1 March 2022

مؤسسة الإمارات للتعليم المدرسي
EMIRATES SCHOOLS ESTABLISHMENT

بسم الله الرحمن الرحيم

السلام عليكم ورحمة الله وبركاته

GRADE 12 MATHEMATICS

ADVANCED STREAM

ACADEMIC YEAR 2021/2022 – TERM 2

EOT2 EXAM COVERAGEIN TERM 2 (2021 - 2022)2هيكل امتحانات نهاية الفصل

*	Best 20 answers out of 25 will count. Example: 14 correct answers yield a grade of 70/100, while 20 and 23 correct answers yield a (full) grade of 100/100 each.
*	تحتسب أفضل 20 إجابة من 25. مثال: 14 إجابة صحيحة تعطي علامة 70/100 بينما 20 أو 23 إجابة صحيحة تعطي العلامة الكاملة أي 100/100.
**	Questions might appear in a different order in the actual exam.
**	قد تظهر الأسئلة بترتيب مختلف في الامتحان الفعلي.
***	As it appears in the textbook/LMS/SoW.
***	كما وردت في كتاب الطالب وLMS و الخطة الفصلية.

1	Find the linear approximation of a given function at a given point إيجاد التقريب الخطي لدالة معطاة عند قيمة محددة	(1-6)	236

In exercises 1–6, find the linear approximation to $f(x)$ at $x = x_0$. Use the linear approximation to estimate the given number.

1. $f(x) = \sqrt{x}, x_0 = 1, \sqrt{1.2}$
2. $f(x) = (x + 1)^{1/3}, x_0 = 0, \sqrt[3]{1.2}$
3. $f(x) = \sqrt{2x + 9}, x_0 = 0, \sqrt{8.8}$
4. $f(x) = 2/x, x_0 = 1, 2/0.99$
5. $f(x) = \sin 3x, x_0 = 0, \sin(0.3)$
6. $f(x) = \sin x, x_0 = \pi, \sin(3.0)$

2	Use l'Hopital's rule to compute limits in various cases استخدام نظرية لوبيتال في إيجاد قيمة نهاية معطاة في الحالات المختلفة	(1-6)	247

In exercises 1–40, find the indicated limits.

1. $\lim_{x \to -2} \frac{x+2}{x^2-4}$

2. $\lim_{x \to 2} \frac{x^2-4}{x^2-3x+2}$

3. $\lim_{x \to \infty} \frac{3x^2+2}{x^2-4}$

4. $\lim_{x \to -\infty} \frac{x+1}{x^2+4x+3}$

5. $\lim_{t \to 0} \frac{e^{2t}-1}{t}$

6. $\lim_{t \to 0} \frac{\sin t}{e^{3t}-1}$

3	Use l'Hopital's rule to compute limits in various cases استخدام نظرية لوبيتال في إيجاد قيمة نهاية معطاة في الحالات المختلفة	(21,22,25,29,30)	248

21. $\lim_{x\to\infty} \frac{\ln x}{x^2}$

22. $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

25. $\lim_{t\to 1} \frac{\ln(\ln t)}{\ln t}$

29. $\lim_{x\to 0^+} \frac{\ln x}{\cot x}$

30. $\lim_{x\to 0^+} \frac{\sqrt{x}}{\ln x}$

4	Find the critical points of a given function إيجاد الأعداد الحرجة لدالة معطاة	(5,6)	258

In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.

5. (a) $f(x) = x^3 - 3x^2 + 6x$ (b) $f(x) = -x^3 + 3x^2 - 3x$

6. (a) $f(x) = x^4 - 2x^2 + 1$ (b) $f(x) = x^4 - 3x^3 + 2$

5	Find the absolute extrema of a given function إيجاد القيم القصوى المطلقة لدالة معطاة	(25,26)	258

In exercises 25–34, find the absolute extrema of the given function on each indicated interval.

25. $f(x) = x^3 - 3x + 1$ on (a) $[0, 2]$ and (b) $[-3, 2]$

26. $f(x) = x^4 - 8x^2 + 2$ on (a) $[-3, 1]$ and (b) $[-1, 3]$

7	Find the local extrema of a given function using the First Derivative Test إيجاد القيم القصوى المحلية لدالة معينة باستخدام اختبار المشتقة الأولى	(13,14,25)	267

In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

13. $y = xe^{-2x}$

14. $y = x^2e^{-x}$

In exercises 21–26, approximate the x-coordinates of all extrema and sketch graphs showing global and local behavior of the function.

25. $y = (x^2 + x + 0.45)e^{-2x}$

26. $y = x^5 \ln 8x^2$

EXAMPLE 5.1 Determining Concavity

Determine where the graph of $f(x) = 2x^3 + 9x^2 - 24x - 10$ is concave up and concave down, and draw a graph showing all significant features of the function.

Solution Here, we have $f'(x) = 6x^2 + 18x - 24$

and from our work in example 4.3, we have

$$f'(x)\begin{cases} > 0 \text{ on } (-\infty, -4) \cup (1, \infty) & f \text{ increasing.} \\ < 0 \text{ on } (-4, 1). & f \text{ decreasing.} \end{cases}$$

Further, we have $f''(x) = 12x + 18 \begin{cases} > 0, \text{ for } x > -\frac{3}{2} & \text{Concave up.} \\ < 0, \text{ for } x < -\frac{3}{2}. & \text{Concave down.} \end{cases}$

Using all of this information, we are able to draw the graph shown in Figure 4.56. Notice that at the point $\left(-\frac{3}{2}, f\left(-\frac{3}{2}\right)\right)$, the graph changes from concave down to concave up. Such points are called *inflection points,* which we define more precisely in Definition 5.2.

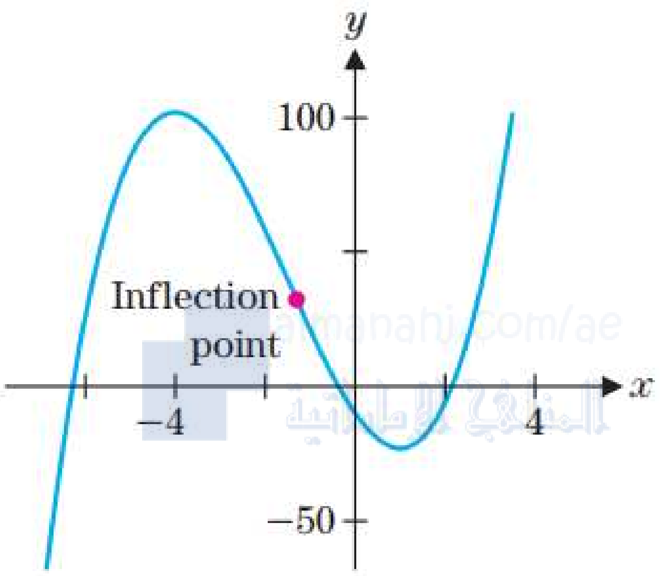


FIGURE 4.56
$y = 2x^3 + 9x^2 - 24x - 10$

8	Learn the notion of an Inflection Point and find one التعريف على مفهوم نقطة الانعطاف وإيجادها	(1-5)	276

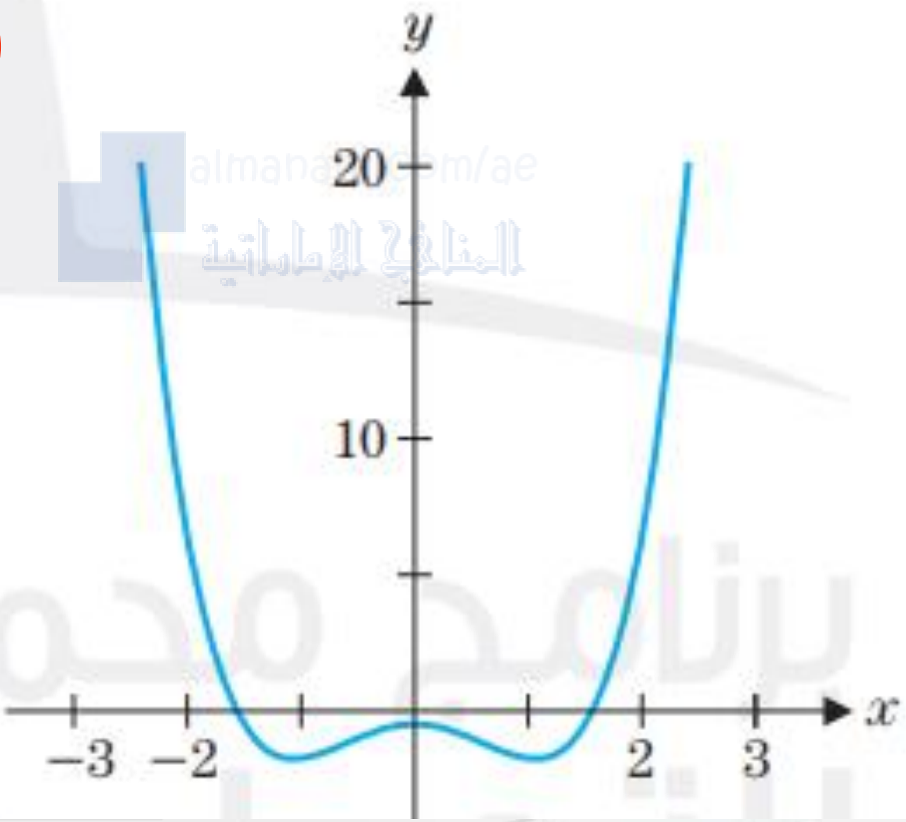
In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.

1. $f(x) = x^3 - 3x^2 + 4x - 1$
2. $f(x) = x^4 - 6x^2 + 2x + 3$
3. $f(x) = x + 1/x$
4. $f(x) = x + 3(1 - x)^{1/3}$
5. $f(x) = \sin x - \cos x$

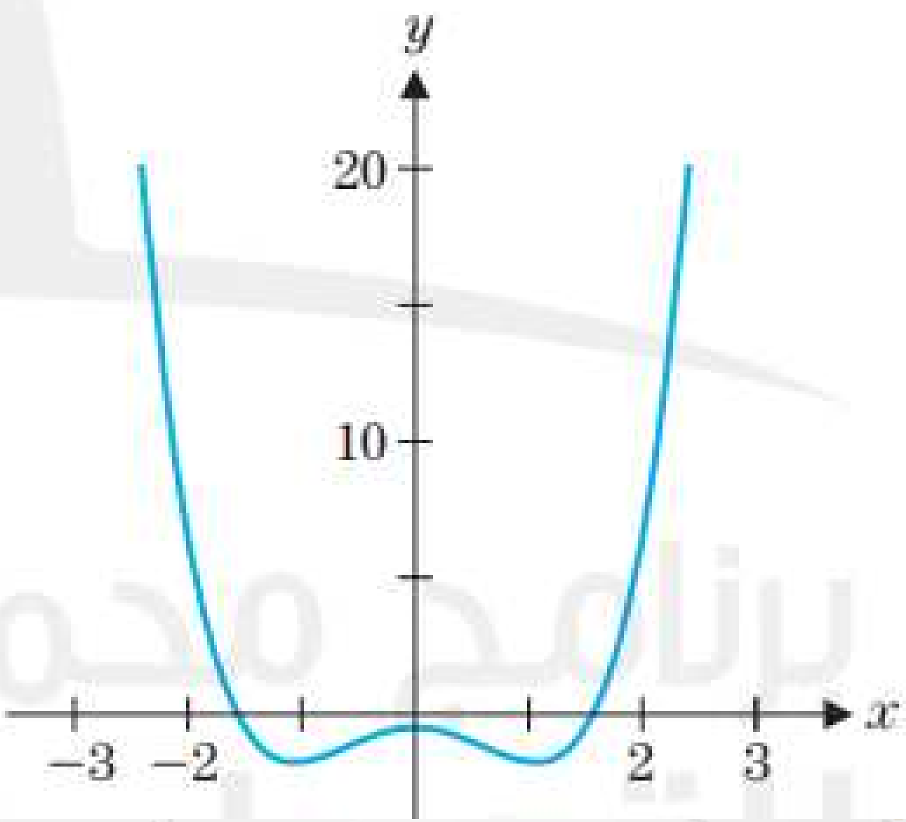
6	Identify increasing and decreasing functions التعرف على مفهومي الدالة المتناقصة والدالة المتزايدة	(45,46)	276

In exercises 45 and 46, estimate the intervals of increase and decrease, the locations of local extrema, intervals of concavity and locations of inflection points.

45.



45.



In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.

6. $f(x) = \dfrac{x^2 - 1}{x}$

7. $f(x) = \dfrac{x^2 + 4}{x^3}$

8. $f(x) = \dfrac{x - 4}{x^3}$

9. $f(x) = \dfrac{2x}{x^2 - 1}$

10. $f(x) = \dfrac{3x^2}{x^2 + 1}$

11	Solve mathematical and real-life optomization problems حل مسائل رياضية وحياتية على القيم القصوى لإيجاد القيم المثلى	(1-7)	296

1. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 ft^2. Find the minimum perimeter and the dimensions of the corresponding enclosure.

2. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There is 96 feet of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.

3. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 ft of fencing available, what dimensions of the corral will maximize the enclosed area?

4. A showroom for a department store is to be rectangular with walls on three sides, 6-ft door openings on the two facing sides and a 10-ft door opening on the remaining wall. The showroom is to have 800 ft^2 of floor space. What dimensions will minimize the length of wall used?

5. Show that the rectangle of maximum area for a given perimeter P is always a square.

6. Show that the rectangle of minimum perimeter for a given area A is always a square.

7. A box with no top is to be built by taking a 6 in-by-10 in sheet of cardboard, cutting x-in squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

12	Solve mathematical and real-life problems on related rates حل مسائل رياضية وحياتية على المعدلات المرتبطة	(1-7)	303

1. Oil spills out of a tanker at the rate of 120 gl/min per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}''$. Given that 1 ft^3 equals 7.5 gallons, determine the rate at which the radius of the spill is increasing when the radius reaches (a) 100 ft and (b) 200 ft. Explain why the rate decreases as the radius increases.

2. Oil spills out of a tanker at the rate of 90 gallon per minute. The oil spreads in a circle with a thickness of $\frac{1}{8}''$. Determine the rate at which the radius of the spill is increasing when the radius reaches 100 feet.

3. Oil spills out of a tanker at the rate of g gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}''$. (a) Given that the radius of the spill is increasing at a rate of 0.6 ft/min when the radius equals 100 feet, determine the value of g. (b) If the thickness of the oil is doubled, how does the rate of increase of the radius change?

4. Assume that the infected area of an injury is circular. (a) If the radius of the infected area is 3 mm and growing at a rate of 1 mm/hr, at what rate is the infected area increasing? (b) Find the rate of increase of the infected area when the radius reaches 6 mm. Explain in commonsense terms why this rate is larger than that of part (a).

5. Suppose that a raindrop evaporates in such a way that it maintains a spherical shape. Given that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and its surface area is $A = 4\pi r^2$, if the radius changes in time, show that $V' = Ar'$. If the rate of evaporation (V') is proportional to the surface area, show that the radius changes at a constant rate.

6. Suppose a forest fire spreads in a circle with radius changing at a rate of 5 ft/min. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

7. A 10 ft ladder leans against the side of a building as in example 8.2. If the bottom of the ladder is pulled away from the wall at the rate of 3 ft/sec and the ladder remains in contact with the wall, (a) find the rate at which the top of the ladder is dropping when the bottom is 6 ft from the wall. (b) Find the rate at which the angle between the ladder and the horizontal is changing when the bottom of the ladder is 6 ft from the wall.

EXAMPLE 9.1 Analyzing the Marginal Cost of Producing a Commercial Product

Suppose that

$$C(x) = 0.02x^2 + 2x + 4000$$

is the total cost (in AED) for a company to produce x units of a certain product. Compute the marginal cost at $x = 100$ and compare this to the actual cost of producing the 100th unit.

Solution The marginal cost function is the derivative of the cost function:

$$C'(x) = 0.04x + 2$$

and so, the marginal cost at $x = 100$ is $C'(100) = 4 + 2 = 6$ AED per unit. On the other hand, the actual cost of producing item number 100 would be $C(100) - C(99)$. (Why?) We have

$$\begin{aligned} C(100) - C(99) &= 200 + 200 + 4000 - (196.02 + 198 + 4000) \\ &= 4400 - 4394.02 = 5.98 \text{ AED.} \end{aligned}$$

Note that this is very close to the marginal cost of AED 6. Also notice that the marginal cost is easier to compute. ■

14	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة	(13,15,16,23)	329

In exercises 5–28, find the general antiderivative.

13. $\int 2\sec x\tan x\,dx$

15. $\int 5\sec^2 x\,dx$

16. $\int 4\frac{\cos x}{\sin^2 x}\,dx$

23. $\int \frac{\cos x}{\sin x}\,dx$

15	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة	(21,25)	329

21. $\int \frac{4x}{x^2 + 4}\, dx$

25. $\int \frac{e^x}{e^x + 3}\, dx$

16	Understand the notion of indefinite integral as an finding an intiderivative التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة	(45-48)	330

45. Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

48. Determine the position function if the acceleration function is $a(t) = t^2 + 1$, the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 0$.

17	Use the sigma notation to compute basic summation استخدام رمز المجموع سيجما لإيجاد المجاميع البسيطة	(6,8,16)	337

In exercises 5–8, write out all terms and compute the sums.

6. $\sum_{i=3}^{7}(i^2+i)$

8. $\sum_{i=6}^{8}(i^2+2)$

In exercises 9–18, use summation rules to compute the sum.

16. $\sum_{i=4}^{20}(i-3)(i+3)$

18	Estimate the area under a curve on a given interval using rectangles تقدير المساحة تحت المنحنى لدالة في فترة محددة باستخدام المستطيلات	(35-38)	345

In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

35.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	2.0	2.4	2.6	2.7	2.6	2.4	2.0	1.4	0.6

36.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$f(x)$	2.0	2.2	1.6	1.4	1.6	2.0	2.2	2.4	2.0

37.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.8	1.4	1.1	0.7	1.2	1.4	1.8	2.4	2.6

38.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$	0.0	0.4	0.6	0.8	1.2	1.4	1.2	1.4	1.0

19	Understand the notion of a definite integral التعرف على مفهوم التكامل المحدود	(15-18)	356

In exercises 15–20, write the given (total) area as an integral or sum of integrals.

15. The area above the x-axis and below $y = 4 - x^2$

16. The area above the x-axis and below $y = 4x - x^2$

17. The area below the x-axis and above $y = x^2 - 4$

18. The area below the x-axis and above $y = x^2 - 4x$

19. The area between $y = \sin x$ and the x-axis for $0 \le x \le \pi$

20. The area between $y = \sin x$ and the x-axis for $-\frac{\pi}{2} \le x \le \frac{\pi}{4}$.

20	Apply the Integral Mean Value Theorem تطبيق نظرية القيمة المتوسطة في التكامل	(25-28)	356

In exercises 25–28, compute the average value of the function on the given interval.

25. $f(x) = 2x + 1, [0, 4]$

26. $f(x) = x^2 + 2x, [0, 1]$

27. $f(x) = x^2 - 1, [1, 3]$

28. $f(x) = 2x - 2x^2, [0, 1]$

21	Learn the properties of definite integrals التعرف على خصائص التكامل المحدود	(35,36)	356

In exercises 35 and 36, use Theorem 4.2 to write the expression as a single integral.

35. (a) $\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$ (b) $\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx$

36. (a) $\int_0^2 f(x)\,dx + \int_2^1 f(x)\,dx$ (b) $\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$

22	Learn the properties of definite integrals التعرف على خصائص التكامل المحدود	(37,38)	356

In exercises 37 and 38, assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

37. (a) $\displaystyle\int_1^3 [f(x) + g(x)]\,dx$ (b) $\displaystyle\int_1^3 [2f(x) - g(x)]\,dx$

38. (a) $\displaystyle\int_1^3 [f(x) - g(x)]\,dx$ (b) $\displaystyle\int_1^3 [4g(x) - 3f(x)]\,dx$

23	Learn the Fundamental Theorem of Calculsu (Part I) and use it to compute various definite integrals التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة	(1-6)	366

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

1. $\int_0^2 (2x-3)\,dx$

2. $\int_0^3 (x^2-2)\,dx$

3. $\int_{-1}^1 (x^3+2x)\,dx$

4. $\int_0^2 (x^3+3x-1)\,dx$

5. $\int_1^4 \left(x\sqrt{x}+\frac{3}{x}\right)\,dx$

6. $\int_1^2 \left(4x-\frac{2}{x^2}\right)\,dx$

In exercises 25–32, find the derivative $f'(x)$.

29. $f(x) = \int_{e^x}^{2-x} \sin t^2 \, dt$

30. $f(x) = \int_{2-x}^{xe^x} e^{2t} \, dt$

31. $f(x) = \int_{x^2}^{x^3} \sin(3t) \, dt$

32. $f(x) = \int_{3x}^{\sin x} (t^2 + 4) \, dt$

25	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات	(11-19)	376

In exercises 5–30, evaluate the indicated integral.

11. $\int xe^{x^2+1}\,dx$

12. $\int e^x\sqrt{e^x+4}\,dx$

13. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

14. $\int \frac{\cos(1/x)}{x^2}\,dx$

15. $\int \frac{\sqrt{\ln x}}{x}\,dx$

16. $\int \sec^2 x\sqrt{\tan x}\,dx$

17. $\int \frac{1}{\sqrt{u}\,(\sqrt{u}+1)}\,du$

18. $\int \frac{v}{v^2+4}\,dv$

19. $\int \frac{4}{x(\ln x+1)^2}\,dx$