1.84 \text{ kg})(9.81 \text{ m/s}^2)(1.49 \text{ m} - 0.87 \text{ m}) = 11.2 \text{ J}$

**ROUND:** Since the least precise value is given to two significant figures, the result is  $E_{\text{lost}} = 11 \text{ J}$ .

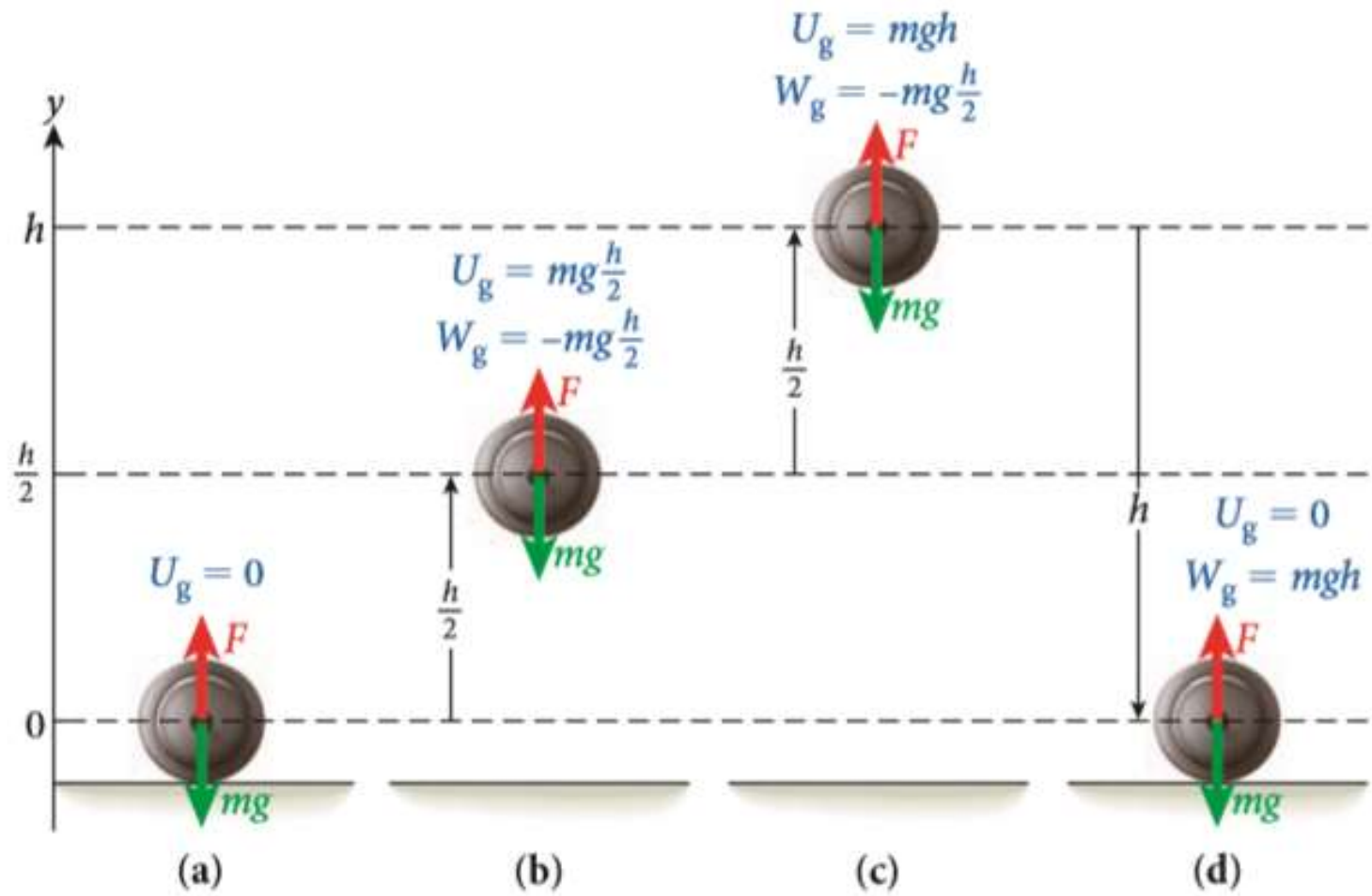
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if  $\Delta U_g$  is positive, there exists the potential (hence the name *potential energy*) to allow  $\Delta U_g$  to be negative in the future, thereby extracting positive work, since  $W_g = -\Delta U_g$ .



Calculate the work done by friction force for an object sliding across a horizontal surface between two points:  $W_f = \vec{f} \cdot \Delta \vec{x} = -f \cdot (x - x_o) = -\mu_k mg \cdot (x - x_o)$

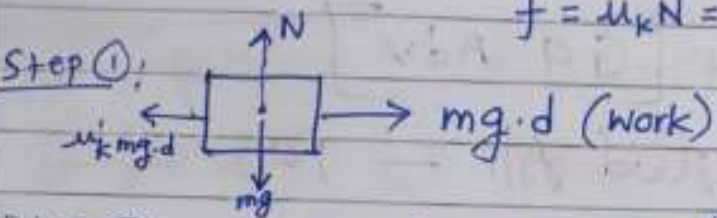
## Concept Check 6.1

A person pushes a box of mass  $m$  a distance  $d$  across a floor. The coefficient of kinetic friction between the box and the floor is  $\mu_k$ . The person then picks up the box, raises it to a height  $h$ , carries it back to the starting point, and puts it back down on the floor. How much work has the person done on the box?

- a) zero
- b)  $\mu_k mgd$
- c)  $\mu_k mgd + 2mgh$
- d)  $\mu_k mgd - 2mgh$
- e)  $2\mu_k mgd + 2mgh$

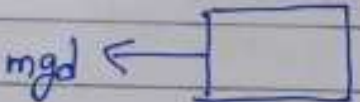
**b**

$f = \mu_k N = \mu_k mg$

Step ①: 

Step ②: case ② No displacement  $W=0$  (raising)

Step ③: No friction (Box is lifted)



Total work:

$$(\cancel{mg \cdot d} - \mu_k mg \cdot d) + 0 + (-\cancel{mgd})$$

$$W_f = -\mu_k mg \cdot d$$

$$W = \mu_k mg \cdot d$$

7

Determine the change in potential energy due to spring force:

$$\Delta U_s = U_s(y) - U_s(y_0) = \frac{1}{2}kx^2 - \frac{1}{2}kx_0^2$$

Student Book  
MCQ. 6.1/6.5/6.6/6.9

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6.1 A block of mass 5.0 kg slides without friction at a speed of 8.0 m/s on a horizontal table surface until it strikes and sticks to a horizontal spring (with spring constant of  $k = 2000$ . N/m and very small mass), which in turn is attached to a wall. How far is the spring compressed before the mass comes to rest?

- a) 0.40 m                      c) 0.30 m                      e) 0.67 m  
b) 0.54 m                      d) 0.020 m

6.1) a

6.5 Which of the following is *not* a valid potential energy function for the spring force  $F = -kx$ ?

- a)  $(\frac{1}{2})kx^2$                       c)  $(\frac{1}{2})kx^2 - 10$  J                      e) None of the above  
b)  $(\frac{1}{2})kx^2 + 10$  J                      d)  $-(\frac{1}{2})kx^2$                       is valid.

6.5) d

6.6 You use your hand to stretch a spring to a displacement  $x$  from its equilibrium position and then slowly bring it back to that position. Which is true?

- a) The spring's  $\Delta U$  is positive.      d) The hand's  $\Delta U$  is negative.  
b) The spring's  $\Delta U$  is negative.      e) None of the above statements is true.  
c) The hand's  $\Delta U$  is positive.

6.6) e

6.9 A spring has a spring constant of 80. N/m. How much potential energy does it store when stretched by 1.0 cm?

- a)  $4.0 \cdot 10^{-3}$  J      c) 80 J      e) 0.8 J  
b) 0.40 J      d) 800 J

Use  
$$U = \frac{1}{2}kx^2$$

6.9) a



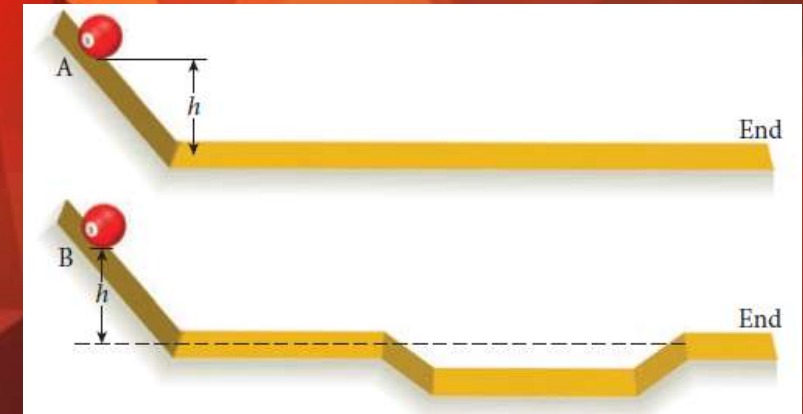
6.2 A pendulum swings in a vertical plane. At the bottom of the swing, the kinetic energy is 8 J and the gravitational potential energy is 4 J. At the highest position of the swing, the kinetic and gravitational potential energies are

- a) kinetic energy = 0 J and gravitational potential energy = 4 J.
- b) kinetic energy = 12 J and gravitational potential energy = 0 J.
- c) kinetic energy = 0 J and gravitational potential energy = 12 J.
- d) kinetic energy = 4 J and gravitational potential energy = 8 J.
- e) kinetic energy = 8 J and gravitational potential energy = 4 J.

6.2) c

6.18 Two identical billiard balls start at the same height and the same time and roll along different tracks, as shown in the figure.

- Which ball has the highest speed at the end?
- Which one will get to the end first?



(a) Assuming both billiard balls have the same mass,  $m$ , the initial energies,  $E_{Ai}$  and  $E_{Bi}$  are given by  $E_{Ai} = mgh$  and  $E_{Bi} = mgh$ . The final energy is all due to kinetic energy, so the final energies are  $E_{Af} = (mv_A^2)/2$  and  $E_{Bf} = (mv_B^2)/2$ . By conservation of energy (assuming no loss due to friction),  $E_i = E_f$ . For each ball the initial and final energies are equal. This means  $mgh = (mv_A^2)/2 \Rightarrow v_A = \sqrt{2gh}$  and  $mgh = (mv_B^2)/2 \Rightarrow v_B = \sqrt{2gh}$ . Therefore,  $v_A = v_B$ . The billiard balls have the same speed at the end.

(b) Ball B undergoes an acceleration of  $a$  and a deceleration of  $-a$  due to the dip in the track. The effects of the acceleration and deceleration ultimately cancel. However, the ball rolling on track B will have a greater speed over of the lowest section of track. Therefore, ball B will win the race.



## DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

kilogram · meter/second (kg · m/s)



10

Relate momentum to kinetic energy  $K = \frac{p^2}{2m}$ Student Book  
Exercises/ Q./7.25190  
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## Momentum and Kinetic Energy

In Chapter 5, we established the relationship,  $K = \frac{1}{2}mv^2$  (equation 5.1), between the kinetic energy  $K$ , the speed  $v$ , and the mass  $m$ . We can use  $p = mv$  to obtain

$$K = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m}.$$

This equation gives us an important relationship between kinetic energy, mass, and momentum:

$$K = \frac{p^2}{2m}. \quad (7.3)$$

**7.25** A car of mass 1200. kg, moving with a speed of 72.0 mph on a highway, passes a small SUV with a mass  $1\frac{1}{2}$  times bigger, moving at  $\frac{2}{3}$  the speed of the car.

- What is the ratio of the momentum of the SUV to that of the car?
- What is the ratio of the kinetic energy of the SUV to that of the car?

**THINK:** Compute the ratios of the momenta and kinetic energies of the car and SUV.

$$m_{\text{car}} = 1200. \text{ kg}, m_{\text{SUV}} = 1.5m_{\text{car}} = \frac{3}{2}m_{\text{car}}, v_{\text{car}} = 72.0 \text{ mph}, \text{ and } v_{\text{SUV}} = \frac{2}{3}v_{\text{car}}.$$

**RESEARCH:**

(a)  $p = mv$

(b)  $K = \frac{1}{2}mv^2$

**SKETCH:**



## SIMPLIFY:

$$(a) \frac{p_{\text{SUV}}}{p_{\text{car}}} = \frac{m_{\text{SUV}} v_{\text{SUV}}}{m_{\text{car}} v_{\text{car}}} = \frac{(3/2)m_{\text{car}} (2/3)v_{\text{car}}}{m_{\text{car}} v_{\text{car}}}$$

$$(b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = \frac{(1/2)m_{\text{SUV}} v_{\text{SUV}}^2}{(1/2)m_{\text{car}} v_{\text{car}}^2} = \frac{(3/2)m_{\text{car}} \left( (2/3)v_{\text{car}} \right)^2}{m_{\text{car}} v_{\text{car}}^2}$$

## CALCULATE:

$$(a) \frac{p_{\text{SUV}}}{p_{\text{car}}} = \frac{(3/2)(2/3)}{1} = 1$$

$$(b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = \frac{(3/2)(2/3)^2}{1} = \frac{(3/2)(4/9)}{1} = 2/3 = 0.6667$$

$$\text{ROUND: (a) } \frac{p_{\text{SUV}}}{p_{\text{car}}} = 1.0 \quad (b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = 0.67$$



Calculate the linear momentum of a particle as the product of the particle's mass and velocity ( $\vec{p}=m\vec{v}$ )

7.24 Rank the following objects from highest to lowest in terms of momentum and from highest to lowest in terms of energy.

- an asteroid with mass  $10^6$  kg and speed 500 m/s
- a high-speed train with a mass of 180,000 kg and a speed of 300 km/h
- a 120-kg linebacker with a speed of 10 m/s
- a 10-kg cannonball with a speed of 120 m/s
- a proton with a mass of  $2 \cdot 10^{-27}$  kg and a speed of  $2 \cdot 10^8$  m/s

	$m$	$v$
(a)	$10^6$ kg	500 m/s
(b)	180,000 kg	300 km/h
(c)	120 kg	10 m/s
(d)	10 kg	120 m/s
(e)	$2 \cdot 10^{-27}$ kg	$2 \cdot 10^8$ m/s

**RESEARCH:**  $E = \frac{1}{2}mv^2$ ,  $p = mv$

**CALCULATE:**

$$(a) E = \frac{1}{2}(10^6 \text{ kg})(500 \text{ m/s})^2 = 1.3 \cdot 10^{11} \text{ J}, \quad p = (10^6 \text{ kg})(500 \text{ m/s}) = 5.0 \cdot 10^8 \text{ kg m/s},$$

$$(b) E = \frac{1}{2}(1.8 \cdot 10^5 \text{ kg}) \left( (300 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) \right)^2 = 6.3 \cdot 10^8 \text{ J},$$

$$p = (1.8 \cdot 10^5 \text{ kg}) \left( (300 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) \right) = 1.5 \cdot 10^7 \text{ kg m/s}$$

$$(c) p_y = \sqrt{2(49.5 \text{ J})(0.442 \text{ kg})} \sin 58.0^\circ = 5.610 \text{ kg m/s}, \quad p = (120 \text{ kg})(10 \text{ m/s}) = 1200 \text{ kg m/s}$$

$$(d) E = \frac{1}{2}(10 \text{ kg})(120 \text{ m/s})^2 = 7.2 \cdot 10^4 \text{ J}, \quad p = (10 \text{ kg})(120 \text{ m/s}) = 1200 \text{ kg m/s}$$

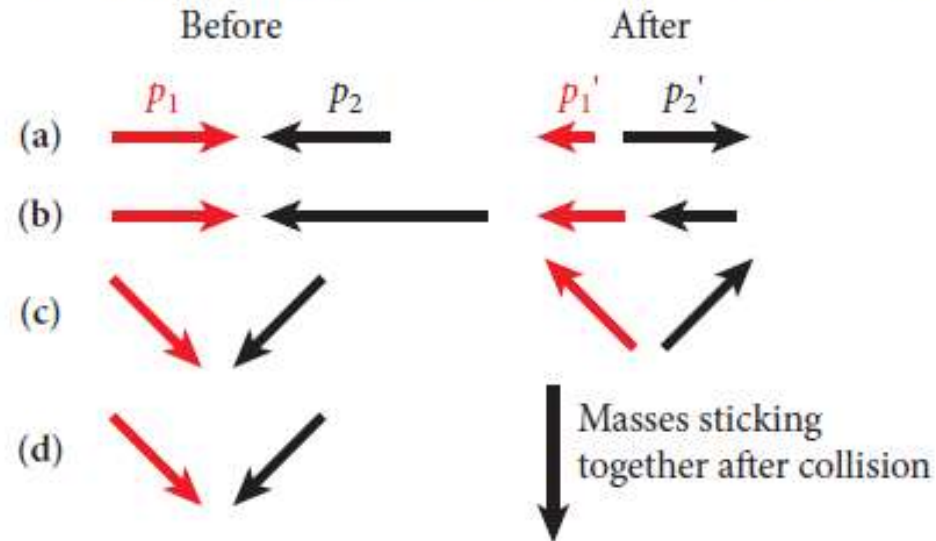
$$(e) E = \frac{1}{2}(2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s})^2 = 4 \cdot 10^{-11} \text{ J}, \quad p = (2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s}) = 4 \cdot 10^{-19} \text{ kg m/s}$$

	$E$ [J]	$p$ [kg m/s]
(a)	$1 \cdot 10^{11}$	$5 \cdot 10^8$
(b)	$6 \cdot 10^8$	$2 \cdot 10^7$
(c)	$6 \cdot 10^3$	$1 \cdot 10^3$
(d)	$7 \cdot 10^4$	$1 \cdot 10^3$
(e)	$4 \cdot 10^{-11}$	$4 \cdot 10^{-19}$

**DOUBLE-CHECK:** In order from largest to smallest energy: (a), (b), (d), (c), (e); and momentum: (a), (b), (d) = (c), (e).

Apply the conservation of linear momenta for an isolated system of particles to relate the initial momenta of the particles to their final momenta at any later instant

7.3 The figure shows sets of possible momentum vectors before and after a collision, with no external forces acting. Which sets could actually occur?



7.3) b, d

7.4 The value of the momentum for a system is the same at a later time as at an earlier time if there are no

- collisions between particles within the system.
- inelastic collisions between particles within the system.
- changes of momentum of individual particles within the system.
- internal forces acting between particles within the system.
- external forces acting on particles of the system.

7.4) e



7.11 For a totally elastic collision between two objects, which of the following statements is (are) true?

- a) The total mechanical energy is conserved.
- b) The total kinetic energy is conserved.
- c) The total momentum is conserved.
- d) The momentum of each object is conserved.
- e) The kinetic energy of each object is conserved.

**7.11) a,b,c**

7.12 For a totally inelastic collision between two objects, which of the following statements is (are) true?

- a) The total mechanical energy is conserved.
- b) The total kinetic energy is conserved.
- c) The total momentum is conserved.
- d) The total momentum after the collision is always zero.
- e) The total kinetic energy after the collision can never be zero.

**7.12) c**

14	Solve problems related to elastic collisions in one dimension.
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**7.10** A red curling stone moving with a speed of 2.0 m/s collides head-on with a yellow curling stone at rest (totally elastic collision). What are the speeds of the two curling stones just after the collision?

- a) The red stone is at rest, and the yellow stone is moving with a speed of 2.0 m/s.
- b) The red stone and the yellow stone are both moving with a speed of 1.0 m/s.
- c) The red stone bounces off the yellow stone and moves with a speed of 2.0 m/s, and the yellow stone remains at rest.

**7.10) a**



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Apply the conservation laws of momentum and total kinetic energy for elastic collisions in one dimension for the special case of equal masses and show that the two objects simply exchange their momenta and velocities where:

$$P_{f(1,x)} = P_{i(2,x)} \quad / \quad P_{f(2,x)} = P_{i(1,x)} \quad \text{and} \quad v_{f(1,x)} = v_{i(2,x)} \quad / \quad v_{f(2,x)} = v_{i(1,x)}$$

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*in an elastic collision, the sum of the kinetic energies has to remain constant.*

for conservation of kinetic energy can be written as

$$\frac{P_{f1,x}^2}{2m_1} + \frac{P_{f2,x}^2}{2m_2} = \frac{P_{i1,x}^2}{2m_1} + \frac{P_{i2,x}^2}{2m_2}$$

The equation for conservation of momentum

$$P_{f1,x} + P_{f2,x} = P_{i1,x} + P_{i2,x}$$

the components of the final momentum vectors:

$$P_{f1,x} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) P_{i1,x} + \left( \frac{2m_2}{m_1 + m_2} \right) P_{i2,x}$$

$$P_{f2,x} = \left( \frac{2m_1}{m_1 + m_2} \right) P_{i1,x} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) P_{i2,x}$$

we can also obtain expressions for the final velocities by using  $p_x = mv_x$ :

$$v_{f1,x} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{i1,x} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{i2,x}$$

$$v_{f2,x} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{i1,x} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{i2,x}$$



$$p_{f1,x} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) p_{i1,x} + \left( \frac{2m_1}{m_1 + m_2} \right) p_{i2,x}$$

$$p_{f2,x} = \left( \frac{2m_2}{m_1 + m_2} \right) p_{i1,x} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) p_{i2,x}.$$

$$v_{f1,x} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{i1,x} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{i2,x}$$

$$v_{f2,x} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{i1,x} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{i2,x}.$$

## Special Case 1: Equal Masses

If  $m_1 = m_2$ , the general expressions in equation 7.12 simplify considerably, because the terms proportional to  $m_1 - m_2$  are equal to zero and the ratios  $2m_1/(m_1 + m_2)$  and  $2m_2/(m_1 + m_2)$  become unity. We then obtain the extremely simple result

$$p_{f1,x} = p_{i2,x} \quad (\text{for the special case where } m_1 = m_2) \quad (7.15)$$

$$p_{f2,x} = p_{i1,x}.$$

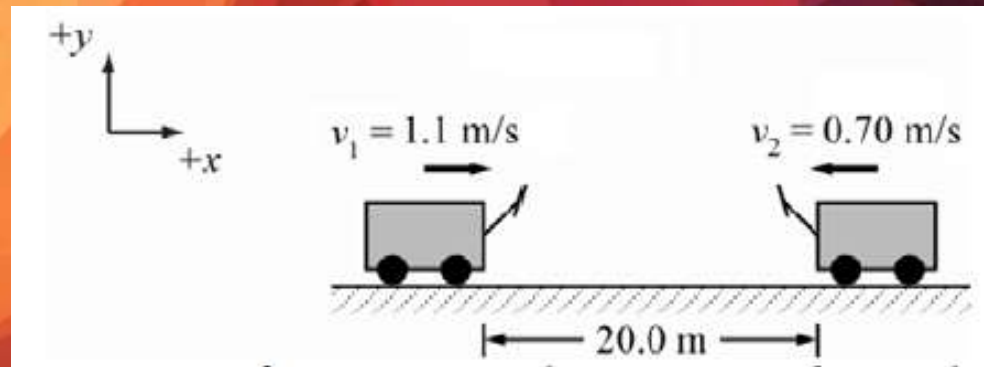
The initial momentum of object 1 becomes the final momentum of object 2. The same is true for the velocities:

$$v_{f1,x} = v_{i2,x} \quad (\text{for the special case where } m_1 = m_2) \quad (7.16)$$

$$v_{f2,x} = v_{i1,x}.$$

•7.51 You notice that a shopping cart 20.0 m away is moving with a velocity of 0.700 m/s toward you. You launch an identical cart with a velocity of 1.10 m/s directly at the other cart in order to intercept it. When the two carts collide elastically, they remain in contact for 0.200 s. Graph the position, velocity, and force for both carts as a function of time.

**THINK:** Two carts, separated by a distance  $x_0 = 20.0$  m, are travelling towards each other with speeds  $v_1 = 1.10$  m/s and  $v_2 = 0.700$  m/s. They collide for  $\Delta t = 0.200$  s. This is an elastic collision. I need to plot  $x$  vs.  $t$ ,  $v$  vs.  $t$  and  $F$  vs.  $t$ .



**RESEARCH:** Use the conservation of momentum and energy to get the speeds after collision. Then use the impulse  $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$  to get the force.



**SIMPLIFY:** First, need the position of the collision. Using  $x = x_0 + v_0 t \Rightarrow x_1 = 0 + v_1 t$  and  $x_2 = x_0 - v_2 t$ ,  
 $x_1 = x_2 = v_1 t = x_0 - v_2 t \Rightarrow t = x_0 / (v_1 + v_2)$ . Conservation of momentum:

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$$

$$v_{i1}^2 - v_{f1}^2 = v_{f2}^2 - v_{i2}^2$$

$$(v_{i1} - v_{f1})(v_{i1} + v_{f1}) = (v_{f2} - v_{i2})(v_{f2} + v_{i2})$$

$$v_{i1} + v_{f1} = v_{f2} + v_{i2}$$

$$v_{f2} = v_{i1} + v_{f1} - v_{i2}$$



Substituting back into (1):

$$v_{i1} - v_{f1} = v_{i1} + v_{f1} - v_{i2} - v_{f2} \Rightarrow 2v_{f1} = 2v_{i2} \Rightarrow v_{f1} = v_{i2} \text{ and } v_{f2} = v_{i1}.$$

The change of momentum is  $\Delta p_2 = m(v_{f2} - v_{i2}) = m(v_{i1} - v_{i2})$ . The force on the other cart is

$$F_2 \Delta t = \Delta p_2 \Rightarrow F_2 = \frac{\Delta p_2}{\Delta t} = \frac{m(v_{i1} - v_{i2})}{\Delta t}.$$

The force on your car is equal and opposite.

**CALCULATE:** The time for the collision to occur is  $t = \frac{20.0 \text{ m}}{0.700 \text{ m/s} + 1.10 \text{ m/s}} = 11.11 \text{ s}$  and during this

time the other cart has moved  $x = (0.700 \text{ m/s})(11.11 \text{ s}) = 7.78 \text{ m}$ .

**ROUND:** For the two calculations shown above three significant figures are required:  $t = 11.1 \text{ s}$  and  $x = 7.78 \text{ m}$ .

**PART 2**

**TOTAL QUESTIONS 4**

**FREE-RESPONSE QUESTIONS(FRQ)**

**MARKS 40**



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- (1) Relate the work done by the gravitational force and the gravitational potential energy for an object lifted from rest to a height  $h$  as:  $\Delta U_g = -W_g$ .
- (2) Calculate the change in gravitational potential energy of a mass as:  $\Delta U_g = U_g(y) - U_g(y_o) = mg(y - y_o) = mgh$ .
- (3) Determine the change in potential energy due to spring force:  $\Delta U_s = U_s(y) - U_s(y_o) = \frac{1}{2}kx^2 - \frac{1}{2}kx_o^2$

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### EXAMPLE 6.1 Weightlifting

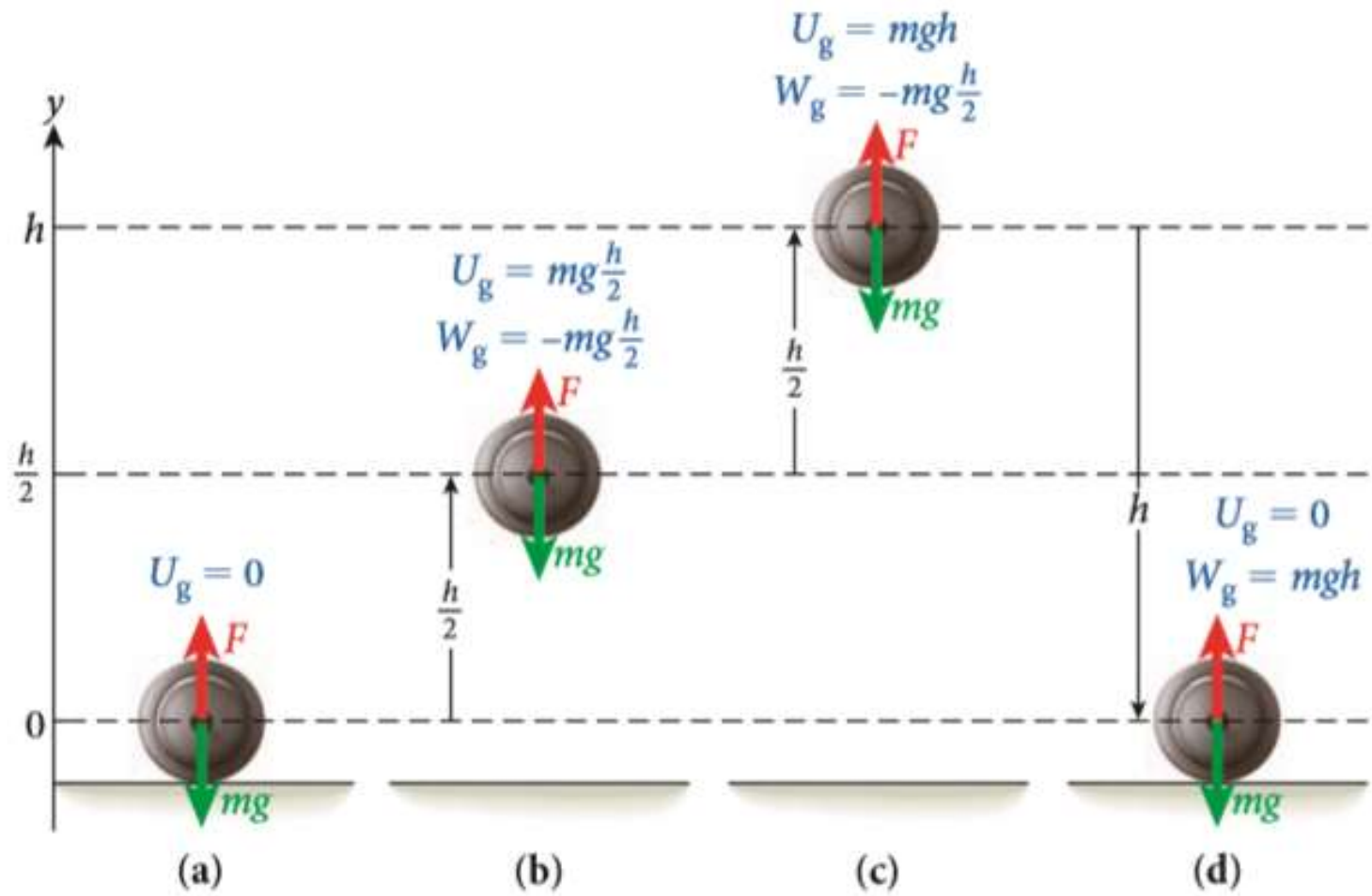
#### PROBLEM

Let's consider the gravitational potential energy in a specific situation: a weightlifter lifting a barbell of mass  $m$ . What is the gravitational potential energy and the work done during the different phases of lifting the barbell?

#### SOLUTION

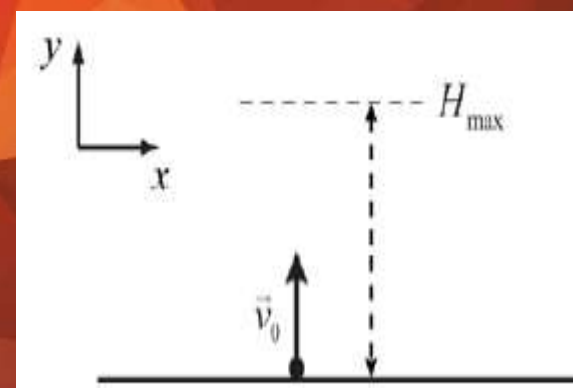
The weightlifter starts with the barbell on the floor, as shown in Figure 6.2a. At  $y = 0$ , the gravitational potential energy can be defined to be  $U_g = 0$ . The weightlifter then picks up the barbell, lifts it to a height of  $y = h/2$ , and holds it there, as shown in Figure 6.2b. The gravitational potential energy is now  $U_g = mgh/2$ , and the work done by gravity on the barbell is  $W_g = -mgh/2$ . The weightlifter next lifts the barbell over his head to a height of  $y = h$ , as shown in Figure 6.2c. The gravitational potential energy is now  $U_g = mgh$ , and the work done by gravity during this part of the lift is again  $W_g = -mgh/2$ . Having completed the lift, the weightlifter lets go of the barbell, and it falls to the floor, as illustrated in Figure 6.2d. The gravitational potential energy of the barbell on the floor is again  $U_g = 0$ , and the work done by gravity during the fall is  $W_g = mgh$ .





if  $\Delta U_g$  is positive, there exists the potential (hence the name *potential energy*) to allow  $\Delta U_g$  to be negative in the future, thereby extracting positive work, since  $W_g = -\Delta U_g$ .

6.41 A ball is thrown up in the air, reaching a height of 5.00 m. Using energy conservation considerations, determine its initial speed.



**THINK:** The maximum height achieved is  $H_{\max} = 5.00 \text{ m}$ , while the initial height  $h_0$  is zero. The speed of the ball when it reaches its maximum height is  $v = 0$ . Determine the initial speed.

**RESEARCH:** In an isolated system with only conservative forces,  $\Delta E_{\text{mec}} = 0$ . Then,  $\Delta K = -\Delta U$ . Use  $U = mgH_{\max}$  and  $K = mv^2 / 2$ .

**SIMPLIFY:**  $K_f - K_i = -(U_f - U_i) = U_i - U_f$ , so  $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh_0 - mgH_{\max}$ .

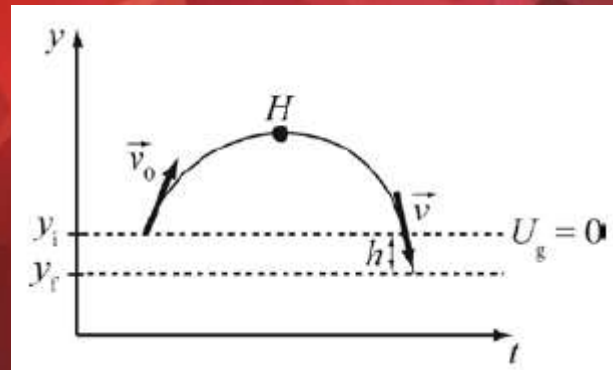
Substituting  $v = 0$  and  $h_0 = 0$  gives the equation  $-\frac{1}{2}mv_0^2 = -mgH_{\max}$ . Therefore,  $v_0 = \sqrt{2gH_{\max}}$ .

**CALCULATE:**  $v_0 = \sqrt{2(9.81 \text{ m/s}^2)(5.00 \text{ m})} = 9.9045 \text{ m/s}$

**ROUND:** With three significant figures in  $H_{\max}$ ,  $v_0 = 9.90 \text{ m/s}$ .



- 6.44 A classmate throws a 1.00-kg book from a height of 1.00 m above the ground straight up into the air. The book reaches a maximum height of 3.00 m above the ground and begins to fall back. Assume that 1.00 m above the ground is the reference level for zero gravitational potential energy. Determine
- the gravitational potential energy of the book when it hits the ground.
  - the velocity of the book just before hitting the ground.



**THINK:** The book's mass is  $m = 1.0$  kg. The initial height is  $y_0 = 1.0$  m, where  $U_g = 0$ , the maximum height is  $H = 3.0$  m, and the final height is  $y_f = 0$  m. Determine (a) the potential energy of the book when it hits the ground,  $U_g$ , and (b) the velocity of the book as it hits the ground,  $v_f$ . The book is thrown straight up into the air, so the launch angle is vertical. The sketch is not a plot of the trajectory of the book, but a plot of height versus time.



## RESEARCH:

- (a) Gravitational potential energy is given by  $U_g = mgh$ . To compute the final energy, consider the height relative to the height of zero potential,  $y_i = 1.0$  m.
- (b) To determine  $v_f$ , consider the initial point to be at  $y = H$  (where  $v = 0$ ), and the final point to be at the point of impact  $y = y_f = 0$ . Assume there are only conservative forces, so that  $\Delta K = -\Delta U$ .  $\Delta U$  between  $H$  and  $y_f$  is unaffected by the choice of reference point.

## SIMPLIFY:

- (a) Relative to  $U_g = 0$  at  $y_i$ , the potential energy of the book when it hits the ground is given by

$$U_g = mgh = mg(y_f - y_i).$$

- (b)  $\Delta K = -\Delta U \Rightarrow K_f - K_i = -(U_f - U_i)$ . With  $v = 0$  at the initial point,  $K_f = U_i - U_f$  and  $(1/2)mv^2 = mgH - mgy_f = mgH$ . Solving for  $v_f$  gives the equation:  $v_f = -\sqrt{2gH}$ . The negative root is chosen because the book is falling.

**CALCULATE:**

(a)  $U_g = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0 - 1.0 \text{ m}) = -9.81 \text{ J}$

(b)  $v_f = -\sqrt{2(9.81 \text{ m/s}^2)(3.0 \text{ m})} = -7.6720 \text{ m/s}$

**ROUND:** With two significant figures in  $m$ ,  $y_i$  and  $H$ :

(a)  $U_g = -9.8 \text{ J}$

(b)  $v_f = -7.7 \text{ m/s}$ , or  $7.7 \text{ m/s}$  downward.

**DOUBLE-CHECK:**  $U_g$  should be negative at  $y_f$ , relative to  $U_g = 0$  at  $y_0$ , because there should be a loss of potential energy. Also, it is sensible for the final velocity of the book to be directed downward.



•6.82 A 1.00-kg block compresses a spring for which  $k = 100. \text{ N/m}$  by 20.0 cm; the spring is then released, and the block moves across a horizontal, frictionless table, where it hits and compresses another spring, for which  $k = 50.0 \text{ N/m}$ . Determine

- the total mechanical energy of the system,
- the speed of the mass while moving freely between springs, and
- the maximum compression of the second spring.

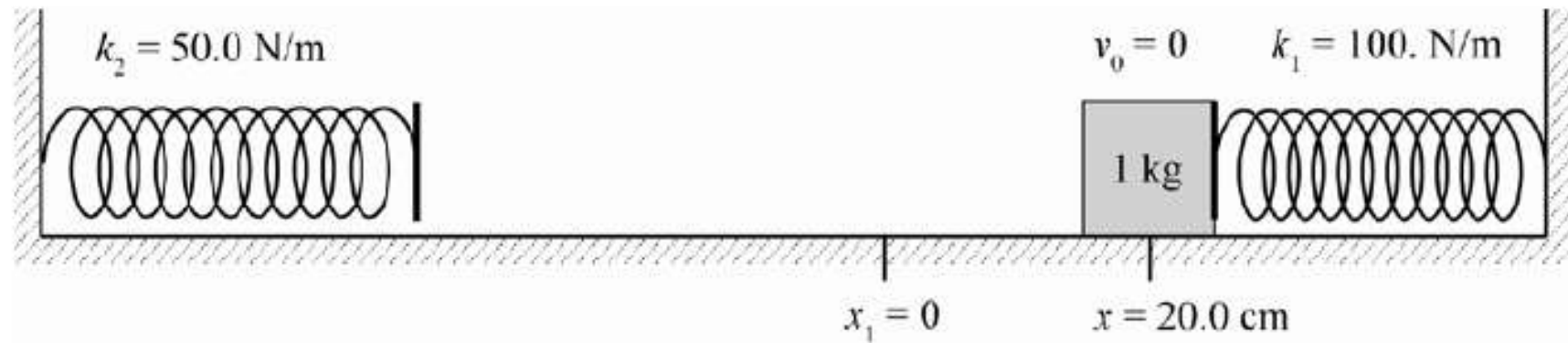
**THINK:** A 1.00 kg block is moving between two springs with constants  $k_1 = 100. \text{ N/m}$  and  $k_2 = 50.0 \text{ N/m}$ . If the block is compressed against spring 1 by 20.0 cm, determine

- the total energy in the system,
- the speed of the block as it moves from one spring to the other and
- the maximum compression on spring 2.

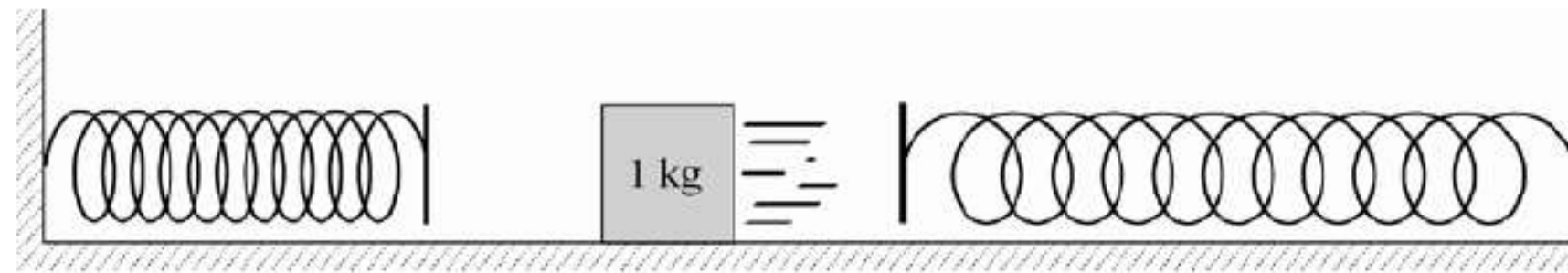


SKETCH:

(a)



(b)



(c)



## RESEARCH:

(a) The total mechanical energy can be determined by recalling that in a conservative system

$$E_{\text{tot}} = \text{constant} = U_{\text{max}} = K_{\text{max}}. \quad U_{\text{max}} \text{ can be determined from spring 1: } U_{\text{max}} = \frac{1}{2}k_1x_{\text{max}}^2 = E_{\text{tot}}.$$

(b)  $K_{\text{max}} = U_{\text{max}} \Rightarrow (mv_{\text{max}}^2)/2 = (k_1v_{\text{max},1}^2)/2$ . Since the system is conservative, the speed of the block is  $v_{\text{max}}$  anytime it is not touching a spring.

(c) The compression on spring 2 can be determined by the following relation:

$$U_{\text{max},2} = K_{\text{max}} \Rightarrow \frac{1}{2}k_2v_{\text{max},2}^2 = K_{\text{max}}.$$

## SIMPLIFY:

$$(a) \quad E_{\text{tot}} = \frac{1}{2}k_1x_{\text{max},1}^2$$

$$(b) \quad v_{\text{max}} = \sqrt{\frac{k_1}{m}x_{\text{max},1}^2} = x_{\text{max},1}\sqrt{\frac{k_1}{m}}$$

$$(c) \quad x_{\text{max},2} = \sqrt{\frac{2K_{\text{max}}}{k_2}}$$

## CALCULATE:

$$(a) \quad E_{\text{tot}} = \frac{1}{2}(100. \text{ N/m})(20.0 \cdot 10^{-2} \text{ m})^2 = 2.00 \text{ J}$$

$$(b) \quad v_{\text{max}} = (20.0 \cdot 10^{-2} \text{ m})\sqrt{\frac{(100. \text{ N/m})}{1.00 \text{ kg}}} = 2.00 \text{ m/s}$$

$$(c) \quad x_{\text{max},2} = \sqrt{\frac{2(2.00 \text{ J})}{50.0 \text{ N/m}}} = 2.83 \cdot 10^{-1} \text{ m} = 28.3 \text{ cm}$$



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- (1) Define mechanical energy as the sum of kinetic energy and potential energy ( $E = K + U$ ).
- (2) State the law of conservation of mechanical energy: "For a mechanical process that occurs inside an isolated system and involves only conservative forces, the total mechanical energy is conserved;  $\Delta E_{mech} = \Delta K + \Delta U = 0$  or  $K + U = K_0 + U_0$ ."
- (3) Apply the work-kinetic energy theorem to relate the work done by a force and the resulting change in kinetic energy.
- (4) Calculate the work done by friction force for an object sliding across a horizontal surface between two points:  $W_f = \vec{f} \cdot \Delta \vec{x} = -f \cdot (x - x_0) = -\mu mg \cdot (x - x_0)$

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- The mechanical energy,  $E$ , is the sum of the kinetic energy and the potential energy:  $E = K + U$ .

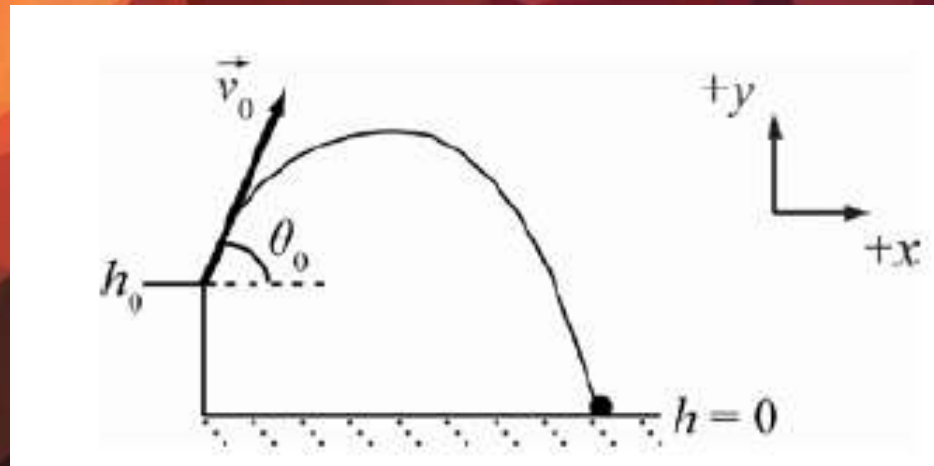
The total mechanical energy is conserved for any mechanical process inside an isolated system that involves only conservative forces:  $\Delta E = \Delta K + \Delta U = 0$ . An alternative way of expressing this law of conservation of mechanical energy is  $K + U = K_0 + U_0$ .

•6.45 Suppose you throw a 0.0520-kg ball with a speed of 10.0 m/s and at an angle of  $30.0^\circ$  above the horizontal from a building 12.0 m high.

- What will be its kinetic energy when it hits the ground?
- What will be its speed when it hits the ground?

**THINK:** The ball's mass is  $m = 0.0520$  kg. The initial speed is  $v_0 = 10.0$  m/s. The launch angle is  $\theta_0 = 30.0^\circ$ . The initial height is  $h_0 = 12.0$  m. Determine:

- kinetic energy of the ball when it hits the ground,  $K_f$  and
- the ball's speed when it hits the ground,  $v$ .





**RESEARCH:** Assuming only conservative forces act on the ball (and neglecting air resistance),  $\Delta K = -\Delta U$ .  $K_f$  can be determined using the equations  $\Delta K = -\Delta U$ ,  $K = mv^2 / 2$  and  $U = mgh$ . Note that  $U_f = 0$ , as  $h = 0$ . With  $K_f$  known,  $v$  can be determined.

**SIMPLIFY:**

$$(a) \Delta K = -\Delta U \Rightarrow K_f - K_i = U_i - U_f = U_i \Rightarrow K_f = U_i + K_i = mgh_0 + \frac{1}{2}mv_0^2$$

$$(b) K_f = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2K_f / m}$$

**CALCULATE:**

$$(a) K_f = (0.0520 \text{ kg})(9.81 \text{ m/s}^2)(12.0 \text{ m}) + \frac{1}{2}(0.0520 \text{ kg})(10.0 \text{ m/s})^2 = 6.121 \text{ J} + 2.60 \text{ J} = 8.721 \text{ J}$$

$$(b) v = \sqrt{2(8.721 \text{ J}) / (0.0520 \text{ kg})} = 18.32 \text{ m/s}$$

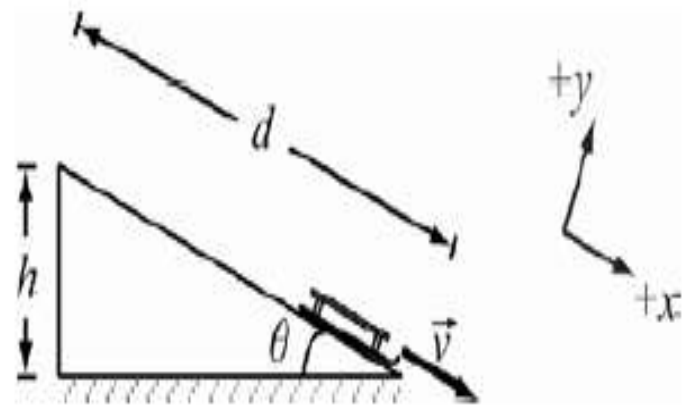
**ROUND:** With  $m$  having three significant figures,  $K_f = 8.72 \text{ J}$  and  $v = 18.3 \text{ m/s}$ .

- 6.47 a) If you are at the top of a toboggan run that is 40.0 m high, how fast will you be going at the bottom, provided you can ignore friction between the sled and the track?
- b) Does the steepness of the run affect how fast you will be going at the bottom?
- c) If you do not ignore the small friction force, does the steepness of the track affect the value of the speed at the bottom?

**THINK:** The initial height is  $h = 40.0$  m. Determine:

- (a) the speed  $v_f$  at the bottom, neglecting friction,  
(b) if the steepness affects the final speed; and  
(c) if the steepness affects the final speed when friction is considered.





## RESEARCH:

- (a) With conservative forces,  $\Delta K = -\Delta U$ .  $v$  can be determined from  $K = (mv_f^2)/2$  and  $U = mgh$ .
- (b and c) Note that the change in the angle  $\theta$  affects the distance,  $d$ , traveled by the toboggan: as  $\theta$  gets larger (the incline steeper),  $d$  gets smaller.
- (c) The change in thermal energy due to friction is proportional to the distance traveled:  $\Delta E_{\text{th}} = \mu_k Nd$ . The total change in energy of an isolated system is  $\Delta E_{\text{tot}} = 0$ , where  $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}}$ , and  $\Delta E_{\text{th}}$  denotes the non-conservative energy of the toboggan-hill system (in this case, friction).

### SIMPLIFY:

(a) With  $K_i = 0$  (assuming  $v_0 = 0$ ) and  $U_f = 0$  (taking the bottom to be  $h = 0$ ):

$$K_f = U_i \Rightarrow \frac{1}{2}mv_f^2 = mgh \Rightarrow v_f = \sqrt{2gh}$$

(b) The steepness does not affect the final speed, in a system with only conservative forces, the distance traveled is not used when conservation of mechanical energy is considered.

(c) With friction considered, then for the toboggan-hill system,

$$\Delta E = \Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow \Delta K = -\Delta U - \Delta E_{\text{th}} \Rightarrow K_f = U_i - \Delta E_{\text{th}} = mgh - \mu_k Nd$$

The normal force  $N$  is given by  $N = mg\cos\theta$ , while on the hill. With  $d = h / \sin\theta$ ,

$$K_f = mgh - \mu_k (mg\cos\theta) \left( \frac{h}{\sin\theta} \right) = mgh(1 - \mu_k \cot\theta).$$

The steepness of the hill does affect  $K_f$  and therefore  $v$  at the bottom of the hill.

### CALCULATE:

$$(a) v_f = \sqrt{2(9.81 \text{ m/s}^2)(40.0 \text{ m})} = 28.01 \text{ m/s}$$

**ROUND:** Since  $h$  has three significant figures,  $v = 28.0 \text{ m/s}$ .

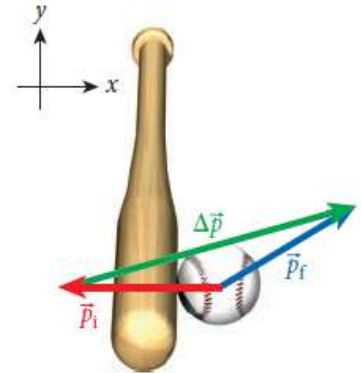


- (1) Calculate the change in momentum (due to change in velocity) as the difference between the final and initial momenta.  $(\Delta\vec{p} = \vec{p}_f - \vec{p}_i) = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$
- (2) Apply the relationship between impulse, change in momentum, average force, and the time interval over which the impulse acts on the object to calculate unknown physical quantities.

### EXAMPLE 7.1 Baseball Home Run

A Major League pitcher throws a fastball that crosses home plate with a speed of 90.0 mph (40.23 m/s) and an angle of  $5.0^\circ$  below the horizontal. A batter slugs it for a home run, launching it with a speed of 110.0 mph (49.17 m/s) at an angle of  $35.0^\circ$  above the horizontal (Figure 7.4). The mass of a baseball is required to be between 5 and 5.25 oz; let's say that the mass of the ball hit here is 5.10 oz (0.145 kg).

– Continued



### PROBLEM 1

What is the magnitude of the impulse the baseball receives from the bat?

### SOLUTION 1

The impulse is equal to the momentum change of the baseball. Unfortunately, there is no shortcut; we must calculate  $\Delta\vec{v} \equiv \vec{v}_f - \vec{v}_i$  for the  $x$ - and  $y$ -components separately, add them as vectors, and finally multiply by the mass of the baseball:

$$\Delta v_x = (49.17 \text{ m/s})(\cos 35.0^\circ) - (40.23 \text{ m/s})(\cos 185.0^\circ) = 80.35 \text{ m/s}$$

$$\Delta v_y = (49.17 \text{ m/s})(\sin 35.0^\circ) - (40.23 \text{ m/s})(\sin 185.0^\circ) = 31.71 \text{ m/s}$$

$$\Delta v = \sqrt{\Delta v_x^2 + \Delta v_y^2} = \sqrt{(80.35)^2 + (31.71)^2} \text{ m/s} = 86.38 \text{ m/s}$$

$$\Delta p = m\Delta v = (0.145 \text{ kg})(86.38 \text{ m/s}) = 12.5 \text{ kg m/s.}$$

## PROBLEM 2

High-speed video shows that the ball-bat contact lasts only about 1 ms (0.001 s). Suppose, for the home run we're considering, that the contact lasted 1.20 ms. What was the magnitude of the average force exerted on the ball by the bat during that time?

## SOLUTION 2

The force can be calculated by simply using the formula for the impulse:

$$\Delta \vec{p} = \vec{J} = \vec{F}_{\text{ave}} \Delta t$$
$$\Rightarrow F_{\text{ave}} = \frac{\Delta p}{\Delta t} = \frac{12.5 \text{ kg m/s}}{0.00120 \text{ s}} = 10.4 \text{ kN.}$$



**FIGURE 7.5** A baseball being compressed as it is hit by a baseball bat.

This force is approximately the same as the weight of an entire baseball team! The collision of the bat and the ball results in significant compression of the baseball, as shown in Figure 7.5.



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(1) Relate momentum to kinetic energy.

(2) Combine the equations from momentum and kinetic energy conservation for an elastic collision to obtain expressions for final velocities:

$$P_{f(1,x)} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \times P_{i(1,x)} + \left( \frac{2m_2}{m_1 + m_2} \right) \times P_{i(2,x)} \quad / \quad P_{f(2,x)} = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) \times P_{i(2,x)} + \left( \frac{2m_1}{m_1 + m_2} \right) \times P_{i(1,x)}$$

**7.10** A red curling stone moving with a speed of 2.0 m/s collides head-on with a yellow curling stone at rest (totally elastic collision). What are the speeds of the two curling stones just after the collision?

- a) The red stone is at rest, and the yellow stone is moving with a speed of 2.0 m/s.
- b) The red stone and the yellow stone are both moving with a speed of 1.0 m/s.
- c) The red stone bounces off the yellow stone and moves with a speed of 2.0 m/s, and the yellow stone remains at rest.

**7.10) a**

**MCQ 7.10**  
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**7.24/7.25**  
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**Exercise Que.**  
**7.51**  
**PAGE NO. 219**

7.24 Rank the following objects from highest to lowest in terms of momentum and from highest to lowest in terms of energy.

- a) an asteroid with mass  $10^6$  kg and speed 500 m/s
- b) a high-speed train with a mass of 180,000 kg and a speed of 300 km/h
- c) a 120-kg linebacker with a speed of 10 m/s
- d) a 10-kg cannonball with a speed of 120 m/s
- e) a proton with a mass of  $2 \cdot 10^{-27}$  kg and a speed of  $2 \cdot 10^8$  m/s

	$m$	$v$
(a)	$10^6$ kg	500 m/s
(b)	180,000 kg	300 km/h
(c)	120 kg	10 m/s
(d)	10 kg	120 m/s
(e)	$2 \cdot 10^{-27}$ kg	$2 \cdot 10^8$ m/s

RESEARCH:  $E = \frac{1}{2}mv^2$ ,  $p = mv$



**CALCULATE:**

$$(a) E = \frac{1}{2}(10^6 \text{ kg})(500 \text{ m/s})^2 = 1.3 \cdot 10^{11} \text{ J}, \quad p = (10^6 \text{ kg})(500 \text{ m/s}) = 5.0 \cdot 10^8 \text{ kg m/s},$$

$$(b) E = \frac{1}{2}(1.8 \cdot 10^5 \text{ kg}) \left( (300 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) \right)^2 = 6.3 \cdot 10^8 \text{ J},$$

$$p = (1.8 \cdot 10^5 \text{ kg}) \left( (300 \text{ km/h}) \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) \right) = 1.5 \cdot 10^7 \text{ kg m/s}$$

$$(c) p_y = \sqrt{2(49.5 \text{ J})(0.442 \text{ kg})} \sin 58.0^\circ = 5.610 \text{ kg m/s}, \quad p = (120 \text{ kg})(10 \text{ m/s}) = 1200 \text{ kg m/s}$$

$$(d) E = \frac{1}{2}(10 \text{ kg})(120 \text{ m/s})^2 = 7.2 \cdot 10^4 \text{ J}, \quad p = (10 \text{ kg})(120 \text{ m/s}) = 1200 \text{ kg m/s}$$

$$(e) E = \frac{1}{2}(2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s})^2 = 4 \cdot 10^{-11} \text{ J}, \quad p = (2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s}) = 4 \cdot 10^{-19} \text{ kg m/s}$$

	$E$ [J]	$p$ [kg m/s]
(a)	$1 \cdot 10^{11}$	$5 \cdot 10^8$
(b)	$6 \cdot 10^8$	$2 \cdot 10^7$
(c)	$6 \cdot 10^3$	$1 \cdot 10^3$
(d)	$7 \cdot 10^4$	$1 \cdot 10^3$
(e)	$4 \cdot 10^{-11}$	$4 \cdot 10^{-19}$

**DOUBLE-CHECK:** In order from largest to smallest energy: (a), (b), (d), (c), (e); and momentum: (a), (b), (d) = (c), (e).

**7.25** A car of mass 1200. kg, moving with a speed of 72.0 mph on a highway, passes a small SUV with a mass  $1\frac{1}{2}$  times bigger, moving at  $\frac{2}{3}$  the speed of the car.

- What is the ratio of the momentum of the SUV to that of the car?
- What is the ratio of the kinetic energy of the SUV to that of the car?

**THINK:** Compute the ratios of the momenta and kinetic energies of the car and SUV.

$$m_{\text{car}} = 1200. \text{ kg}, m_{\text{SUV}} = 1.5m_{\text{car}} = \frac{3}{2}m_{\text{car}}, v_{\text{car}} = 72.0 \text{ mph}, \text{ and } v_{\text{SUV}} = \frac{2}{3}v_{\text{car}}.$$

**RESEARCH:**

(a)  $p = mv$

(b)  $K = \frac{1}{2}mv^2$

**SKETCH:**





## SIMPLIFY:

$$(a) \frac{p_{\text{SUV}}}{p_{\text{car}}} = \frac{m_{\text{SUV}} v_{\text{SUV}}}{m_{\text{car}} v_{\text{car}}} = \frac{(3/2)m_{\text{car}} (2/3)v_{\text{car}}}{m_{\text{car}} v_{\text{car}}}$$

$$(b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = \frac{(1/2)m_{\text{SUV}} v_{\text{SUV}}^2}{(1/2)m_{\text{car}} v_{\text{car}}^2} = \frac{(3/2)m_{\text{car}} \left( (2/3)v_{\text{car}} \right)^2}{m_{\text{car}} v_{\text{car}}^2}$$

## CALCULATE:

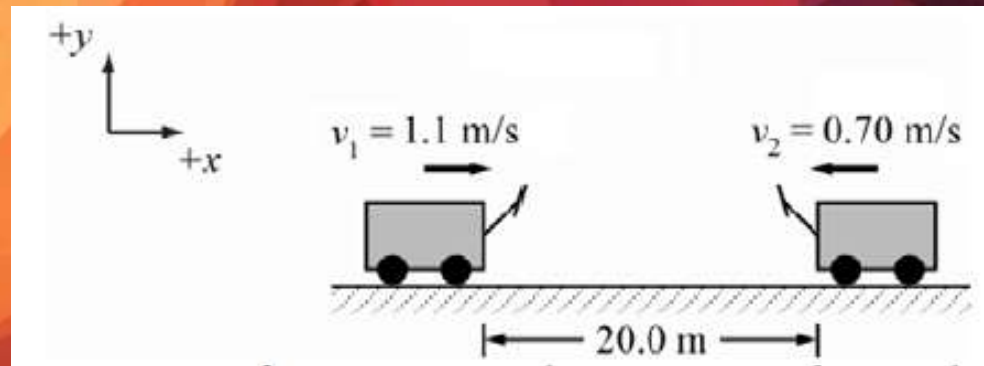
$$(a) \frac{p_{\text{SUV}}}{p_{\text{car}}} = \frac{(3/2)(2/3)}{1} = 1$$

$$(b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = \frac{(3/2)(2/3)^2}{1} = \frac{(3/2)(4/9)}{1} = 2/3 = 0.6667$$

$$\text{ROUND: (a) } \frac{p_{\text{SUV}}}{p_{\text{car}}} = 1.0 \quad (b) \frac{K_{\text{SUV}}}{K_{\text{car}}} = 0.67$$

•7.51 You notice that a shopping cart 20.0 m away is moving with a velocity of 0.700 m/s toward you. You launch an identical cart with a velocity of 1.10 m/s directly at the other cart in order to intercept it. When the two carts collide elastically, they remain in contact for 0.200 s. Graph the position, velocity, and force for both carts as a function of time.

**THINK:** Two carts, separated by a distance  $x_0 = 20.0$  m, are travelling towards each other with speeds  $v_1 = 1.10$  m/s and  $v_2 = 0.700$  m/s. They collide for  $\Delta t = 0.200$  s. This is an elastic collision. I need to plot  $x$  vs.  $t$ ,  $v$  vs.  $t$  and  $F$  vs.  $t$ .



**RESEARCH:** Use the conservation of momentum and energy to get the speeds after collision. Then use the impulse  $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$  to get the force.



**SIMPLIFY:** First, need the position of the collision. Using  $x = x_0 + v_0 t \Rightarrow x_1 = 0 + v_1 t$  and  $x_2 = x_0 - v_2 t$ ,  
 $x_1 = x_2 = v_1 t = x_0 - v_2 t \Rightarrow t = x_0 / (v_1 + v_2)$ . Conservation of momentum:

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$$

$$v_{i1}^2 - v_{f1}^2 = v_{f2}^2 - v_{i2}^2$$

$$(v_{i1} - v_{f1})(v_{i1} + v_{f1}) = (v_{f2} - v_{i2})(v_{f2} + v_{i2})$$

$$v_{i1} + v_{f1} = v_{f2} + v_{i2}$$

$$v_{f2} = v_{i1} + v_{f1} - v_{i2}$$

Substituting back into (1):

$$v_{i1} - v_{f1} = v_{i1} + v_{f1} - v_{i2} - v_{f2} \Rightarrow 2v_{f1} = 2v_{i2} \Rightarrow v_{f1} = v_{i2} \text{ and } v_{f2} = v_{i1}.$$

The change of momentum is  $\Delta p_2 = m(v_{f2} - v_{i2}) = m(v_{i1} - v_{i2})$ . The force on the other cart is

$$F_2 \Delta t = \Delta p_2 \Rightarrow F_2 = \frac{\Delta p_2}{\Delta t} = \frac{m(v_{i1} - v_{i2})}{\Delta t}.$$

The force on your car is equal and opposite.

**CALCULATE:** The time for the collision to occur is  $t = \frac{20.0 \text{ m}}{0.700 \text{ m/s} + 1.10 \text{ m/s}} = 11.11 \text{ s}$  and during this

time the other cart has moved  $x = (0.700 \text{ m/s})(11.11 \text{ s}) = 7.78 \text{ m}$ .

**ROUND:** For the two calculations shown above three significant figures are required:  $t = 11.1 \text{ s}$  and  $x = 7.78 \text{ m}$ .



**KEEP  
CALM  
AND  
STUDY  
HARD**



Thank  
You!