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موقع المناهج الإماراتية

الملف مراجعة وأوراق عمل الوحدة الرابعة

موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ رياضيات ⇦ الفصل الأول

روابط مواقع التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم

روابط مواد الصف الحادي عشر المتقدم على تلغرام

التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الأول

1	مراجعة لامتحان منتصف الفصل الأول
2	حساب المثلثات القائمة الزاوية
3	مراجعة في وحدة القوى
4	نموذج الاجابة لامتحان الوزارة
5	التوزيع الزمني للفصل الاول

مدرسة عبد القادر الجزائري للتعليم الثانوي

قسم الرياضيات

دائـــرة التـعـلـيـم والـمـعـرفـة
DEPARTMENT OF EDUCATION AND KNOWLEDGE

CHAPTER 4

Trigonometric Identities and Equations

math 2020

الاسم/ ....

المدرسة/ ....

الشعبة/ ....

الفصل الدراسي الاول 2019/2018

الصف الحادي عشر (متقدم)

ملحوظة هامة

هذه الأوارق مجرد عامل مساعد للطلاب ولايمكن إهمال كتاب المدرسة حيث أن التدريب عليه يمنحك التفوق

أ / قاسم علي العتوم

LESSON 1 : Basic Trigonometric Identities

NewVocabulary identity trigonometric identity cofunction

KeyConcept Reciprocal and Quotient Identities

Reciprocal Identities			Quotient Identities
$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Example 1 : Use Reciprocal and Quotient Identities

a. If $\csc\theta = \frac{7}{4}$, find $\sin\theta$.

b. If $\cot x = \frac{2}{5\sqrt{5}}$ and $\sin x = \frac{\sqrt{5}}{3}$, find $\cos x$.

Practice 1 :

A If $\sec x = \frac{5}{3}$, find $\cos x$.

B If $\csc\beta = \frac{25}{7}$ and $\sec\beta = \frac{25}{24}$, find $\tan\beta$.

KeyConcept Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

Example 2: Use Pythagorean Identities

If $\tan\theta = -8$ and $\sin\theta > 0$, find $\sin\theta$ and $\cos\theta$.

Practice 2 : **Find the value of each expression using the given information.**

A. $\csc\theta$ and $\tan\theta$; $\cot\theta = -3$, $\cos\theta < 0$

B. $\cot x$ and $\sec x$; $\sin x = \frac{1}{6}$, $\cos x > 0$

KeyConcept Cofunction Identities

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$ $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

KeyConcept Odd-Even Identities

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

Example 3: Use Cofunction and Odd-Even Identities

If $\tan\theta = 1.28$, find $\cot\left(\theta - \frac{\pi}{2}\right)$.

Practice 3 : If $\sin x = -0.37$, find $\cos\left(x - \frac{\pi}{2}\right)$.

Example 4; Simplify by Rewriting Using Only Sine and Cosine

Simplify $\csc\theta\sec\theta - \cot\theta$. **Solve Algebraically**

Practice 4:

Simplify $\sec x - \tan x \sin x$.

Example 5; Simplify by Factoring

Simplify $\sin^2 x \cos x - \sin\left(\frac{\pi}{2} - x\right)$.

Practice 5 : Simplify $-\csc\left(\frac{\pi}{2} - x\right) - \tan^2 x \sec x$.

Example 6; Simplify by Combining Fractions

Simplify $\frac{\sin x \cos x}{1 - \sin x} - \frac{1 + \sin x}{\cos x}$.

Practice 6: Simplify each expression.

A $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

B $\frac{\csc x}{1 + \sec x} + \frac{\csc x}{1 - \sec x}$

Example 7; Rewrite to Eliminate Fractions

Rewrite $\frac{1}{1+\cos x}$ as an expression that does not involve a fraction.

Practice 6: Rewrite as an expression that does not involve a fraction.

A $\frac{\cos^2 x}{1-\sin x}$

B $\frac{4}{\sec x+\tan x}$

LESSON 2 : Verifying Trigonometric Identities

NewVocabulary verify an identity

Example 1: Verify a Trigonometric Identity

Verify that $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$.

Practice 1 : Verify each identity.

A $\sec^2 \theta \cot^2 \theta - 1 = \cot^2 \theta$

B $\tan^2 \alpha = \sec \alpha \csc \alpha \tan \alpha - 1$

Example 2: Verify a Trigonometric Identity by Combining Fractions

Verify that $2 \csc x = \dfrac{1}{\csc x + \cot x} + \dfrac{1}{\csc x - \cot x}$.

Practice 2 :

Verify that $\dfrac{\cos \alpha}{1 + \sin \alpha} + \dfrac{\cos \alpha}{1 - \sin \alpha} = 2 \sec \alpha$.

Example 3: Verify a Trigonometric Identity by Multiplying

Verify that $\dfrac{\sin \alpha}{1 - \cos \alpha} = \csc \alpha + \cot \alpha$.

Practice 3 :

Verify that $\frac{\tan x}{\sec x + 1} = \csc x - \cot x$.

Example 4: Verify a Trigonometric Identity by Factoring

Verify that $\cot \theta \sec \theta \csc^2 \theta - \cot^3 \theta \sec \theta = \csc \theta$.

Practice 4

Verify that $\sin^2 x \tan^2 x \csc^2 x + \cos^2 x \tan^2 x \csc^2 x = \sec^2 x$.

Example 5: **Verify an Identity by Working Each Side Separately**

Verify that $\frac{\tan^2 x}{1+\sec x} = \frac{1-\cos x}{\cos x}$.

Practice 5:

Verify that $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$.

ConceptSummary Strategies for Verifying Trigonometric Identities

- Start with the more complicated side of the identity and work to transform it into the simpler side, keeping the other side of the identity in mind as your goal.
- Use reciprocal, quotient, Pythagorean, and other basic trigonometric identities.
- Use algebraic operations such as combining fractions, rewriting fractions as sums or differences, multiplying expressions, or factoring expressions.
- Convert a denominator of the form $1 \pm u$ or $u \pm 1$ to a single term using its conjugate and a Pythagorean Identity.
- Work each side separately to reach a common intermediate expression.
- If no other strategy presents itself, try converting the entire expression to one involving only sines and cosines.

Example 6: Determine Whether an Equation is an Identity

Use a graphing calculator to test whether each equation is an identity. If it appears to be an identity, verify it. If not, find a value for which both sides are defined but not equal.

$$\frac{\cos\beta + 1}{\tan^2\beta} = \frac{\cos\beta}{\sec\beta + 1}$$

LESSON 3: Solving Trigonometric Equations

Example 1: Solve by Isolating Trigonometric Expressions

Solve $2 \tan x - \sqrt{3} = \tan x$.

Practice 1:

Solve $4 \sin x = 2 \sin x + \sqrt{2}$.

Example 2: Solve by Taking the Square Root of Each Side

Solve $4 \sin^2 x + 1 = 4$.

Practice 2:

Solve $3\cot^2 x + 4 = 7$.

Example 3: Solve by Factoring

Find all solutions of each equation on the interval $[0, 2\pi)$.

a. $\cos x \sin x = 3\cos x$

b. $\cos^4 x + \cos^2 x - 2 = 0$

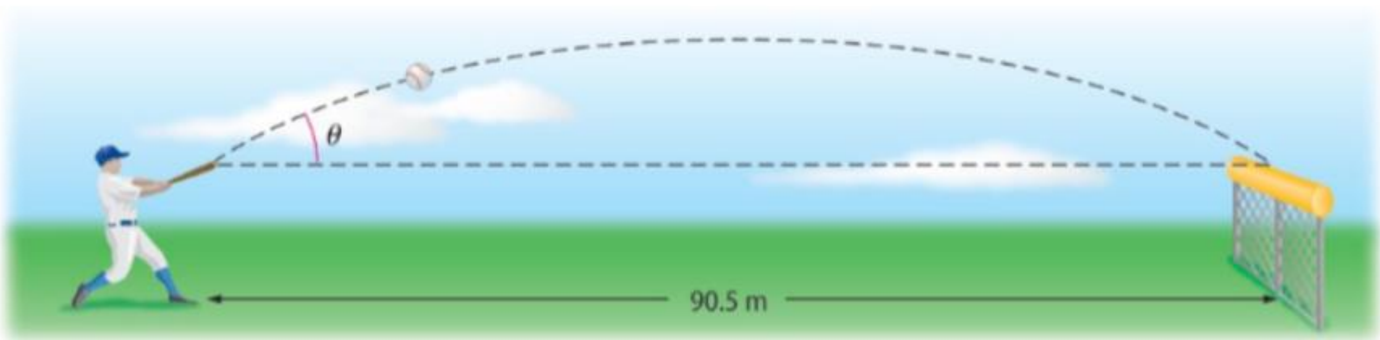
Practice 3:

A $2 \sin x \cos x = \sqrt{2} \cos x$

B $4 \cos^2 x + 2 \cos x - 2\sqrt{2} \cos x = \sqrt{2}$

Example 4: Real-World Trigonometric Functions of Multiple Angles

BASEBALL A baseball leaves a bat with an initial speed of 30 meters per second and clears a fence 90.5 meters away. The height of the fence is the same height as the initial height of the batted ball. If the distance the ball traveled is given by $d = \frac{v_0^{\,2} \sin 2\theta}{9.8}$, where 9.8 is in meters per second squared, find the interval of possible launch angles of the ball.



Example 5: Solve by Rewriting Using a Single Trigonometric Function

Find all solutions of $2\cos^2 x - \sin x - 1 = 0$ on the interval $[0, 2\pi)$

practice 5: **Find all solutions of each equation on the interval $[0, 2\pi)$.**

A $1 - \cos x = 2\sin^2 x$

A $\cot^2 x \csc^2 x + 2\csc^2 x - \cot^2 x = 2$

Example 6: Solve by Squaring

Find all solutions of $\csc x - \cot x = 1$ on the interval $[0, 2\pi]$.

Practice 6:

A $\sec x + 1 = \tan x$

A $\cos x = \sin x - 1$

LESSON 4: Sum and Difference Identities

New Vocabulary reduction identity

Key Concept Sum and Difference Identities

Sum Identities

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

Difference Identities

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Example 1: Evaluate a Trigonometric Expression

Find the exact value of each trigonometric expression

a. $\sin 15^\circ$

b. $\tan \dfrac{7\pi}{12}$

practice 1:

A $\cos 15^\circ$

A $\sin \dfrac{5\pi}{12}$

Example 2: Real-World Use a Sum or Difference Identity

ELECTRICITY An alternating current i in amperes in a certain circuit can be found after t seconds using $i = 3\ (\sin 165)t$, where 165 is a degree measure.

a. Rewrite the formula in terms of the sum of two angle measures.

b. Use a sine sum identity to find the exact current after 1 second.

Example 3:

a. Find the exact value of $\dfrac{\tan 32^\circ + \tan 13^\circ}{1 - \tan 32^\circ \tan 13^\circ}$.

b. Simplify $\sin x \sin 3x - \cos x \cos 3x$.

practice 3:

A Find the exact value of $\cos \frac{7\pi}{8} \cos \frac{5\pi}{24} + \sin \frac{7\pi}{8} \sin \frac{5\pi}{24}$.

A Simplify $\frac{\tan 6x - \tan 7x}{1 + \tan 6x \tan 7x}$.

Example 4: Write as an Algebraic Expression

Write $\sin(\arctan \sqrt{3} + \arcsin x)$ as an algebraic expression of x that does not involve trigonometric functions.

practice 4:

Write each trigonometric expression as an algebraic expression.

A $\cos(\arcsin 2x + \arccos x)$

A $\sin\left(\arctan x - \arccos \frac{1}{2}\right)$

Example 5: Verify Cofunction Identities

Verify $\sin\left(\frac{\pi}{2} - x\right) = \cos x$..

practice 5: **Verify each cofunction identity using a difference identity.**

A $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

A $\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$

Example 6: Verify Reduction Identities

a. $\sin\left(\theta + \frac{3\pi}{2}\right) = -\cos \theta$

b. $\tan(x - 180°) = \tan x$

practice 6: **Verify each cofunction identity.**

A $\cos(360^{\circ} - \theta) = \cos\theta$

B $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

Example 7: Solve a Trigonometric Equation

Find the solutions of $\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$ on the interval $[0, 2\pi]$.

practice 7:

Find the solutions of $\cos(x + \pi) - \sin(x - \pi) = 0$ on the interval $[0, 2\pi]$.

LESSON 5:

Multiple-Angle and Product-to-Sum Identities

NewVocabulary

KeyConcept Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Proof Double-Angle Identity for Sine

$\sin 2\theta = \sin(\theta+\theta)$	$2\theta = \theta + \theta$
$= \sin\theta\cos\theta + \cos\theta\sin\theta$	Sine Sum Identity where $\alpha = \beta = \theta$
$= 2\sin\theta\cos\theta$	Simplify.

Example 1: Evaluate Expressions Involving Double Angles

If $\sin\theta = -\frac{7}{25}$ on the interval $\left(\pi, \frac{3\pi}{2}\right)$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

practice 1: If $\cos\theta = \frac{3}{5}$ on the interval $\left(0, \frac{\pi}{2}\right)$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Example 2: Solve an Equation Using a Double-Angle Identity

Solve $\sin 2\theta - \sin\theta = 0$ on the interval $[0, 2\pi]$.

practice 2: Solve each equation on the interval $[0, 2\pi]$..

A $\cos 2\alpha = -\sin^2 \alpha$

B $\tan 2\beta = 2 \tan \beta$

KeyConcept Power-Reducing Identities

$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ $\quad\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ $\quad\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$

Proof Power-Reducing Identity for Sine

$\frac{1 - \cos 2\theta}{2} = \frac{1 - (1 - 2\sin^2 \theta)}{2}$ Cosine Double-Angle Identity

$= \frac{2\sin^2 \theta}{2}$ Subtract.

$= \sin^2 \theta$ Simplify.

Example 3: Use an Identity to Reduce a Power

Rewrite $\sin^4 x$ in terms with no power greater than 1.

practice 3:

Rewrite each expression in terms with no power greater than 1.

A $\cos^4 x$

B $\sin^3 \theta$

Example 4: Solve an Equation Using a Power-Reducing Identity

Solve $\cos^2 x - \cos 2x = \frac{1}{2}$.

practice 4: **Solve each equation.**

A $\cos^4 \alpha - \sin^4 \alpha = \frac{1}{2}$

B $\sin^2 3\beta = \sin^2 \beta$

KeyConcept Half-Angle Identities

$\sin \frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos \theta}{2}}$

$\cos \frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos \theta}{2}}$

$\tan \frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$

$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$

$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$

Proof Half-Angle Identity for Cosine

$\pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1 + \cos\left(2 \cdot \frac{\theta}{2}\right)}{2}}$ Rewrite θ as $2 \cdot \frac{\theta}{2}$.

$= \pm\sqrt{\frac{1 + \cos 2x}{2}}$ Substitute $x = \frac{\theta}{2}$.

$= \pm\sqrt{\cos^2 x}$ Cosine Power-Reducing Identity

$= \cos x$ Simplify.

$= \cos \frac{\theta}{2}$ Substitute.

Example 5: **Evaluate an Expression Involving a Half Angle**

Find the exact value of $\cos 112.5°$

practice 5: **Find the exact value of each expression.**

A $\sin 75°$

B $\tan \frac{7\pi}{12}$

Example 6: Solve an Equation Using a Half-Angle Identity

Solve $\sin^2 x = 2\cos^2 \frac{x}{2}$ on the interval $[0, 2\pi]$.

practice 6:

Solve each equation on the interval $[0, 2\pi]$.

A $2\sin^2 \frac{x}{2} + \cos x = 1 + \sin x$

B $8\tan \frac{x}{2} + 8\cos x \tan \frac{x}{2} = 1$

KeyConcept Product-to-Sum Identities

$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\cos\alpha \sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Proof Product-to-Sum Identity for $\sin\alpha\cos\beta$

$\frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$	More complicated side of identity
$= \frac{1}{2}(\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta)$	Sum and Difference Identities
$= \frac{1}{2}(2\sin\alpha\cos\beta)$	Combine like terms.
$= \sin\alpha\cos\beta$	Multiply.

Example 7: Use an Identity to Write a Product as a Sum or Difference

Rewrite $\cos 5x \sin 3x$ as a sum or difference.

practice 7:

A $\sin 4\theta \cos\theta$

B $\sin 7x \sin 6x$

KeyConcept Sum-to-Product Identities

$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right)$

$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right)$

$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha+\beta}{2}\right) \sin \left(\frac{\alpha-\beta}{2}\right)$

$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha+\beta}{2}\right) \sin \left(\frac{\alpha-\beta}{2}\right)$

Proof Sum-to-Product Identity for $\sin \alpha + \sin \beta$

$2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right)$

$= 2 \sin x \cos y$ — Substitute $x = \frac{\alpha+\beta}{2}$ and $y = \frac{\alpha-\beta}{2}$.

$= 2\left\{\frac{1}{2}[\sin (x+y) + \sin (x-y)]\right\}$ — Product-to-Sum Identity

$= \sin \left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) + \sin \left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right)$ — Substitute and simplify.

$= \sin \left(\frac{2\alpha}{2}\right) + \sin \left(\frac{2\beta}{2}\right)$ — Combine fractions.

$= \sin \alpha + \sin \beta$ — Simplify.

Example 8: Use a Product-to-Sum or Sum-to-Product Identity

Find the exact value of $\sin \frac{5\pi}{12} + \sin \frac{\pi}{12}$.

Practice8: Find the exact value of each expression.

A $3 \cos 37.5^\circ \cos 187.5^\circ$

B $\cos \frac{7\pi}{12} - \cos \frac{\pi}{12}$

Example 9:

Solve $\cos 4x + \cos 2x = 0$..

Practice 9: Solve each equation.

A $\sin x + \sin 5x = 0$

B $\cos 3x - \cos 5x = 0$