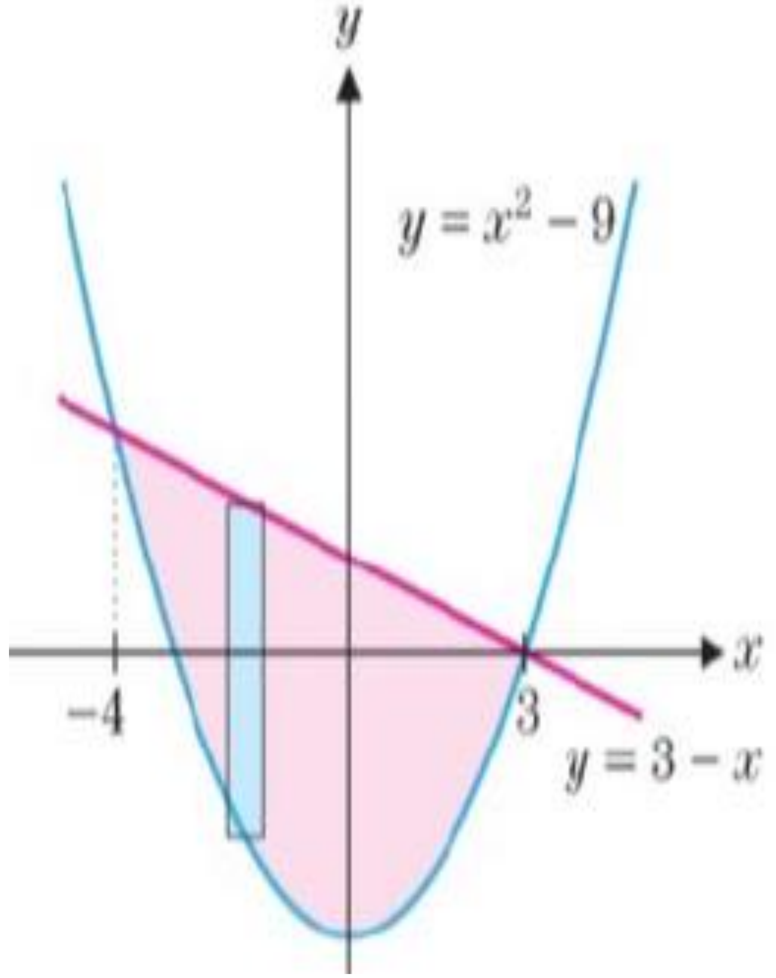
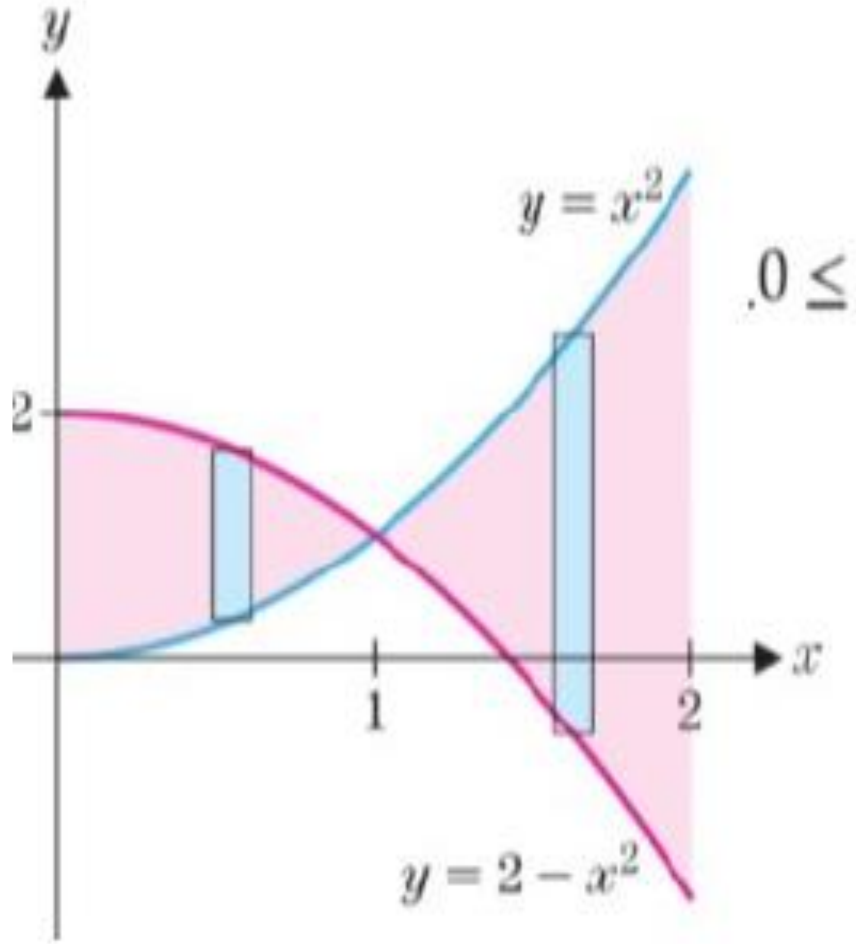


المثال 1.1 إيجاد مساحة منطقة بين منحنيين

أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $y = 3 - x$ و $y = x^2 - 9$.

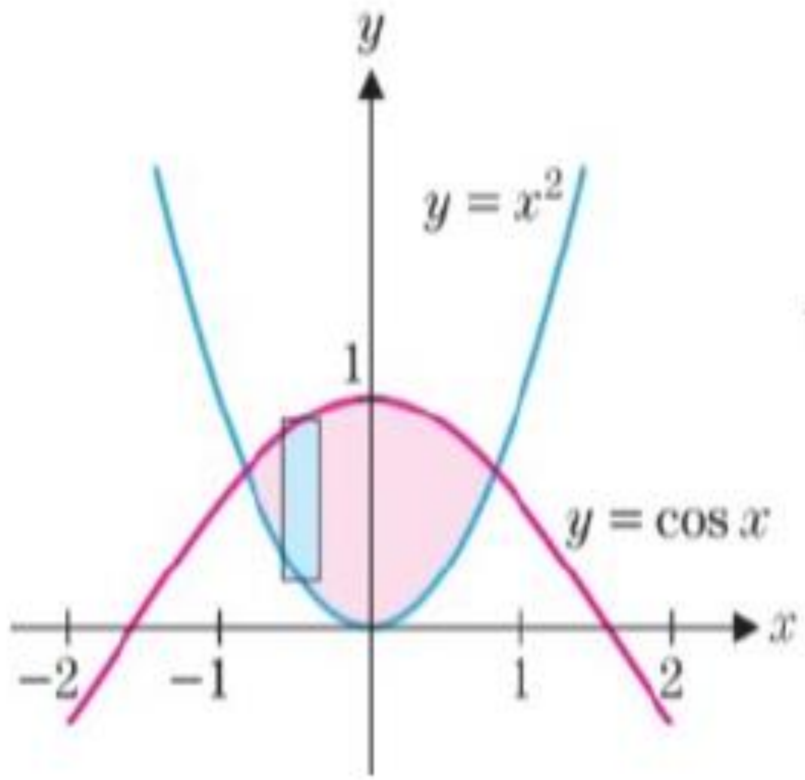
المثال 1.2 إيجاد مساحة منطقة بين منحنيين متقاطعين

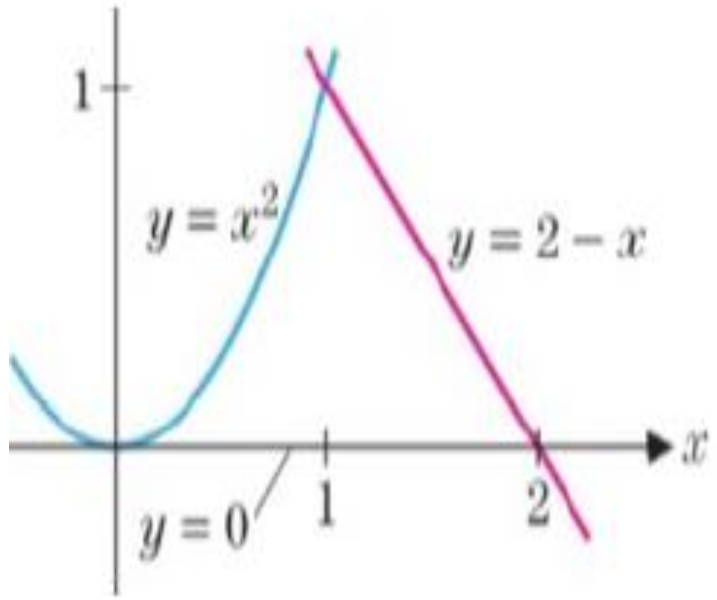
أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $y = x^2$ و $y = 2 - x^2$ لأجل $0 \le x \le 2$.



المثال 1.3 **حالة تكون فيها نقاط التقاطع معروفة تقريبيًا فقط**

أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $y = \cos x$ و $y = x^2$.





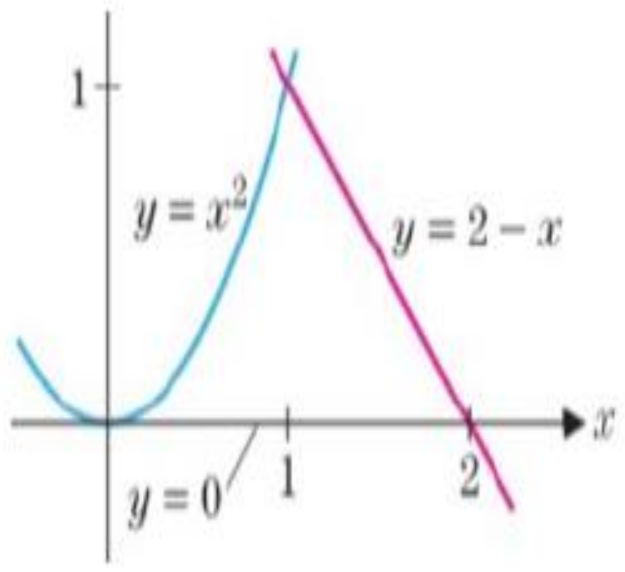
المثال 1.4 مساحة منطقة تحدّدها ثلاثة منحنيات

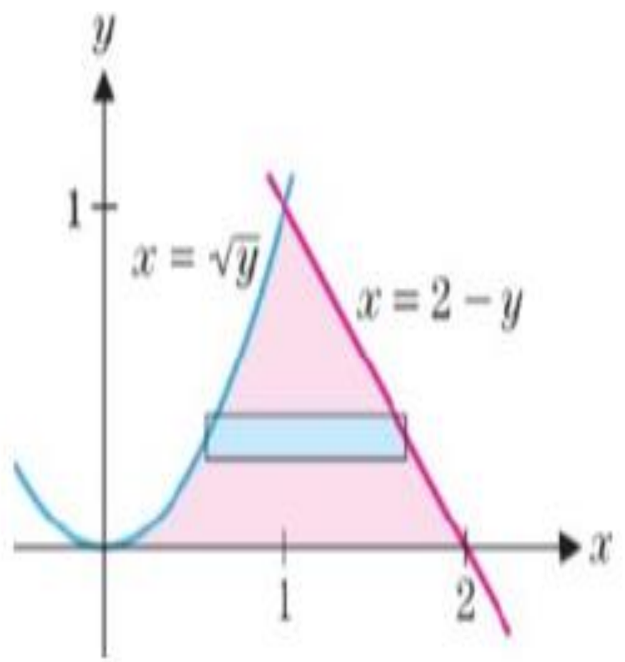
أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $y = x^2$، $y = 2 - x$ و $y = 0$.

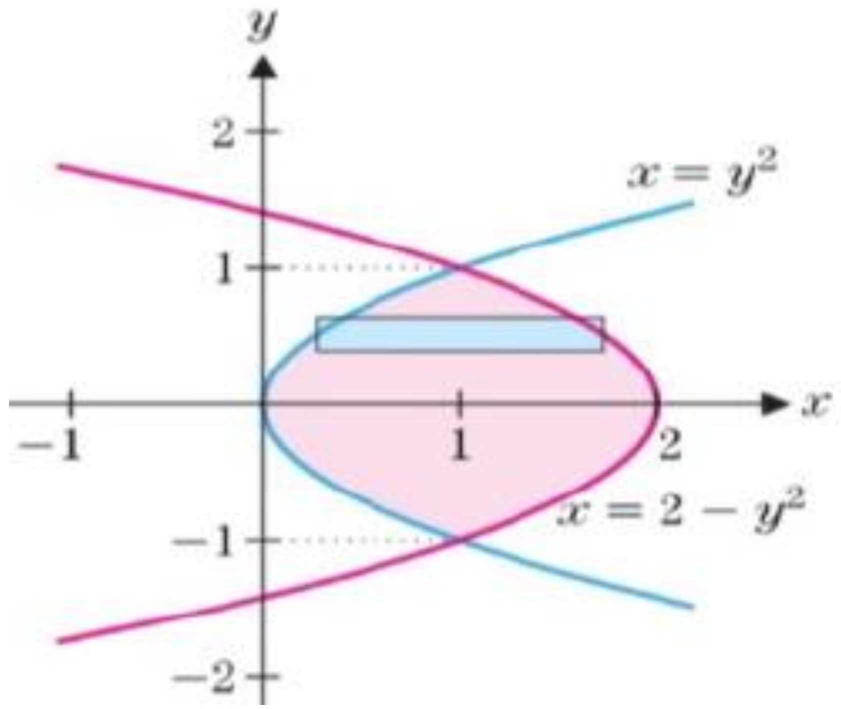
المثال 1.5 مساحة منطقة محسوبة كتكامل بمعلومية y

كرّر المثال 1.4، ولكن التكامل بمعلومية y بدلًا من ذلك.

أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $y = x^2$، $y = 2 - x$ و $y = 0$.







المثال 1.6 مساحة منطقة محدودة بدوال y

أوجد مساحة المنطقة المحدودة بالتمثيلين البيانيين $x = y^2$ و $x = 2 - y^2$.

المثال 1.7 تقدير الطاقة المفقودة بواسطة كرة التنس

على فرض أنّ قياسات الاختبار توفر البيانات التالية أثناء اصطدام كرة التنس بالمضرب. قدّر نسبة الطاقة المفقودة أثناء الاصطدام.

x (cm)	0.0	0.25	0.50	0.75	1
$f_c(x)$ (N)	0	110	220	400	700
$f_e(x)$ (N)	0	100	200	300	700

x	0.0	0.25	0.50	0.75	1
$f_c(x) - f_e(x)$	0	10	20	100	0

في التمارين 4—1، أوجد المساحة بين المنحنيين على الفترة المُعطاة.

1. $y = x^3, y = x^2 - 1, 1 \leq x \leq 3$

2. $y = \cos x, y = x^2 + 2, 0 \leq x \leq 2$

3. $y = e^x, y = x - 1, -2 \leq x \leq 0$

4. $y = e^{-x}, y = x^2, 1 \leq x \leq 4$

في التمارين 12–5، ارسم وأوجد مساحة المنطقة التي تحدّدها تقاطعات المنحنيات.

5. $y = x^2 - 1, y = 7 - x^2$

6. $y = x^2 - 1,\ y = \frac{1}{2}x^2$

7. $y = x^3, y = 3x + 2$

8. $y = \sqrt{x}, y = x^2$

9. $y = 4xe^{-x^2}$, $y = |x|$

10. $y = \dfrac{2}{x^2 + 1}, \; y = |x|$

11. $y = \dfrac{5x}{x^2 + 1}, y = x$

12. $y = \sin x \ (0 \leq x \leq 2\pi)$, $y = \cos x$

في التمارين 18–13، ارسم وقدّر المساحة التي تحدّدها تقاطعات المنحنيات.

13. $y = e^x, y = 1 - x^2$

14. $y = x^4, y = 1 - x$

15. $y = \sin x, y = x^2$

16. $y = \cos x, y = x^4$

17. $y = x^4, y = 2 + x$

18. $y = \ln x, y = x^2 - 2$

في التمارين 26–19، ارسم وأوجد مساحة المنطقة المحدودة بالمنحنيات المُعطاة. اختَرْ متغيّر التكامل بحيث تتم كتابة المساحة كتكامل واحد. تحقق من إجاباتك على التمارين 21–19 باستخدام صيغة هندسية أساسية للمساحة.

19. $y = x, y = 2 - x, y = 0$

20. $y = x, y = 2, y = 6 - x, y = 0$

21. $x = y, x = -y, x = 1$

22. $x = 3y, x = 2 + y^2$

23. $y = 2x\ (x > 0), y = 3 - x^2, x = 0$

24. $x = y^2, x = 4$

25. $y = e^x, y = 4e^{-x}, x = 0$

26. $y = \frac{\ln x}{x}, y = \frac{1 - x}{x^2 + 1}, 1 \leq x \leq 4$

28. باستخدام المفهوم نفسه كما في التمرين 27. تُعطى القيم للقوة $f_c(x)$ أثناء إنكماش كرة الجولف والقوة $f_e(x)$ أثناء تمددها من العلاقة

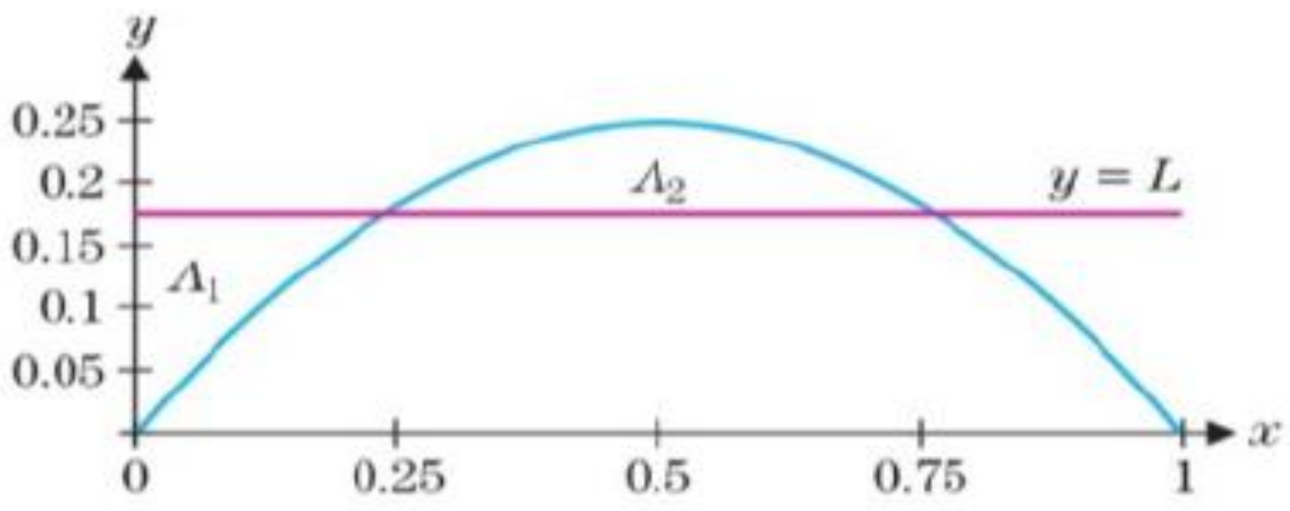
x (cm)	0	1.125	2.250	3.375	4.5
$f_c(x)$ (N)	0	880	2200	4400	7900
$f_e(x)$ (N)	0	550	1540	3000	7900

30. يعمل قوس القدم البشري مثل النابض أثناء المشي والقفز. فيخزّن الطاقة بينما يمتد القدم (أي يصبح القوس مسطحًا) ويعيد الطاقة بينما يرتد القدم. في البيانات، x هي الإزاحة العمودية للقوس و $f_s(x)$ هي القوة على القدم أثناء التمدّد و $f_r(x)$ هي القوة أثناء الارتداد (انظر كتاب ألكسندر *Exploring Biomechanics*)؛

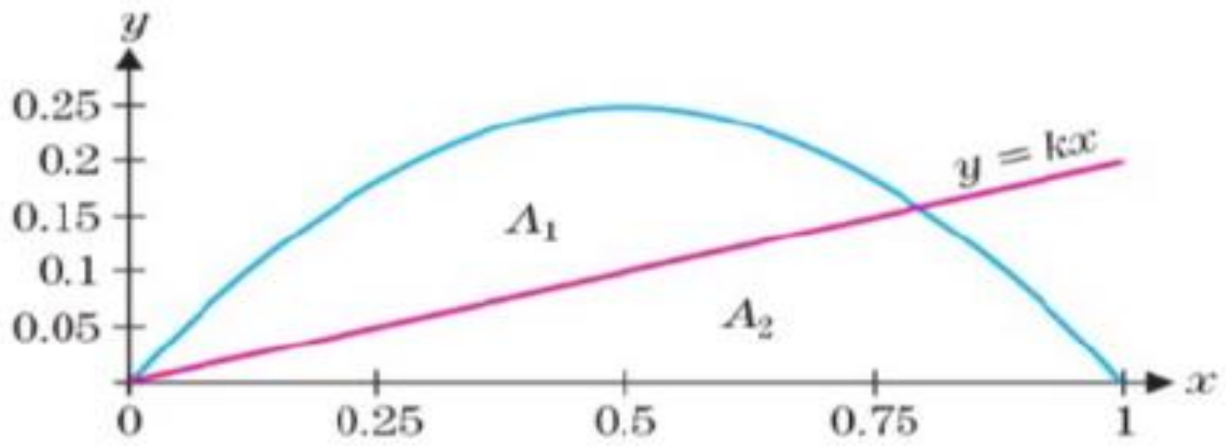
x (mm)	0	2.0	4.0	6.0	8.0
$f_s(x)$ (N)	0	300	1000	1800	3500
$f_r(x)$ (N)	0	150	700	1300	3500

استخدم قاعدة سمبسون لتقدير نسبة الطاقة التي يعيدها القوس.

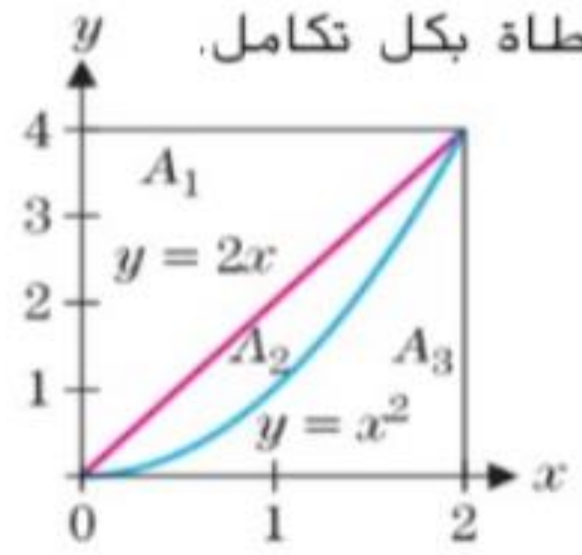
37. لأجل $y = x - x^2$ كما هو مبين. أوجِد قيمة L بحيث تكون $A_1 = A_2$.



38. لأجل $y = x - x^2$ و $y = kx$ كما هو مبين. أوجد k بحيث تكون $A_1 = A_2$.

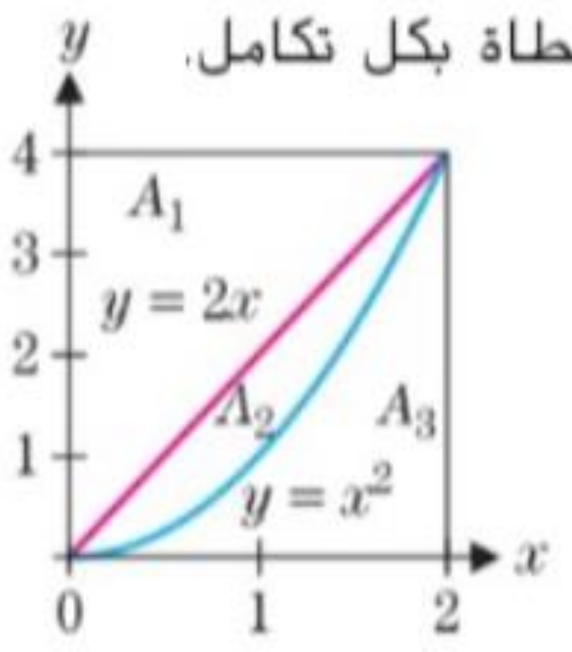


39. بدلالة A_1، A_2 و A_3، حدّد المساحة المُعطاة بكل تكامل.



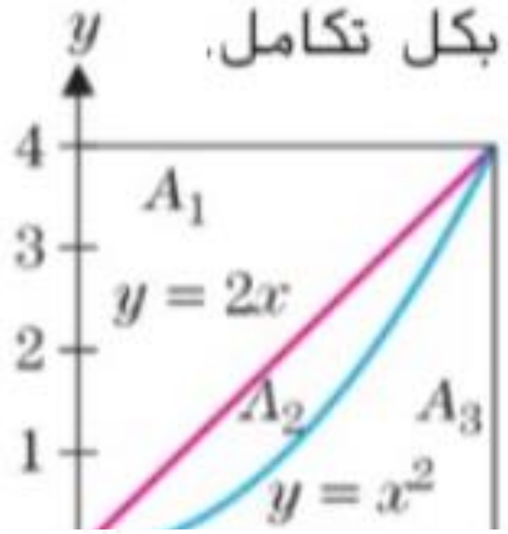
(a) $\int_0^2 (2x - x^2)\, dx$

39. بدلالة A_1، A_2 و A_3، حدّد المساحة المُعطاة بكل تكامل.



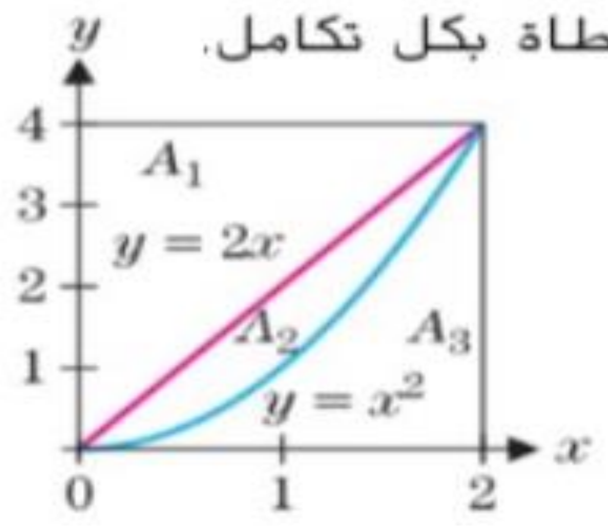
(b) $\int_0^2 (4 - x^2)\,dx$

.39 بدلالة A_1، A_2 و A_3، حدّد المساحة المُعطاة بكل تكامل.



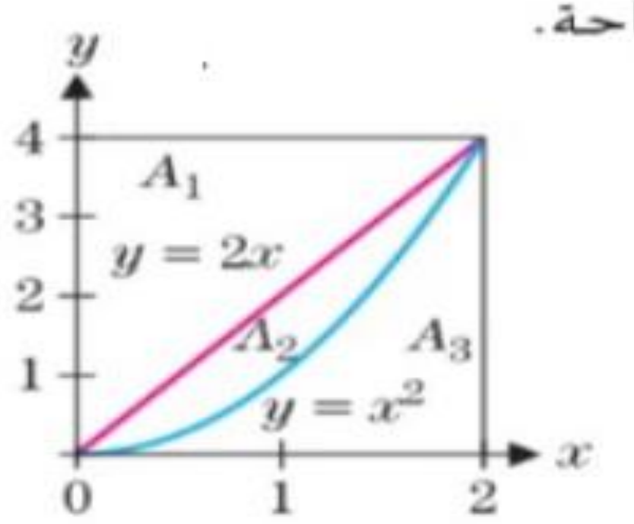
(c) $\int_0^4 (2 - \sqrt{y})\, dy$

39. بدلالة A_1، A_2 و A_3، حدّد المساحة المُعطاة بكل تكامل.



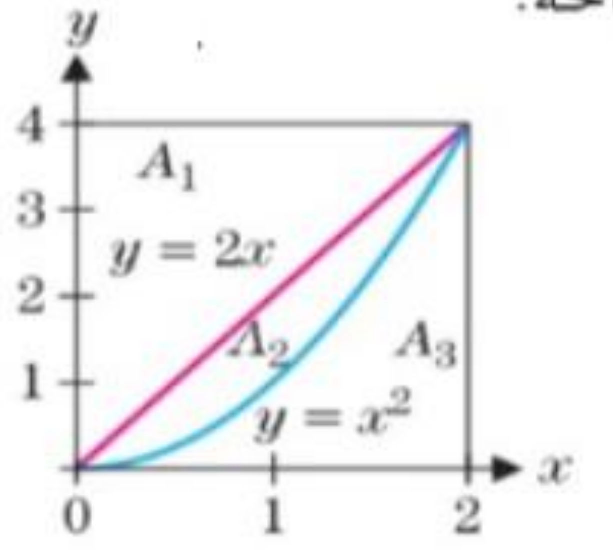
(d) $\int_0^4 (\sqrt{y} - \frac{y}{2})\, dy$

40. أعط تكاملًا مساويًا لكل مساحة.



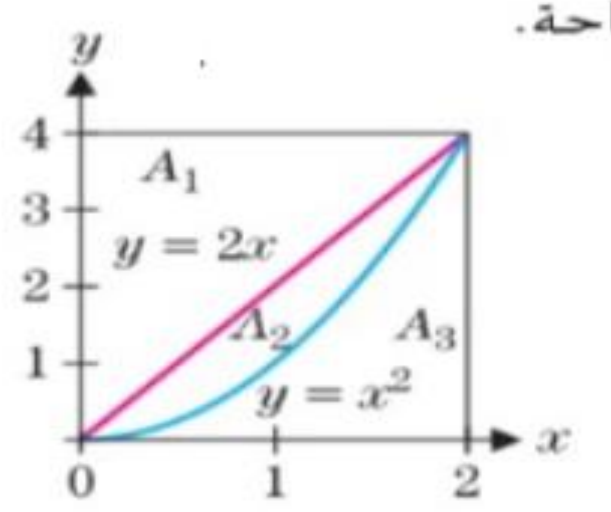
(a) $A_2 + A_3$

40. أعط تكاملًا مساويًا لكل مساحة.



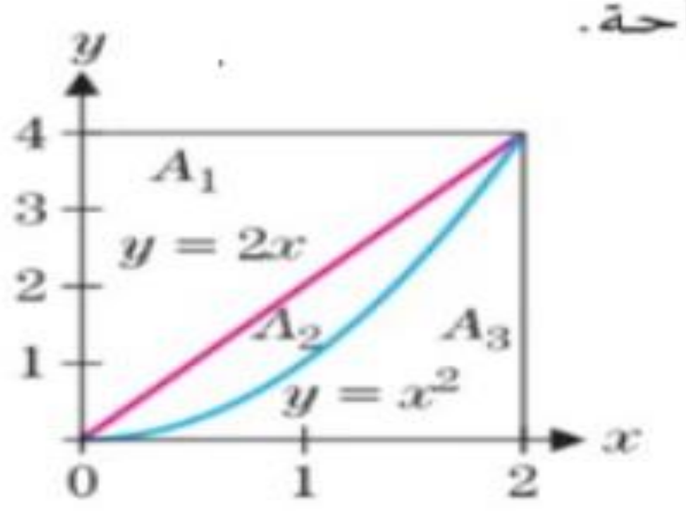
(b) $A_1 + A_2$

40. أعط تكاملًا مساويًا لكل مساحة.



(c) A_1

40. أعط تكاملًا مساويًا لكل مساحة.



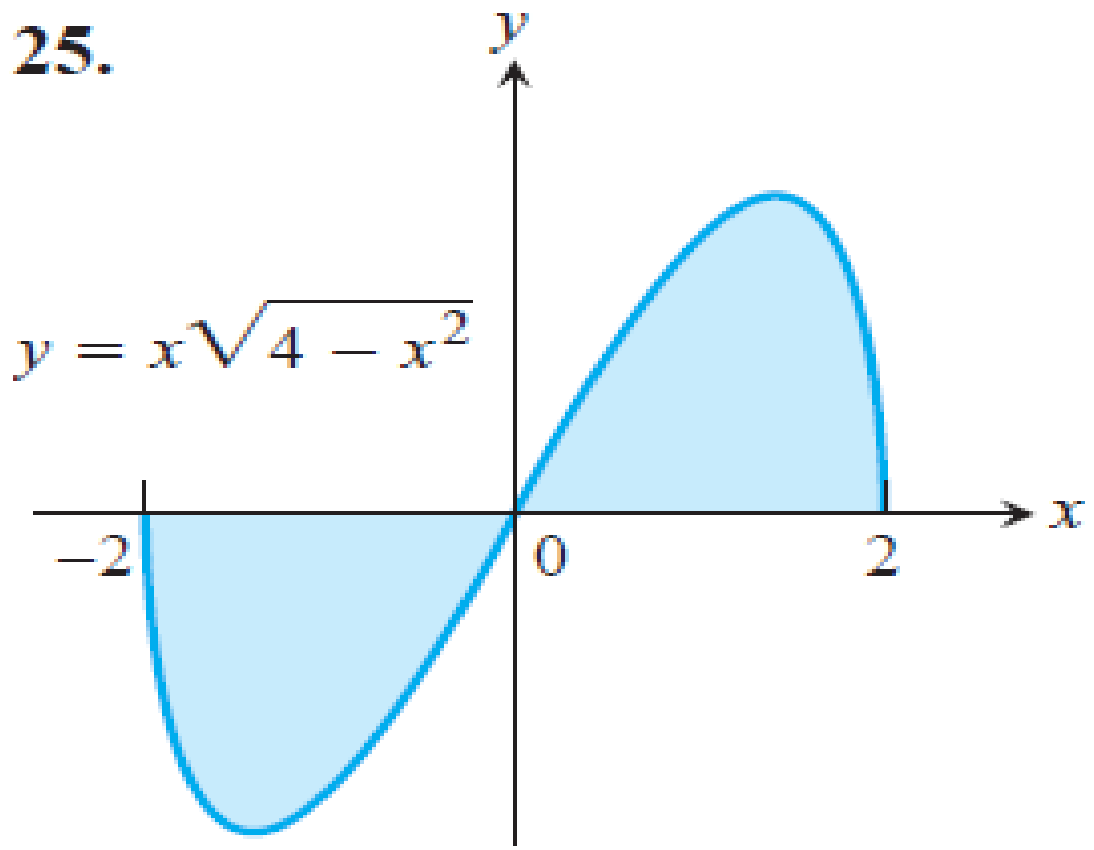
(d) A_3

Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

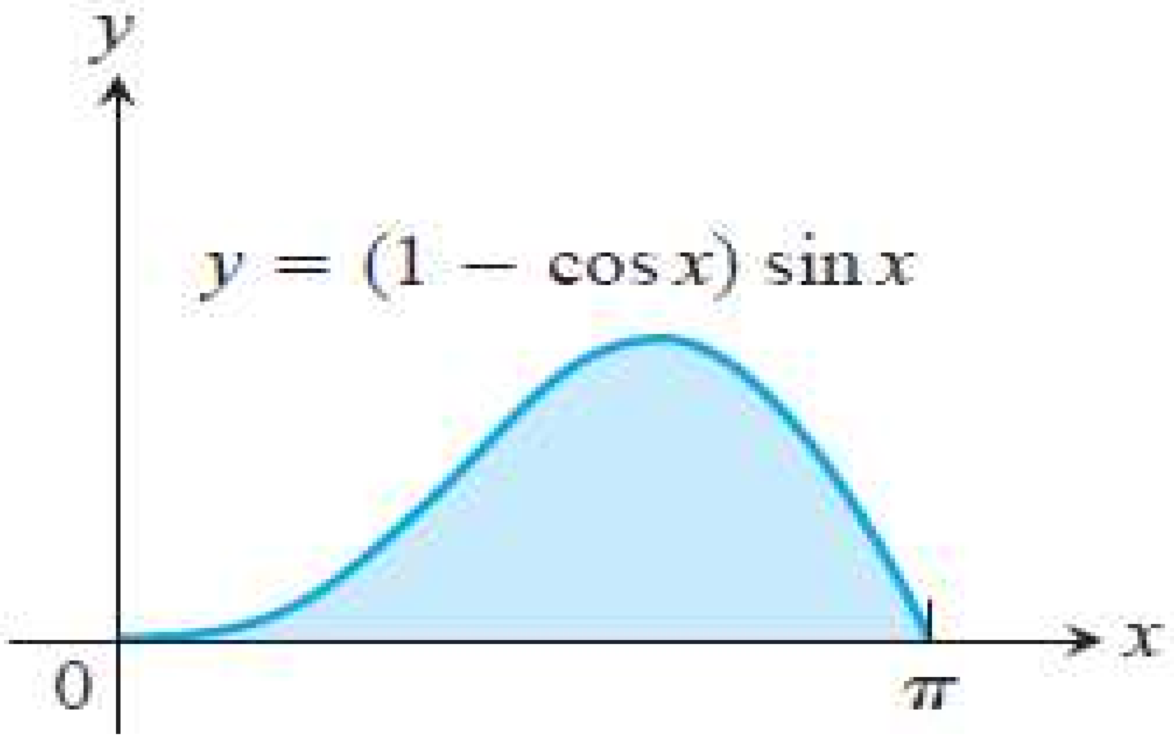
Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

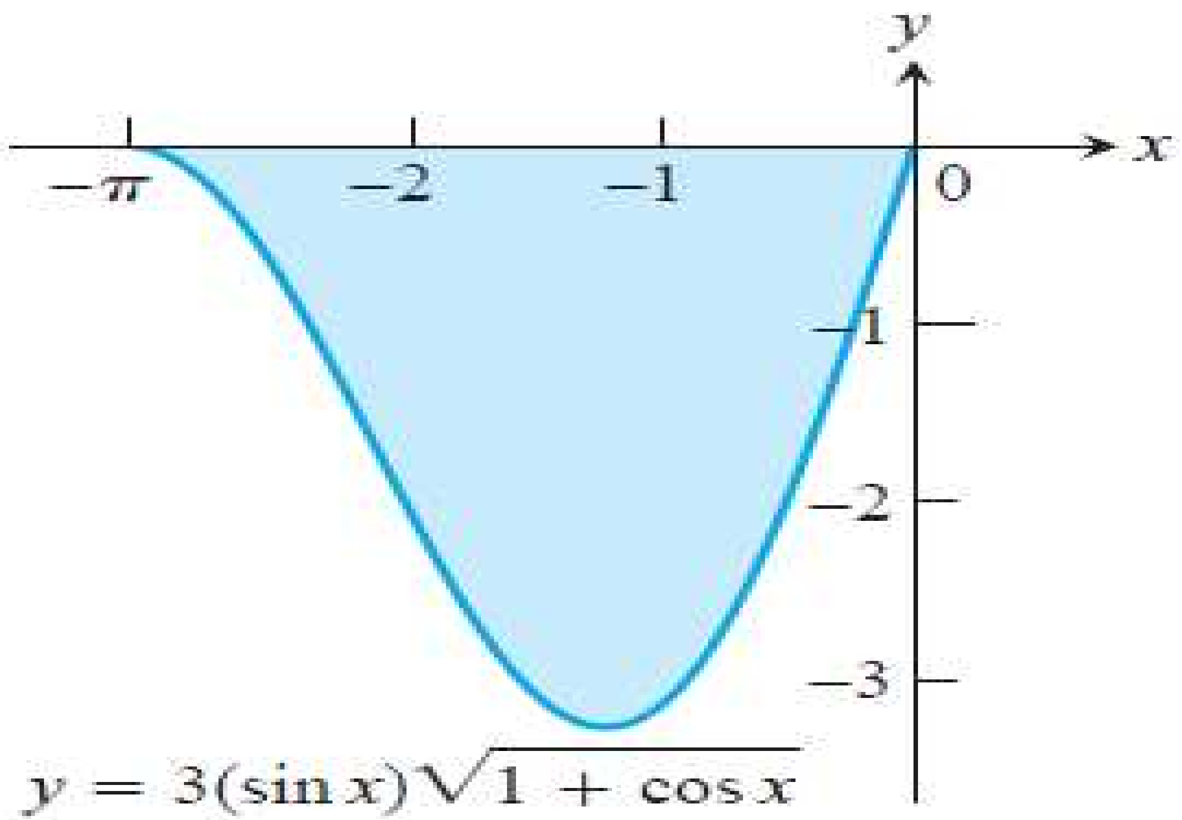
25.



26.

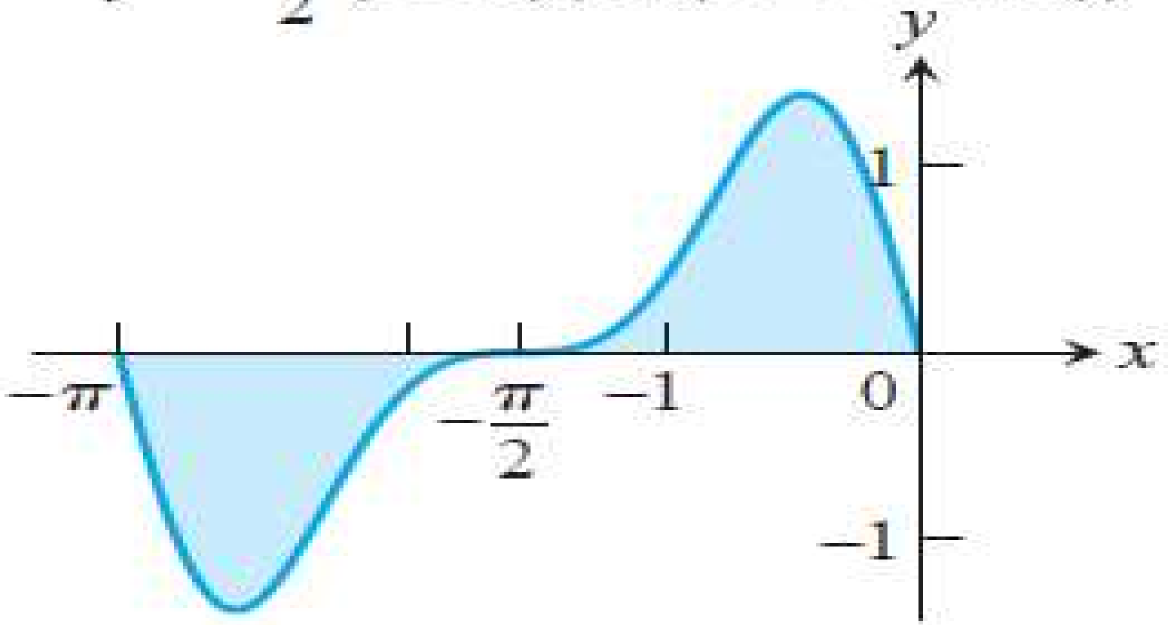


27.



28.

$$y = \frac{\pi}{2}(\cos x)(\sin(\pi + \pi \sin x))$$



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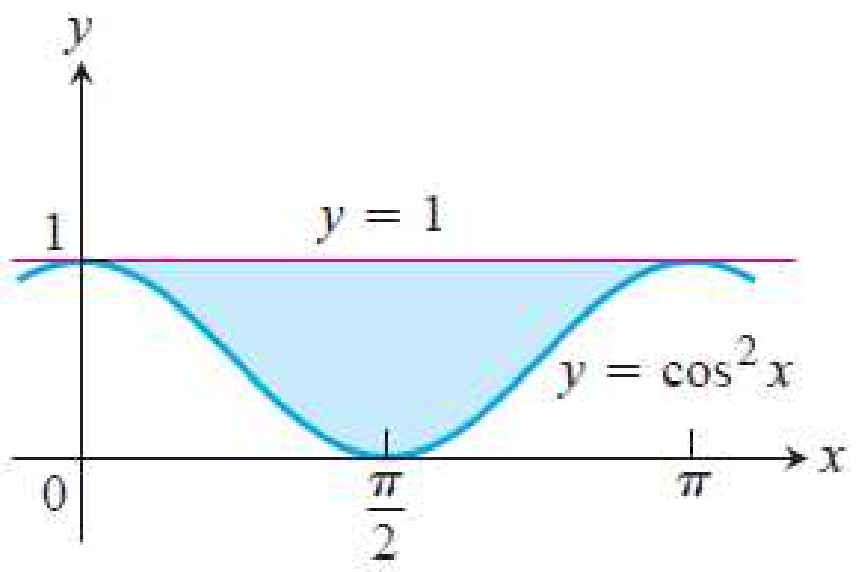

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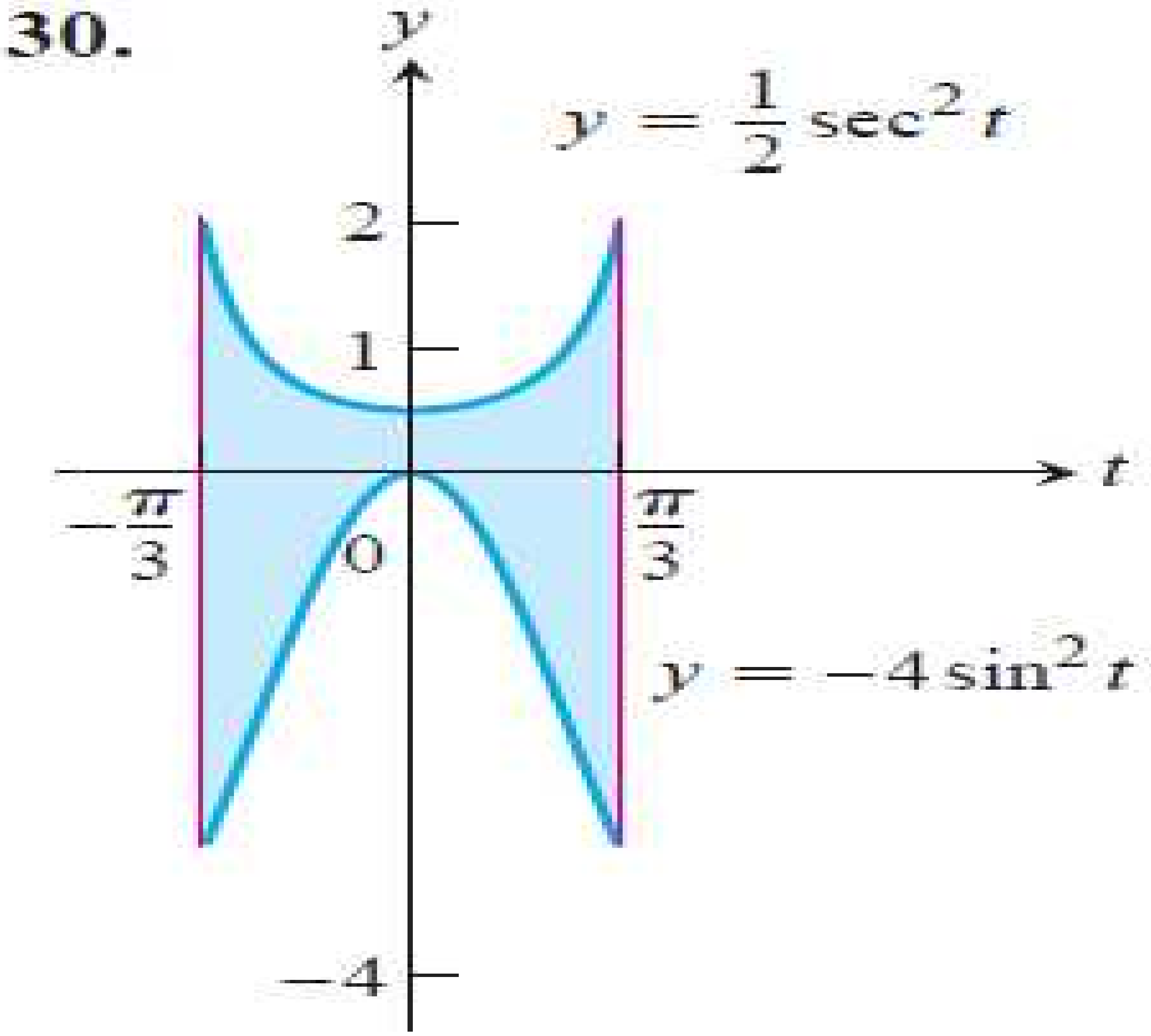

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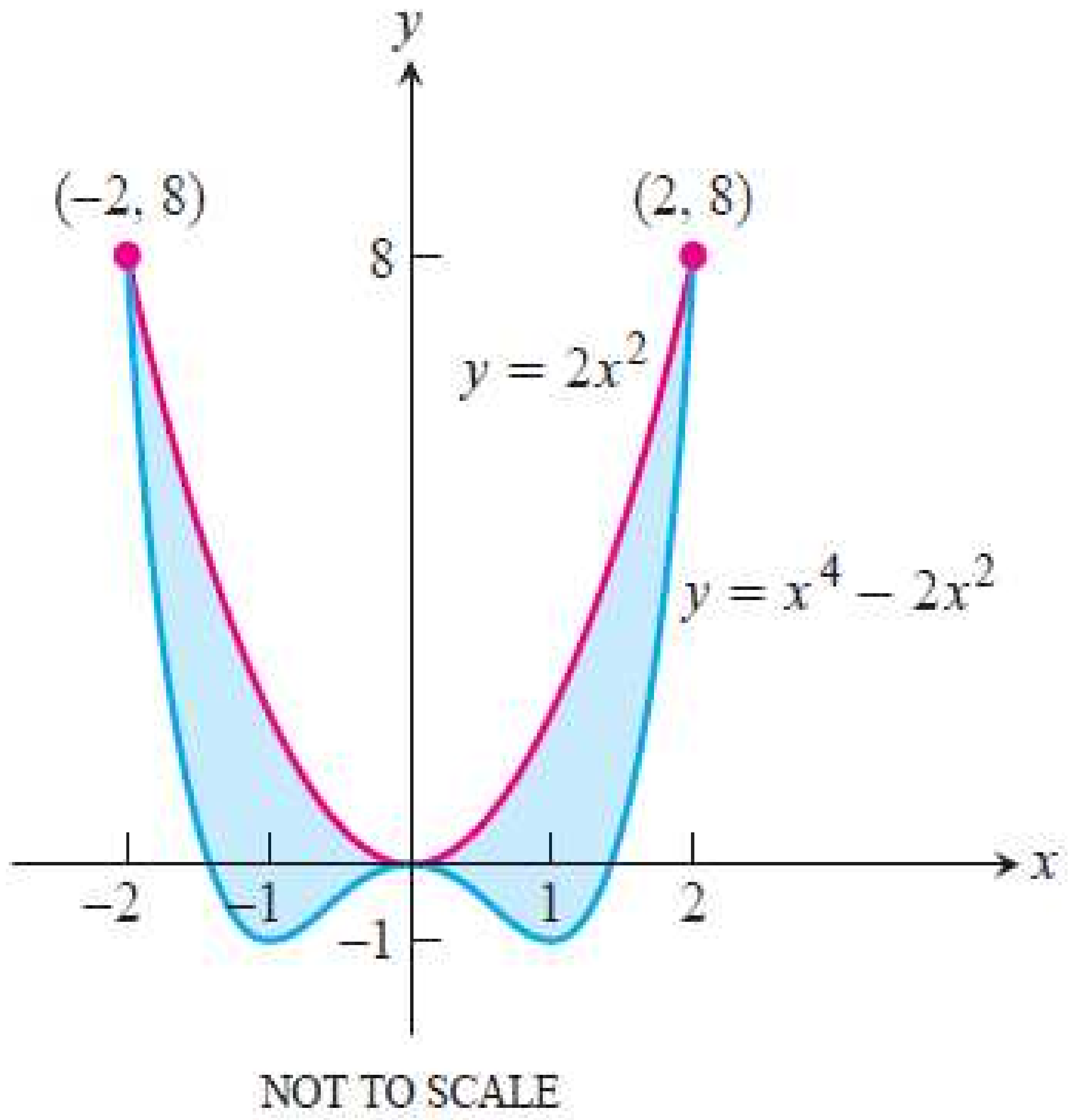
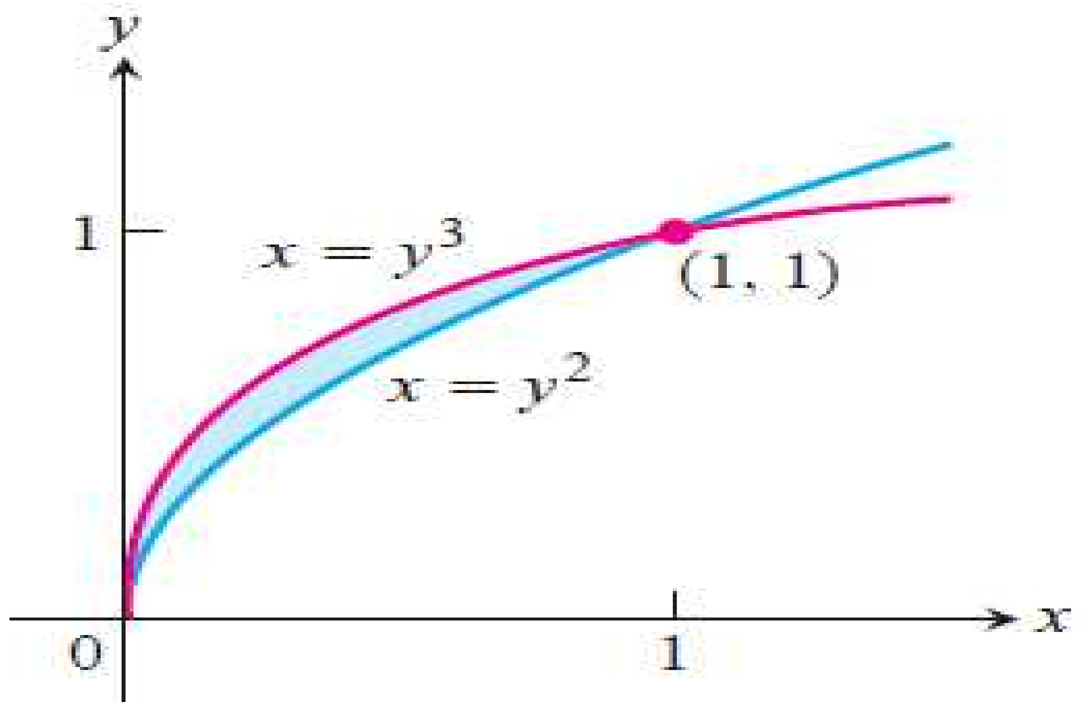
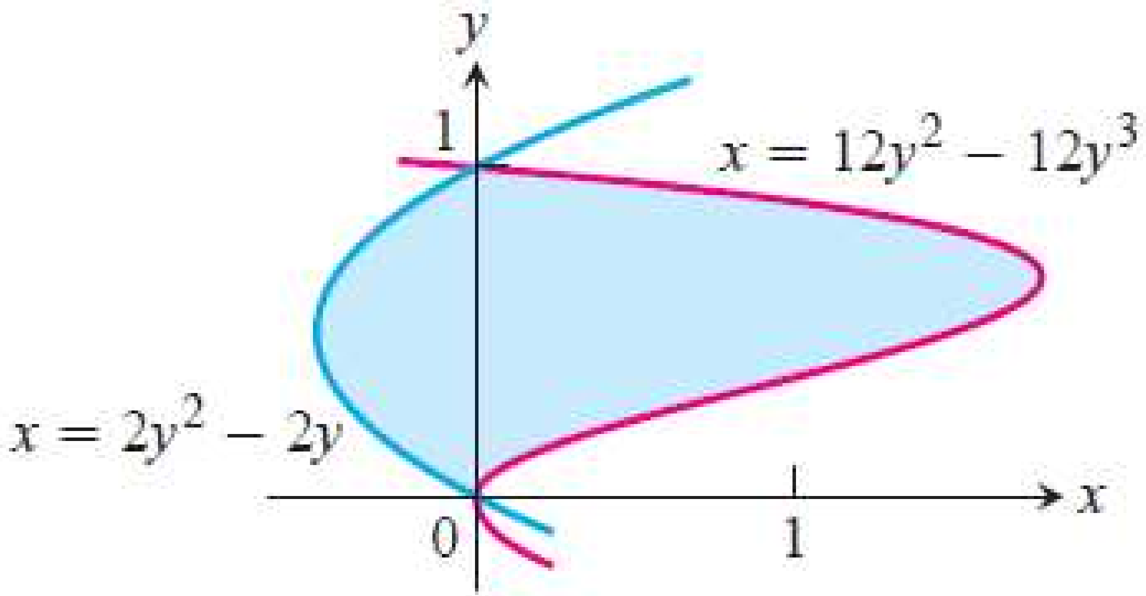


32.



33.



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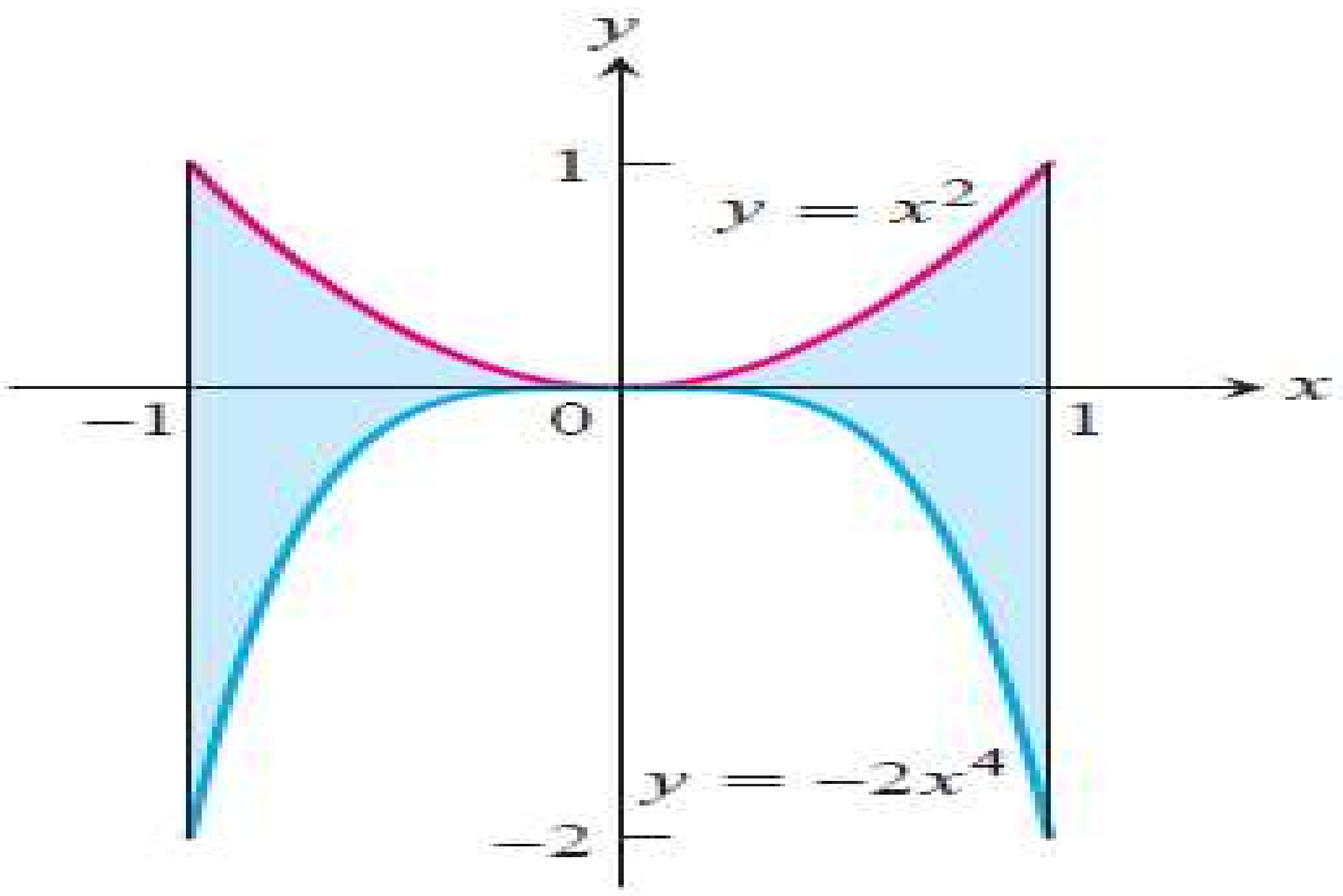

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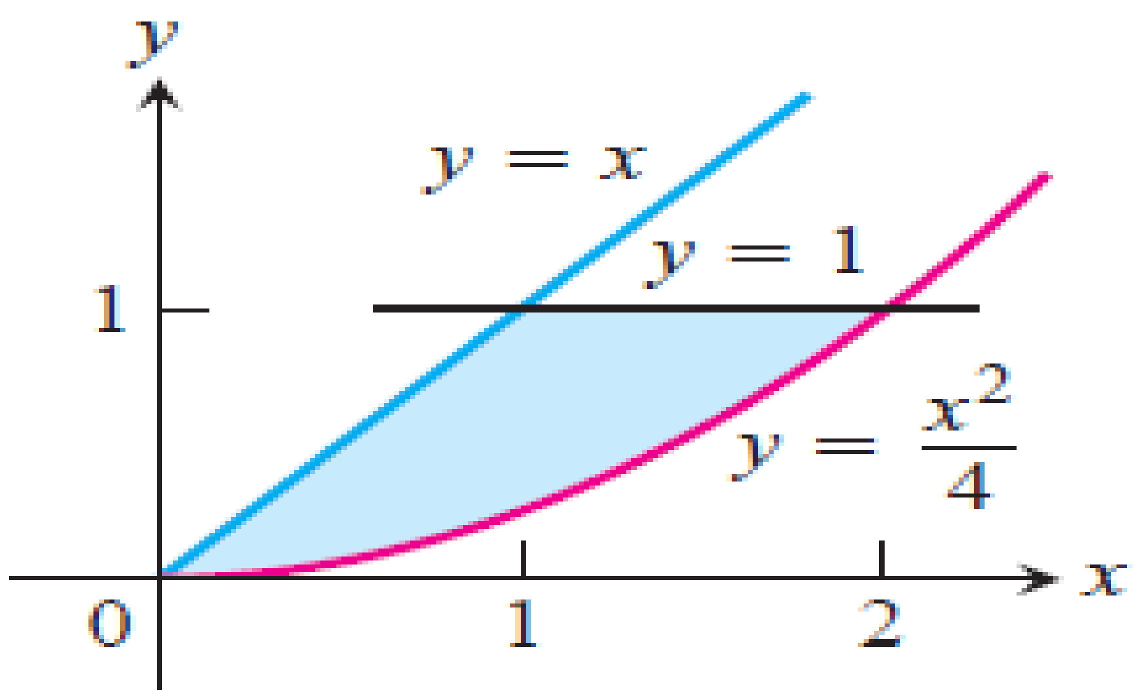
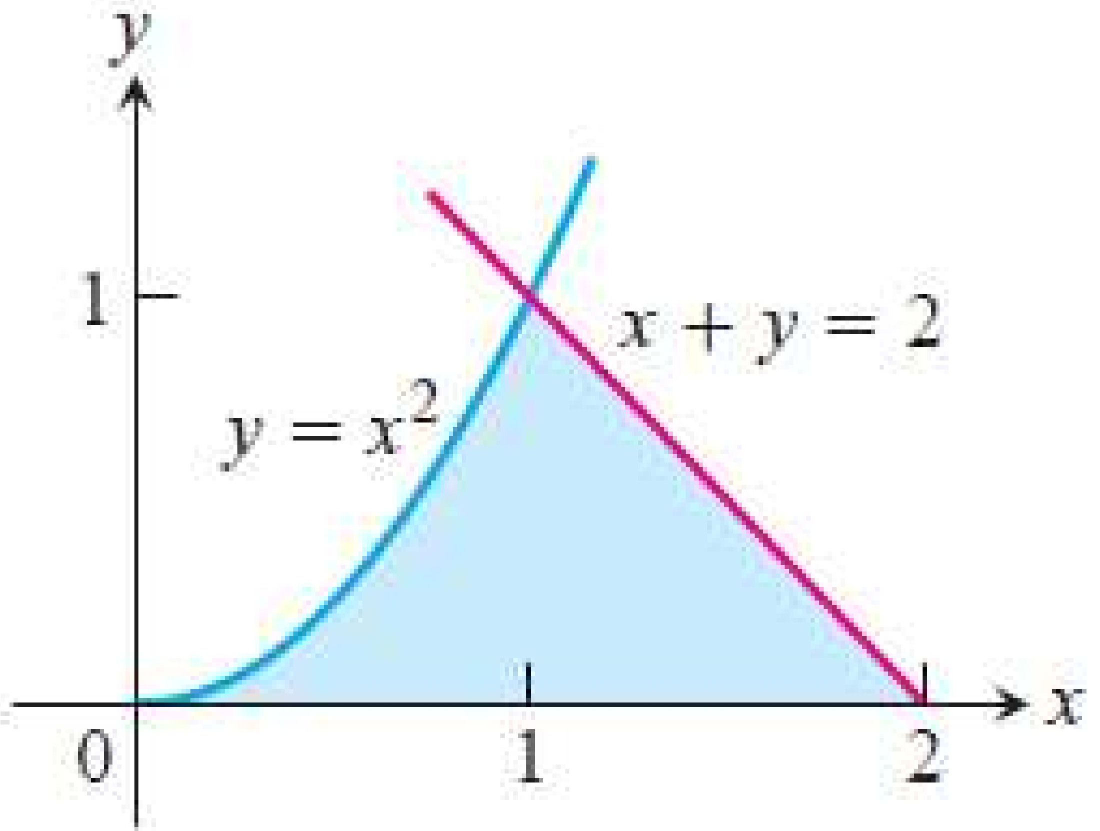


36.



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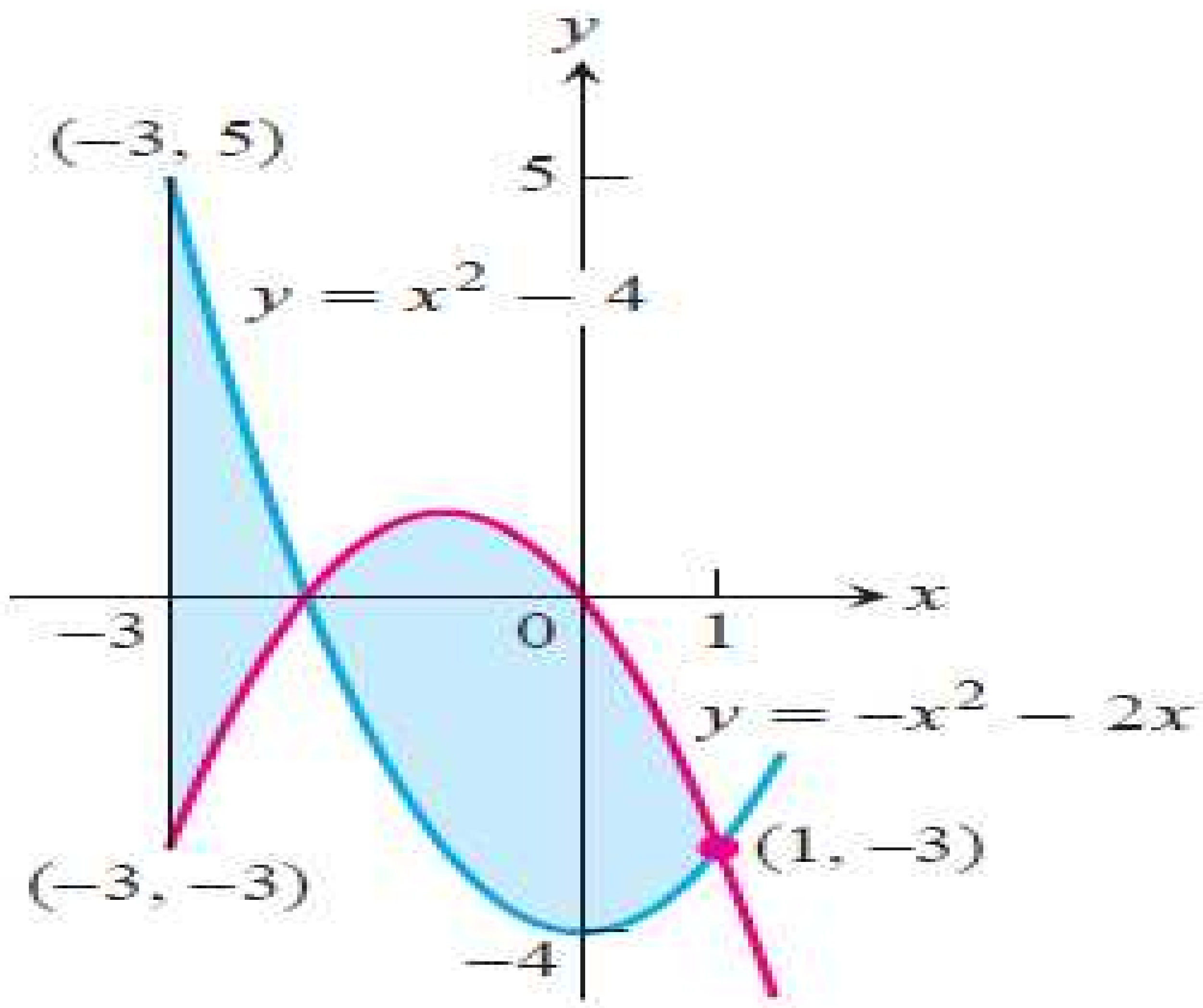

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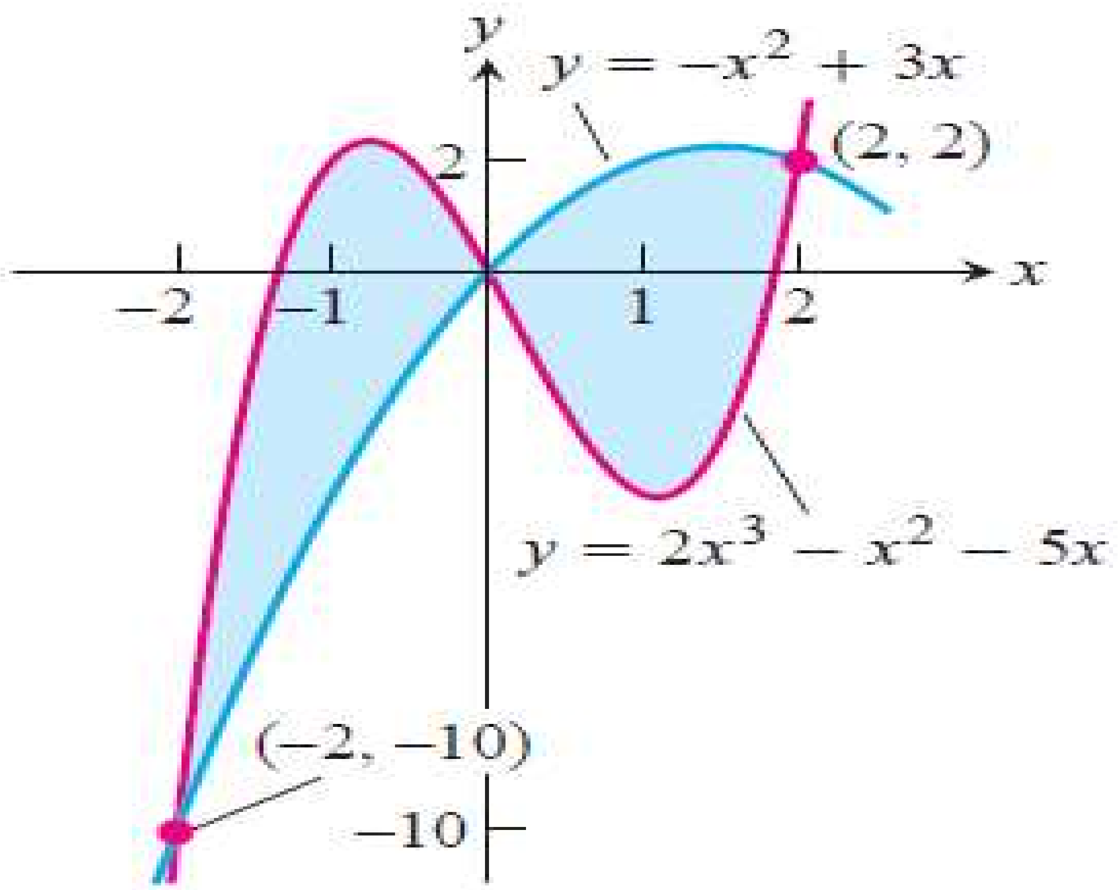

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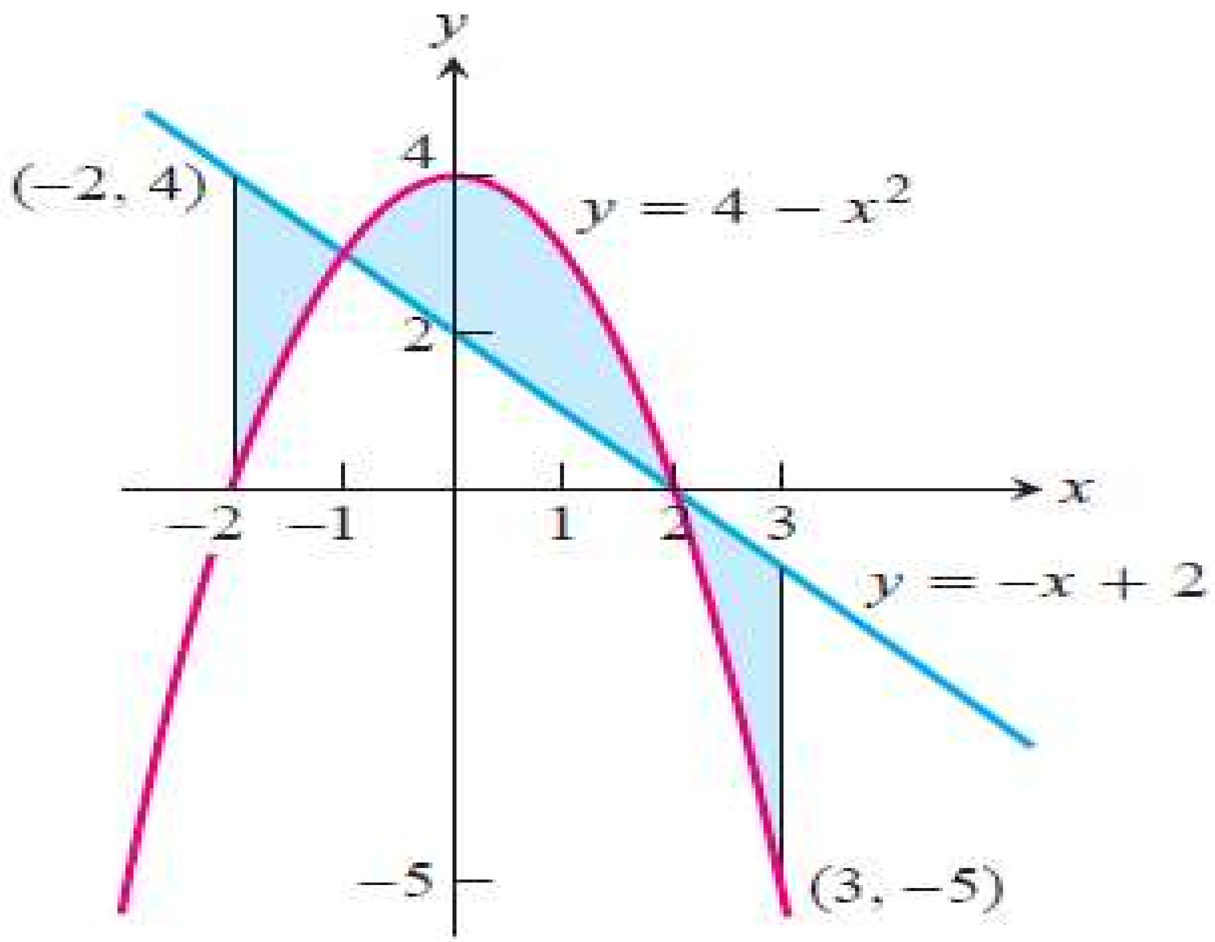

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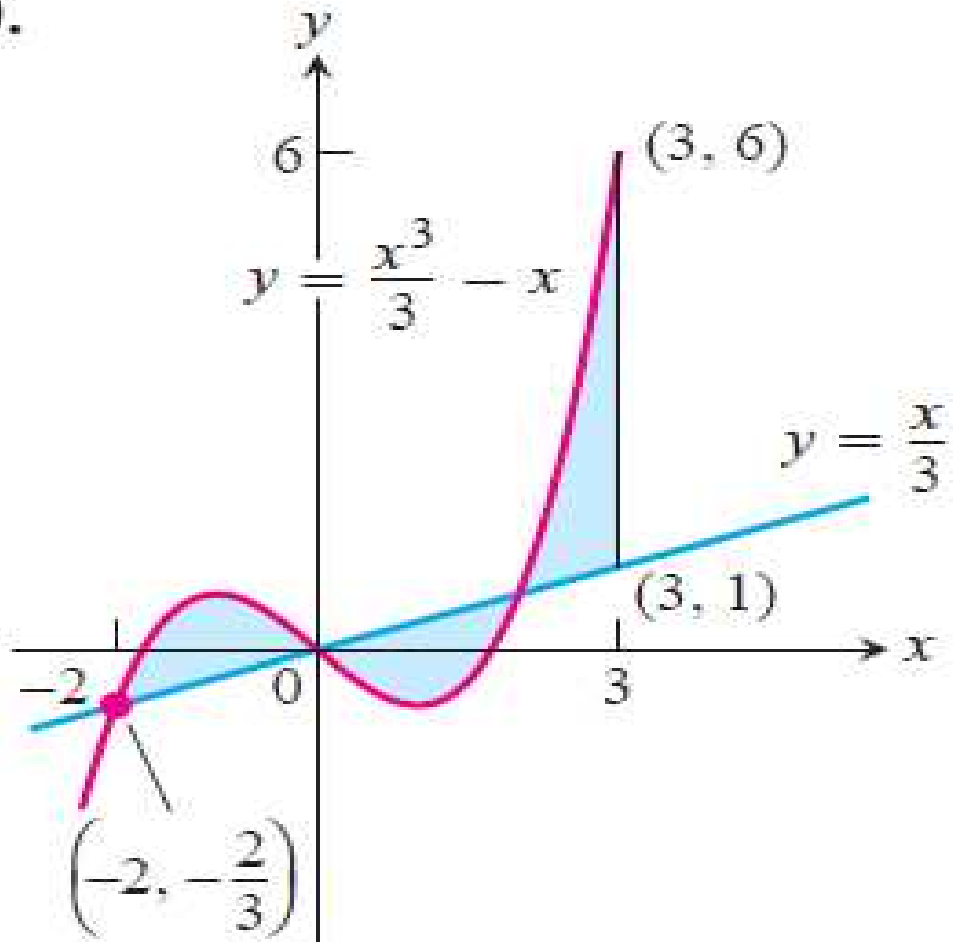


Find the areas of the regions enclosed by the lines and curves in Exercises 41–50.

41. $y = x^2 - 2$ and $y = 2$

42. $y = 2x - x^2$ and $y = -3$

43. $y = x^4$ and $y = 8x$

44. $y = x^2 - 2x$ and $y = x$

45. $y = x^2$ and $y = -x^2 + 4x$

46. $y = 7 - 2x^2$ and $y = x^2 + 4$

47. $y = x^4 - 4x^2 + 4$ and $y = x^2$

48. $y = x\sqrt{a^2 - x^2}$, $a > 0$, and $y = 0$

49. $y = \sqrt{|x|}$ and $5y = x + 6$ (How many intersection points are there?)

50. $y = |x^2 - 4|$ and $y = (x^2/2) + 4$

51. $x = 2y^2$, $x = 0$, and $y = 3$

52. $x = y^2$ and $x = y + 2$

53. $y^2 - 4x = 4$ and $4x - y = 16$

54. $x - y^2 = 0$ and $x + 2y^2 = 3$

55. $x + y^2 = 0$ and $x + 3y^2 = 2$

56. $x - y^{2/3} = 0$ and $x + y^4 = 2$

57. $x = y^2 - 1$ and $x = |y|\sqrt{1 - y^2}$

58. $x = y^3 - y^2$ and $x = 2y$

59. $4x^2 + y = 4$ and $x^4 - y = 1$

60. $x^3 - y = 0$ and $3x^2 - y = 4$

61. $x + 4y^2 = 4$ and $x + y^4 = 1$, for $x \geq 0$

62. $x + y^2 = 3$ and $4x + y^2 = 0$

63. $y = 2\sin x$ and $y = \sin 2x, \quad 0 \le x \le \pi$

64. $y = 8\cos x$ and $y = \sec^2 x, \quad -\pi/3 \le x \le \pi/3$

65. $y = \cos(\pi x/2)$ and $y = 1 - x^2$

66. $y = \sin(\pi x/2)$ and $y = x$

67. $y = \sec^2 x$, $y = \tan^2 x$, $x = -\pi/4$,   and $x = \pi/4$

68. $x = \tan^2 y$ and $x = -\tan^2 y$, $-\pi/4 \le y \le \pi/4$

69. $x = 3\sin y\sqrt{\cos y}$ and $x = 0$, $0 \le y \le \pi/2$

70. $y = \sec^2(\pi x/3)$ and $y = x^{1/3}, \quad -1 \le x \le 1$

71. Find the area of the propeller-shaped region enclosed by the curve $x - y^3 = 0$ and the line $x - y = 0$.

72. Find the area of the propeller-shaped region enclosed by the curves $x - y^{1/3} = 0$ and $x - y^{1/5} = 0$.

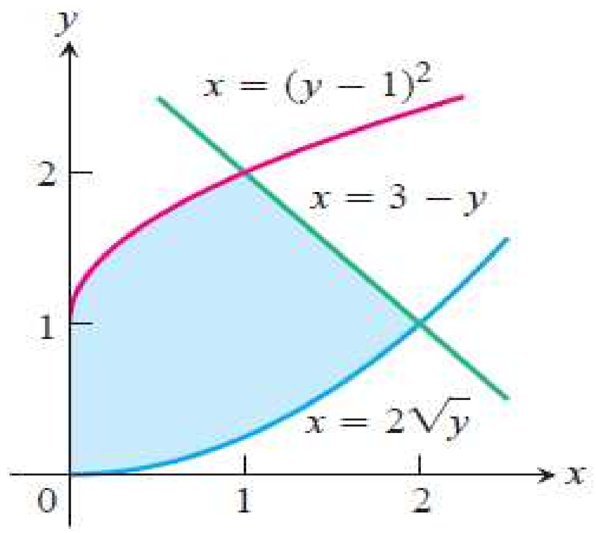
73. Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

74. Find the area of the "triangular" region in the first quadrant bounded on the left by the y-axis and on the right by the curves $y = \sin x$ and $y = \cos x$.

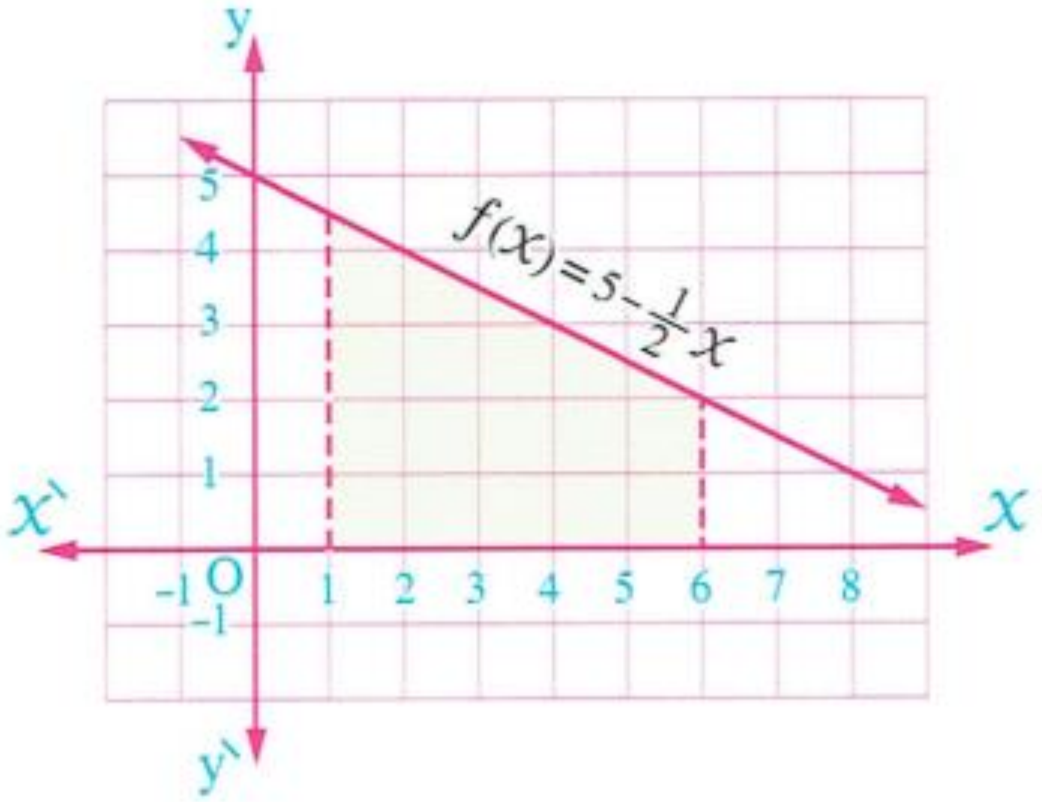
76. Find the area of the region between the curve $y = 3 - x^2$ and the line $y = -1$ by integrating with respect to **a.** x, **b.** y.

77. Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the line $y = x/4$, above left by the curve $y = 1 + \sqrt{x}$, and above right by the curve $y = 2/\sqrt{x}$.

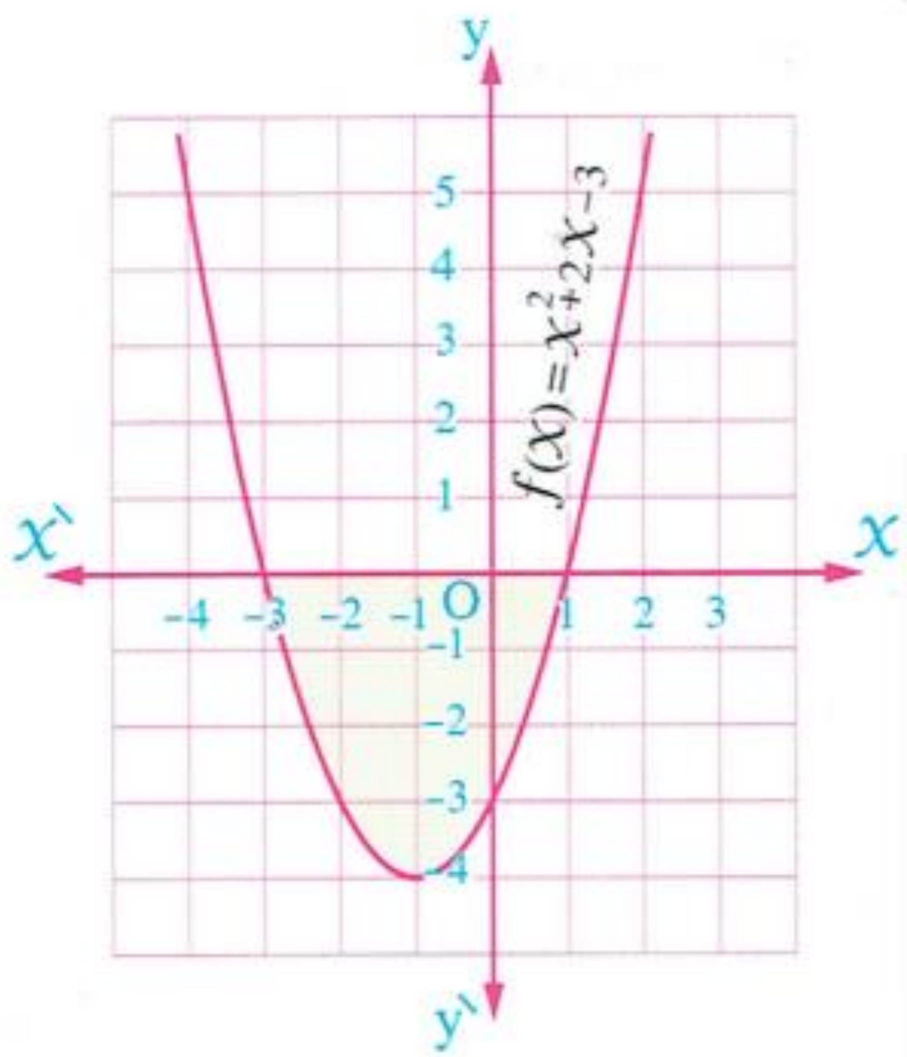
78. Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y - 1)^2$, and above right by the line $x = 3 - y$.



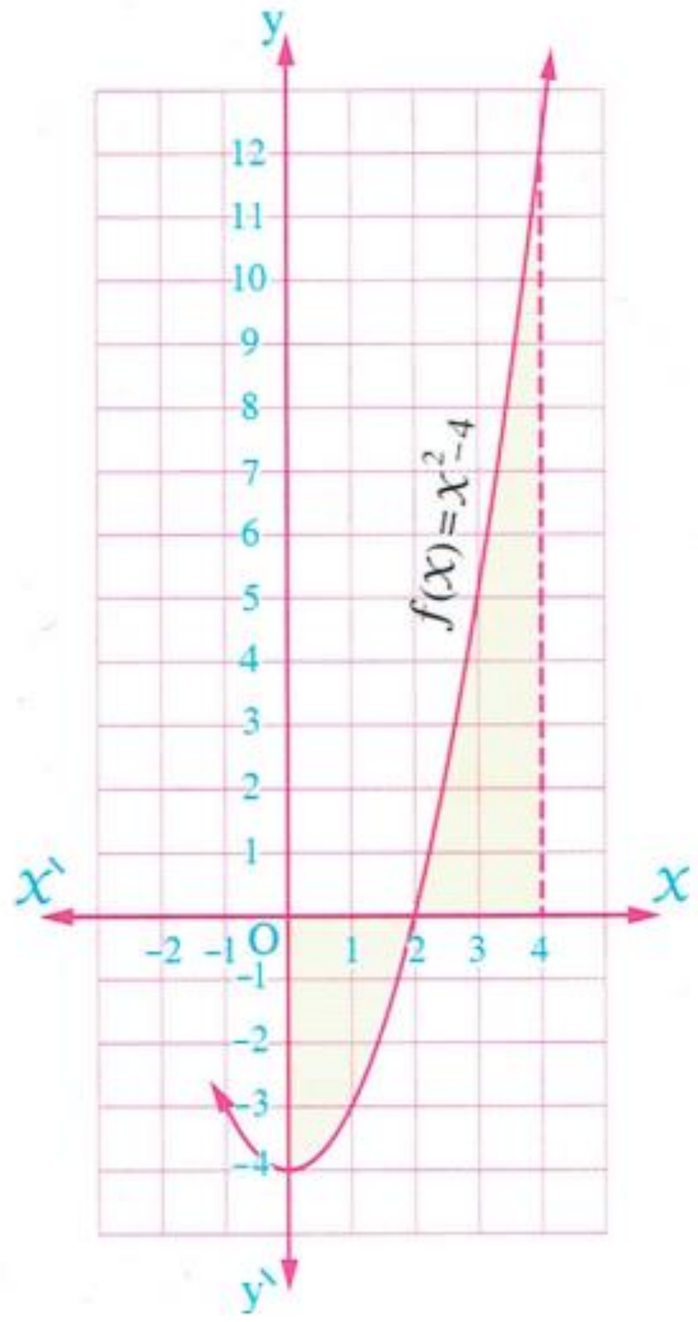
Find the shaded area in each of the following :



Find the shaded area in each of the following :



Find the shaded area in each of the following :



Find the area of the region bounded by the curve of the function $f : f(x) = x^2 + 4$ and the x-axis and the two straight lines : $x = -1$, $x = 2$

Find the area of the region bounded by the curve of the function $f : f(X) = e^{X}$ and the two straight lines $X = 0$, $X = \ln 3$ and X- axis

Find the area of the region bounded by the curve of the function $y = 1 - x^3$ and the straight lines $y = 0$, $x = -1$, $x = 2$

Find the area of the region bounded by the curve of the function $y = x^3 - 4x$ and the x-axis

Find the area of the region bounded by the curve of the function $f : f(\mathcal{X}) = \sqrt[3]{\mathcal{X} + 2}$ and the straight line $\mathcal{X} = 6$, and above $\mathcal{X}$-axis

Find the area of the region bounded by the curve of the function $y = \frac{1}{2}(9 - x^2)$ and the straight lines $y = 0$, $x = -5$, $x = 4$ and lies below the x-axis

An engineer design a hotel reception in form of the curve whose equation $y = 4x - \frac{1}{2}x^2$ where x is in metres. If it is covered by glass , the cost of one square metre is L.E. 1200. Find the cost of the glass.

By using the area under the curve of the function $y = \sqrt{16 - x^2}$, find $\int_0^4 \sqrt{16 - x^2}\, dx$

Find the area of the region bounded by : $y_1 = 6 - x^2$, $y_2 = \frac{1}{2}x^2$ and the two straight lines $x = -1$, $x = 1$

Find the area of the region bounded by the curve $y_1 = 4 - x^2$ and the straight line $y_2 = 3x$

Find the area of the region bounded by the two curves : $y_1 = 12 - x^2$, $y_2 = 2x^2$

Find the area of the region bounded by the curves : $y_1 = \sqrt{x}$ and the straight line $y_2 = x - 2$ and the y-axis.

Find the area of the region bounded by the two curves : $y_1 = \sqrt{2x}$, $y_2 = \frac{1}{2}x^2$

Find the area of the region bounded by the curve of the function f and the curve of the function g where $f(x) = x^3 - 3x^2 + 5$, $g(x) = x + 2$

Find the area bounded by the curves : $y = \frac{2}{x^2}$, $x > 0$, $y = 2x$, $y = \frac{1}{4}x$

Find the area of the region bounded by the curves : $y = \sqrt{6+x}$, $y = \sqrt{12-x}$, $y = 1$

Find the area of the region under the curve of the function y and above the given interval in each of the following :

$y = |x - 3|$ above the interval $[2 , 5]$

Using the area under the curve

$$\int_0^2 \sqrt{4-x^2}\, dx$$

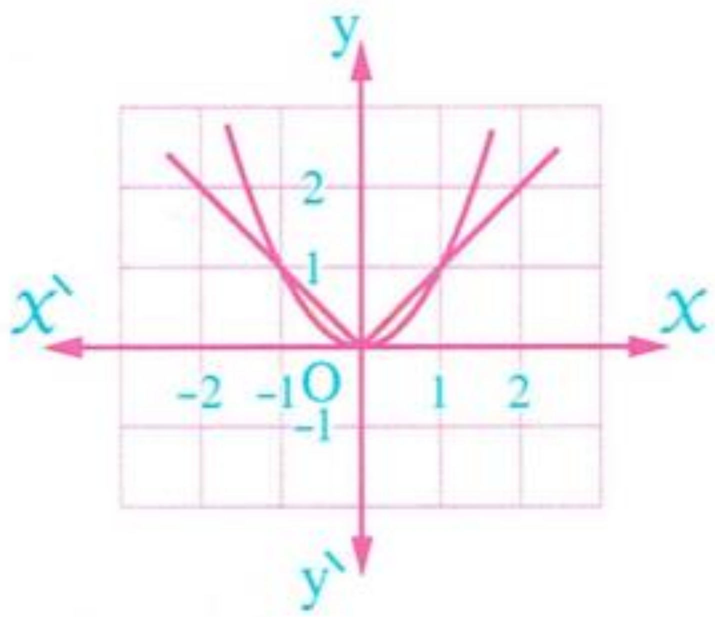
Using the area under the curve

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx$$

(1) In the opposite figure :

The area of the region bounded by the two curves

$y = x^2$, $y = |x|$ equals ……………



(a) $2\int_{-1}^{0} (x^2 - x)\, dx$

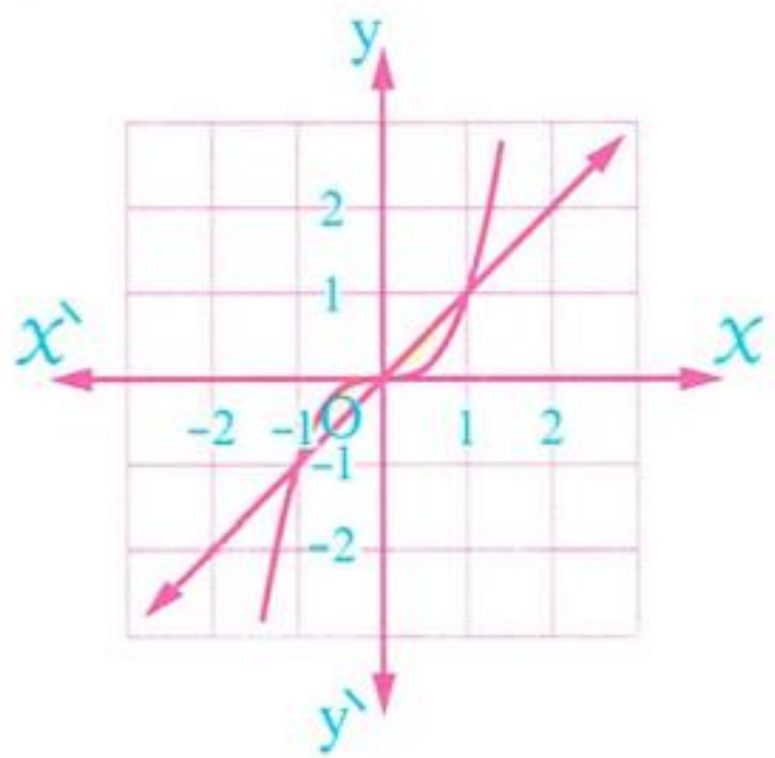
(b) $\int_{0}^{1} (x - x^2)\, dx$

(c) $2\int_{0}^{1} (x - x^2)\, dx$

(d) $\int_{-1}^{1} (x - x^2)\, dx$

(2) In the opposite figure :

The area of the region bounded by the curve $y = x^3$ and the straight line $y = x$ equals ....



(a) $\int_{-1}^{1} (x^3 - x)\, dx$

(b) $2 \int_{0}^{1} (x^3 - x)\, dx$

(c) $\int_{0}^{1} (x - x^3)\, dx$

(d) $2 \int_{0}^{1} (x - x^3)\, dx$

(3) The area of the region bounded by the straight lines

$y = x$, $x = 2$, $y = 0$ equals ……………… square units.

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

(4) The area of the region bounded by the curve $y = x^3$ and the straight lines $y = 0$, $x = 2$ equals ……………… square unit.

(a) 8 (b) 4 (c) 2 (d) 1

(5) The area of the region bounded by the straight lines $y = 2x - 3$, $y = x + 1$, $x = 2$ equals ……………

(a) 2 (b) 3 (c) $\frac{9}{2}$ (d) 6

(6) The area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis equals .... square units.

(a) 2 (b) 4 (c) 2π (d) 4π

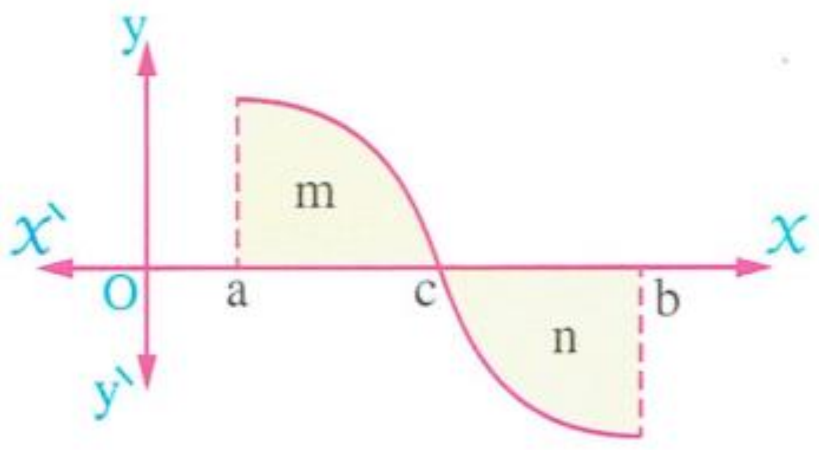
(7) In the opposite figure :

It is a part of a curve $f(X)$ in the interval $[a , b]$

, then if the area m equals 5 square unit

and area n equals 3 square unit

, then $\int_a^b f(X)\, dX =$ ……………

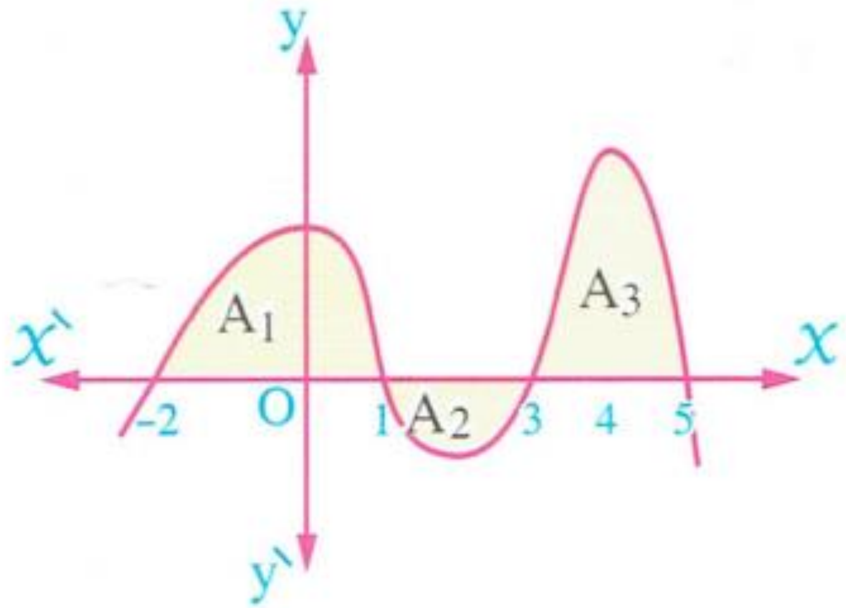


(a) – 5 **(b)** 4 **(c)** 2 **(d)** 8

(8) In the opposite figure :

If $A_1 = 5$ square units , $A_2 = 2$ square units , $A_3 = 8$ square units

, then $\int_{-2}^{5} f(x)\, dx + \int_{-2}^{5} |f(x)|\, dx =$ ……………



(a) 15 (b) 20

(c) 22 (d) 26

(9) In the opposite figure :

If the area of the shaded region = $\frac{8}{3}$ square units ,

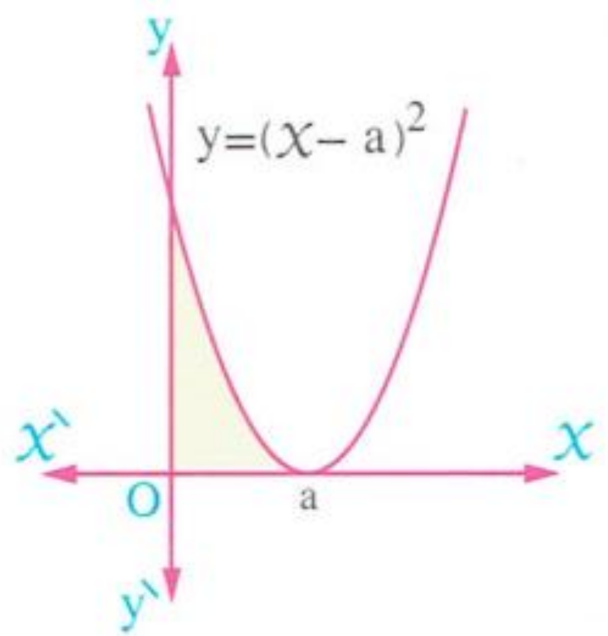
then a = ...

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 2



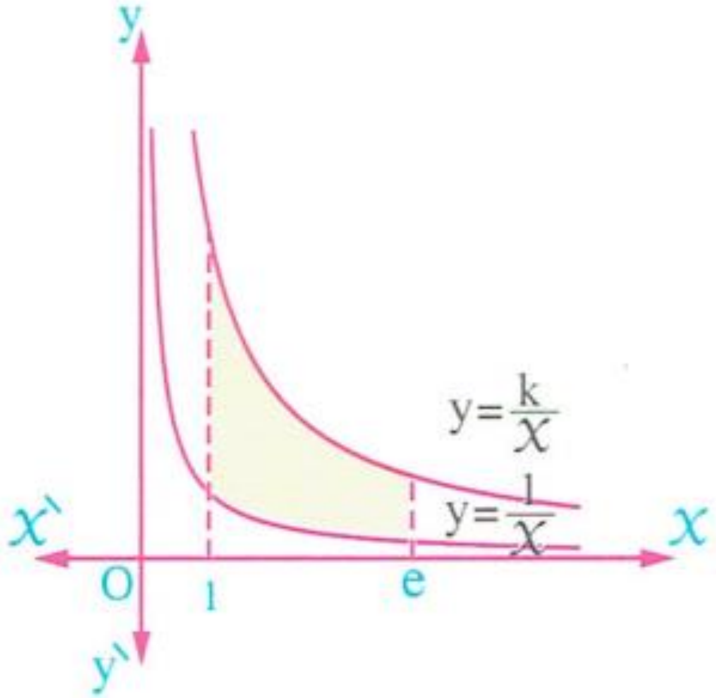
(10) In the opposite figure :

If the area of the shaded region = 2 square units ,

then k = ...

(a) 2 (b) 3

(c) 4 (d) 5



(11) The area of the region bounded by the curve $\mathcal{X}\,y = 4$, $\mathcal{X}$-axis and the two straight lines $\mathcal{X} = 1$, $\mathcal{X} = 3$ equals ....

(a) $2 \ln 3$ (b) $4 \ln 3$ (c) $3 \ln 3$ (d) $3 \ln 4$

7 Find in square units the area of the region bounded by the curve of :

$f(X) = (X - 2)^3$ and the X-axis in the interval $[2 , 4]$ « 4 square units »

8 An architect has designed an arc -like entryway of a hotel whose equation $y = -\frac{1}{2}(\mathcal{X} - 1)(\mathcal{X} - 7)$ where $\mathcal{X}$ in metres. How much does the glass cost if this entry way is covered by the glass which costs L.E. 1500 per square metre ? « L.E. 27000 »

9 If the cost of a squared metre of granite to cover the floor of a hotel corridors is LE 400 and five corridors have been already covered with granite and the area of each is bounded by the curve of the function f and the two straight lines $X = 0$ and $y = 0$ where $f(X) = 12 - \frac{1}{3} X^2$ Find the cost covering the five corridors. « L.E. 96000 »

10 An advertising company produces a poster to market an item. If the poster is shaped as an area bounded by the curve of the two functions f and g where $f(X) = 2X^2$ and $g(X) = X^4 - 2X^2$ and X is approximated in decimetre , calculate the area needed of adhesive paper to produce 1000 posters for this item. « $\frac{25600}{3}$ dm^2 »

11 Find the area of the region above X-axis bounded by the curve of the function

$f : f(X) = \frac{4X}{X^2 + 1}$ and the straight line $X = 4$ « 2 ln (17) square units »

12 Find the area of the region bounded by the curve : $y = x^3 - 6x^2 + 8x$ and the x-axis.

« 8 square units »

14 If : $f(X) = X^3 - 3X + 3$, **find :**

(1) The absolute extrema value of the function f in the interval f $[0 , 2]$ « 1 , 5 »

(2) The area of the region bounded by the curve of the function f and the straight lines

$X = 0$, $X = 2$, $y = 0$ « 4 square unit »

15 Find the area bounded by the curve of the function $f : f(X) = (3 - X)(X - 1)^2$ and the two coordinate axes where $f(X) \geq 0$

« $\frac{9}{4}$ square units »

16 Find the area bounded by the curve of the function $f : f(X) = (X - 1)(X - 2)(X - 3)$ and the two straight lines $X = 4$, $y = 0$ where $f(X) \geq 0$ « $2\frac{1}{2}$ square units »

17 Find the area of the region bounded by the curve : $\sqrt{x} + \sqrt{y} = 1$ and the two straight lines $x = 0$, $y = 0$ « $\frac{1}{6}$ square units »

19 Find the area of the region bounded by the curve of the function f and the curve of the function g where $f(X) = X^3 - 3X^2 + 5$, $g(X) = X + 2$ « 8 square units »