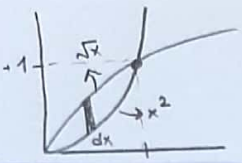
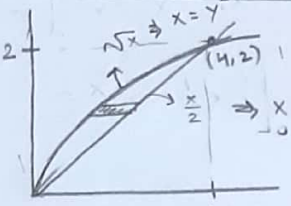
حل تمارين الوحدة 6 <التكامل>

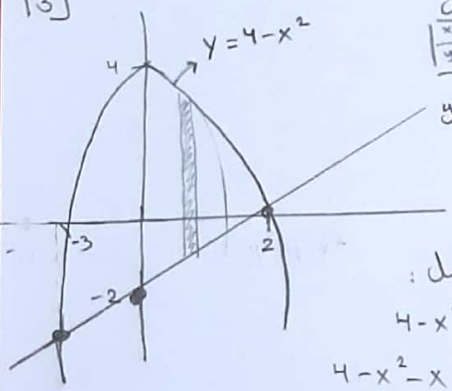
[1] 

$$A = \int_0^1 \sqrt{x} - x^2 \, dx$$ <b> ✓

[2] 

$$A = \int_0^2 2y - y^2 \, dy$$ <c> ✓

يمين ، يسار

[3] 

$y = x - 2$

x	0	2
y	-2	0

$y = x - 2$

حدود التكامل:

$4 - x^2 = x - 2$

$4 - x^2 - x + 2 = 0$

$-x^2 - x + 6 = 0$

$x = 2 \quad x = -3$

$$A = \int_{-3}^{2} 4 - x^2 - (x-2) \, dx$$

$$A = \int_{-3}^{2} 4 - x^2 - x + 2 \, dx$$

$$A = \int_{-3}^{2} -x^2 - x + 6 \, dx$$ <b>

[4] $A = \int 5x - x^2 - 2x$

$$A = \int_0^3 3x - x^2 \, dx$$

$$A = \frac{9}{2}$$ <b>

حدود التكامل:

$5x - x^2 = 2x$

$5x - 2x - x^2 = 0$

$3x - x^2 = 0$

$x = 3 \quad x = 0$

[5] $-y^2 + 10 = (y-2)^2$

$-y^2 + 10 = y^2 - 4y + 4$

$-y^2 - y^2 + 10 + 4y - 4 = 0$

$-2y^2 + 4y + 6 = 0$

$y = 3 \quad y = -1$

$$A = \int_{-1}^{3} -y^2 + 10 - (y-2)^2 \, dy$$

$$A = \frac{64}{3}$$ <b>

[6] $A = \int 6 - x - x^2 \, dx$

حدود التكامل:

$x^2 = 6 - x$

$x^2 + x - 6 = 0$

$x = 2 \quad x = -3$

$$A = \int_{-3}^{2} 6 - x - x^2 \, dx$$

$$A = \frac{125}{6}$$

[7] $\int = 4 - y^2 - y + 2 \, dy$

حدود التكامل:

$4 - y^2 = y - 2$

$4 - y^2 - y + 2 = 0$

$-y^2 - y + 6 = 0$

$y = 2 \quad y = -3$

$$A = \int_{-3}^{2} 6 - y^2 - y \, dy$$

$$A = \frac{125}{6}$$ <d>

[8] $A = 2\int_0^1 (x-1)^3 - (x-1) \, dx$

$A = \frac{1}{2}$

أو

$$A = \int_0^1 (x-1)^3 - (x-1) \, dx + \int_1^2 (x-1) - (x-1)^3 \, dx$$

$A = \frac{1}{2}$

[9] $A_1 = A_2$

$$A_1 = \int_0^{-K+1} x - x^2 - Kx$$

$$A_1 = \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{K}{2}x^2 \Big]_0^{-K+1}$$

$A_1 + A_2 = \int_0^1 x - x^2$

$A_1 + A_2 = \frac{1}{6}$

$A_1 = \frac{1}{6} \div 2 = \frac{1}{12}$

$A_2 = \frac{1}{6} \div 2 = \frac{1}{12}$

$$\frac{1}{12} = \frac{1}{2}(-K+1)^2 - \frac{1}{3}(-K+1)^3 - \frac{K}{2}(-K+1)^2$$

$$\frac{1}{12} = \frac{1}{2}(K^2 - 2K + 1) - \frac{1}{3}(-K+1)^3 - \frac{K}{2}(K^2 - 2K + 1)$$

$$\frac{1}{12} = \frac{1}{2}(K^2 - 2K + 1) - \frac{1}{3}(-K+1) - \frac{K^3}{2} + K^2 - \frac{K}{2}$$

$$\frac{1}{12} = \frac{1}{2}K^2 - K + 1 + \frac{1}{3}K - \frac{1}{3} - \frac{K^3}{2} + K^2 - \frac{K}{2}$$

$$\frac{3}{2}K^2 - \frac{7}{6}K - \frac{K^3}{2} - \frac{1}{3} = \frac{1}{12}$$

$$-\frac{K^3}{2} + \frac{3}{2}K^2 - \frac{7}{6}K - \frac{5}{12} = 0$$

$K = +0.26$ <A>

$Kx = x - x^2$

$Kx - x + x^2 = 0$

$x(K-1) + x^2 = 0$

$(K-1) = \frac{-yx}{x}$

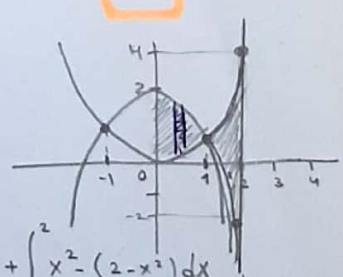
$K - 1 = -x$

$x = -K + 1$

[10] A_2 ✓ <b>

[11] A_3 <c> ✓

[12] $A_1 + A_2$ <d>

[13] 

$$A = \int_0^1 2 - x^2 - x^2 \, dx + \int_1^2 x^2 - (2 - x^2) \, dx$$

$A = \frac{4}{3} + \frac{8}{3} = 4$ <d>

التقاطع:

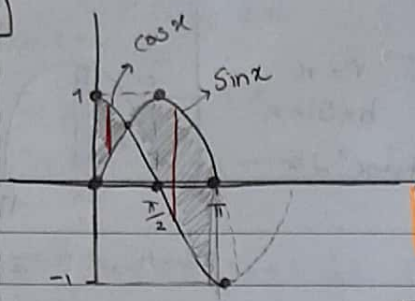
$x^2 = 2 - x^2$

$x^2 + x^2 - 2 = 0$

$2x^2 - 2 = 0$

$x = 1 \quad x = -1$

14



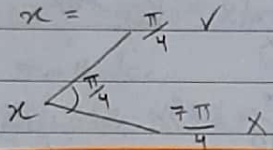
$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$ (S | A / T | C)

$x = \tan^{-1} 1$

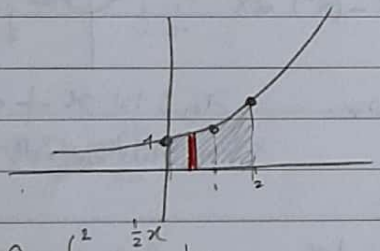
$x = \frac{\pi}{4}$ ✓



$$\int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\frac{\pi}{4}}^{\pi} \sin x - \cos x \, dx$$

⟨C⟩

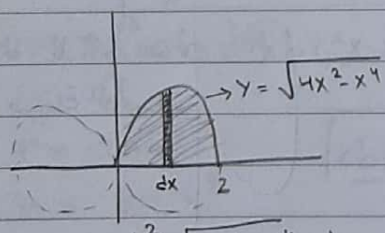
15



$A = \int_0^2 e^{\frac{1}{2}x} \, dx$

$A = 3.43 = \boxed{2e-2}$ ⟨a⟩

16



$A = 4\int_0^2 \sqrt{4x^2 - x^4} \, dx = 10.667 = \frac{32}{3}$

$y^2 = x^2(4 - x^2)$

$y = \sqrt{x^2(4-x^2)}$

$y = x\sqrt{4-x^2}$

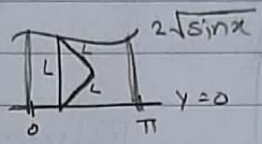
17

$V = \int A(x) \, dx$ (limits: 0 to z, z)

$V = \int_0^{10} \frac{4}{25}(10 - z) \, dz$

$\boxed{V = 8}$ ⟨a⟩ ✓

18



$L = 2\sqrt{\sin x} - 0$

$L = 2\sqrt{\sin x}$

$V = \int A(x) \, dx$

$V = \int \frac{\sqrt{3}}{4} L^2 \, dx$

$V = \int_0^{\pi} \frac{\sqrt{3}}{4} \left(2\sqrt{\sin x}\right)^2 dx$

$V = \int_0^{\pi} \frac{\sqrt{3}}{4}(4\sin x) \, dx$

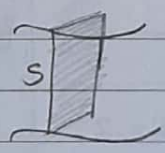
$V = \int_0^{\pi} \sqrt{3}\sin x \, dx$

$V = -\sqrt{3}\cos x \Big]_0^{\pi}$

$V = -\sqrt{3}\cos\pi - -\sqrt{3}\cos 0 =$

$V = \boxed{2\sqrt{3}}$ ⟨b⟩

19



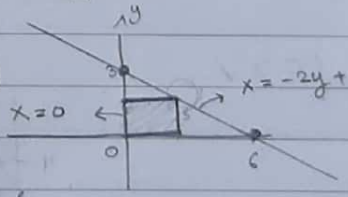
$V = \int_{-1}^{1} A(x) \, dx$

$V = \int_{-1}^{1} S^2 \, dx$

$V = \int_{-1}^{1} (2 - x^2 - x^2)^2 \, dx$

$V = \int_{-1}^{1} (2 - 2x^2)^2 \, dx = \frac{64}{15}$ ⟨b⟩

20



$V = \int A(y) \, dy$ $\quad S = -2y + 6 - 0$

$S = -2y + 6$

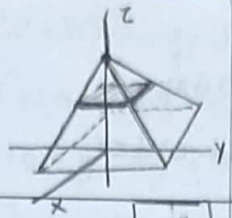
$V = \int S^2 \, dy$

$V = \int_0^3 (-2y + 6)^2 \, dy$

$\boxed{V = 36}$ ⟨b⟩

21 $V = \int_0^H \left(L - \frac{L}{H}x\right)^2 dx$

$V = \int_0^{100} \left(180 - \frac{180}{100}z\right)^2 dz$



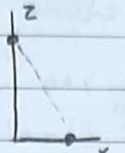
$V = \int A(z)\,dz \qquad S = 2x$

$V = \int S^2\,dz$

$V = \int_0^{100} (2x)^2\,dz$

$V = \int_0^{100} \left(180 - \frac{9}{5}z\right)^2 dz$ ✓ ⟨a⟩

$(x_1, z_1) = (0, 100) \qquad (x_2, z_2) = (90, 0)$



$m = \frac{0-100}{90-0} = -\frac{10}{9}$

$z - 0 = \frac{-10}{9}(x - 90)$

$z = \frac{-10}{9}x + 100$

$z = 100 - \frac{10}{9}x$

$-\frac{10}{9}x = z - 100$

$x = \frac{-9}{10}z + 90$

$2x = -\frac{9}{5}z + 180$

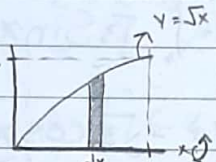
22 $V = \pi \int r_o^2 - r_i^2\,d[x]$

$V = \pi \int_0^4 (\sqrt{x})^2 - 0^2\,dx$

$V = \pi \int_0^4 x\,dx$

$V = \pi \left[\frac{1}{2}x^2\right]_0^4$

$V = 8\pi$ ⟨c⟩ ✓



حلقات إذا: المقاطع عمودية على محور الدوران.

$r_o = \sqrt{x} - 0 = \sqrt{x}$

$r_i = 0 - 0 = 0$

23

$V = \pi \int_0^4 r_o^2 - r_i^2\,dx$

$V = \pi \int_0^4 2^2 - (\sqrt{x})^2\,dx$

$r_o = 2 - 0 = 2$

$r_i = \sqrt{x} - 0 = \sqrt{x}$

$V = 8\pi$ ⟨c⟩ ✓

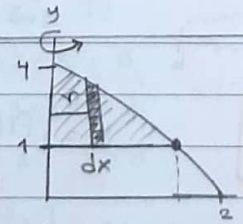
24

$V = 2\pi \int_0^{\sqrt{3}} rh\,dx, \quad r = x$

$h = 4 - x^2 - 1$

$V = 2\pi \int_0^{\sqrt{3}} x(-x^2 + 3)\,dx \qquad h = -x^2 + 3$

$V = \frac{9\pi}{2}$ ✓ ⟨a⟩



dx ∥ محور الدوران

إذاً طريقة الأصداف

الحدود:

$4 - x^2 = 1$

$x = \pm\sqrt{3}$

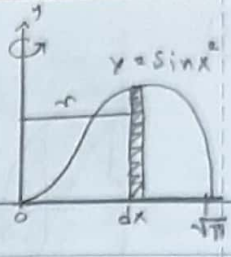
$x = \sqrt{3}$ ✓

25

$V = 2\pi \int rh\,dx \qquad r = x \qquad h = \sin x^2$

$V = 2\pi \int_0^{\sqrt{\pi}} x \sin x^2\,dx$

$V = 2\pi$ ⟨b⟩



dx ∥ محور الدوران

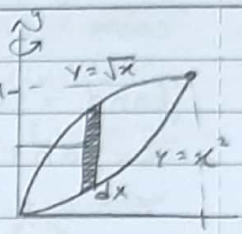
إذاً طريقة الأصداف.

26 $V = 2\pi \int_0^1 rh\,dx$

$r = x \qquad h = \sqrt{x} - x^2$

$V = 2\pi \int_0^1 x(\sqrt{x} - x^2)\,dx$

$V = 0.9424 = \frac{3\pi}{10}$ ⟨a⟩



dx ∥ محور الدوران

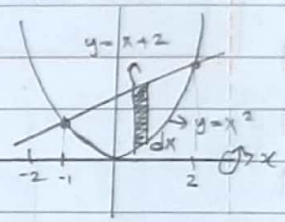
إذاً طريقة الصداف

27 $V = \pi \int_{-1}^2 r_o^2 - r_i^2\,dx$

$r_o = x + 2 \qquad r_i = x^2$

$V = \pi \int_{-1}^2 (x+2)^2 - (x^2)^2\,dx$

$V = \frac{72\pi}{5}$ ⟨a⟩ ✓



dx ⊥ x إذا طريقة الأقراص/الحلقات

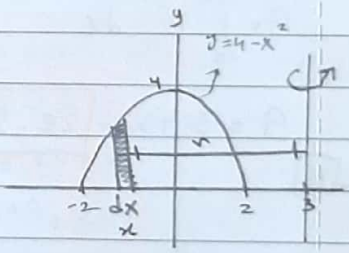
28 $V = 2\pi \int_{-2}^2 rh\,dx$

$r = 3 - x$

$h = 4 - x^2 - 0$

$V = 2\pi \int_{-2}^2 (3-x)(4-x^2)\,dx$

$V = 64\pi$ ⟨d⟩



dx ∥ محور الدوران

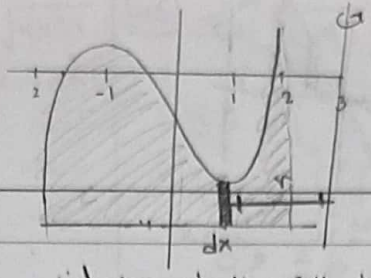
إذاً طريقة الأصداف

29) $V = 2\pi \int_{-2}^{2} rh\,dx$

$r = 3 - x$

$h = x^3 - 3x - 1 - -4$

$V = 2\pi \int_{-2}^{2} (3-x)(x^3 - 3x + 3)\,dx$



dx ∥ محور الدوران ⇐ اهـ اف

$V = \frac{392\pi}{5}$ ⟨a⟩ ✓

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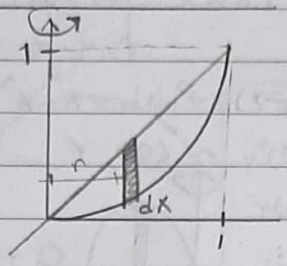
30) $V = 2\pi \int rh\,dx$

$V = 2\pi \int_{0}^{1} rh\,dx$ $r = x$

$h = x - x^2$

$V = 2\pi \int_{0}^{1} x(x - x^2)\,dx$ ⟨c⟩ ✓

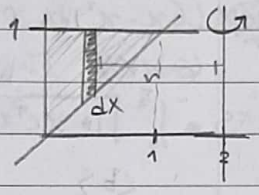


dx ∥ محور الدوران ⇐ اهـ ان

31) $V = 2\pi \int rh\,dx$ $r = 2 - x$

$h = 1 - x$

$V = 2\pi \int_{0}^{1} (2-x)(1-x)\,dx$ ⟨d⟩ ✓



dx ∥ محور الدوران ⇐ اهـ ان

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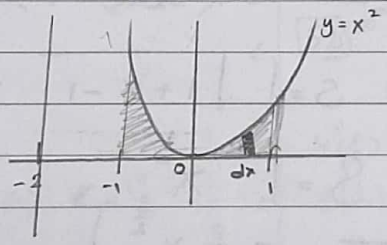
32) $V = 2\pi \int_{-1}^{1} rh\,dx$

$r = x - -2$

$h = x^2 - 0$

$V = 2\pi \int_{-1}^{1} (x+2)(x^2)\,dx$ ⟨b⟩

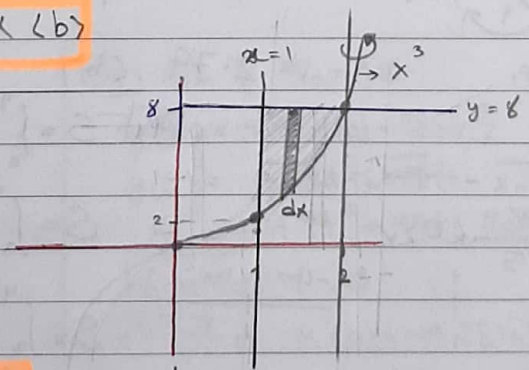


33) $V = 2\pi \int_{1}^{2} rh\,dx$

$r = 2 - x$

$h = 8 - x^3$

$V = 2\pi \int_{1}^{2} (2-x)(8-x^3)\,dx$ ⟨c⟩ ✓



dx ∥ محور الدوران ⇐ اهـ ان

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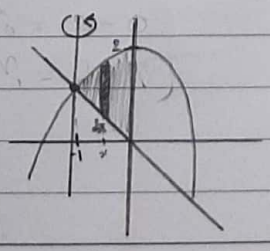
34) $V = \pi \int_{-1}^{0} rh\,dx$

$r = x - (-1) = x + 1$ ⟨d⟩

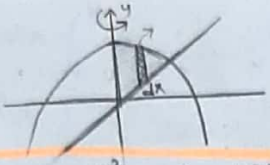


$2 - x^2 = -x$

$2 - x^2 + x = 0$

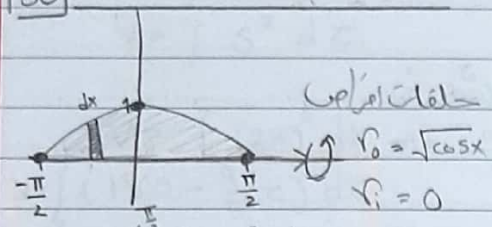
$x = 2 \quad x = -1$

35



$h = 2 - x^2 - x$ ⟨a⟩ ✓

36



$r_o = \sqrt{\cos x}$

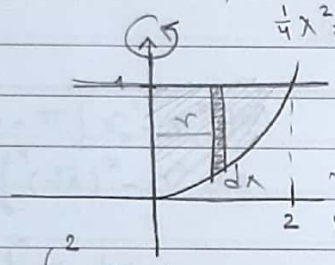
$r_i = 0$

$V = \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r_o^2 - r_i^2 \, dx$

$V = \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = 2\pi$ ⟨b⟩ ✓

37 $y = \frac{1}{4}x^2 \quad x = 0 \quad y = 1$

$\frac{1}{4}x^2 = 1$



$r = x$

$h = 1 - \frac{1}{4}x^2$

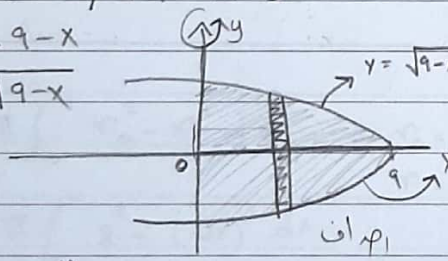
$V = 2\pi \int_0^2 rh \, dx$

$V = 2\pi \int_0^2 (x)(1 - \frac{1}{4}x^2) \, dx = 2\pi$ ⟨b⟩ ✓

38 $x = -y^2 + 9 \quad x = 0$

$y^2 = 9 - x$

$y = \sqrt{9-x}$

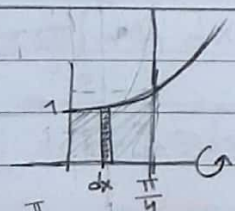


$V = 2\pi \int_0^9 rh \, dx \qquad r = x$

$h = \sqrt{9-x} - -\sqrt{9-x} = 2\sqrt{9-x}$

$V = 2\pi \int_0^9 x(2\sqrt{9-x}) \, dx = \frac{1296\pi}{5}$ ⟨c⟩ ✓

39



$r_o = \sec x$

$r_i = 0$

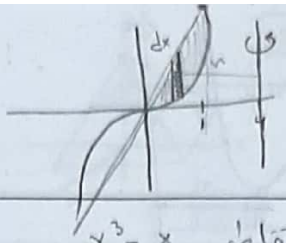
$V = \pi \int_0^{\frac{\pi}{4}} r_o^2 - r_i^2 \, dx$

$V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

$V = \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$

$V = \pi$ ⟨a⟩ ✓

40



$x^3 = x$ نقاط التقاطع

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0 \quad x = 1 \quad x = -1$

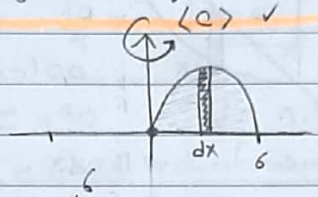
$V = 2\pi \int_0^1 rh \, dx$ الدوران

$r = 4 - x$

$V = 2\pi \int_0^1 (4-x)(x - x^3) \, dx \quad h = x - x^3$

⟨c⟩ ✓

41



الدوران

$V = 2\pi \int_0^6 x(6x - x^2) \, dx \quad r = x$

⟨a⟩ ✓ $\quad h = 6x - x^2$

42 $S = \int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x}$ ⟨b⟩

$f'(x) = \sec^2 x$

$[\,]^2 = \sec^4 x$

43 $f(x) = \frac{2}{3}(x-1)^{\frac{3}{2}} \quad [1, 3]$

$f'(x) = (x-1)^{\frac{1}{2}}$

$]^2 = x - 1$

$S = \int_1^3 \sqrt{1 + x - 1}$

$S = \int_1^3 x^{\frac{1}{2}}$

$= \frac{2}{3} x^{\frac{3}{2}} \Big]_1^3$

$= 2.79$ ⟨b⟩

44 $S = \int_2^4 \sqrt{1 + x^2 - 2x} \, dx$

$S = \int_2^4 \sqrt{(1-x)^2} \, dx$

$S = \int_2^4 |1 - x| \, dx$

$-(1-x)$	$1-x$
1	2 4

$S = x - \frac{1}{2}x^2 \Big]_2^4 = 4$ ⟨b⟩

45 $S = \int_3^5 \sqrt{1+4x^2}\,dx$ $f'(x) = \sqrt{4x^2-1}$

$[\ ]^2 = 4x^2-1$

$S = \int_3^5 2x\,dx$

$S = x^{\frac{1}{2}}\Big]_3^5$

$S = 16$ ⟨c⟩ ✓

46 $S = \int_{-2}^{2} \sqrt{1+[-\sinh\frac{x}{20}]^2}\,dx$ $y = 20\cosh(\frac{x}{20})$

$y' = -20\sinh\frac{x}{20}\cdot\frac{1}{20} = -\sinh\frac{x}{20}$

$S = 2\int_0^2 \sqrt{1+\sinh^2\frac{x}{20}}\,dx$ $(y')^2 = \sinh^2\frac{x}{20}$

$S = 2\int_0^2 \sqrt{\cosh^2\frac{x}{20}}\,dx$ → $S = 2[\sinh\frac{x}{20}\cdot 20]_0^{20}$

$S = 2\int_0^2 \cosh\frac{x}{20}\,dx$ $S = 40\sinh 1$ ⟨a⟩

47 $S = 2\pi\int_1^e \ln x\sqrt{1+\frac{1}{x^2}}\,dx$ $y = \ln x$

$y' = \frac{1}{x}$

$[y']^2 = \frac{1}{x^2}$

⟨C⟩ ✓

48 $S = 2\pi\int_0^3 \frac{1}{9}x^3\sqrt{1+\frac{1}{9}x^4}\,dx$ $\frac{1}{3}x^2 = f(x)$

$\frac{1}{9}x^4 = [f(x)]^2$

⟨d⟩ ✓

49 $V_0 = 4$ ft/s $\theta = 45°$

$y''(t) = -32$ $y'(0) = 4\sin 45 = 2\sqrt{2}$

$y(0) = 0$

$y(t) = -\frac{1}{2}(32)t^2 + 2\sqrt{2}t + 0$

$y(t) = -16t^2 + 2\sqrt{2}t$

$x''(t) = 0$ $x'(0) = 4\cos 45 = 2\sqrt{2}$ $x(0) = 0$

$x(t) = 2\sqrt{2}t + 0$

$t = \frac{x}{2\sqrt{2}}$

$y = -16\left(\frac{x}{2\sqrt{2}}\right)^2 + 2\sqrt{2}\left(\frac{x}{2\sqrt{2}}\right)$

$= -16\frac{x^2}{8} + x$

$y = -2x^2 + x$ ⟨a⟩ ✓

50 $y''(t) = 9.8$ $y'(0) = 98\sin\frac{\pi}{6} = 49$

$y(0) = 0$

$y(t) = -4.9t^2 + 49t$ ⟨d⟩

51 $V_0 = 98$ $\theta = \frac{\pi}{6}$

$x''(t) = 0$ $x'(0) = 98\cos\frac{\pi}{6} = 49\sqrt{3}$

$x(0) = 0$

$x(t) = 49\sqrt{3}t$ ⟨a⟩ ✓

52 $V_0 = 19.6$

$y''(t) = -9.8$ $y'(0) = 19.6$ $y(0) = 0$

$y(t) = -4.9t^2 + 19.6t$

$0 = -4.9t^2 + 19.6t$ $t = 4$ ⟨b⟩ ✓

53 196

$y(t) = 0$

$y''(t) = 9.8$ $y'(0) = 0$ $y(0) = 196$

$y(t) = -4.9t^2 + 196$

$0 = -4.9t^2 + 196$

$t = 6.32$

$y'(t) = -9.8t$

$y'(6.32) = -61.9$ ⟨b⟩ ✓

54 $\theta = \frac{\pi}{6}$ $V_0 = 98$

$y''(t) = 9.8$ $y'(0) = 98\sin\frac{\pi}{6}$ $y(0) = 0$

$y(t) = -4.9t^2 + 49t$

$0 = -4.9t^2 + 49t$

$t = 10$ $t = 0$

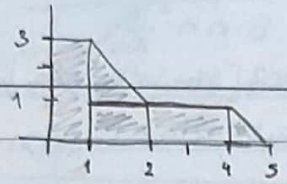
$x''(t) = 0$ $x'(0) = 49\sqrt{3}$ $x(0) = 0$

$x(t) = 49\sqrt{3}t$

$x(10) = 49\sqrt{3}(10) = 848.7$ ⟨c⟩

h(t) = 0

55 $W = \int_0^5 F(x)\,dx$



$W = A = (3\times1)+(1\times1)+(\frac{1}{2}\times2\times1)+(2\times1)+(\frac{1}{2}\times1\times1)$

$= \frac{15}{2} = 7.5$ ✓ ⟨a⟩

56 $W = \int F(x)\,dx$

$W = \int_0^{\frac{1}{2}} 12x\,dx$

$W = \frac{3}{2}$ ⟨d⟩

$F = Kx$

$3 = K(\frac{1}{4})$

$K = 12$

$F = 12x$

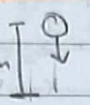
58

$W = \int_0^{1000} 3x+700\,dx =$

$W = 2.2\times10^6$ J

⟨a⟩ ✓

57



$y''(t) = 9.8$ $y'(0) = -1.2$ m/s ✓

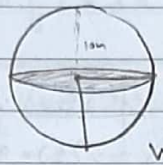
$y(0) = 6$

⟨a⟩ ✓ الجواب

59 $W = \int_0^{20} 62.4\pi x(20-x)^2\,dx$

$W = 2.613\times10^6$ IB

60

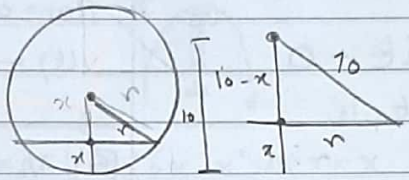


ارتفاع الطبقة — ارتفاع الماء

$W = \int_0^h 9800(A)(H-x)\,dx$

متغير

$W = \int_0^{20} 9800(\pi r^2)(20-x)\,dx$



$r^2 = 10^2 - (10-x)^2$

$r^2 = 100 - (100 - 20x + x^2)$

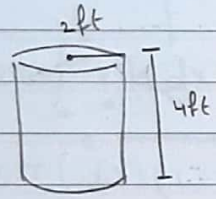
$r^2 = 100 - 100 + 20x - x^2$

$r^2 = 20x - x^2$

$W = \int_0^{20} 9800\pi(20x-x^2)(20-x)\,dx$

$W = 4.1\times10^8$ ✓ ⟨d⟩ ✓

61



$W = \int_0^4 62.4\pi(2)^2(5-x)\,dx$

$W = \frac{14976\pi}{5} = 9409.69$

62

$F(t) = 600(4t-3t^2)$

$= \int_0^{0.01} 600(4t-3t^2)\,dt$

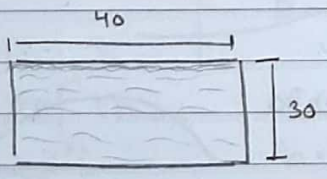
$= 0.12$ Ns ⟨b⟩ ✓

63

$\bar{x} = \frac{M}{m} = \frac{\int_0^1 x(\frac{1}{6}x+2)\,dx}{\int_0^1 \frac{1}{6}x+2\,dx}$

$\bar{x} = \frac{38}{75}$ ⟨a⟩

64



$F = \int_0^{30} \rho g x w\,dx$ (rog.xw)

$F = \int_0^{30} 62.4x(40)\,dx$

$F = 1123200$ ⟨b⟩

65

$W = \int_0^{30000} F(x)\,dx$

$W = \int_0^{30000} 10000 - \frac{x}{15}\,dx$

$W = 2.7\times10^8$ ⟨d⟩

1 lb → 15 ft

1 lb ← $\frac{1}{15}$ ft

lb $\frac{x}{15}$ ← x ft

$F = 10000 - \frac{x}{15}$

66

$W = \int_0^{80} F(x)\,dx$

$W = \int_0^{80} 100 - \frac{x}{2}\,dx$

$W = 6400\ J$ ⟨b⟩ ✓

4ft → 1s

1s → 2Ib

4ft → 2Ib

1ft → $\frac{2}{4}$ Ib

1ft → $\frac{1}{2}$ Ib

xft → $\frac{x}{2}$ Ib

$F(x) = 100 - \frac{x}{2}$

67

1) $f(x) \geq 0$ $[0,1]$

2) $\int_0^1 f(x)dx = 1$

a) 1) $f(x) \geq 0$ ✓

2) $\int_0^1 x\,dx = \frac{1}{2}$ ✗

b) 1) $f(x) \geq 0$ ✓

$\int_0^1 4x^3 dx = 1$ ✓

pdf دالة إذاً هي

c) 1) $f(x) \leq 0$ ✗

d) 1) $f(x) \geq 0$ ✓

$\int_0^1 e^x dx = 1.718$ ✗

⟨b⟩ الجواب

68

$\int_0^{\pi} K \sin x = 1$

$K \int_0^{\pi} \sin x = 1$

$K = \frac{1}{2}$ ⟨d⟩

69

$\int_0^1 \frac{K}{1+x^2} dx = 1$

$K \int_0^1 \frac{1}{1+x^2} dx = 1$

$K \tan^{-1} x \Big]_0^1 = 1$

$K \tan^{-1} 1 = 1$

$\frac{\pi}{4} K = 1$

$K = \frac{4}{\pi}$

70

$\int_0^2 2Ke^{-Kx} dx = 1$

$2K \int_0^2 e^{-Kx} = 1$

$2K \left[\frac{e^{-Kx}}{-K}\right]_0^2 = 1$

$-2e^{-Kx} \Big]_0^2 = 1$

$-2e^{-2K} + 2e^{-K(0)}$

$-2e^{-2K} + 2 = 1$

$-2e^{-2K} = -1$

$e^{-2K} = \frac{1}{2}$

$-2K = \ln \frac{1}{2}$

$K = \frac{\ln \frac{1}{2}}{-2}$

$K = \frac{\ln 1 - \ln 2}{-2}$

$= +\frac{1}{2} \ln 2$

$= \ln 2^{\frac{1}{2}}$

⟨a⟩

71

$f(t) = 4e^{-4t}\,dt$

t بالسنوات

$P(t \leq 1) = P(0 \leq t \leq 1)$

$= \int_0^1 4e^{-4t}\,dt =$

$= 0.98$ ⟨c⟩ ✓

72

$f(t) = 3e^{-3t}$

t بالسنوات

$P(t > 2) = 1 - P(0 \leq t \leq 2)$

$= 1 - \int_0^2 3e^{-3t}\,dt = 2.47 \times 10^{-3} = \frac{1}{e^6}$

⟨b⟩ ✓

73

$\mu = \int_0^1 x(x + 2x^3)\,dx$

$\mu = \frac{11}{15}$ ⟨d⟩ ✓

74

c = الوسيط

$\int_0^c \frac{1}{2} \sin x\,dx = \frac{1}{2}$

$\frac{1}{2} \int_0^c \sin x\,dx = \frac{1}{2}$

$\int_0^c \sin x\,dx = 1$

$-\cos x \Big]_0^c = 1$

$-\cos c + 1 = 1$

$-\cos c = 0$

$\cos c = 0$

$c = \frac{\pi}{2}$ ✓

$\frac{3\pi}{2}$ مرفوضة لأنه خارج الفترة ✗

$c = \frac{\pi}{2}$ ⟨c⟩

75

$\mu = \int_0^1 x \left(\frac{4}{\pi(1+x^2)}\right) dx$

$\mu = \frac{4}{\pi} \int_0^1 \frac{x}{1+x^2}\,dx = 0.44 = \frac{2}{\pi} \ln 2$

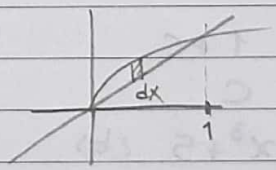
⟨a⟩

76

$\mu = \int_0^1 4t^2 e^{-2t}\,dt$

$\mu = 0.323$ ⟨a⟩

77



الحدود

$x = \sqrt{x}$

$x^2 = x$

$x^2 - x = 0$

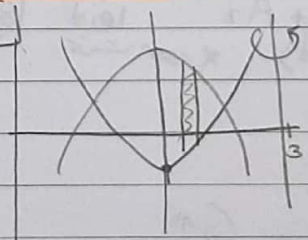
$x(x-1) = 0$

$x = 0$ $x = 1$

$A = \int_0^1 \sqrt{x} - x\,dx$

$A = 0.1666 = \frac{1}{6}$

78



$x^2 - 1$

$h = 1 - x^2 - (x^2 - 1)$

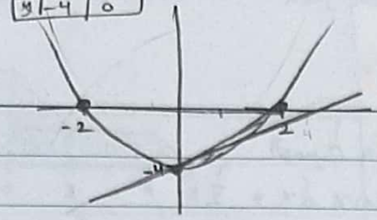
$h = 1 - x^2 - x^2 + 1$

$= 2 - 2x^2$

$= 2(1 - x^2)$ ⟨b⟩ ✓

79 $y = x - 4 \quad y = x^2 - 4$

x	0	4
y	-4	0



$x^2 - 4 = x - 4$

$x^2 - x - 4 + 4 = 0$

$x(x-1) = 0$

$x = 0 \quad x = 1$

$A = \int_0^1 x - 4 - (x^2 - 4)\, dx$

$A = \frac{1}{6}$ ⟨a⟩

80 $S = \int_1^4 \sqrt{1 + 9x^4}\, dx$

$S = \int \sqrt{1 + [f'(x)]^2}\, dx$

$f(1) = 6$

$[f'(x)]^2 = 9x^4$

$f'(x) = \sqrt{9x^4}$

$\int f'(x) = \int \sqrt{9x^4}\, dx$

$f(x) = 3\int x^2\, dx$

$f(x) = \frac{3}{3}x^3 + C$

$f(x) = x^3 + C$

$6 = 1 + C$

$5 = C$

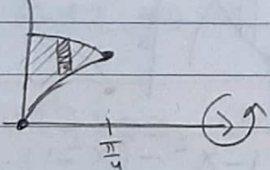
$f(x) = x^3 + 5$ ⟨b⟩

81 $\int_{-4}^{4} f(x)\, dx - 2\int_{1}^{4} f(x)\, dx$

$A_1 + A_2 - 2(-A_2)$ لانها تحت محور x

$A_1 + A_2$ ⟨d⟩

82



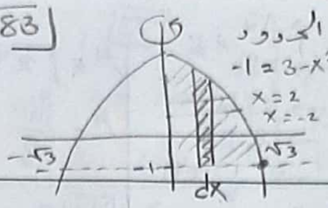
$V = \pi \int r_o^2 - r_i^2\, dx \quad r_o = \cos x$

$r_i = \sin x$

$V = \pi \int_0^{\frac{\pi}{4}} \cos^2 x - \sin^2 x\, dx$

$V = \pi \int_0^{\frac{\pi}{4}} \cos 2x\, dx = \frac{\pi}{2}$

83



$-1 = 3 - x^2$

$x = 2$

$x = -2$

$V = 2\pi \int_0^2 r h\, dx$

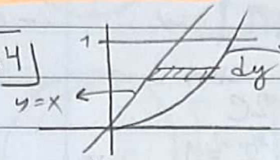
$r = x \quad h = 3 - x^2 + 1$

$h = 4 - x^2$

$V = 2\pi \int_0^2 x(4 - x^2)\, dx$

$V = 8\pi$ ⟨b⟩

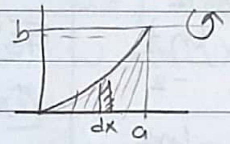
84



$A = \int_0^1 \sqrt{4y} - y\, dy$

⟨a⟩

85

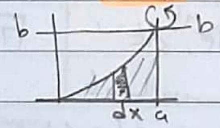


$V = \pi \int_0^a r_o^2 - r_i^2\, dx$

$V = \pi \int_0^a (b)^2 - (b - f(x))^2\, dx$

⟨a⟩

86



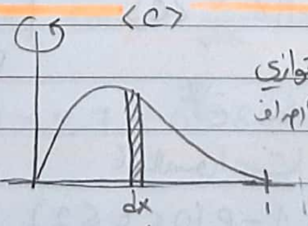
توازي الاطراف

$V = 2\pi \int_0^a r h\, dx$

$V = 2\pi \int_0^4 (a - x)(f(x))\, dx$

⟨c⟩

87



توازي الاطراف

$V = 2\pi \int^1 r h\, dx$

$V = 2\pi \int_0^1 x(x)(x-1)^2\, dx$

⟨c⟩

88 $S = \int \sqrt{1 + \tan^2 x}$

$S = \int^b \sqrt{\sec^2 x}$

$S = \int_0^b \sec x\, dx$

⟨a⟩

$f(x) = \ln \sec x$

$f'(x) = \frac{\sec x \tan x}{\sec x}$

$f'(x) = \tan x$

$[f'(x)]^2 = \tan^2 x$

89 $\int_0^2 f(x) - g(x)$

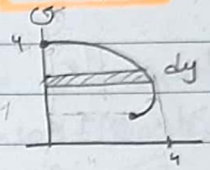
$= \int_0^2 1 - \frac{x}{2} - (-1 + \frac{x}{2})\, dx$

$= \int_0^2 1 - \frac{x}{2} + 1 - \frac{x}{2}\, dx$

$= \int_0^2 2 - x\, dx$

$= 2$ ⟨b⟩

90



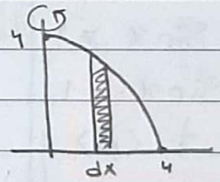
$V = \pi \int_1^4 r_o^2 - r_i^2\, dy$

$r_o = 4y - y^2$

$V = \pi \int_1^4 (4y - y^2)^2\, dy$

$V = \frac{153\pi}{5}$

91 $y = \sqrt{16 - x^2}$



$V = 2\pi \int_0^4 r h\, dx$

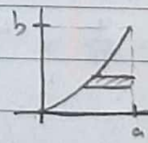
$V = 2\pi \int_0^4 (x)(\sqrt{16 - x^2})\, dx$

$V = \frac{128\pi}{3}$

92 $V = \pi \int_0^b (a^2 - [g(y)]^2\, dy$

(1) حلقة اطراف

(2) عمودي على محور الدوران



∴ محور الدوران هو y

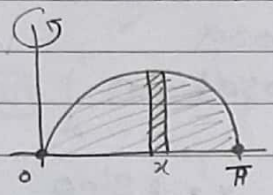
$x = 0$

⟨a⟩

93 $V = \pi \int_0^a \underline{b^2} - [\underline{b} - f(x)]^2 \, dx$

94 جوزي

95 $y = \sin x$ يدور على الفترة $[0, \pi]$ حول المحور

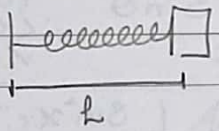


$V = 2\pi \int_0^{\pi} rh \, dx$

$r = 2\pi \int_0^{\pi} x(\sin x) \, dx$

(b) ✓

96 $F = 750 lb$ $\quad x = \frac{1}{4} ft$



$W = \int F(x) \, dx$ $\quad F = Kx$

$W = \int_0^{\frac{1}{2}} Kx \, dx$ $\quad 750 = K \frac{1}{4}$

$K = 3000$

$W = \int_0^{\frac{1}{2}} 3000x \, dx = 375$

(c)

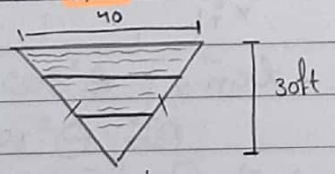
97

$W = \int_0^9 F(x) \, dx =$ الشغل المبذول

أقل شغل ⇐ أقل مسافة

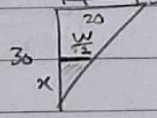
F_2 (a) 3.7×10^5

98



$F(x) = \int_0^{30} \rho g x w \, dx$

$= \int_0^{30} 62.4x \left(\frac{40x}{30} \right)$



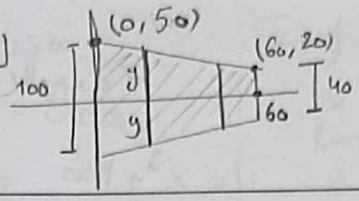
$\frac{30}{20} = \frac{x}{\frac{w}{2}}$ $\quad \frac{30}{20} = \frac{2x}{w}$

$w = \frac{40x}{30}$

$= \int_0^{30} \frac{416}{5} x^2 \, dx = 7.4 \times 10^5$

$=$

99



$w = 2y$

(0,50) (60,20)

x y x y

$m = \frac{20-50}{60-0} = -\frac{1}{2}$

$y = -\frac{1}{2}(x) + 50$

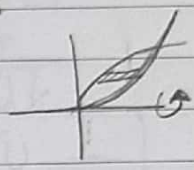
$2y = -x + 100$

$F(x) = \int \rho g x w \, dx$

$F(x) = \int_0^{60} 62.4x(\text{ثابت})$

$F(x) = \int_0^{60} 62.4x(-x+100) \, dx = 6739200 lb$ ✓

100 $\pi \int_0^4 \left[(\sqrt{x})^2 - (\frac{1}{8}x^2)^2 \right] dx$



$y = \sqrt{x}$ $\quad y = \frac{1}{8}x^2$

$y^2 = x$ $\quad \sqrt{8y} = x$

$2\pi \int_0^2 rh \, dy$

$2\pi \int_0^2 y(\sqrt{8y} - y^2) \, dy$

(a) ✓

$y^2 = \sqrt{8y}$

$y^4 = 8y$

$y^4 - 8y = 0$

$y(y^3 - 8) = 0$

$y = 0$ $\quad y^3 - 8 = 0$

$y = 0$ $\quad y = 2$

101 $f(x) = \frac{1}{2}\sin x \quad [0, b]$

$$\int_0^b \frac{1}{2}\sin x \, dx = 1$$

$$-\frac{1}{2}\cos x \Big]_0^b = 1$$

$$-\frac{1}{2}\cos b + \frac{1}{2}\cos(0) = 1$$

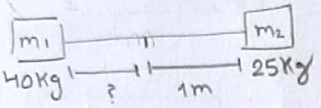
$$-\frac{1}{2}\cos b = \frac{1}{2}$$

$$-\cos b = 1$$

$$\cos b = -1$$

$$\boxed{b = \pi} \quad \langle a \rangle$$

102



$$m_1 d_1 = m_2 d_2$$

$$40(1) = (25)(d_2)$$

$$\frac{40}{25} = d_2$$

$$\boxed{d_2 = 1.6} \quad \langle c \rangle$$

103

$$J = \int_0^6 F(t)\,dt = A$$

$$J = A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(15)$$

$$\boxed{J = 45} \quad \langle c \rangle$$

104

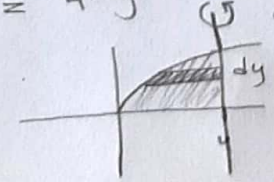
$$\pi \int_0^2 (4 - y^2)^2 \, dy$$

افراص ⇐ الشريحة ⊥ محور الدوران

$$V = \pi \int (r_o^2 - r_i^2)\, dy$$

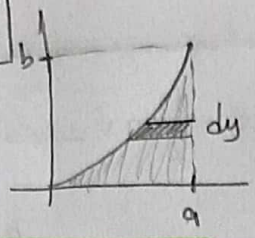
$$V = \pi \int (4 - y^2)^2 - 0 \, dy$$

$r_o = 4 - y^2$ بُعد الدالة الخارجية عن محور الدوران



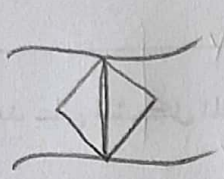
$$\boxed{x = 4} \quad \langle a \rangle$$

105



$$A = \int_0^b (a - g(y))\, dy \quad \langle b \rangle$$

106



قانون حساب قطر المربع $L = \sqrt{2}\, S$

$$S = \frac{L}{\sqrt{2}}$$

$$A = S^2 = \frac{\ell^2}{2}$$

$$\ell = \sqrt{2\sin x}$$

$$V = \int_0^{\pi} \frac{\ell^2}{2}\, dx$$

$$V = \int_0^{\pi} \frac{2\sin x}{2}\, dx$$

$$V = 2 \quad \langle b \rangle$$

107

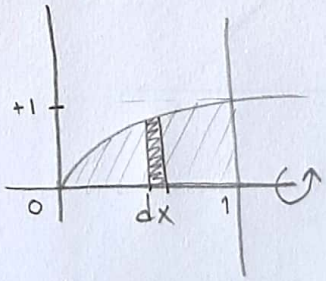
$$V = \pi \int_0^1 r_o^2 - r_i^2\, dx \qquad r_o = \sqrt{\frac{2x}{x^2+1}}$$

$$r_i = 0$$

$$V = \pi \int_0^1 \left(\sqrt{\frac{2x}{x^2+1}}\right)^2 - 0\, dx$$

$$V = \pi \int_0^1 \frac{2x}{x^2+1}\, dx$$

$$V = 2.17 = \pi \ln(2) \quad \langle b \rangle$$



تقاطع دائرة $\perp dx$ محور الدوران

108 $f(x) = 4t^3$ بالسنوات $[0,1]$

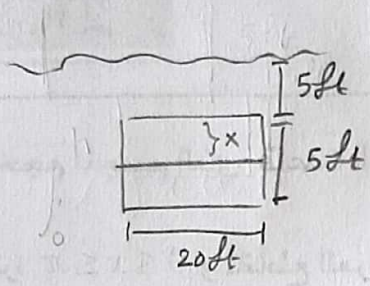
$$\mu = \int_0^1 (t)(4t^3)\,dt = \frac{4}{5}$$

$$t > \frac{4}{5}$$

$$P = 1 - \int_0^{4/5} 4t^3\,dt$$

$$= 0.5904 \times 100 \approx \boxed{60\%} \quad \langle c \rangle$$

109

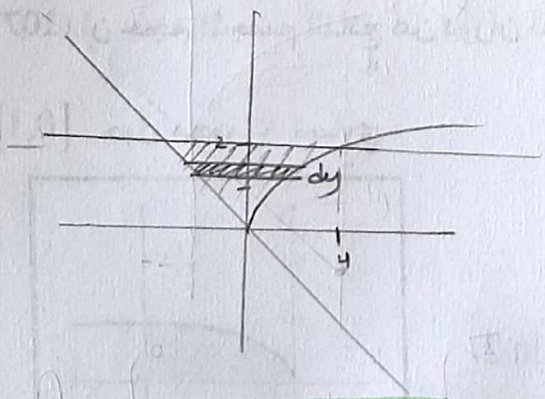


$$F = \int_0^{10} 62.4\, x\, w\,dx$$

متغير (بعد الشريحة عن السطح) ثابت

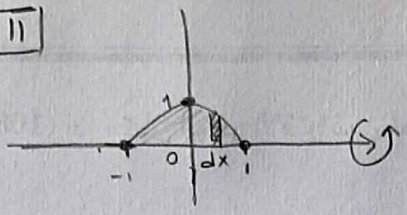
$$F = \int_0^{10} 62.4\,(x+5)(20)\,dx \quad \langle d \rangle$$

110



$$\boxed{A = \int_0^2 y^2 + y\,dy} \quad \langle a \rangle$$

111



$$f(x) = \sqrt{1-x^2}$$

$$= \frac{-2x}{2\sqrt{1-x^2}}$$

$$= \left[\frac{-x}{\sqrt{1-x^2}}\right]^2$$

$$= \frac{x^2}{1-x^2}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{1-x^2}\ \sqrt{1+\frac{x^2}{1-x^2}}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{1-x^2}\ \sqrt{\frac{1-x^2+x^2}{1-x^2}}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{1-x^2}\ \sqrt{\frac{1}{1-x^2}}$$

$$S = 2\pi \int_{-1}^{1} \cancel{\sqrt{1-x^2}}\ \frac{1}{\cancel{\sqrt{1-x^2}}}\,dx = \boxed{4\pi}\ \langle d\rangle$$

112 $S = \int_0^2 \sqrt{1+[f'(x)]^2}$

$$f(x) = \sqrt{4-x^2}$$

$$f'(x) = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$$

$$= \frac{x^2}{4-x^2}$$

$$S = \int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\,dx$$

$$S = \int_0^2 \sqrt{\frac{4-x^2+x^2}{4-x^2}}$$

$$S = \int_0^2 \sqrt{\frac{4}{4-x^2}}\,dx = \int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx = 2\int_0^2 \frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

$$u = \frac{x}{2}$$

$$\frac{du}{dx} = \frac{1}{2}$$

$$dx = 2\,du$$

$$\int_{\square\to 0}^{\square\to 1}$$

$$S = 2\int_0^1 \frac{1}{\sqrt{1-u^2}}\,du = 2\sin^{-1}u\Big]_0^1 = \boxed{\pi}\ \langle a\rangle$$

113 $\int_0^b Ke^{-4x}\,dx = 1$

$$K\int_0^b e^{-4x}\,dx = 1$$

$$K\left[-\frac{1}{4}e^{-4x}\right]_0^b = 1$$

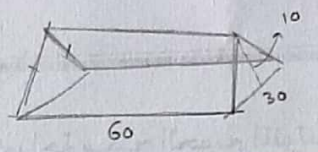
$$K\left[-\frac{1}{4}e^{-4(b)} + \frac{1}{4}e^{-4(0)}\right] = 1$$

$$K\left[-\frac{1}{4}e^{-4b} + \frac{1}{4}\right] = 1$$

$$K = \frac{1}{-\frac{1}{4}e^{-4b}+\frac{1}{4}} = \frac{1}{\frac{1}{4}(-e^{-4b}+1)} = \boxed{\frac{4}{1-e^{-4b}}}\ \langle a\rangle$$

$$\frac{1}{K} - \frac{1}{4} = -\frac{1}{4}e^{-4kb}$$

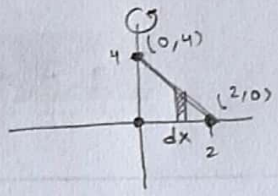
114



المقاطع العرضية المكعبات

$$V = \int_0^{60} \frac{1}{2} \times 30 \times 10 \; dx \quad \langle a \rangle$$

115



$(0,4)\ (2,0)$ $\quad m = \frac{0-4}{2-0} = -2$

$y - 4 = -2(x - 0)$

$y = -2x + 4$

$V = 2\pi \int rh \, dx \qquad r = x$

$h = -2x + 4$

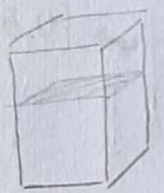
$$V = 2\pi \int_0^2 x(-2x+4)\, dx \quad \langle b \rangle$$

116

$$W = \int_0^{0.4} F(x)\, dx = A$$

$$W = \frac{1}{2}hb = \frac{1}{2}(4)(0.4) = 0.8 \quad \langle d \rangle$$

117



ثابتة

$W = \int \rho g A (H - x)$

الشغل الكلي

$$W = \int_0^{10} \rho g A x$$

$$W = \rho g A \int_0^{10} x = \frac{\rho g A x^2}{2}\Big]_0^{10} = 50 \rho g A$$

شغل العامل الأول ← العمق الذي سيصل إليه

$$\frac{1}{2}W = \int_0^S \rho g A x \, dx$$

$$\frac{1}{2}(50 \rho g A) = \frac{\rho g A x^2}{2}\Big]_0^S$$

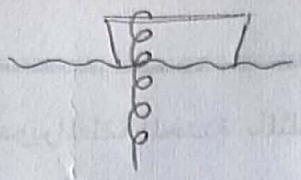
$$\frac{1}{2}(50 \rho g A) = \frac{\rho g A S^2}{2}$$

$$25 = \frac{S^2}{2}$$

$$S^2 = 50$$

$$S = \sqrt{50} = 7.07 \quad \langle c \rangle$$

118



40ft → 1000 Ib

1ft → $\frac{1000}{40}$ Ib

1ft → 25 Ib

xft → 25x

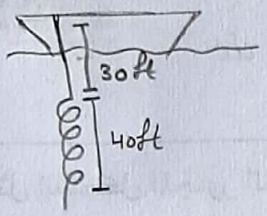
F = 1000 − 25x

الوزن متغير

$W = \int F(x)\,dx$

$W = \int_0^{40} (1000 - 25x)\,dx$ ⟨d⟩

119



40ft → 1000 Ib

1ft → 25 Ib

F = 1000 − 25x

$W_1 + W_2$

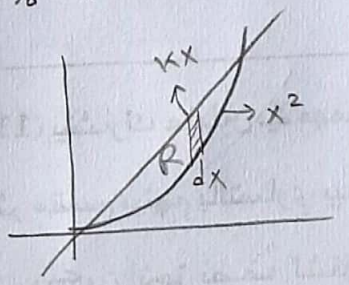
$\int_0^{30} 1000\,dx + \int_0^{40} 1000 - 25x\,dx$ ⟨A⟩

120 $y^2 = 4x^2 - x^4$

$y = \sqrt{4x^2 - x^4}$

$A = 4\int_0^2 \sqrt{4x^2 - x^4}\,dx = \frac{32}{3}$ ⟨b⟩

121



$kx = x^2$

$k = x$

$A = \int_0^k kx - x^2\,dx$

$\frac{4}{3} = \frac{k}{2}x^2 - \frac{1}{3}x^3\Big]_0^k$

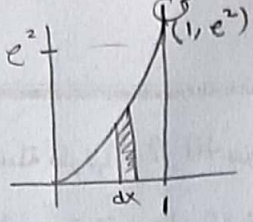
$\frac{4}{3} = \frac{k^3}{2} - \frac{k^3}{3}$

$\frac{4}{3} = \frac{1}{6}k^3$

$8 = k^3$

$k = 2$ ⟨b⟩

122



$r = 1 - x$

$h = e^{2x}$

$$V = 2\pi \int_0^1 rh\,dx$$

$$\boxed{V = 2\pi \int_0^1 (1-x)e^{2x}\,dx} \quad \langle c \rangle$$

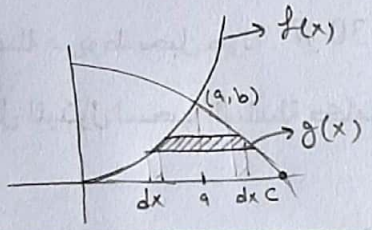
123 $\int_{-1}^{4} f(x)\,dx + \int_{-1}^{4} 2\,dx$

$A - B + C + 10$

Limits

$1 - 2 + 3 + 10 = \boxed{12}$ $\langle c \rangle$

124



$$\boxed{A = \int_0^b g^{-1}(y) - f^{-1}(y)\,dx} \quad \langle d \rangle$$

125 H ft

$y''(t) = -32 \quad y'(0) = 0 \quad y(0) = H$

$y = -16t^2 + V_0 t + Y_0$

$y = -16t^2 + H$

$0 = -16t^2 + H$

$16t^2 = H$

$t^2 = \frac{H}{16} \Rightarrow \boxed{t = \frac{1}{4}\sqrt{H}}$ $\langle D \rangle$

126 $V_0 = 48$, 160 ft

$y''(t) = -32 \quad y'(0) = -48 \quad y(0) = 160$

$y = -16t^2 - 48t + 160$

$0 = -16t^2 - 48t + 160$

$\boxed{t = 2} \quad \cancel{t = -5}$

$y' = -32t - 48$

$\boxed{y' = -112 \text{ m/s}}$ $\langle a \rangle$