تم تحميل هذا الملف من موقع المناهج الإماراتية	
	almanahj.com/ae موقع المناهج الإماراتية

الملف حل مراجعة أسئلة وفق الهيكل الوزاري
موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ رياضيات ⇦ الفصل الثالث

روابط مواقع التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم			
			
روابط مواد الصف الحادي عشر المتقدم على تلغرام			
التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثالث	
1	أسئلة الاختبار التجريبي الأول نخبة
2	حل أسئلة وفق الهيكل الوزاري نخبة
3	مراجعة نهائية مكونة من ثلاثة أجزاء
4	أسئلة الامتحان النهائي
5	نموذج أسئلة وفق الهيكل الوزاري الجديد

مؤسسة الإمارات للتعليم المدرسي
EMIRATES SCHOOLS ESTABLISHMENT

United Arab Emirates

مدرسة سلامة بنت بطي -Salamah bint buti School

مراجعة

أسئلة الهيكل

المادة: رياضيات

الصف الحادي عشر متقدم

الفصل الدراسي الثالث 2023/2022

إعداد المعلمة: فاطمة السعيدي

طالباتي العزيزات، العلم هو الوسيلة الوحيدة التي يرتفع بها شأن الإنسان إلى مراتب الكرامة والشرف.

Academic Year العام الدراسي	2022/2023
Term الفصل	3
Subject	Mathematics/Reveal+Bridge
المادة	الرياضيات/ريفيل+ بريدج
Grade الصف	11
Stream	Advanced
المسار	المتقدم
Number of Main Questions عدد الأسئلة الأساسية	Part (1) - 10
	Part (2) - 10
	Part (3) - 3
Marks per Main Question الدرجات لكل سؤال أساسي	Part (1) - 3
	Part (2) - 5
	Part (3) - (6-7)
****Number of Bonus Questions عدد الأسئلة الإضافية	2
Marks per Bonus Question الدرجات لكل سؤال إضافي	5
*** Type of All Questions نوع كافة الأسئلة	Part(1 and 2) MCQ
	Part (3) FRQ
* Maximum Overall Grade *الدرجة القصوى الممكنة	110
Exam Duration - مدة الامتحان	150 minutes
Mode of Implementation -طريقة التطبيق	SwiftAssess & Paper-Based
Calculator	Allowed
الآلة الحاسبة	مسموحة

Part (1) - 10

1	Graph points with polar coordinates التمثيل البياني للنقاط باستخدام إحداثيات القطب

Find three different pairs of polar coordinates that name the given point if $-360° \leq \theta \leq 360°$ or $-2\pi \leq \theta \leq 2\pi$.

14. $(1, 150°)$

$(1, 150° - 360°) = (1, -210°)$

$(1, 150° + 180°) = (-1, 330°)$

$(1, 150° - 180°) = (-1, -30°)$

15. $(-2, 300°)$

$(2, 300° - 180°) = (2, 120°)$

$(-2, 300° - 360°) = (-2, -60°)$

$(2, -60° - 180°) = (2, -240°)$

16. $\left(4, -\frac{7\pi}{6}\right)$

$\left(4, -\frac{7\pi}{6} + 2\pi\right) = (4, \frac{5\pi}{6})$

$\left(-4, -\frac{7\pi}{6} + \pi\right) = (-4, -\frac{\pi}{6})$

$\left(-4, -\frac{\pi}{6} + 2\pi\right) = (-4, \frac{11\pi}{6})$

17. $\left(-3, \frac{2\pi}{3}\right)$

$\left(3, \frac{2\pi}{3} + \pi\right) = (3, \frac{5\pi}{3})$

$\left(3, \frac{2\pi}{3} - \pi\right) = (3, -\frac{\pi}{3})$

$\left(-3, \frac{2\pi}{3} - 2\pi\right) = (-3, -\frac{4\pi}{3})$

18. $\left(5, \frac{11\pi}{6}\right)$

$\left(5, \frac{11\pi}{6} - 2\pi\right) = (5, -\frac{\pi}{6})$

$\left(-5, \frac{11\pi}{6} - \pi\right) = (-5, \frac{5\pi}{6})$

$\left(-5, \frac{5\pi}{6} - 2\pi\right) = (-5, -\frac{7\pi}{6})$

19. $\left(-5, -\frac{4\pi}{3}\right)$

$\left(-5, -\frac{4\pi}{3} + 2\pi\right) = (-5, \frac{2\pi}{6})$

$\left(5, -\frac{4\pi}{3} + \pi\right) = (5, -\frac{\pi}{3})$

$\left(5, -\frac{\pi}{3} + 2\pi\right) = (5, \frac{5\pi}{3})$

20. $(2, -30°)$

$(2, -30° + 360°) = (2, 330°)$

$(-2, -30° + 180°) = (-2, 150°)$

$(-2, -30° - 180°) = (-2, -210°)$

21. $(-1, -240°)$

$(1, -240° - 180°) = (1, 300°)$

$(1, -240° + 180°) = (1, -60°)$

$(-1, -240° - 360°) = (-1, 120°)$

Find a different pair of polar coordinates for each point such that $0 \leq \theta \leq 180°$ or $0 \leq \theta \leq \pi$.

44. $(5, 960°)$

We need to subtract $960°$ by $180k$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$$(-5, 960° - 5(180°)) = (-5, 60°)$$

45. $\left(-2.5, \frac{5\pi}{2}\right)$

We need to subtract $\frac{5\pi}{3}$ from $k\pi$, such that the result is between 0 and π. Test multiples of π.

$$\left(-2.5, \frac{5\pi}{3} - 2\pi\right) = \left(-2.5, \frac{\pi}{2}\right)$$

46. $\left(4, \frac{11\pi}{4}\right)$

We need to subtract $\frac{11\pi}{4}$ from $k\pi$, such that the result is between 0 and π. Test multiples of π.

$$\left(4, \frac{11\pi}{4} - 2\pi\right) = \left(4, \frac{3\pi}{4}\right)$$

47. $(1.25, -920°)$

We need to add $180k$ to $-920°$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$$(1.25, -920° + 6(180°)) = (1.25, 160°)$$

48. $\left(-1, -\frac{21\pi}{8}\right)$

We need to add $k\pi$ to $-\frac{21\pi}{8}$, such that the result is between 0 and π. Test multiples of π.

$$\left(1, -\frac{21\pi}{8} + 3\pi\right) = \left(1, \frac{3\pi}{8}\right)$$

49. $(-6, -1460°)$

We need to add $180k$ to $-1460°$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$$(6, -1460° + 9(180°)) = (6, 160°)$$

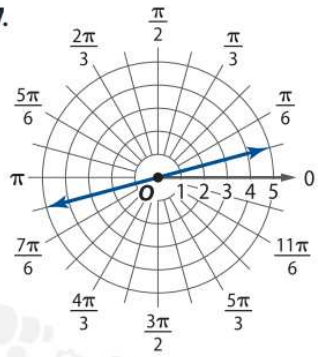
Important Note:

- If k is odd, we need to replace r with $-r$ and $-r$ with r to obtain the correct polar coordinates.
- If k is even, use r obtain the correct polar coordinates.

2	Graph simple polar equations
	التمثيل البياني للمعادلات القطبية البسيطة

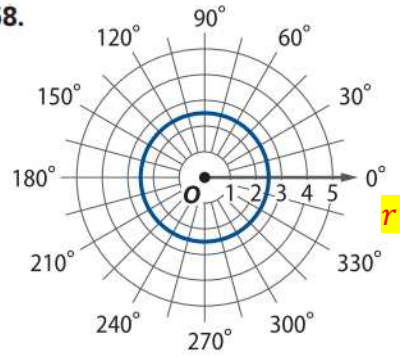
Write an equation for each polar graph.

57.



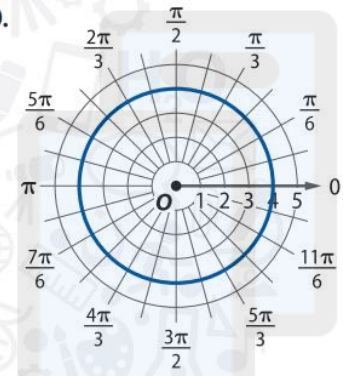
$\theta = \frac{\pi}{12}$

58.



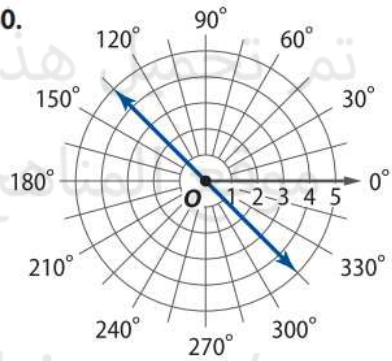
$r = 2.5$ or $r = -2.5$

59.



$r = 4$ or $r = -4$

60.

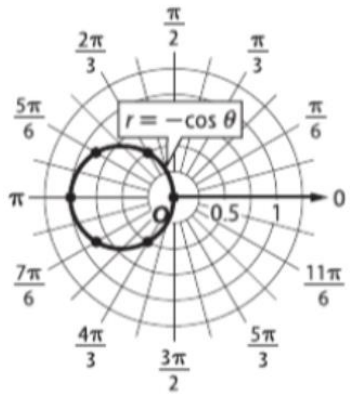


$\theta = 135°$

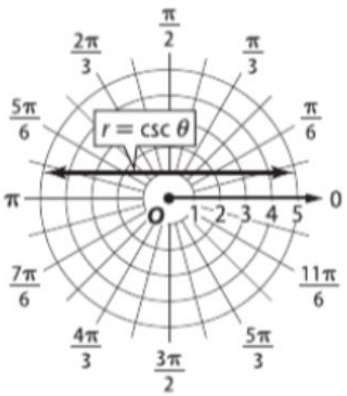
3	Graph polar equations
	تمثيل المعادلات القطبية بيانيا

Graph each equation by plotting points.

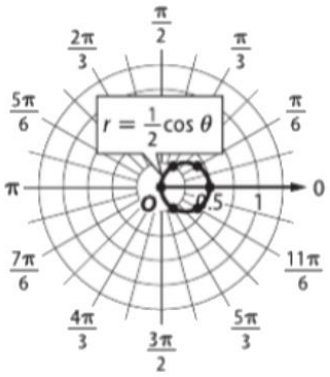
1. $r = -\cos\theta$



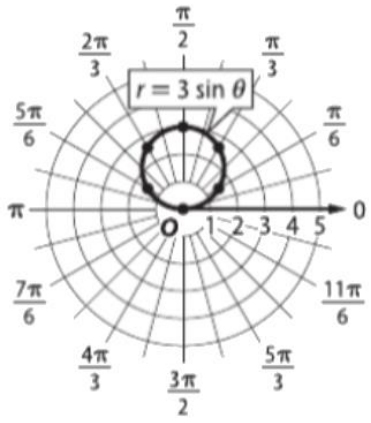
2. $r = \csc\theta$



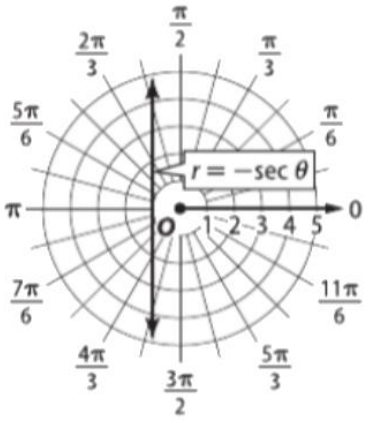
3. $r = \frac{1}{2}\cos\theta$



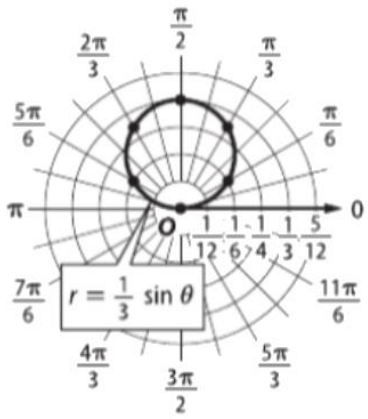
4. $r = 3 \sin \theta$



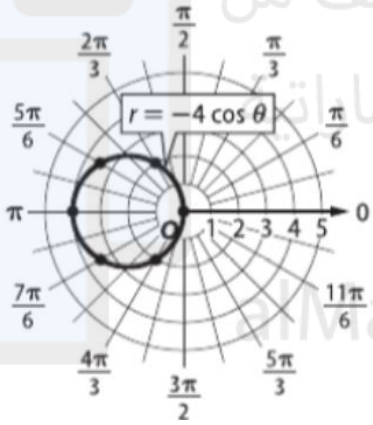
5. $r = -\sec \theta$



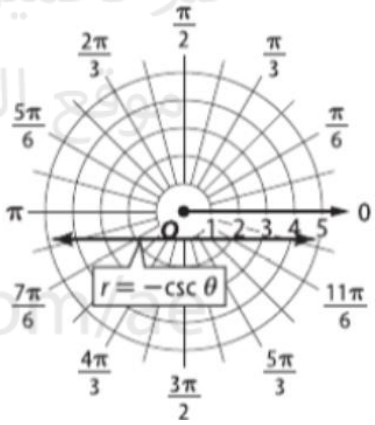
6. $r = \frac{1}{3} \sin \theta$



7. $r = -4 \cos \theta$



8. $r = -\csc \theta$



4	Convert between polar and rectangular coordinates
	التحويل بين الإحداثيات القطبية والديكارتية

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest hundredth, if necessary. $x = r \cos \theta$ $y = r \sin \theta$

1. $\left(2, \frac{\pi}{4}\right)$ $r = 2$ and $\theta = \frac{\pi}{4}$

$x = 2 \cos \frac{\pi}{4} = 2\sqrt{2}$

$y = 2 \sin \frac{\pi}{4} = 2\sqrt{2}$

$(2\sqrt{2}, 2\sqrt{2})$

2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$ $r = \frac{1}{4}$ and $\theta = \frac{\pi}{2}$

$x = \frac{1}{4} \cos \frac{\pi}{2} = 0$

$y = \frac{1}{4} \sin \frac{\pi}{2} = \frac{1}{4}$

$(0, \frac{1}{4})$

3. $(5, 240°)$ $r = 5$ and $\theta = 240°$

$x = 5\cos 240° = -\frac{5}{2}$

$y = 5\sin 240° = -\frac{5\sqrt{3}}{2}$

$(-\frac{5}{2}, -\frac{5\sqrt{3}}{2})$

4. $(2.5, 250°)$ $r = 2$ and $\theta = 250°$

$x = 2.5\cos 250° = -0.86$

$y = 2.5\sin 250° = -2.35$

$(-0.86, -2.35)$

5. $\left(-2, \frac{4\pi}{3}\right)$ $r = -2$ and $\theta = 240°$

$x = -2\cos\frac{4\pi}{3} = 1$

$y = -2\sin\frac{4\pi}{3} = \sqrt{3}$

$(1, \sqrt{3})$

6. $(-13, -70°)$ $r = -13$ and $\theta = -70°$

$x = -13\cos(-70°) = -4.45$

$y = -13\sin(-70°) = 12.22$

$(-4.45, 12.22)$

7. $\left(3, \frac{\pi}{2}\right)$ $r = 3$ and $\theta = \frac{\pi}{2}$

$x = 3\cos\frac{\pi}{2} = 0$

$y = 3\sin\frac{\pi}{2} = 3$

$(0, 3)$

8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$ $r = \frac{1}{2}$ and $\theta = \frac{3\pi}{4}$

$x = \frac{1}{2}\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{4}$

$y = \frac{1}{2}\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{4}$

$(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$

9. $(-2, 270°)$ $r = -2$ and $\theta = 270°$

$x = -2\cos 270° = 0$

$y = -2\sin 270° = 2$

$(0, 2)$

10. $(4, 210°)$ $r = 4$ and $\theta = 210°$

$x = 4\cos 210° = -2\sqrt{3}$

$y = 4\sin 210° = -2$

$(-2\sqrt{3}, -2)$

11. $\left(-1, -\frac{\pi}{6}\right)$ $r = -1$ and $\theta = -\frac{\pi}{6}$

$x = -1\cos -\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$y = -1\sin -\frac{\pi}{6} = \frac{1}{2}$

$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

12. $\left(5, \frac{\pi}{3}\right)$ $r = 5$ and $\theta = \frac{\pi}{3}$

$x = 5\cos\frac{\pi}{3} = \frac{5}{2}$

$y = 5\sin\frac{\pi}{3} = \frac{5\sqrt{3}}{2}$

$(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

5	Relate arithmetic sequences to linear functions
	ربط المتتاليات الحسابية بالدوال الخطية

Determine whether each sequence is arithmetic. Write yes or no. $d = a_2 - a_1$

1. 8, −2, −12, −22, ***yes***

$d = -2 - 8 = -10$

$d = -12 - (-2) = -10$

$d = -22 - (-12) = -10$

2. −19, −12, −5, 2, 9 ***yes***

$d = -12 - (-19) = 7$

$d = -5 - (-12) = 7$

$d = 2 - (-5) = 7$

$d = 9 - (2) = 7$

3. 1, 2, 4, 8, 16 ***no***

$d = 2 - 1 = 1$

$d = 4 - 2 = 2$

4. 0.6, 0.9, 1.2, 1.8, … ***no***

$d = 0.9 - 0.6 = 0.3$

$d = 1.2 - 0.9 = 0.3$

$d = 1.8 - 1.2 = 0.6$

21. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots$ ***no***

$d = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

$d = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$

22. −9, −3, 0, 3, 9 ***no***

$d = -3 - (-9) = 6$

$d = 0 - (-3) = 3$

23. 14, −5, −19, … ***no***

$d = -5 - 14 = -19$

$d = -19 - (-5) = -14$

24. $\frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{11}{9}, \cdots$ ***yes***

$d = \frac{5}{9} - \frac{2}{9} = \frac{1}{3}$

$d = \frac{8}{9} - \frac{5}{9} = \frac{1}{3}$

$d = \frac{11}{9} - \frac{8}{9} = \frac{1}{3}$

6	Relate geometricsequences to exponential functions
	ربط المتتاليات الهندسية بالدوال الأسية

Find the next three terms of each geometric sequence. Then graph the sequence.

$$r = \frac{a_2}{a_1}$$

14. 8, 12, 18, 27, ...

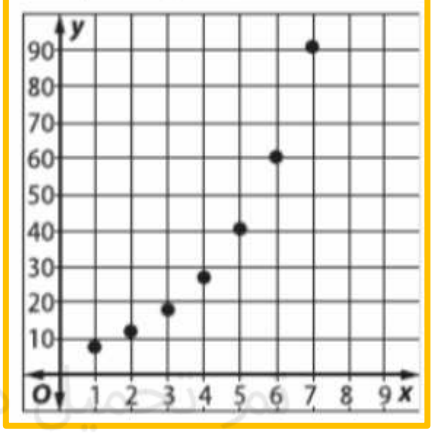
$$r = \frac{12}{8} = 1.5$$

$$a_5 = 27 \times 1.5 = 40.5$$

$$a_6 = 40.5 \times 1.5 = 60.75$$

$$a_7 = 60.75 \times 1.5 = 91.125$$

40.5, 60.75, 91.125



15. 8, 16, 32, 64, ...

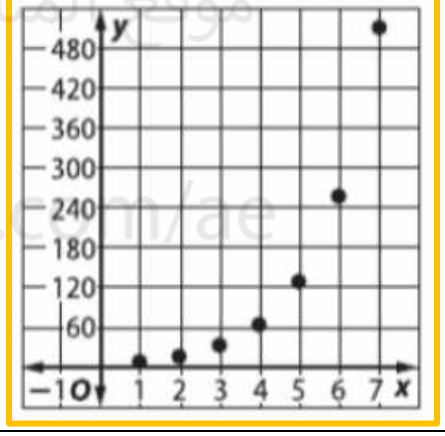
$$r = \frac{16}{8} = 2$$

$$a_5 = 64 \times 2 = 128$$

$$a_6 = 128 \times 2 = 256$$

$$a_7 = 256 \times 2 = 512$$

128, 256, 512



16. 250, 50, 10, 2, ...

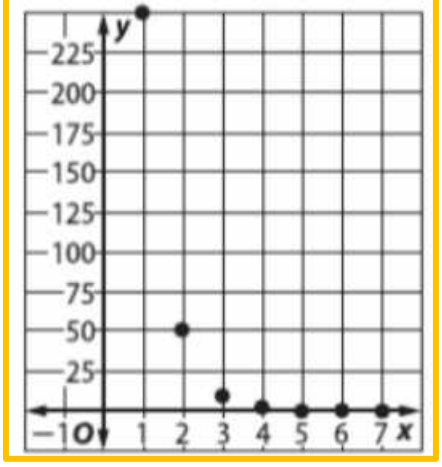
$$r = \frac{50}{250} = 0.2$$

$$a_5 = 2 \times 0.2 = \frac{2}{5}$$

$$a_6 = \frac{2}{5} \times 0.2 = \frac{2}{25}$$

$$a_7 = \frac{2}{25} \times 0.2 = \frac{2}{125}$$

$\frac{2}{5}, \frac{2}{25}, \frac{2}{125}$



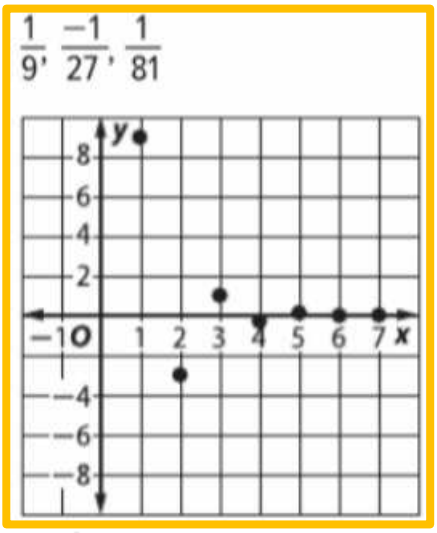
17. $9, -3, 1, -\frac{1}{3}, \ldots$

$$r = \frac{-3}{9} = -\frac{1}{3}$$

$$a_5 = -\frac{1}{3} \times -\frac{1}{3} = \frac{1}{9}$$

$$a_6 = \frac{1}{9} \times -\frac{1}{3} = -\frac{1}{27}$$

$$a_7 = -\frac{1}{27} \times -\frac{1}{3} = \frac{1}{81}$$



7	Find the nth term and arithmetic means of arithmetic sequences إيجاد الحد النوني والوسط الحسابي للمتتاليات الحسابية

Write an equation for the nth term of each arithmetic sequence. $a_n = a_1 + (n-1)d$

20. $24, 35, 46, \ldots$ $a_n = 11n + 13$	**21.** $31, 17, 3, \ldots$ $a_n = -14n + 45$	**22.** $a_9 = 45, d = -3$ $a_n = -3n + 72$
23. $a_7 = 21, d = 5$ $a_n = 5n - 14$	**24.** $a_4 = 12, d = 0.25$ $a_n = 0.25n + 11$	**25.** $a_5 = 1.5, d = 4.5$ $a_n = 4.5n - 21$
26. $9, 2, -5, \ldots$ $a_n = -7n + 16$	**27.** $a_6 = 22, d = 9$ $a_n = 9n - 32$	**28.** $a_8 = -8, d = -2$ $a_n = -2n + 8$
29. $a_{15} = 7, d = \frac{2}{3}$ $a_n = \frac{2}{3}n - 3$	**30.** $-12, -17, -22, \ldots$ $a_n = -5n - 7$	**31.** $a_3 = -\frac{4}{5}, d = \frac{1}{2}$ $a_n = \frac{1}{2}n - \frac{23}{10}$

8	Find the nth term and geometric means of geometric sequences
	إيجاد الحد النوني والوسط الحسابي للمتتاليات الهندسية

Find a_n for each geometric sequence. $a_n = a_1 r^{n-1}$

18. $a_1 = 2400, r = \frac{1}{4}, n = 7$

$a_n = a_1 r^{n-1}$

$a_n = 2400(\frac{1}{4})^{7-1}$

$a_n = \frac{75}{128}$

19. $a_1 = 800, r = \frac{1}{2}, n = 6$

$a_n = a_1 r^{n-1}$

$a_n = 800(\frac{1}{2})^{6-1}$

$a_n = 25$

20. $a_1 = \frac{2}{9}, r = 3, n = 7$

$a_n = a_1 r^{n-1}$

$a_n = \frac{2}{9}(3)^{7-1}$

$a_n = 162$

21. $a_1 = -4, r = -2, n = 8$

$a_n = a_1 r^{n-1}$

$a_n = -4(-2)^{8-1}$

$a_n = 512$

9	Write repeating decimals as fractions
	كتابة الكسور العشرية المتكررة في صورة كسور اعتيادية

Write each repeating decimal as a fraction. $S_n = \frac{a_1}{1-r}$

14. $0.\overline{35}$

$0.\overline{35} = 0.35 + 0.0035 + \ldots$

$a_1 = 0.35 \qquad r = \frac{0.0035}{0.35} = 0.01$

$S_n = \frac{0.35}{1-0.01} = \frac{35}{99}$

15. $0.\overline{642}$

$0.\overline{642} = 0.642 + 0.000642$

$a_1 = 0.642 \qquad r = \frac{0.000642}{0.642} = 0.01$

$S_n = \frac{0.642}{1-0.01} = \frac{214}{333}$

Write $0.\overline{63}$ as a fraction.

$S_n = \frac{0.63}{1-0.01} = \frac{7}{11}$

Write $0.\overline{21}$ as a fraction.

$S_n = \frac{0.21}{1-0.01} = \frac{7}{33}$

10	Recognize recursive functions
	التعرف على دوال التكرار

Find the first three iterates of each function for the given initial value.

8. $f(x) = 5x + 2, x_0 = 8$	$x_1 = f(x_0) = 5(8) + 2 = 42$ $x_2 = f(x_1) = 5(42) + 2 = 212$ $x_3 = f(x_2) = 5(212) + 2 = 1062$
9. $f(x) = -4x + 2, x_0 = 5$	$x_1 = f(x_0) = -4(5) + 2 = -18$ $x_2 = f(x_1) = -4(-18) + 2 = 74$ $x_3 = f(x_2) = -4(74) + 2 = -294$
10. $f(x) = 6x + 3, x_0 = -4$	$x_1 = f(x_0) = 6(-4) + 3 = -21$ $x_2 = f(x_1) = 6(-21) + 3 = -123$ $x_3 = f(x_2) = 6(-123) + 3 = -735$
11. $f(x) = 8x - 4, x_0 = -6$	$x_1 = f(x_0) = 8(-6) - 4 = -52$ $x_2 = f(x_1) = 8(-52) - 4 = -420$ $x_3 = f(x_2) = 8(-420) - 4 = -3364$
33. $f(x) = 12x + 8, x_0 = 4$	$x_1 = f(x_0) = 12(4) + 8 = 56$ $x_2 = f(x_1) = 12(56) + 8 = 680$ $x_3 = f(x_2) = 12(680) + 8 = 8168$
34. $f(x) = -9x + 1, x_0 = -6$	$x_1 = f(x_0) = -9(-6) + 1 = 55$ $x_2 = f(x_1) = -9(55) + 1 = -494$ $x_3 = f(x_2) = -9(-494) + 1 = 4447$

35. $f(x) = -6x + 3, x_0 = 8$

$$x_1 = f(x_0) = -6(8) + 3 = -45$$
$$x_2 = f(x_1) = -6(-45) + 3 = 273$$
$$x_3 = f(x_2) = -6(273) + 3 = -1635$$

36. $f(x) = 8x + 3, x_0 = -4$

$$x_1 = f(x_0) = 8(-4) + 3 = -29$$
$$x_2 = f(x_1) = 8(-29) + 3 = -229$$
$$x_3 = f(x_2) = 8(-229) + 3 = -1829$$

37. $f(x) = -3x^2 + 9, x_0 = 2$

$$x_1 = f(x_0) = -3(2)^2 + 9 = -3$$
$$x_2 = f(x_1) = -3(-3)^2 + 9 = -18$$
$$x_3 = f(x_2) = -3(-18)^2 + 9 = -963$$

38. $f(x) = 4x^2 + 5, x_0 = -2$

$$x_1 = f(x_0) = 4(-2)^2 + 5 = 21$$
$$x_2 = f(x_1) = 4(21)^2 + 5 = 1769$$
$$x_3 = f(x_2) = 4(1769)^2 + 5 = 12517449$$

39. $f(x) = 2x^2 - 5x + 1, x_0 = 6$

$$x_1 = f(x_0) = 2(6)^2 - 5(6) + 1 = 43$$
$$x_2 = f(x_1) = 2(43)^2 - 5(43) + 1 = 3484$$
$$x_3 = f(x_2) = 2(3484)^2 - 5(3484) + 1 = 24259093$$

40. $f(x) = -0.25x^2 + x + 6, x_0 = 8$

$$x_1 = f(x_0) = -0.25(8)^2 + (8) + 6 = -2$$
$$x_2 = f(x_1) = -0.25(-2)^2 + (-2) + 6 = 3$$
$$x_3 = f(x_2) = -0.25(3)^2 + (3) + 6 = 6.75$$

41. $f(x) = x^2 + 2x + 3, x_0 = \frac{1}{2}$

$x_1 = f(x_0) = (0.5)^2 + 2(0.5) + 3 = 4.25$

$x_2 = f(x_1) = (4.25)^2 + 2(4.25) + 3 = 29.5625$

$x_3 = f(x_2) = (29.5625)^2 + 2(29.5625) + 3 = 936.0664$

42. $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$

$x_1 = f(x_0) = 2(-0.5)^2 + (-0.5) + 1 = 1$

$x_2 = f(x_1) = (1)^2 + 2(1) + 3 = 4$

$x_3 = f(x_2) = (4)^2 + 2(4) + 3 = 37$

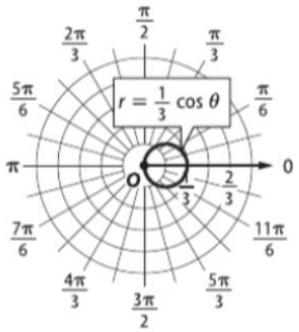
Part (2) - 10

11	Identify and graph classical curves
	تحديد المنحنيات الكلاسيكية وتمثيلها بيانيا

Identify the type of curve given by each equation. Then use symmetry, zeros, and maximum r-values to graph the function.

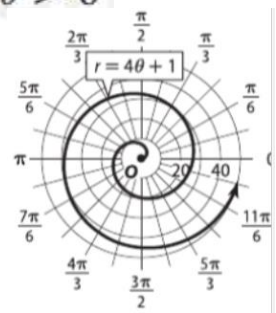
26. $r = \frac{1}{3}\cos\theta$

Circle



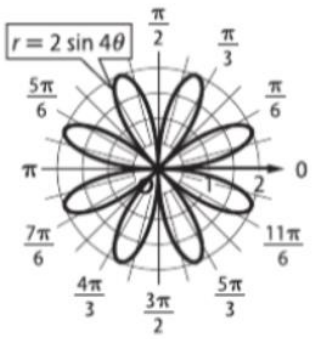
27. $r = 4\theta + 1; \theta > 0$

Spirals of Archimedes



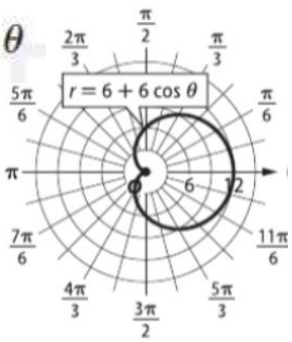
28. $r = 2\sin 4\theta$

Rose



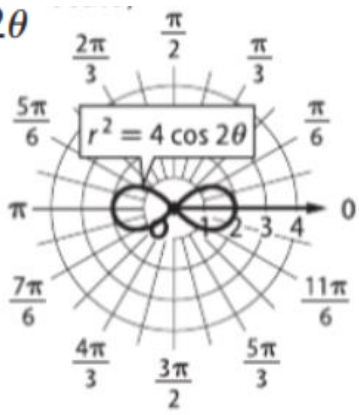
29. $r = 6 + 6\cos\theta$

Cardioid



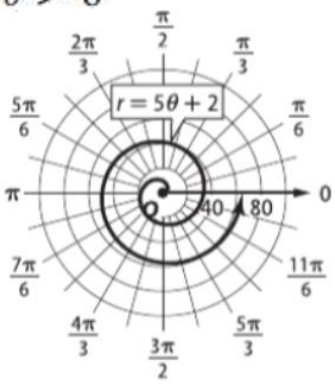
30. $r^2 = 4\cos 2\theta$

Lemniscate



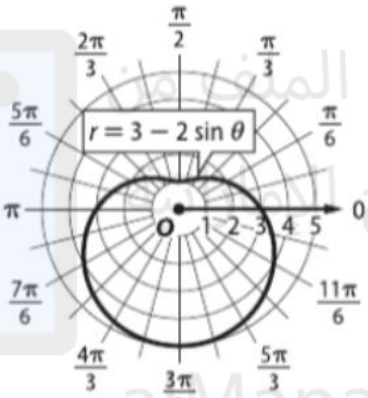
31. $r = 5\theta + 2;\ \theta > 0$

Spirals of Archimedes



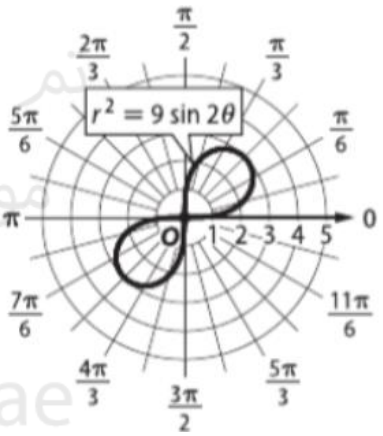
32. $r = 3 - 2\sin\theta$

Limaçon



33. $r^2 = 9\sin 2\theta$

Lemniscate



12	Convert between polar and rectangular equations
	التحويل بين المعادلات القطبية والديكارتية

Write each equation in rectangular form, and then identify its graph. Support your answer by graphing the polar form of the equation $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $r = x^2 + y^2$

36. $r = 3\sin\theta$

$$r = 3\sin\theta$$

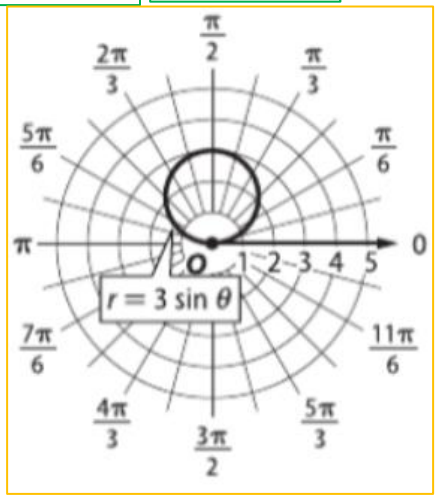
$$r \cdot r = 3\frac{y}{r} \cdot r$$

$$r^2 = 3y$$

$$x^2 + y^2 = 3y$$

$$x^2 + y^2 - 3y = 0$$

Circle



37. $\theta = -\frac{\pi}{3}$

$$\theta = -\frac{\pi}{3}$$

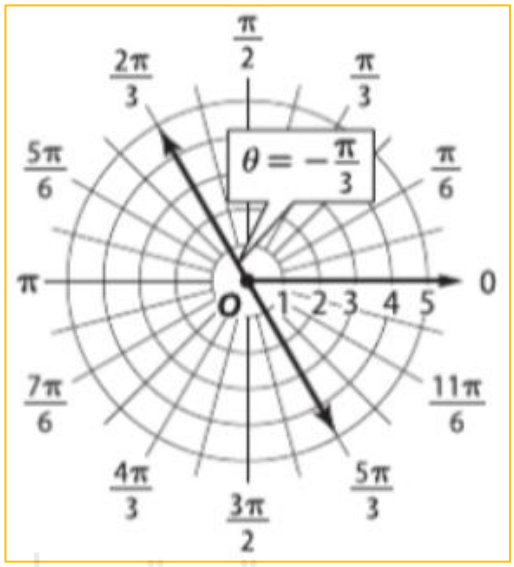
$$\tan\theta = \tan(-\frac{\pi}{3})$$

Line

$$\tan\theta = -\sqrt{3}$$

$$x.\frac{y}{x} = -\sqrt{3}.x$$

$$y = -\sqrt{3}x$$



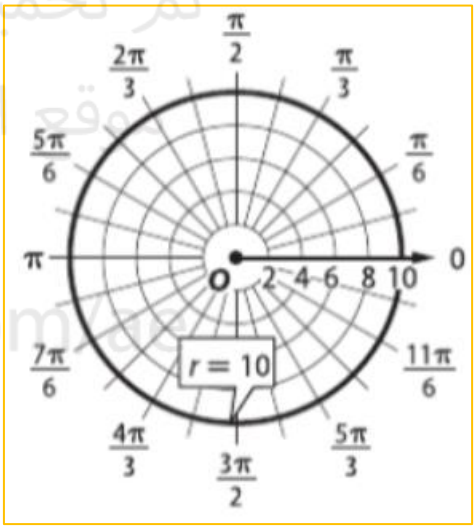
38. $r = 10$

$$r = 10$$

$$r^2 = 100$$

Circle

$$x^2 + y^2 = 100$$



39. $r = 4\cos\theta$

$$r = 4\cos\theta$$

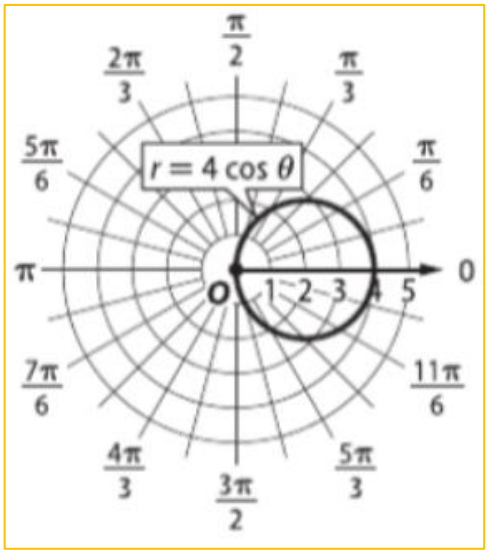
$$r.r = 4\frac{x}{r}.r$$

Circle

$$r^2 = 4x$$

$$x^2 + y^2 = 4x$$

$$x^2 + y^2 - 4x = 0$$



40. $\tan\theta = 4$

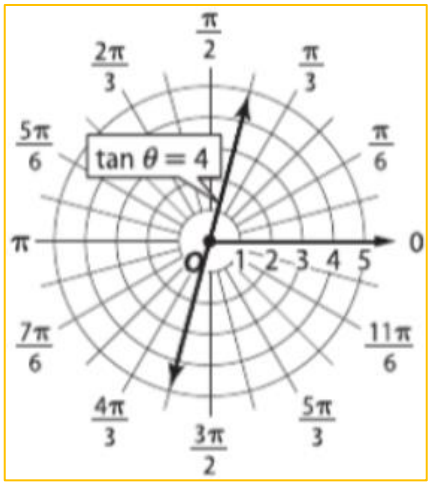
$$\tan\theta = 4$$

$$\frac{y}{x} = 4$$

$$x.\frac{y}{x} = 4x$$

$$y = 4x$$

Line



41. $r = 8\csc\theta$

$$r = 8\csc\theta$$

$$\frac{1}{r}.r = 8\frac{r}{x}.\frac{1}{r}$$

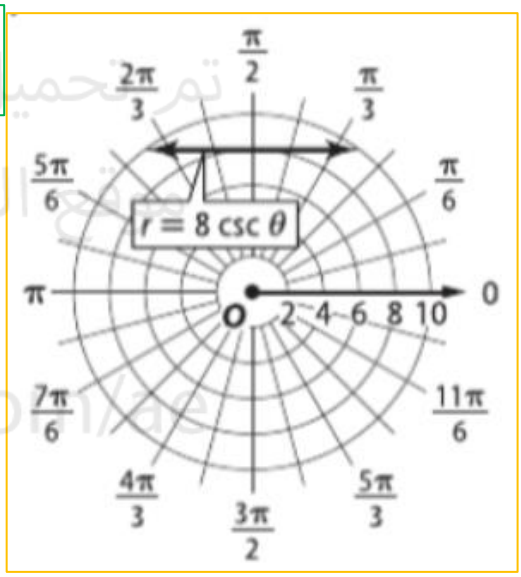
$$1 = \frac{8}{x}$$

$$x.1 = \frac{8}{x}.x$$

$$x = 8$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y}$$

Line



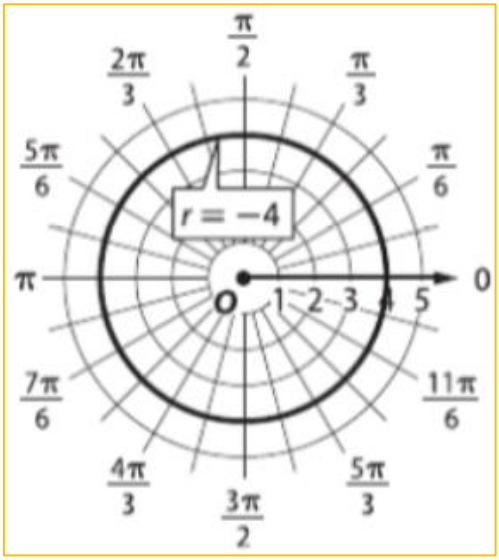
42. $r = -4$

$$r = -4$$

$$r^2 = 16$$

$$x^2 + y^2 = 16$$

Circle



43. $\cot\theta = -7$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y}$$

$$\cot\theta = -7$$

$$\frac{x}{y} = -7$$

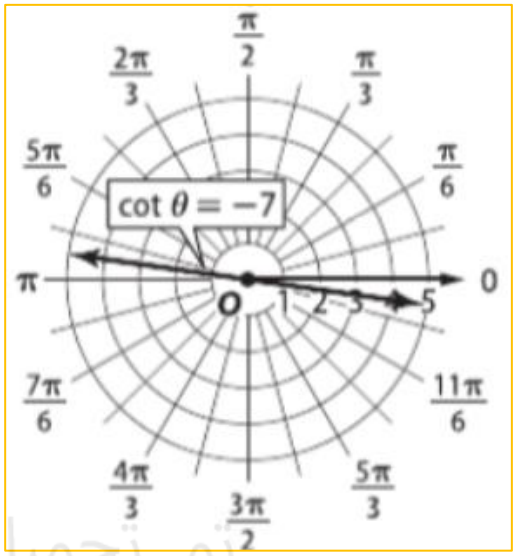
$$\frac{1}{x}.\frac{x}{y} = -7.\frac{1}{x}$$

$$\frac{1}{y} = \frac{-7}{x}$$

$$\frac{1}{-7}.x = -7.y.\frac{1}{-7}$$

$$y = \frac{1}{-7}x$$

Line



44. $\theta = \frac{3\pi}{4}$

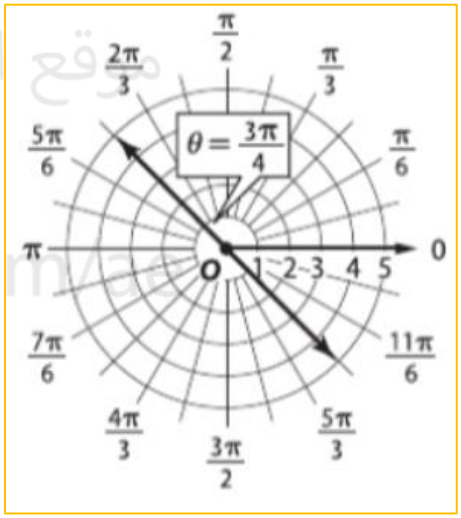
$$\tan\theta = \tan(\frac{3\pi}{4})$$

$$\tan\theta = -1$$

$$x.\frac{y}{x} = -x$$

$$y = -x$$

Line



45. $r = \sec\theta$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x}$$

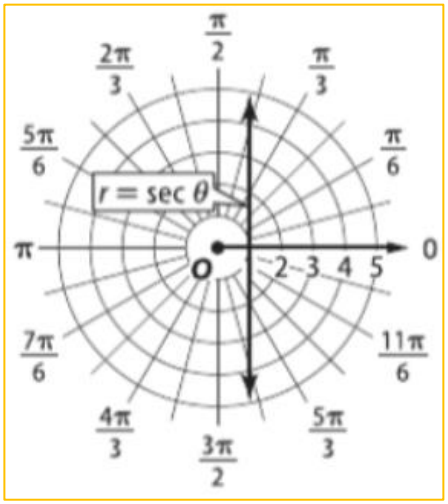
$$r = \frac{r}{x}$$

$$\frac{1}{r}.r = \frac{r}{x}.\frac{1}{r}$$

$$x.1 = \frac{1}{x}.x$$

$$x = 1$$

Line



Express each complex number in polar form.

$r = \sqrt{a^2 + b^2}$ $\quad \theta = tan^{-1}(\frac{b}{a})$ $\quad \theta = tan^{-1}\left(\frac{b}{a}\right) + \pi \ if \ a < 0$ $\quad z = r\,cos\theta + i\,r sin\theta$

10. $4 + 4i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (4)^2} = 4\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{4}{4}\right) = \frac{\pi}{4}$$

$$z = 4\sqrt{2}\,cos\frac{\pi}{4} + i\,4\sqrt{2}sin\frac{\pi}{4} \quad OR \quad z = 4\sqrt{2}\,(cos\frac{\pi}{4} + i\,sin\frac{\pi}{4})$$

11. $-2 + i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{5}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{1}{-2}\right) + \pi = 2.68$$

$$z = \sqrt{5}\,cos2.68 + i\,\sqrt{5}sin2.68 \quad OR \quad z = \sqrt{5}\,(cos2.68 + i\,sin2.68)$$

12. $4 - \sqrt{2}i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (-\sqrt{2})^2} = 3\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{-\sqrt{2}}{4}\right) = -0.34$$

$$z = 3\sqrt{2}\,cos(-0.34) + i\,3\sqrt{2}sin(-0.34) \quad OR \quad z = 3\sqrt{2}\,(cos - 0.34 + i\,sin - 0.34)$$

13. $2 - 2i$

Note: you can add $360°$ or 2π if theta was negative

$$r = \sqrt{a^2 + b^2} = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{-2}{2}\right) + 2\pi = \frac{7\pi}{4}$$

$$z = 2\sqrt{2}\,cos\frac{7\pi}{4} + i\,2\sqrt{2}sin\frac{7\pi}{4} \quad OR \quad z = \sqrt{5}\,(cos\frac{7\pi}{4} + i\,sin\frac{7\pi}{4})$$

14. $4 + 5i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{41}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{5}{4}\right) = 0.90$$

$$z = \sqrt{41}\, cos0.90 + i\sqrt{41} sin0.90 \quad OR \quad z = \sqrt{41}\,(cos0.90 + i\, sin0.90)$$

15. $-2 + 4i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-2)^2 + (4)^2} = 2\sqrt{5}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{4}{-2}\right) + \pi = 2.03$$

$$z = 2\sqrt{5}\, cos2.03 + i\, 2\sqrt{5} sin2.03 \quad OR \quad z = 2\sqrt{5}\,(cos2.03 + i\, sin2.03)$$

16. $-1 - \sqrt{3}i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) + \pi = \frac{4\pi}{3}$$

$$z = 2cos\frac{4\pi}{3} + i\, 2sin\frac{4\pi}{3} \quad OR \quad z = 2(cos\frac{4\pi}{3} + i\, sin\frac{4\pi}{3})$$

17. $3 + 3i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$$

$$z = 3\sqrt{2}\, cos\frac{\pi}{4} + i\, 3\sqrt{2} sin\frac{\pi}{4} \quad OR \quad z = 3\sqrt{2}\,(cos\frac{\pi}{4} + i\, sin\frac{\pi}{4})$$

14	Investigate several different types of sequences
	استكشاف عدة أنواع مختلفة من المتتاليات

Determine whether each sequence is convergent or divergent.

- If a sequence has a limit such that the terms approach a unique number, then it is said to converge.
- If not, the sequence is said to diverge.

18. $a_1 = 4,\ 1.5a_{n-1},\ n \geq 2$ Divergent	**20.** $a_n = -n^2 - 8n + 106$ Divergent	**19.** $a_n = \frac{5}{10^n}$ Convergent
21. $a_1 = -64,\ \frac{3}{4}a_{n-1},\ n \geq 2$ Convergent	**22.** $a_1 = 1,\ a_n = 4 - a_{n-1},\ n \geq 2$ Divergent	**23.** $a_n = n^2 - 3n + 1$ Divergent
24. $a_n = \frac{n^2 + 4}{3 + n}$ Divergent	**26.** $a_n = \frac{5n + 6}{n}$ Convergent	**27.** $a_n = \frac{5n}{5^n} + 1$ Convergent
25. $a_1 = 9,\ a_n = \frac{a_{n-1} + 3}{2},\ n \geq 2$ Convergent		

15	Find sums of arithmetic series
	إيجاد مجاميع المتسلسلات الحسابية

Find the sum of each arithmetic series. $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

57. $\sum_{k=1}^{16} (4k - 2)$ $\quad n = 16 - 1 + 1 = 16$ $\quad a_1 = 4(1) - 2 = 2$

$a_n = 4(16) - 2 = 62$

$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 16\left(\frac{2 + 62}{2}\right) = 512$ $\quad S_{16} = 512$

58. $\sum_{k=4}^{13}(4k+1)$

$n = 13 - 4 + 1 = 10$ $\quad a_1 = 4(4) + 1 = 17$

$a_n = 4(13) + 1 = 53$

$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 10\left(\frac{17 + 53}{2}\right) = 350$ $\quad S_{10} = 350$

59. $\sum_{k=5}^{16}(2k+6)$

$n = 16 - 5 + 1 = 12$ $\quad a_1 = 2(5) + 6 = 16$

$a_n = 2(16) + 6 = 38$

$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 12\left(\frac{16 + 38}{2}\right) = 324$ $\quad S_{12} = 324$

60. $\sum_{k=0}^{12}(-3k+2)$

$n = 12 - 0 + 1 = 13$ $\quad a_1 = -3(0) + 2 = 2$

$a_n = -3(12) + 2 = -34$

$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 13\left(\frac{2 + (-34)}{2}\right) = -202$ $\quad S_{13} = -202$

16	Find the nth term and geometric means of geometric sequences
	إيجاد الحد النوني والوسط الحسابي للمتتاليات الهندسية

Find the geometric means of each sequence. $a_n = a_1 r^{n-1}$

35. 810, ?, ?, ?, 10

$n = 5$ $\quad a_1 = 810$ $\quad a_n = 10$

$a_n = a_1 r^{n-1} \rightarrow 10 = 810 r^{5-1} \rightarrow r = \pm\frac{1}{3}$

$a_2 = 810\left(\frac{1}{3}\right)^{2-1} = 270$ $\quad a_2 = 810\left(-\frac{1}{3}\right)^{2-1} = -270$

$a_3 = 810\left(\frac{1}{3}\right)^{3-1} = 90$ $\quad a_3 = 810\left(-\frac{1}{3}\right)^{3-1} = 90$

$a_4 = 810\left(\frac{1}{3}\right)^{4-1} = 30$ $\quad a_4 = 810\left(-\frac{1}{3}\right)^{4-1} = -30$

36. 640, ?, ?, ?, 2.5 $n = 5$ $a_1 = 640$ $a_n = 2.5$

$$a_n = a_1 r^{n-1} \longrightarrow 2.5 = 640 r^{5-1} \longrightarrow r = \pm 0.25$$

$a_2 = 640(0.25)^{2-1} = 160$ $a_2 = 640(-0.25)^{2-1} = -160$

$a_3 = 640(0.25)^{3-1} = 40$ $a_3 = 640(-0.25)^{3-1} = 40$

$a_4 = 640(0.25)^{4-1} = 10$ $a_4 = 640(-0.25)^{4-1} = -10$

37. $\frac{7}{2}$, ?, ?, ?, $\frac{56}{81}$ $n = 5$ $a_1 = \frac{7}{2}$ $a_n = \frac{56}{81}$

$$a_n = a_1 r^{n-1} \longrightarrow \frac{56}{81} = \frac{7}{2} r^{5-1} \longrightarrow r = \pm \frac{2}{3}$$

$a_2 = \frac{7}{2}(\frac{2}{3})^{2-1} = \frac{7}{3}$ $a_2 = \frac{7}{2}(-\frac{2}{3})^{2-1} = -\frac{7}{3}$

$a_3 = \frac{7}{2}(\frac{2}{3})^{3-1} = \frac{14}{9}$ $a_3 = \frac{7}{2}(-\frac{2}{3})^{3-1} = \frac{14}{9}$

$a_4 = \frac{7}{2}(\frac{2}{3})^{4-1} = \frac{28}{27}$ $a_4 = \frac{7}{2}(-\frac{2}{3})^{4-1} = -\frac{28}{27}$

38. $\frac{729}{64}$, ?, ?, ?, $\frac{324}{9}$ $n = 5$ $a_1 = \frac{729}{64}$ $a_n = \frac{324}{9}$

$$a_n = a_1 r^{n-1} \longrightarrow \frac{324}{9} = \frac{729}{64} r^{5-1} \longrightarrow r = \pm \frac{4}{3}$$

$a_2 = \frac{729}{64}(\frac{4}{3})^{2-1} = \frac{243}{16}$ $a_2 = \frac{729}{64}(-\frac{4}{3})^{2-1} = -\frac{243}{16}$

$a_3 = \frac{729}{64}(\frac{4}{3})^{3-1} = \frac{81}{4}$ $a_3 = \frac{729}{64}(-\frac{4}{3})^{3-1} = \frac{81}{4}$

$a_4 = \frac{729}{64}(\frac{4}{3})^{4-1} = 27$ $a_4 = \frac{729}{64}(-\frac{4}{3})^{4-1} = -27$

39. Find two geometric means between 3 and 375.

$n = 4 \qquad a_1 = 3 \qquad a_n = 375$

$a_n = a_1 r^{n-1} \longrightarrow 375 = 3r^{4-1} \longrightarrow r = 5$

$a_2 = 3(5)^{2-1} = 15$

$a_3 = 3(5)^{3-1} = 75$

40. Find two geometric means between 16 and −2.

$n = 4 \qquad a_1 = 16 \qquad a_n = -2$

$a_n = a_1 r^{n-1} \longrightarrow -2 = 16r^{4-1} \longrightarrow r = -0.5$

$a_2 = 16(-0.5)^{2-1} = -8$

$a_3 = 16(-0.5)^{3-1} = 4$

17	Find sums of geometric series
	إيجاد مجاميع المتسلسلات الهندسية

Find a_1 for each geometric series described.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \qquad S_n = \frac{a_1 - a_n r}{1 - r}$$

51. $S_n = -2912, r = 3, n = 6$

$S_n = \frac{a_1 - a_1 r^n}{1 - r} \longrightarrow -2912 = \frac{a_1 - a_1(3)^6}{1 - (3)}$

$\longrightarrow -2912 = \frac{a_1(1 - (3)^6)}{1 - (3)} \longrightarrow -2912 = \frac{-728a_1}{-2}$

$\longrightarrow -2912 = 364a_1 \longrightarrow a_1 = -8$

52. $S_n = -10{,}922,\ r = 4,\ n = 7$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \longrightarrow -10922 = \frac{a_1 - a_1(4)^7}{1 - (4)}$$

$$\longrightarrow -10922 = \frac{a_1(1 - (4)^7)}{1 - (4)} \longrightarrow -10922 = \frac{-16383a_1}{-3}$$

$$\longrightarrow -10922 = 5461a_1 \longrightarrow a_1 = -2$$

53. $S_n = 1330,\ a_n = 486,\ r = \frac{3}{2}$

$$S_n = \frac{a_1 - a_n r}{1 - r} \longrightarrow 1330 = \frac{a_1 - (486)(\frac{3}{2})}{1 - (\frac{3}{2})}$$

$$\longrightarrow 1330 = \frac{a_1 - 729}{-0.5} \longrightarrow 1330(-0.5) = a_1 - 729$$

$$\longrightarrow -665 = a_1 - 729 \longrightarrow -665 + 729 = a_1 \longrightarrow a_1 = -2$$

54. $S_n = 4118,\ a_n = 128,\ r = \frac{2}{3}$

$$S_n = \frac{a_1 - a_n r}{1 - r} \longrightarrow 4118 = \frac{a_1 - (128)(\frac{2}{3})}{1 - (\frac{2}{3})}$$

$$\longrightarrow 4118 = \frac{a_1 - \frac{256}{3}}{\frac{1}{3}} \longrightarrow 4118(\frac{1}{3}) = a_1 - \frac{256}{3}$$

$$\longrightarrow \frac{4118}{3} = a_1 - \frac{256}{3} \longrightarrow \frac{4118}{3} + \frac{256}{3} = a_1 \longrightarrow a_1 = 1458$$

55. $a_n = 1024, r = 8, n = 5$

$$a_n = a_1 r^{n-} \longrightarrow 1024 = a_1 8^{5-1} \longrightarrow a_1 = 0.25$$

56. $a_n = 1875, r = 5, n = 7$

$$a_n = a_1 r^{n-} \longrightarrow 1875 = a_1 5^{7-1} \longrightarrow a_1 = \frac{3}{25}$$

18	Find sums of infinite geometric series إيجاد مجاميع المتسلسلات الهندسية اللانهائية

Find the sum of each infinite series, if it exists.

$$S = \frac{a_1}{1 - r}$$

- If $|r| < 1$, the series has sum
- If $|r| \geq 1$, the series has no sum

22. $18 + 21.6 + 25.92 + \ldots$

$$r = \frac{21.6}{18} = 1.2 \geq 1$$

No sum exists

23. $-3 - 4.2 - 5.88 - \ldots$

$$r = \frac{-4.2}{-3} = 1.4 \geq 1$$

No sum exists

24. $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \ldots$

$$r = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3} < 1$$

Sum exists

$$a_1 = \frac{1}{2} \qquad S = \frac{a_1}{1 - r}$$

$$S = \frac{\frac{1}{2}}{1 - \frac{1}{3}} = \frac{3}{4}$$

25. $\frac{12}{5} + \frac{6}{5} + \frac{3}{5} + \ldots$

$$r = \frac{6}{5} \div \frac{12}{5} = \frac{1}{2} < 1$$

Sum exists

$$a_1 = \frac{12}{5} \qquad S = \frac{a_1}{1 - r}$$

$$S = \frac{\frac{12}{5}}{1 - \frac{1}{2}} = \frac{24}{5}$$

26. $21 + 14 + \frac{28}{3} + \ldots$

$$r = \frac{14}{21} = \frac{2}{3} < 1$$

Sum exists

$a_1 = 21$

$$S = \frac{a_1}{1 - r} \qquad S = \frac{21}{1 - \frac{2}{3}} = 63$$

27. $32 + 40 + 50 + \ldots$

$$r = \frac{40}{32} = 1.25 \geq 1$$

No sum exists

19	Recognize and use special sequences التعرف على المتتاليات الخاصة واستخدامها

Write a recursive formula for each sequence.

Arithmetic: $a_n = a_{n-1} + d$ | **Geometric:** $a_n = ra_{n-1}$

5. 3, 8, 18, 38, 78, ...

Not Arithmetic or Geometric

$2(3) + 2 = 8$

$2(8) + 2 = 18$

$2(18) + 2 = 38$

$a_n = 2a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 2a_{n-1} + 2, a_1 = 3$

6. 5, 14, 41, 122, 365, ...

Not Arithmetic or Geometric

$3(5) - 1 = 14$

$3(14) - 1 = 41$

$3(41) - 1 = 122$

$a_n = 3a_{n-1} - 1$

A recursive formula for the sequence is

$a_n = 3a_{n-1} - 1, a_1 = 5$

24. 16, 10, 7, 5.5, 4.75, ...

Not Arithmetic or Geometric

$0.5(16) + 2 = 10$

$0.5(10) + 2 = 7$

$0.5(7) + 2 = 5.5$

$a_n = 0.5a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 0{,}5a_{n-1} + 2, a_1 = 16$

25. 32, 12, 7, 5.75, ...

Not Arithmetic or Geometric

$0.25(32) + 4 = 12$

$0.25(12) + 4 = 7$

$0.25(7) + 4 = 5.75$

$a_n = 2a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 0.25a_{n-1} + 4, a_1 = 32$

26. 4, 15, 224, 50,175, ...

Not Arithmetic or Geometric

$(4)^2 - 1 = 15$

$(15)^2 - 1 = 224$

$(224)^2 - 1 = 50175$

$a_n = (a_{n-1})^2 - 1$

A recursive formula for the sequence is

$a_n = (a_{n-1})^2 - 1, a_1 = 4$

27. 1, 2, 9, 730, ...

Not Arithmetic or Geometric

$(1)^3 + 1 = 2$

$(2)^3 + 1 = 9$

$(9)^3 + 1 = 730$

$a_n = (a_{n-1})^3 + 1$

A recursive formula for the sequence is

$a_n = (a_{n-1})^3 + 1, a_1 = 1$

28. 9, 33, 129, 513, ...

Not Arithmetic or Geometric

$4(9) - 3 = 33$

$4(33) - 3 = 129$

$4(129) - 3 = 513$

$a_n = 4a_{n-1} - 3$

A recursive formula for the sequence is

$a_n = 4a_{n-1} - 3, a_1 = 9$

29. 480, 128, 40, 18, ...

Not Arithmetic or Geometric

$0.25(480) + 8 = 128$

$0.25(128) + 8 = 40$

$0.25(40) + 8 = 18$

$a_n = 0.25a_{n-1} + 8$

A recursive formula for the sequence is

$a_n = 0.25a_{n-1} + 8, a_1 = 480$

30. 393, 132, 45, 16, ...

Not Arithmetic or Geometric

$\frac{1}{3}(393) + 1 = 132$

$\frac{1}{3}(132) + 1 = 45$

$\frac{1}{3}(45) + 1 = 16$

$a_n = \frac{1}{3}a_{n-1} + 1$

A recursive formula for the sequence is

$a_n = \frac{1}{3}a_{n-1} + 1, a_1 = 393$

31. 68, 104, 176, 320, ...

Not Arithmetic or Geometric

$2(68) - 32 = 104$

$2(104) - 32 = 176$

$2(176) - 32 = 320$

$a_n = 2a_{n-1} - 32$

A recursive formula for the sequence is

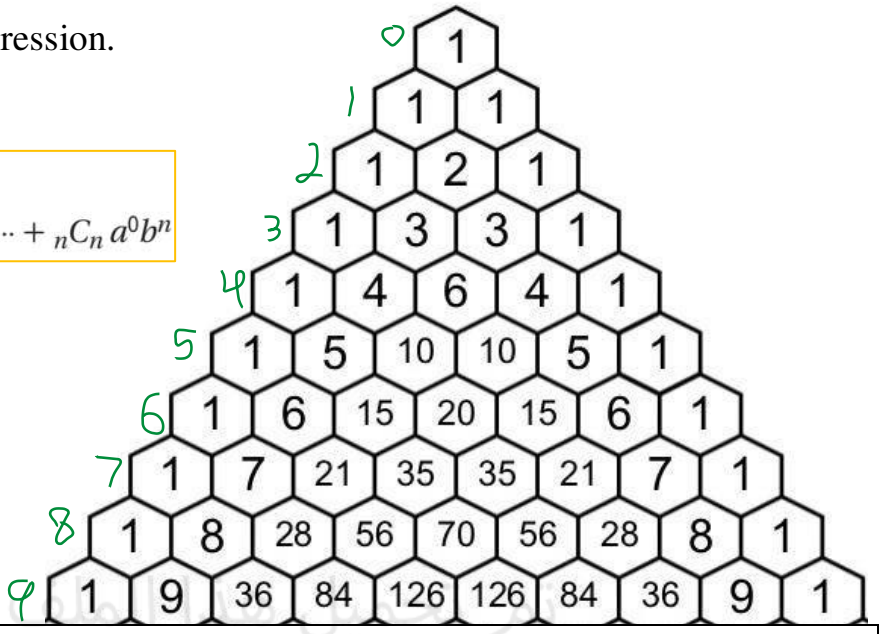
$a_n = 2a_{n-1} - 32, a_1 = 68$

20	Use the Binomial Theorem to expand powers of binomials استخدام نظرية ذات الحدين لتفكيك أسس ذوات الحدين

Find the indicated term of each expression.

If n is a natural number, then $(a+b)^n =$

$${}_nC_0\,a^nb^0 + {}_nC_1\,a^{n-1}b^1 + {}_nC_2\,a^{n-2}b^2 + \cdots + {}_nC_n\,a^0b^n$$



23. third term of $(x+2z)^7$

3 - 1 = 2

7 6 5 4 3 2 1 0

0 1 2 3 4 5 6 7

$${}_7C_2\,a^5b^2$$

$$21(x)^5(2z)^2 \rightarrow 21x^5 4z^2 \rightarrow \boxed{84x^5z^2}$$

24. fourth term of $(y-3x)^6$

6 5 4 3 2 1 0

0 1 2 3 4 5 6

$${}_6C_3\,a^3b^3$$

$$20(y)^3(-3x)^3 \rightarrow 20y^3(-27)z^3 \rightarrow \boxed{-540y^5z^2}$$

25. seventh term of $(2a-2b)^8$

8 7 6 5 4 3 2 1 0

0 1 2 3 4 5 6 7 8

$${}_8C_6\,a^2b^6$$

$$28(2a)^2(2b)^6 \rightarrow 28(4)a^2(64)b^6 \rightarrow \boxed{7168a^2b^6}$$

26. sixth term of $(4x+5y)^6$

6 5 4 3 2 1 0

0 1 2 3 4 5 6

$${}_6C_5\,a^1b^5$$

$$6(4x)^1(5y)^5 \rightarrow 6(4x)(3125)y^5 \rightarrow \boxed{75000xy^5}$$

27. fifth term of $(x-4)^9$

9 8 7 6 **5** 4 3 2 1 0

0 1 2 3 **4** 5 6 7 8 9

$${}_9C_4 a^5 b^4$$

$$126x^5(-4)^4 \rightarrow 126x^5(256) \rightarrow 32256x^5$$

28. fourth term of $(c+6)^8$

8 7 6 **5** 4 3 2 1 0

0 1 2 **3** 4 5 6 7 8

$${}_8C_3 a^5 b^3$$

$$56(c)^5(6)^3 \rightarrow 56c^5(216) \rightarrow 12096c^5$$

Part (3) - 3

21	Find products, quotients, powers, and roots of complex numbers in polar form
	إيجاد ناتج ضرب الأعداد المركبة وناتج قسمتها وأسسها والجذور في الصورة القطبية

Find each product or quotient, and express it in rectangular form.

Product Formula $z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$

Quotient Formula $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)]$, where z_2 and $r_2 \neq 0$

26. $6\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\cdot 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ Product

$$(6)(4)\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+isin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right]$$

$$24\left[\cos\left(\frac{3\pi}{4}\right)+isin\left(\frac{3\pi}{4}\right)\right]$$

$$24\left[-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)\right]$$

\

Rectangular form: $-12\sqrt{2}+12\sqrt{2}i$

27. $5(\cos 135^\circ + i \sin 135^\circ) \cdot 2(\cos 45^\circ + i \sin 45^\circ)$ Product

$$(5)(2)[\cos(135^\circ + 45^\circ) + i\sin(135^\circ + 45^\circ)]$$

$$10[\cos(180^\circ) + i\sin(180^\circ)]$$

$$10[-1 + i0)]$$

Rectangular form: -10

28. $3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \div \frac{1}{2}(\cos \pi + i \sin \pi)$ Quotien

$$\frac{3}{0.5}\left[\cos\left(\frac{3\pi}{4} - \pi\right) + i\sin\left(\frac{3\pi}{4} - \pi\right)\right]$$

$$6\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

$$6\left[\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right]$$

Rectangular form: $3\sqrt{2} - 3\sqrt{2}i$

29. $2(\cos 90^\circ + i \sin 90^\circ) \cdot 2(\cos 270^\circ + i \sin 270^\circ)$ Product

$$(2)(2)[\cos(90^\circ + 270^\circ) + i\sin(90^\circ + 270^\circ)]$$

$$4[\cos(360^\circ) + i\sin(360^\circ)]$$

$$4[1 + i0)]$$

Rectangular form: 4

30. $3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\div 4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$ Quotien

$$\frac{3}{4}\left[\cos\left(\frac{\pi}{6}-\frac{2\pi}{3}\right)+isin(\frac{\pi}{6}-\frac{2\pi}{3})\right]$$

$$\frac{3}{4}\left[\cos\left(-\frac{\pi}{2}\right)+isin(-\frac{\pi}{2})\right]$$

$$\frac{3}{4}[0-i)]$$

Rectangular form: $-\frac{3}{4}i$

31. $4\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)\div 2\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$ Quotien

$$\frac{4}{2}\left[\cos\left(\frac{9\pi}{4}-\frac{3\pi}{2}\right)+isin(\frac{9\pi}{4}-\frac{3\pi}{2})\right]$$

$$2\left[\cos\left(\frac{3\pi}{4}\right)+isin(\frac{3\pi}{4})\right]$$

$$2\left[-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2})\right]$$

Rectangular form: $-\sqrt{2}+\sqrt{2}i$

32. $\frac{1}{2}(\cos 60^\circ+i\sin 60^\circ)\cdot 6(\cos 150^\circ+i\sin 150^\circ)$ Product

$$(0.5)(6)[\cos(60^\circ+150^\circ)+isin(60^\circ+150^\circ)]$$

$$3[\cos(210^\circ)+isin(210^\circ)]$$

$$3\left[-\frac{\sqrt{3}}{2}-i\frac{1}{2})\right]$$

Rectangular form: $-\frac{3\sqrt{3}}{2}-\frac{3}{2}i$

33. $6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\div 2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ Quotien

$$\frac{6}{2}\left[\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)+isin(\frac{3\pi}{4}-\frac{\pi}{4})\right]$$

$$3\left[\cos\left(\frac{\pi}{2}\right)+isin(\frac{\pi}{2})\right]$$

$$3[0+i)]$$

Rectangular form: $3i$

34. $5(\cos 180^\circ + i\sin 180^\circ)\cdot 2(\cos 135^\circ + i\sin 135^\circ)$ Product

$$(5)(2)[\cos(180^\circ+135^\circ)+isin(180^\circ+135^\circ)]$$

$$10[\cos(315^\circ)+isin(315^\circ)]$$

$$10\left[-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2})\right]$$

Rectangular form: $-5\sqrt{2}+5\sqrt{2}i$

35. $\frac{1}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\div 3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ Quotien

$$\frac{0.5}{3}\left[\cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+isin(\frac{\pi}{3}-\frac{\pi}{6})\right]$$

$$\frac{1}{6}\left[\cos\left(\frac{\pi}{6}\right)+isin(\frac{\pi}{6})\right]$$

$$\frac{1}{6}\left[\frac{\sqrt{3}}{2}+\frac{1}{2}i)\right]$$

Rectangular form: $\frac{\sqrt{3}}{2}+\frac{1}{12}i$

22	Recognize and use special sequences
	التعرف على المتتاليات الخاصة واستخدامها

FINANCIAL LITERACY Nasser had AED 15,000 in credit card debt when he graduated from college. The balance increased by 2% each month due to interest (murabaha), and Nasser could only make payments of AED 400 per month. Write a recursive formula for the balance of his account each month. Then determine the balance after five months.

initial balance + balance times 2% − monthly payment

$a_2 = a_1 + (a_1 \times 0.02) - 400$

$a_2 = 1.02a_1 - 400$ The formula is $a_n = 1.02a_{n-1} - 400,\ a_1 = 15{,}000$

$a_n = 1.02a_{n-1} - 400$

$a_1 = 15{,}000$ $a_2 = (15{,}000 \times 1.02) - 400$ or 14,900

$a_2 = 14{,}900$ $a_3 = (14{,}900 \times 1.02) - 400$ or 14,798

$a_3 = 14{,}798$ $a_4 = (14{,}798 \times 1.02) - 400$ or 14,693.96

$a_4 = 14{,}693.96$ $a_5 = (14{,}693.96 \times 1.02) - 400$ or 14,587.84

$a_5 = 14{,}587.84$ $a_6 = (14{,}587.84 \times 1.02) - 400$ or 14,479.60

Write a recursive formula for a AED 10,000 debt, at 2.5% interest (murabaha) per month, with a AED 600 monthly payment. Then find the first five balances.

initial balance + balance times 2.5% − monthly payment

$a_2 = a_1 + (a_1 \times 0.025) - 600$

$a_2 = 1.025a_1 - 600$ The formula is $a_n = 1.025a_{n-1} - 600,\ a_1 = 10{,}000$

$a_n = 1.025a_{n-1} - 600$

$a_1 = 10{,}000$ $a_2 = 1.025(10{,}000) - 600 = 9650$

$a_2 = 9650$ $a_3 = 1.025(9650) - 600 = 9291.25$

$a_3 = 9291.25$ $a_4 = 1.025(9291.25) - 600 = 8923.53$

$a_4 = 8923.53$ $a_5 = 1.025(8923.53) - 600 = 8546.62$

FINANCING Faris financed a AED 1,500 rowing machine to help him train for the college rowing team. He could only make a AED 100 payment each month, and his bill increased by 1% due to interest (murabaha) at the end of each month.

a. Write a recursive formula for the balance owed at the end of each month.

initial balance + balance times 1% − monthly payment

$$a_2 = a_1 + (a_1 \times 0.01) - 100$$

$$a_2 = 1.01a_1 - 100$$

The formula is $a_n = 1.01a_{n-1} - 100,\ a_1 = 1500$

b. Find the balance owed after the first four months.

$$a_n = 1.01a_{n-1} - 100$$

$a_1 = 1500$ $\quad a_2 = 1.01(1500) - 100 = 1415$

$a_2 = 1415$ $\quad a_3 = 1.01(1415) - 100 = 1329.15$

$a_3 = 1329.15$ $\quad a_4 = 1.01(1329.15) - 100 = 1242.4415$

$a_4 = 1242.4415$ $\quad a_5 = 1.01(1242.4415) - 100 = 1154.865915$

c. How much interest (murabaha) has accumulated after the first six months?

$a_5 = 1242.44$ $\quad a_6 = 1.01(1154.865915) - 100 = 1066.414574$

Interest

$$= (1500 \times 0.01) + (1415 \times 0.01) + (1329.15 \times 0.01) + (1242.4415 \times 0.01) + (1154.865915 \times 0.01) + (1066.414574 \times 0.01) = 77.1$$

FINANCIAL LITERACY Mr. Adnan and his company deposit $AED\ 20{,}000$ into his retirement account at the end of each year. The account earns 8% interest (murabaha) before each deposit.

a. Write a recursive formula for the balance in the account at the end of each year.

initial balance + balance times 1% + monthly payment

$$a_2 = a_1 + (a_1 \times 0.08) + 20000$$

$$a_2 = 1.08a_1 + 20000$$ The formula is $a_n = 1.08a_{n-1} + 20000,\ a_1 = 20000$

b. Determine how much is in the account at the end of each of the first 8 years.

$$a_n = 1.08a_{n-1} + 20000$$

$a_1 = 20{,}000$	$a_2 = 1.08(20000) + 20000 = 41600$
$a_2 = 41600$	$a_3 = 1.08(41600) + 20000 = 64928$
$a_3 = 64928$	$a_4 = 1.08(64928) + 20000 = 90122.24$
$a_4 = 90122.24$	$a_5 = 1.08(90122.24) + 20000 = 117332.02$
$a_5 = 90122.24$	$a_6 = 1.08(117332.02) + 20000 = 146718.58$
$a_6 = 146718.58$	$a_7 = 1.08(146718.58) + 20000 = 178456.07$
$a_7 = 178456.07$	$a_8 = 1.08(178456.07) + 20000 = 212732.56$

23	Use the Binomial Theorem to expand powers of binomials
	استخدام نظرية ذات الحدين لتفكيك أسس ذوات الحدين

Expand each binomial.

1. $(c+d)^5$

$$(c+d)^5 = {}_5C_0c^5 + {}_5C_1c^4d + {}_5C_2c^3d^2 + {}_5C_3c^2d^3 + {}_5C_4cd^4 + {}_5C_5d^5$$

$$= c^5 + 5c^4d + 10c^3d^2 + 10c^2d^3 + 5cd^4 + d^5$$

2. $(g+h)^7$

$$(g+h)^7 = {}_7C_0\, g^7 + {}_7C_1\, g^6h + {}_7C_2\, g^5h^2 + {}_7C_3\, g^4h^3 + {}_7C_4\, g^3h^4 + {}_7C_5\, g^2h^5 + {}_7C_6\, gh^6 + {}_7C_7\, h^7$$

$$= g^7 + 7g^6h + 21\, g^5h^2 + 35g^4h^3 + 35\, g^3h^4 + 21\, g^2h^5 + 7gh^6 + h^7$$

3. $(x-4)^6$

$$(x-4)^6 = {}_6C_0\, x^6 + {}_6C_1\, x^5(-4) + {}_6C_2\, x^4(-4)^2 + {}_6C_3\, x^3(-4)^3 + {}_6C_4\, x^2(-4)^4 + {}_6C_5\, x(-4)^5 + {}_6C_6\,(-4)^6$$

$$= x^6 + 6x^5(-4) + 15x^4(-4)^2 + 20x^3(-4)^3 + 15x^2(-4)^4 + 6x(-4)^5 + (-4)^6$$

$$= x^6 - 24x^5 + 240x^4 - 1280x^3 + 3840x^2 - 6144x + 4096$$

4. $(2y-z)^5$

$$(2y-z)^5 = {}_5C_0(2y)^5 + {}_5C_1(2y)^4(-z) + {}_5C_2(2y)^3(-z)^2 + {}_5C_3(2y)^2(-z)^3 + {}_5C_4(2y)(-z)^4 + {}_5C_5(-z)^5$$

$$= 32y^5 + 5(16)y^4(-z) + 10(8)y^3z^2 + 10(4)y^2(-z)^3 + 5\,(2)yz^4 + (-z)^5$$

$$= 32y^5 - 80y^4z + 80y^3z^2 - 40y^2z^3 + 10yz^4 - z^5$$

5. $(x+3)^5$

$$(x+3)^5 = {}_5C_0\, x^5 + {}_5C_1\, x^4(3) + {}_5C_2\, x^3(3)^2 + {}_5C_3\, x^2(3)^3 + {}_5C_4\, x(3)^4 + {}_5C_5\,(3)^5$$

$$= x^5 + 5x^4(3) + 10x^3(3)^2 + 10x^2(3)^3 + 5x(3)^4 + (3)^5$$

$$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$$

6. $(y-4z)^4$

$$(y-4z)^4 = {}_4C_0\, y^4 + {}_4C_1\, y^3(-4z) + {}_4C_2\, y^2(-4z)^2 + {}_4C_3\, y(-4z)^3 + {}_4C_4\,(-4z)^4$$

$$= y^4 + 4y^3(-4)z + 6y^2(16)z^2 + 4y(-64)z^3 + (256)z^4$$

$$= y^4 - 16y^3z + 96y^2z^2 - 256yz^3 + 256z^4$$

15. $(a-b)^6$

$$(a-b)^6 = {}_6C_0\,a^6 + {}_6C_1 a^5(-b) + {}_6C_2\,a^4(-b)^2 + {}_6C_3\,a^3(-b)^3 + {}_6C_4\,a^2(-b)^4 + {}_6C_5\,a(-b)^5 + {}_6C_6\,(-b)^6$$

$$= a^6 + 6a^5(-b) + 15a^4(-b)^2 + 20a^3(-b)^3 + 15a^2(-b)^4 + 6a(-b)^5 + (-b)^6$$

$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

16. $(c-d)^7$

$$(c-d)^7 = {}_7C_0\,c^7 + {}_7C_1 c^6(-d) + {}_7C_2\,c^5(-d)^2 + {}_7C_3\,c^4(-d)^3 + {}_7C_4\,c^3(-d)^4 + {}_7C_5\,c^2(-d)^5 + {}_7C_6\,c(-d)^6 + {}_7C_7(-d)^7$$

$$= c^7 + 7c^6(-d) + 21c^5(-d)^2 + 35c^4(-d)^3 + 35c^3(-d)^4 + 21c^2(-d)^5 + 7c(-d)^6 + (-d)^7$$

$$= c^7 - 7c^6d + 21c^5d^2 - 35c^4d^3 + 35c^3d^4 - 21c^2d^5 + 7cd^6 - d^7$$

17. $(x+6)^6$

$$(x+6)^6 = {}_6C_0\,x^6 + {}_6C_1 x^5(6) + {}_6C_2\,x^4(6)^2 + {}_6C_3\,x^3(6)^3 + {}_6C_4\,x^2(6)^4 + {}_6C_5\,x(6)^5 + {}_6C_6(6)^6$$

$$= x^6 + 6x^5(6) + 15x^4(6)^2 + 20x^3(6)^3 + 15x^2(6)^4 + 6x(6)^5 + (6)^6$$

$$= x^6 + 36x^5 + 540x^4 + 4320x^3 + 19440x^2 + 46656x + 46656$$

18. $(y-5)^7$

$$(y-5)^7 = {}_7C_0\,y^7 + {}_7C_1 y^6(-5) + {}_7C_2\,y^5(-5)^2 + {}_7C_3\,y^4(-5)^3 + {}_7C_4\,y^3(-5)^4 + {}_7C_5\,y^2(-5)^5 + {}_7C_6\,y(-5)^6 + {}_7C_7(-5)^7$$

$$= y^7 + 7y^6(-5) + 21y^5(-5)^2 + 35y^4(-5)^3 + 35y^3(-5)^4 + 21y^2(-5)^5 + 7y(-5)^6 + (-5)^7$$

$$= y^7 - 35y^6 + 525y^5 - 4375y^4 + 21875y^3 - 65625y^2 + 109375y - 78125$$

19. $(2a+4b)^4$

$$(2a+4b)^4 = {}_4C_0(2a)^4 + {}_4C_1(2a)^3(4b) + {}_4C_2(2a)^2(4b)^2 + {}_4C_3(2a)(4b)^3 + {}_4C_4(4b)^4$$

$$= (16)a^4 + 4(8)a^3(4)b + 6(4)a^2(16)b^2 + 4(2)a(64)b^3 + (256)b^4$$

$$= 16a^4 + 128a^3b + 384a^2b^2 + 512ab^3 + 256b^4$$

20. $(3a-4b)^5$

$(3a-4b)^5 = {}_5C_0(3a)^5 + {}_5C_1(3a)^4(-4b) + {}_5C_2(3a)^3(-4b)^2 + {}_5C_3(3a)^2(-4b)^3 + {}_5C_4(3a)(-4b)^4 + {}_5C_5(-4b)^5$

$= (243)a^5 + 5(81)a^4(-4)b + 10(27)a^3(16)b^2 + 10(9)a^2(-64)b^3 + 5(3)a(256)b^4 + (-1024)b^5$

$= 243a^5 - 1620a^4b + 4320a^3b^2 - 5760a^2b^3 + 3840ab^4 - 1024b^5$

24	A learning outcome from the SoW ناتج من الخطة الفصلية
25	A learning outcome from the SoW ناتج من الخطة الفصلية