University Physics

with

Modern Physics

الحل التفصيلي

Solutions Manual

الوحدة 28

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الوحدة 28 - الحل التفصيلي

Chapter 28: Magnetic Fields of Moving Charges

Concept Checks

28.1. c **28.2.** a **28.3.** a **28.4.** b **28.5.** e **28.6.** d **28.7.** d **28.8.** d

Multiple-Choice Questions

28.1. b **28.2.** c **28.3.** c **28.4.** a **28.5.** d **28.6.** a **28.7.** c **28.8.** c **28.9.** a **28.10.** d **28.11.** a **28.12.** a **28.13.** d

28.14. b

Conceptual Questions

28.15. The wires are twisted in order to cancel out the magnetic fields generated by these wires.

28.16. Since the currents running through the wire generate magnetic fields, these fields may overpower the magnetic field of the Earth and make the compass give a false direction.

28.17. No, an ideal solenoid cannot exist, since we cannot have an infinitely long solenoid. To a certain extent, yes, it renders the derivation void. However, the derivation is an approximation and is an important theoretical example.

28.18. In Example 28.1, the right hand rule implies that the magnetic dipole of the loop points out of the page. Application of the right hand rule to the straight wire tells us that the magnetic field produced by wire points out of the page. Assume the angle between the dipole moment and the field remains fixed. Since the dipole strength is also constant, the only quantity left to vary is field strength. If the potential energy is to be reduced, the loop must move towards a region of smaller magnetic field strength. That is, the loop must move away from the straight current-carrying wire.

28.19. By Coulomb's Law, the electric force between the particles has magnitude $|F_e| = q^2/\left(4\pi\varepsilon_0 d^2\right)$. For the magnetic force, the version of the Biot-Savart Law given in the text can be adapted to describe the magnetic field produced by a moving particle via the replacement $Idl \Rightarrow \left(dq/dt\right)dl \Rightarrow dq\left(dl/dt\right) \Rightarrow qv_s$ with q charge and v_s, the velocity of the source particle. The magnetic field produced by one particle at the location of the other can be written as $B = \mu_0 qvd/\left(4\pi d^3\right)$ with v, common velocity and d, the separate of the particles. The magnitude of the magnetic force one particle is given by $|F_e| = |qvB| = \left(\mu_0/\left(4\pi\right)\right)\left|qv\cdot(qvd)/d^3\right| = \mu_0 q^2 v^2/\left(4\pi d^2\right)$. Since the vectors v, d and $v\cdot d$ are mutually perpendicular (the site of the angle between any two of them is unity) the ratio of forces is $|F_m/F_e| = \mu_0\varepsilon_0 v^2$ which also $|F_m/F_e| = v^2/c^2$, where c is the speed of light.

28.20. The field is given by Ampere's law $B\left(2\pi\right)\left(a+b\right)/2 = \mu_0 i_{enc}$. Current density is then given by:

$$J = i/\left(\pi\left(b^2 - a^2\right)\right)$$

The area of interest is:

$$\pi\left[\left(a+b\right)/2\right]^2 = A$$
$$B(2\pi)\left[\left(a+b\right)/2\right] = \mu_0 AJ$$

$$B=\frac{\mu_0}{\pi(a+b)}\pi\left[\left(\frac{a+b}{2}\right)^2-a^2\right]\cdot\frac{i}{\pi\left(b^2-a^2\right)}$$

$$=\frac{\mu_0 i}{\pi\left(a+b\right)}\cdot\frac{\left(\frac{a+b}{2}\right)^2-a^2}{\left(b^2-a^2\right)}.$$

28.21. The magnetic field at point P would be zero. The contribution from part A would be zero since P lies along the axis of A. The currents through B and C points in opposite directions and yield magnetic fields that cancel out at P .

28.22. Ampere's law states that, $\oint_C Bdl=\mu_0 i,$ but since B is constant the integral must be zero. If so, i is zero everywhere and consequently $J=0$ everywhere.

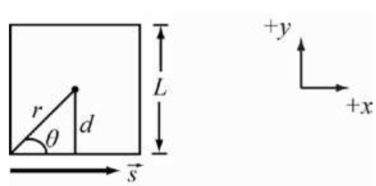
28.23. (a) Since molecular hydrogen is diamagnetic, the molecules must have no intrinsic dipole moment. Since the nuclear spins cannot cancel the electron spins, the electron spins must be opposite to cancel each other.
(b) With only a single electron, the hydrogen atoms must have an intrinsic magnetic moment. Atomic hydrogen gas, if it could be maintained, would have to exhibit paramagnetic or ferromagnetic behavior. But ferromagnetism would require inter atomic interactions strong enough to align the atoms in domains, which is not consistent with the gaseous state. Hence one would expect atomic hydrogen to be paramagnetic.

28.24. The saturation of magnetizations for paramagnetic and ferromagnetic materials is of comparable magnitude. In both types of materials the intrinsic magnetic moments of the atoms arise from a few unpaired electron spins. Magnetization effects in ferromagnetic materials are more pronounced at low applied fields because the atoms come pre-aligned in their domains, but once both types of atoms have been forced into essentially uniform alignment, the magnetization they produce is comparable. For either type of material maximum magnetizations of order 10^6 A m^2 / m^3 $=10^6$ A/m magnetic dipole moment per unit volume are typical.

28.25. The wire carries a current which produces a magnetic field. This magnetic field will deflect the electron by the Lorentz force in the left direction.

28.26. Each side of the loop will create the same magnetic field at the center of the loop. The total field is 4 times the field of one side. The field at the center is given by the Biot-Savart Law:

$$\left|d\vec{B}\right|=\left|\frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}\right|=\frac{\mu_0 i}{4\pi r^2}\sin\theta ds.$$



Since $\sin\theta=d/r$, the differential element of magnetic field is

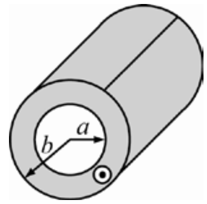
$$dB=\frac{\mu_0 i}{4\pi}\frac{d}{r^3}ds=\frac{\mu_0 id}{4\pi}\frac{ds}{(d^2+s^2)^{3/2}}.$$

Integration gives

$$B=\frac{\mu_0 id}{4\pi}\int_{-d}^{d}\frac{ds}{(d^2+s^2)^{3/2}}=\frac{2\mu_0 id}{4\pi}\int_0^d\frac{ds}{(d^2+s^2)^{3/2}}=\frac{\mu_0 id}{2\pi}\left[\frac{s}{d^2\sqrt{d^2+s^2}}\right]_0^d=\frac{\mu_0 i}{2\pi d}\frac{d}{\sqrt{2}d}=\frac{\mu_0 i}{2\sqrt{2}\pi d}.$$

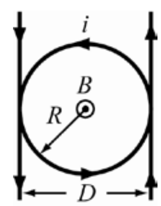
The total field is then $B_{\text{tot}} = 4B = \dfrac{\sqrt{2}\mu_0 i}{\pi d} = \dfrac{\sqrt{2}\mu_0 i}{\pi(L/2)} = \dfrac{2\sqrt{2}\mu_0 i}{\pi L}.$

28.27.



The current that flows through a ring of radius r which lies in the region $a<r<b$ is given by $i = \int J_0 dA = \int_a^r J_0 2\pi\rho d\rho = J_0\pi\rho^2 \Big|_a^r = J_0\pi(r^2 - a^2)$. To find the magnetic field employ Ampere's Law $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$. For a cylinder this becomes $B(2\pi r) = \mu_0 i_{\text{enc}}$ or $B = \mu_0 i_{\text{enc}}/(2\pi r)$. If $r<a$ then $B_{r<a} = 0$, thus if $a<r<b$ then $i_{\text{enc}} = J_0\pi(r^2-a^2)$ and $B_{a<r<b} = \dfrac{\mu_0 J_0\pi(r^2-a^2)}{2\pi r} = \dfrac{\mu_0 J_0(r^2-a^2)}{2r}$. If $r>b$ then $i_{\text{enc}} = J_0\pi(b^2-a^2)$ and $B_{r>b} = \dfrac{\mu_0 J_0(b^2-a^2)}{2r}$. Note that if $r=b$ then $B_{a<r<b} = \dfrac{\mu_0 J_0(b^2-a^2)}{2b} = B_{r>b}$.

28.28. The loop creates a magnetic field of $B_1 = \mu_0 i/(2R)$ at its center and is directed upwards. Out of the page. Both wires contribute a magnetic field of $B_{\text{w}} = \mu_0 i/(2\pi R)$ pointing out of the page. The total fields is then $B_{\text{tot}} = B_1 + 2B_{\text{w}} = \dfrac{\mu_0 i}{2R}(1+\dfrac{2}{\pi})$, and points out of the page.



28.29. **THINK:** Ampere's Law can be used to determine the magnitude of the magnetic field in the two regions.
SKETCH: A sketch is included at the end of the SIMPLIFY step, once the two equations have been found.
RESEARCH: The current with the conductor is given by $i = \int J(r)dA$. The magnetic field is found using Ampere's Law $\oint \vec{B}\bullet d\vec{s} = \mu_0 i_{\text{enclosed}}$ or $B = \dfrac{\mu_0 i_{\text{enc}}}{2\pi r}$.

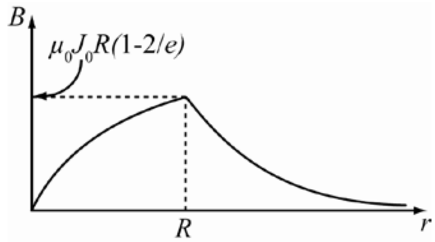
SIMPLIFY: $i = \int J(r)dA = 2\pi\int_0^r J(r')r'dr' = 2\pi J_0 \int_0^r r'e^{-r'/R}dr' = 2\pi J_0\left[-R(R+r')e^{-r'/R}\right]\Big|_0^r$

$$= \left(\left(-R(R+r)e^{-r/R}\right) - \left(-R(R+0)e^{-0/R}\right)\right)2\pi J_0$$

$$= \left(R^2(1-e^{-r/R}) - Rre^{-r/R}\right)2\pi J_0$$

If $r<R$ then $B_{r<R} = \dfrac{\mu_0}{2\pi r}\left[R^2 - R(R+r)e^{-r/R}\right]2\pi J_0 = \dfrac{\mu_0 J_0}{r}[R^2 - R(R+r)e^{-r/R}]$.

If $r>R$ then $B_{r>R} = \dfrac{\mu_0}{2\pi r}\left[R^2 - R(R+R)e^{-R/R}\right]2\pi J_0 = \dfrac{\mu_0 J_0}{r}\left[R^2 - 2R^2e^{-1}\right] = \dfrac{\mu_0 J_0 R^2}{r}[1^2 - 2e^{-1}]$.



CALCULATE: There are no values to substitute.
ROUND: There are no values to round.
DOUBLE-CHECK: Note that the two computed formulas agree when $r = R$.

Exercises

28.30. The force of wire 1 on wire 2 is $F_{1\to 2} = i_2 LB = i_2 L\left[\mu_0 i_1/(2\pi d)\right] = \mu_0 i_1 i_2 L/(2\pi d)$. Since $2i_1 = i_2$, $F_{1\to 2} = \mu_0 i_1^{\,2} L/(\pi d)$. Solving for the current i_1 gives $i_1 = \sqrt{\dfrac{\pi d F_{1\to 2}}{\mu_0 L}} = \sqrt{\dfrac{\pi(0.0030\text{ m})(7.0\cdot 10^{-6}\text{ N})}{(4\pi\cdot 10^{-7}\text{ T m/A})(1.0\text{ m})}} = 0.23\text{ A}.$

The current on the other wire is $i_2 = 0.46$ A.

28.31. The magnetic field created by the wire is given by the Biot-Savart Law $B = \mu_0 i/(2\pi r)$. The force on the electron is given by the Lorentz force $F = qvB = qv\mu_0 i/(2\pi r)$. The acceleration of the electron is

$$a = \frac{F}{m} = \frac{qv\mu_0 i}{2\pi mr} = \frac{(1.602\cdot 10^{-19}\text{ C})(4.0\cdot 10^5\text{ m/s})(4\pi\cdot 10^{-7}\text{ T m/A})(15\text{ A})}{2\pi(9.109\cdot 10^{-31}\text{ kg})(0.050\text{ m})} = 4.2\cdot 10^{12}\text{ m/s}^2$$

The direction of the acceleration is radially away from the wire.

28.32. The magnitude of the magnetic field created by a moving charge along it is line of motion is zero. By the Biot-Savart Law,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{qd\vec{v}\times\hat{r}}{r^2} = 0,$$

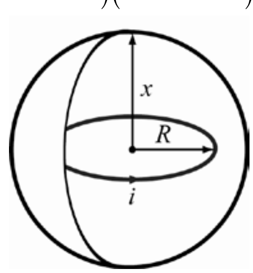
since the angle between the angle between the velocity and the position vector $\hat{r}$ is zero. The situation is the same for an electron and a proton.

28.33. The field along the axis of a current loop of radius R as measured at a distance x from the center of the loop is

$$B = \frac{\mu_0 i}{2}\frac{R^2}{\left(x^2+R^2\right)^{3/2}}.$$

The current of the loop must be

$$i = \frac{2\left(x^2+R^2\right)^{3/2} B}{\mu_0 R^2} = \frac{2\left[\left(2.00\cdot 10^6\text{ m}\right)^2 + \left(6.38\cdot 10^6\text{ m}\right)^2\right]^{3/2}}{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(2.00\cdot 10^6\text{ m}\right)^2}\left(6.00\cdot 10^{-5}\text{ T}\right) = 7.14\cdot 10^9\text{ A}.$$



28.34. What does it mean to have an "average value of the magnetic field measured in the sides"? The answer is that the average value is: $\overline{B} = \oint \vec{B} \bullet d\vec{s} \, / \oint ds.$ And $\oint ds$ is just the total length of the closed path around the loop, in this case $\oint ds = 4l.$ For the integral above we can simple use Ampere's Law and find (see equation 28.10):

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{\text{enc}}$$

We found above that $\overline{B} = \oint \vec{B} \bullet d\vec{s} \, / \oint ds = \oint \vec{B} \bullet d\vec{s} \, / \, 4l.$ Inserting Ampere's Law and solving for the enclosed current then yields:

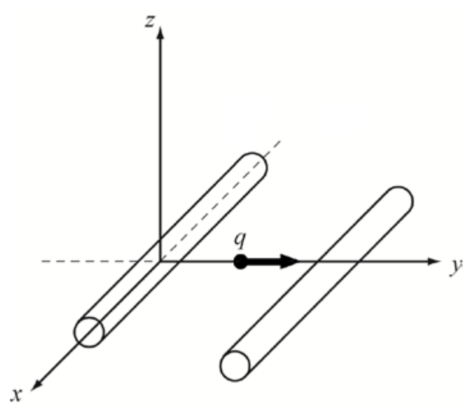
$$\overline{B} = \mu_0 i_{\text{enc}} / 4l \Rightarrow i_{\text{enc}} = 4l\overline{B} / \mu_0$$

Numerically we find $i_{\text{enc}} = 4(0.0300 \text{ m})(3.00 \cdot 10^{-4} \text{ T}) / (4\pi \cdot 10^{-7}) = 28.64789 \text{ A},$ which we round to $i_{\text{enc}} = 28.6 \text{ A}.$

We can also see if our solution makes sense. We have calculated the magnetic field from a long straight wire as a function of the distance to the wire in equation 28.4 and found $B = \mu_0 i / 2\pi r_\perp$. With our value of the current computed above, we can calculate the value of the magnetic field at the corners of the loop (furthest from the wire) and middle of the sides (closest to the wire) and see that these two values of the magnetic field are below and above the average value of *B* that was given in the problem. For the middle of the sides we find ($r_\perp = l/2$): $B = 3.82 \cdot 10^{-4} \text{ T}$, and for the corners we find ($r_\perp = l/\sqrt{2}$): $B = 2.70 \cdot 10^{-4} \text{ T}$. This gives us confidence that we have the right solution.

28.35. **THINK:** A force due to the magnetic field generated by a current carrying wire acts on a moving particle. In order for the net force on the particle to be zero, a second force of equal magnitude and opposite direction must act on the particle. Such a force can be generated by another current carrying wire placed near the first wire. Assume the second wire is to be parallel to the first and has the same magnitude of current. The wire along the *x*-axis has a current of 2 A oriented along the *x*-axis. The particle has a charge of $q = -3 \text{ μC}$ and travels parallel to the *y*-axis through point $(x, y, z) = (0, 2, 0).$

SKETCH:



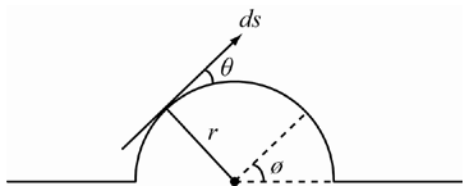
RESEARCH: The magnetic field produced by the current is given by the $B = \mu_0 i / (2\pi r).$ The force on the particle is given by the Lorentz force, $F = qv_0 B.$

SIMPLIFY: If the wires carry the same current then the new wire must be equidistant from the point that the particle passes through the *xy*-plane. Only then will the magnetic force on the particle due to each wire be equal. By the right hand rule, the currents will be in the same direction. This means that $r_1 = r_2.$

CALCULATE: The requirement $r_1 = r_2$ means that the second wire should be placed parallel to the first wire (parallel to the *x*-axis) so that it passes through the point $(x, y, z) = (0, 4, 0).$

ROUND: Not necessary.
DOUBLE-CHECK: It is reasonable that two wires carrying the same current need to be equidistant from a point in order for the magnitude of the force to be the same.

28.36. **THINK:** The current through the wire creates a magnetic field by the Biot-Savart Law. The straight part of the wire only creates a magnetic field at points perpendicular to it. Therefore this part of the wire can be ignored. The magnetic field at the center of the semicircle is created by the charge moving through the semicircle.
SKETCH:



RESEARCH: The Biot-Savart Law can be employed in the form $dB = \frac{\mu_0}{4\pi}\frac{i\sin\theta}{r^2}ds$. Going around the semicircle, the angle ϕ can be related to the current element by $ds = rd\phi$.
SIMPLIFY: Performing the integration gives

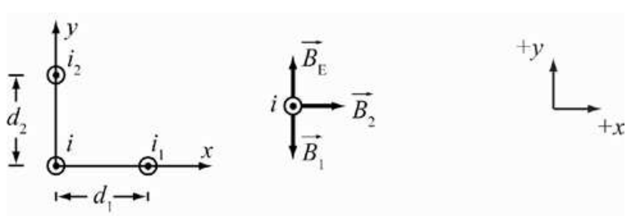
$$B = \frac{\mu_0 i}{4\pi}\int_0^{\pi}\frac{\sin\theta}{r^2}Rd\phi = \left[\frac{\mu_0 i\sin\theta}{4\pi r}\phi\right]_0^{\pi} = \frac{\mu_0 i\sin\theta\pi}{4\pi r} = \frac{\mu_0 i\sin\theta}{4r}.$$

The angle θ between the current and the radial vector $\hat{r}$ is $90°$ for the loop, thus $B = \mu_0 i/(4r)$.

CALCULATE: $B = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(12.0\text{ A}\right)}{4\left(0.100\text{ m}\right)} = 3.76991\cdot 10^{-5}\text{ T}$

ROUND: The values are given to three significant figures, thus the magnetic field produced by the wire is $B = 3.77\cdot 10^{-5}\text{ T}$ and points into the page.
DOUBLE-CHECK: The magnetic field is very small, as would be expected from a real-world point of view.

28.37. **THINK:** Each of the wires creates a magnetic field at the origin. The sum of these fields and the Earth's magnetic field will produce a force on the compass, causing it to align with the total field. The wires carry a current of $i_1 = i_2 = 25.0$ A. The Earth's magnetic field is $\vec{B}_{\text{E}} = 2.6\cdot 10^{-5}\,\hat{y}$ T.
SKETCH:



RESEARCH: The magnetic field produced by a wire is $B = \mu_0 i/(2\pi d)$.

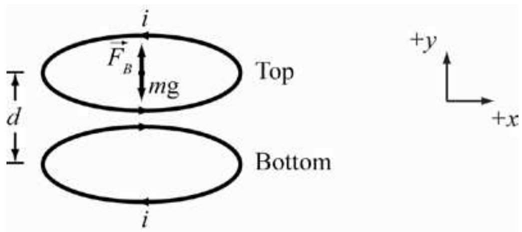
SIMPLIFY: The magnetic field of wire 1 is $\vec{B}_1 = \mu_0 i_1(-\hat{y})/(2\pi d_1)$. Wire 2 produces a magnetic field of $\vec{B}_2 = \mu_0 i_2\hat{x}/(2\pi d_2)$. The sum of the magnetic fields is $\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 + \vec{B}_{\text{E}} = -\frac{\mu_0 i_1}{2\pi d_1}\hat{y} + \frac{\mu_0 i_2}{2\pi d_2}\hat{x} + \vec{B}_{\text{E}}$.

CALCULATE:
$$\vec{B}_{\text{net}} = -\frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(25.0\text{ A}\right)}{2\pi\left(0.15\text{ m}\right)}\hat{y} + \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(25.0\text{ A}\right)}{2\pi\left(0.090\text{ m}\right)}\hat{x} + 2.6\cdot 10^{-5}\text{ T}\hat{y}$$
$$= 5.5555\cdot 10^{-5}\text{ T}\hat{x} - 7.3333\cdot 10^{-6}\text{ T}\hat{y}$$

The direction of the field is $\theta = \tan^{-1}\left(\dfrac{-7.3333\cdot 10^{-6}\text{ T}}{5.5555\cdot 10^{-5}\text{ T}}\right) = -7.5196°.$

ROUND: The angle is accurate to two significant figures. The compass points $7.5°$ below the x-axis.
DOUBLE-CHECK: This is a reasonable answer. The compass points towards the east if $\hat{y}$ is north.

28.38. **THINK:** The coil will levitate if the force from the magnetic field cancels the force of gravity. The coils have radii of $R = 20.0$ cm. The current of the bottom coil is i and travels in the clockwise direction. By the right hand rule the top coil has a current of the same magnitude, moving in a counter clockwise direction. The mass of the coils is $m = 0.0500$ kg. The distance between the coils is $d = 2.00$ mm.
SKETCH:



RESEARCH: The force of gravity is $F_g = mg$. The magnetic force on the top coil due to the bottom coil is $F_B = \mu_0 i_1 i_2 L/(2\pi d) = \mu_0 i_1 i_2 2\pi R/(2\pi d).$

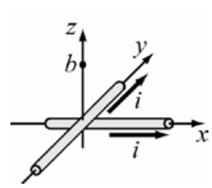
SIMPLIFY: Equating the two forces give $mg = \dfrac{\mu_0 i_1 i_2 2\pi R}{2\pi d} = \dfrac{\mu_0 i^2 R}{d}$. The amount of current is $i^2 = \dfrac{mgd}{\mu_0 R}$ or $i = \sqrt{\dfrac{mgd}{\mu_0 R}}.$

CALCULATE: $i = \sqrt{\dfrac{(0.0500\text{ kg})(9.81\text{ m/s}^2)(0.00200\text{ m})}{(4\pi\cdot 10^{-7}\text{ T m/A})(0.200\text{ m})}} = 62.476\text{ A}$

ROUND: Reporting to 3 significant figures, the current in the coils is 62.5 A and travel in opposite directions.
DOUBLE-CHECK: Dimensional analysis provides a check:

$$i = \sqrt{\frac{[\text{kg}][\text{m/s}^2][\text{m}]}{[T][\text{m/A}][\text{m}]}} = \sqrt{\frac{[\text{kg}][\text{m}][\text{m}][\text{A}]}{[\text{N/(A m)}][\text{m}][\text{m}][\text{s}^2]}} = \sqrt{\frac{[\text{kg}][\text{m}][\text{m}][\text{A}][\text{A}][\text{s}^2][\text{m}]}{[\text{kg}][\text{m}][\text{m}][\text{m}][\text{s}^2]}} = [\text{A}].$$

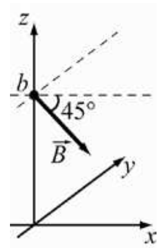
28.39. **THINK:** The current carrying wires along the x- and y-axes will each generate a magnetic field. The superposition of these fields generates a net field. The magnitude and direction of this net field at a point on the z-axis is to be determined.
SKETCH:



RESEARCH: Both currents produce a magnetic field with magnitude $B = \mu_0 i/(2\pi r)$. The magnetic field produced by the wire along the x-axis gives $\overline{B}_1 = \mu_0 i(-\hat{y})/(2\pi b)$. The wire along the y-axis creates a magnetic field of $\overline{B}_2 = \mu_0 i\hat{x}/(2\pi b)$.

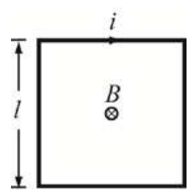
SIMPLIFY: The total magnetic field is then $\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{2\pi b}\hat{x} - \frac{\mu_0 i}{2\pi b}\hat{y}$. The magnitude of the field is $B = \frac{\mu_0 i}{2\pi b}\sqrt{1^2 + (-1)^2} = \frac{\sqrt{2}\mu_0 i}{2\pi b} = \frac{\mu_0 i}{\sqrt{2}\pi b}$. The direction of the field is $\theta = \tan^{-1}\left(\frac{-\mu_0 i}{\sqrt{2}\pi b}\bigg/\frac{\mu_0 i}{\sqrt{2}\pi b}\right)$ in the *x*-*y* plane at a height of *b*.

CALCULATE: $\tan^{-1}(-1) = -45°$ in the *x*-*y* plane at point *b*.



ROUND: Not applicable.
DOUBLE CHECK: Both the right hand rule and the symmetry of the problem indicates that the net field should be in the fourth quadrant.

28.40. **THINK:** The loop creates a magnetic field at its center by the Biot-Savart Law. The loop has side length $l = 0.100$ m and carries a current of $i = 0.300$ A.
SKETCH:



RESEARCH: The Biot-Savart Law states $dB = \frac{\mu_0}{4\pi}\cdot\frac{i\sin\theta ds}{r^2}$. The angle θ is found by using the equations: $\sin\theta = d/r$, $r = \sqrt{s^2 + d^2}$, and $d = l/2$.

SIMPLIFY: The field due to one side of the loop is $dB = \frac{\mu_0 i}{4\pi}\cdot\frac{d}{r^2}ds = \frac{\mu_0 id}{4\pi}\frac{ds}{\left(s^2+d^2\right)^{3/2}}$. Since there are four sides, the total loop is four times this value. The total magnetic field is then

$$B = \int dB = 4\int_{-d}^{d}\frac{\mu_0 id}{4\pi}\cdot\frac{ds}{\left(s^2+d^2\right)^{3/2}} = 4\int_0^d \frac{\mu_0 id}{4\pi}\frac{2ds}{\left(s^2+d^2\right)^{3/2}}$$

$$= \frac{2\mu_0 id}{\pi}\left[\frac{s}{d^2\sqrt{s^2+d^2}}\right]_0^d = \frac{2\mu_0 i}{\pi d}\left(\frac{d}{\sqrt{2d^2}} - \frac{0}{\sqrt{d^2}}\right) = \frac{2\mu_0 i}{\sqrt{2}\pi d} = \frac{\sqrt{8}\mu_0 i}{\pi l}$$

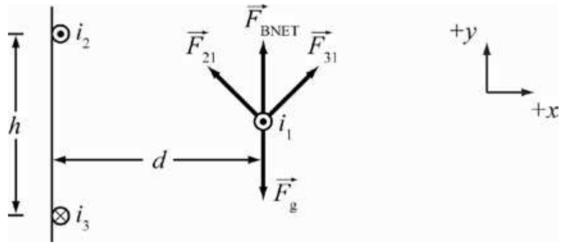
CALCULATE: $B = \frac{\sqrt{8}\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(0.300\text{ A}\right)}{\pi\left(0.100\text{ m}\right)} = 3.394\cdot 10^{-6}\text{ T}$

ROUND: To three significant figures, the magnetic field at the center of the loop is $B = 3.39\cdot 10^{-6}$ T.
DOUBLE-CHECK: The current is small, so the magnetic field it generates is expected to be small. This is a reasonable value.

28.41. **THINK:** In order for wire 1 to levitate, the forces on it must cancel. Both wire 2 and 3 will create magnetic fields that will interact with wire 1. Both wires create forces with horizontal and vertical components. The horizontal components will add destructively. The vertical components however will add constructively.

Therefore, only the vertical components need be calculated. Wires 2 and 3 each carry a current of $i = 600.$ A. All three wires have a linear mass density of $\lambda = 100.$ g/m. The wires are arranged as shown in the figure.

SKETCH:



RESEARCH: The force of gravity on the wire is $F_g = mg$. The force between two wires carrying current is $F_{21} = \mu_0 i_1 i_2 L / (2\pi d)$.

SIMPLIFY: The vertical component of the magnetic force for one wire is $F_{31} = \dfrac{\mu_0 i_3 i_1 L}{2\pi(h/2)} = \dfrac{\mu_0 i_3 i_1 L}{\pi h}$. The total force due to the wires is then $F_B = 2F_{31} = \dfrac{2\mu_0 i_3 i_1 L}{\pi h}$. Equating this to the force of gravity gives: $mg = \lambda L g = \dfrac{2\mu_0 i_3 i_1 L}{\pi h}$. Solving for the current i_1 gives: $i_1 = \dfrac{\pi h \lambda g}{2\mu_0 i_3}$.

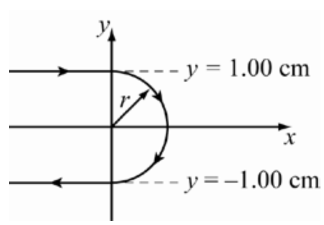
CALCULATE: $i_1 = \dfrac{\pi(0.100 \text{ m})(100.\cdot 10^{-3} \text{ kg/m})(9.81 \text{ m/s}^2)}{2\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)(600. \text{ A})} = 204.375 \text{ A}$

ROUND: The current of wire 1 required to levitate is $i_1 = 204$ A.

DOUBLE-CHECK: The current in wire 1 is on the same order of magnitude as the other currents. This is a reasonable answer.

28.42. **THINK:** The net field is a superposition of the fields created by the top wire, the bottom wire and the loop. The wires are 2.00 cm apart and carry a current of $i = 3.00$ A. The radius of the loop is $r = 1.00$ cm.

SKETCH:



RESEARCH: The magnetic field produced by an infinite wire is $B = \mu_0 i / (2\pi r)$. A semi-infinite wire is half this value, $B = \mu_0 i / (4\pi r)$. A full loop produces a magnetic field of $B = \mu_0 i / (2r)$. The half loop produces half of this, $B = \mu_0 i / (4r)$.

SIMPLIFY: By the right hand rule, the magnetic field points into the page. The magnetic field is the sum of all the fields.

$$B_{\text{net}} = B_{\text{top}} + B_{\text{bottom}} + B_{\text{loop}} = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{4r}\left(\frac{2}{\pi} + 1\right)$$

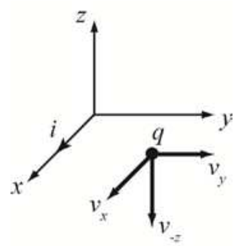
CALCULATE: $B_{\text{net}} = \dfrac{\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)(3.00 \text{ A})}{4(0.0100 \text{ m})}\left(\dfrac{2}{\pi} + 1\right) = 1.54 \cdot 10^{-4}$ T. It is directed in the negative z direction.

ROUND: To 3 significant figures, the magnetic field at the origin is $-1.54 \cdot 10^{-4} \text{ T}\hat{z}$.

DOUBLE-CHECK: The field due to a single infinite wire similar to the wires in the problem would be $B=\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(3.00\text{ A}\right)/\left(2\pi\left(0.0100\text{ m}\right)\right)=6.00\cdot 10^{-5}\text{ T},$ which is similar to the result. Therefore, the result is reasonable.

28.43. **THINK:** The wire creates a magnetic field that produces a Lorentz force on the moving charged particle. The question asked for the force if the particle travels in various directions. The velocity is 3000 m/s in various directions.

SKETCH:



RESEARCH: The magnetic field produced by an infinite wire is $B=\mu_0 i/\left(2\pi d\right)$. By the right hand rule the field points in the positive z-direction. The force produced by the magnetic field is $\vec{F}=q\vec{v}\times\vec{B}$.

SIMPLIFY: The force is given by $\vec{F}=q\vec{v}\times\vec{B}=\dfrac{q\mu_0 i}{2\pi d}\vec{v}\times\hat{z}=\dfrac{q\mu_0 i}{2\pi d}\left(\left|\vec{v}\right|\cdot\hat{n}\times\hat{z}\right)$ where $\hat{n}$ is the direction of the particle.

CALCULATE: $\vec{F}=\dfrac{\left(9.00\text{ C}\right)\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(7.00\text{ A}\right)}{2\pi\left(2.00\text{ m}\right)}\left(3000.\text{ m/s}\cdot\hat{n}\times\hat{z}\right)=1.89\cdot 10^{-2}\text{ N}\left(\hat{n}\times\hat{z}\right)$

Note that $\hat{x}\times\hat{z}=-\hat{y},\;\;\hat{y}\times\hat{z}=\hat{x},\;$ and $-\hat{z}\times\hat{z}=0.$

ROUND: The force should be reported to 3 significant figures.

(a) The force is $\vec{F}=-1.89\cdot 10^{-2}\text{ N}\,\hat{y}$ if the particle travels in the positive x-direction.

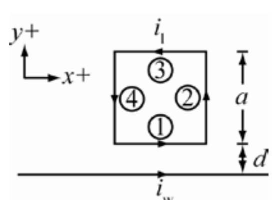
(b) The force is $\vec{F}=1.89\cdot 10^{-2}\text{ N}\,\hat{x}$ if the particle travels in the positive y-direction.

(c) The force is $F=0$ if the particle travels in the negative z-direction.

DOUBLE-CHECK: The right hand rule confirms the directions of the forces for each direction of motion of the particle.

28.44. **THINK:** The wire produces a magnetic field that creates a force on the loop. The wire has current of $i_\text{w}=10.0\text{ A}$ and is $d=0.500\text{ m}$ away from the bottom wire of the loop. The loop carries a current of $i_1=2.00\text{ A}$ and has sides of length $a=1.00$ m.

SKETCH:



RESEARCH: The force on two wires carrying a current is $F=\mu_0 i_1 i_2 L/\left(2\pi d\right)$. The torque is given by $\vec{\tau}=\vec{r}\times\vec{F}.$

SIMPLIFY: The forces on part ② and ④ cancel each other. The force on ① is $F_1=\mu_0 i_\text{w} i_1 a/\left(2\pi d\right)$ and points towards the long wire. The force on ③ is $F_3=\mu_0 i_\text{w} i_1 a/\left[2\pi(d+a)\right]$ and points away from the long wire. The total force is then $F_{\text{net}} = \vec{F}_1 + \vec{F}_3 = \dfrac{\mu_0 i_{\text{w}} i_1 a}{2\pi}\left(\dfrac{1}{d} - \dfrac{1}{d+a}\right)$ and points towards the long wire. Because the force and the length between the axis of rotation are parallel there is no torque on the loop.

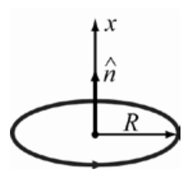
CALCULATE:

$$F_{\text{net}} = \frac{\left(4\pi \cdot 10^{-7} \text{ Tm/A}\right)(10.0 \text{ A})(2.00 \text{ A})\left(1.00 \text{ m}\right)}{2\pi}\left(\frac{1}{0.500 \text{ m}} - \frac{1}{1.50 \text{ m}}\right) = -5.33333 \cdot 10^{-6}\,\hat{y} \text{ N}$$

ROUND: The force is reported to three significant figures. (a) The net force between the loop and the wire is $F = -5.33 \cdot 10^{-6}\,\hat{y}$ N. (b) There is no net torque on the loop.

DOUBLE-CHECK: The force between the long wire and the lower arm of the loop is attractive, because the currents are in the same direction. The currents of the long wire and the upper arm of the loop are in opposite directions, therefore the force is repulsive. Since the lower arm is closer to the long wire, the attractive force dominates, and the net force is in the negative y direction, as calculated.

28.45. The magnetic field at the center of the box is the sum of the fields produced by the coils. A coil produces a magnetic field of $B = \dfrac{\mu_0 N i R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{n}.$

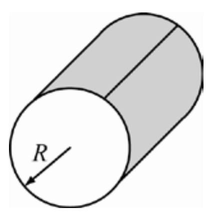


The magnetic field produced by the coil on the $x - z$ plane is

$$B_{\text{xz}} = \frac{\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)(30.0)(5.00 \text{ A})(0.500 \text{ m})^2}{2\left[(0.500 \text{ m})^2 + (0.500 \text{ m})^2\right]^{3/2}}(+\hat{y}) = 6.66 \cdot 10^{-5} \text{ T}\hat{y}$$

The magnetic field produced by the other coil has the same magnitude but points in the negative x-direction. Therefore $B_{\text{tot}} = 6.66 \cdot 10^{-5}$ T$[-\hat{x} + \hat{y}]$. The magnitude of the field is $\sqrt{2} \cdot 6.66 \cdot 10^{-5}$ T, or $9.42 \cdot 10^{-5}$ T. The direction of the field is at an angle of 45° from the negative x-direction towards the positive y-axis.

28.46.



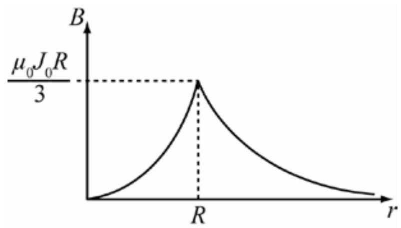
The current within a loop of radius $\rho \le R$ is given by

$$i = \int J(r)dA = 2\pi\int_0^r J(r')r'dr' = 2\pi J_0 \int_0^r \frac{r'}{R}r'dr' = \frac{2\pi J_0}{R}\int_0^r r'^2 dr' = \frac{2\pi J_0}{R}\frac{r'^3}{3}\bigg|_0^r = \frac{2\pi J_0 r^3}{3R}.$$

The magnetic field is given by Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = B\left(2\pi r\right) = \mu_0 i_{\text{enc}} \;\Rightarrow\; B = \frac{\mu_0 i_{\text{enc}}}{2\pi r}.$$

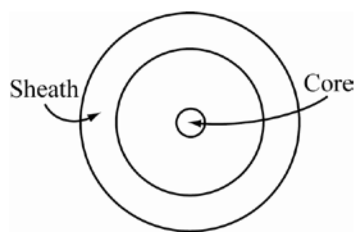
The magnetic field in the region $r<R$ is $B=\frac{\mu_0}{2\pi r}\left(\frac{2\pi J_0 r^3}{3R}\right)=\frac{\mu_0 J_0 r^2}{3R}$. The magnetic field in the region $r>R$ is $B=\frac{\mu_0}{2\pi r}\left(\frac{2\pi J_0 R^3}{3R}\right)=\frac{\mu_0 J_0 R^2}{3r}$.



28.47. Using Ampere's Law, the magnetic field at various points can be determined. $\oint \vec{B}\bullet d\vec{s}=\mu_0 i_{\text{enclosed}}$. For the cylinder, assuming the current is distributed evenly, $B2\pi r=\mu_0 i_{\text{enc}}$ or $B=\mu_0 i_{\text{enc}}/(2\pi r)$. The field at $r=r_{\text{a}}=0$ is zero since is does not enclose any current $B_{\text{a}}=0$. The field at $r=r_{\text{b}}<R$ is $B_{\text{b}}=\frac{\mu_0 i_{\text{enc}}}{2\pi r_{\text{b}}}=\frac{\mu_0}{2\pi r_{\text{b}}}\left(i_{\text{tot}}\frac{\pi r_{\text{b}}^2}{\pi R^2}\right)=\frac{\mu_0 i r_{\text{b}}}{2\pi R^2}=\frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(1.35\text{ A})(0.0400\text{ m})}{2\pi(0.100\text{ m})^2}=1.08\cdot 10^{-6}\text{ T}$. Note that i is equal to the fraction of total area of the conductor's cross section and the total current. The field at $r_{\text{c}}=R$ is $B_{\text{c}}=\frac{\mu_0 i_{\text{enc}}}{2\pi r_{\text{c}}}=\frac{\mu_0 i_{\text{tot}}}{2\pi R}=\frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(1.35\text{ A}\right)}{2\pi\left(0.100\text{ m}\right)}=2.70\cdot 10^{-6}\text{ T}$. The field at $r_{\text{d}}>R$ is $B_{\text{d}}=\frac{\mu_0 i_{\text{enc}}}{2\pi r_{\text{d}}}=\frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(1.35\text{ A}\right)}{2\pi\left(0.160\text{ m}\right)}=1.69\cdot 10^{-6}\text{ T}$. By inspection it can be seen that the magnetic field at r_b, r_c and r_d the magnetic field will point to the right.

28.48. **THINK:** The magnetic field is the sum of the field produced by the wire core B_{C} and the sheath B_{S}. The wire has a radius of $a=1.00$ mm. The sheath has an inner radius of $b=1.50$ mm and outer radius of $c=2.00$ mm. The current of the outer sheath opposes the current in the core.

SKETCH:



RESEARCH: The current density of the core is $J_{\text{C}}=i/(\pi a^2)$ and the current density of the sheath is $J_{\text{S}}=-i/[\pi(c^2-b^2)]$. The enclosed current is calculated by $i_{\text{enclosed}}=\int J dA$. The magnetic field is derived using Ampere's Law: $\oint B\cdot ds=\mu_0 i_{\text{enclosed}}$.

SIMPLIFY: When the radius is within the core , $r\le a$, the magnetic field is

$$\oint B\cdot ds=B_{r\le a}2\pi r=\mu_0 i_{\text{enc}}=\mu_0\int J dA=\mu_0\frac{i}{\pi a^2}\int_0^r\int_0^{2\pi} r d\theta dr$$
$$=\frac{\mu_0 i}{\pi a^2}\frac{2\pi r^2}{2}=\mu_0 i\frac{r^2}{a^2}$$

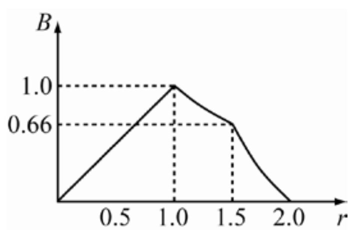
or $B_{r\le a} = \frac{\mu_0 i}{2\pi r}\frac{r^2}{a^2} = \frac{\mu_0 ir}{2\pi a^2}$. If the radius is between the core and the sheath, $a < r < b$, $\oint \vec{B}\bullet d\vec{s} = B_{a<r\le b} 2\pi r = \mu_0 i$ or $B_{a<r\le b} = \mu_0 i/(2\pi r)$. Within the sheath, $b < r < c$, the magnetic field is $\oint \vec{B}\bullet d\vec{s} = B_{b<r<c} 2\pi r = \mu_0 i_{\text{enc}} = \mu_C\left(i + \int_b^r\int_0^{2\pi}\frac{-i}{\pi(c^2-b^2)}rd\theta dr\right) = \mu_0\left(i - \frac{i}{(c^2-b^2)}\frac{2\pi r^2}{\pi 2}\Big|_b^r\right) = \mu_0 i\left[1-\frac{r^2-b^2}{c^2-b^2}\right]$

$$B_{b<r<c} = \frac{\mu_0 i}{2\pi r}\left[1-\frac{r^2-b^2}{c^2-b^2}\right]$$

If the radius is outside of the cable, $r \ge c$, then the magnetic field is $\oint B\cdot ds = B_{r\ge c}2\pi r = \mu_0 i_{\text{enc}} = \mu_0(i-i) = 0$ or $B_{r\ge c} = 0$. In summary the magnetic fields of various regions are

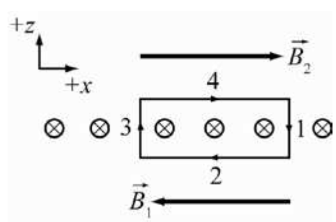
$$B_{r\le a} = \frac{\mu_0 ir}{2\pi a^2},\ B_{a<r\le b} = \frac{\mu_0 i}{2\pi r},\ B_{b<r<c} = \frac{\mu_0 i}{2\pi r}\left[1-\frac{r^2-b^2}{c^2-b^2}\right],\ B_{r\ge c} = 0.$$

CALCULATE: In order to graph the behavior of the magnetic field as a function of the radius, set the magnetic field in units of $\frac{u_0 i}{2\pi a}$.



ROUND: There is no need to round.
DOUBLE-CHECK: Note that the magnetic field outside of the coaxial cable is zero. These cables are used when equipment that is sensitive to magnetic fields needs current.

28.49. **THINK:** To find the magnetic field above the center of the surface of a current carrying sheet, use Ampere's Law. The path taken should be far from the edges and should be rectangular as shown in the diagram. The current density of the sheet is $J = 1.5$ A/cm.
SKETCH:



RESEARCH: The direction of the magnetic field is found using the right hand rule to be $+x$ above the surface of the conductor. Ampere's Law states $\oint \vec{B}\bullet d\vec{s} = B2\pi r = \mu_0 i_{\text{enclosed}}$.

SIMPLIFY: Note that sections 1 and 3 are perpendicular the field. $B\cdot ds = 0$ for these two sections. If the path of 4 and 2 has a length of L, then by Ampere's Law, $\oint B\cdot ds = B_1 L + B_2 L = \mu_0 i_{\text{enclosed}} = \mu_0 JL$. By symmetry $B_1 = B_2$. Thus, $2B_1 = \mu_0 J$ or $B_1 = \mu_0 J/2$.

CALCULATE: $B_1 = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(1.5\text{ A/cm})(100\text{ cm/m})}{2} = 9.42478\cdot 10^{-5}\text{ T}$

ROUND: The magnetic field is accurate to two significant figures. The magnetic field near the surface of the conductor is $B_1 = 9.4\cdot 10^{-5}$ T.

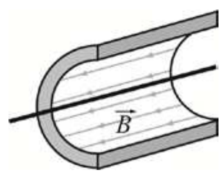
DOUBLE-CHECK: The form for the magnetic field is similar to that of a solenoid. It is divided by a factor of 2, which makes sense when considering the setup of a solenoid. The form of the equation is similar to that of question 28.12. This makes sense because the magnetic field inside a solenoid is generated by a current carrying wire on both sides of the Amperian loop, whereas the field generated by the flat conducting surface originates on one side of the Amperian loop only. In effect, the flat conductor can be seen as similar to half a solenoid, flattened out. See figure 28.21 in the text for a visual.

28.50. The magnetic field in a solenoid is given by the equation:

$$B=\mu_0 ni=\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(\frac{1000}{0.400\text{ m}}\right)\left(2.00\text{ A}\right)=6.28\cdot 10^{-3}\text{ T}.$$

28.51. The magnetic field in a solenoid is given by $B=\mu_0 in.$ Let the magnetic field of solenoid B be $B_B=\mu_0 in.$ The magnetic field of solenoid A is $B_A=\mu_0 i\left(4\,N\right)/\left(3\,L\right)=\left(4/3\right)\mu_0 in=\left(4/3\right)B_B.$ The ratio of solenoid A magnetic field to that of solenoid B is 4:3.

28.52. The magnetic field at a point $r=1.00$ cm from the axis of the solenoid will be the sum of the field due to the solenoid and the field produced by the wire. The solenoid has a magnetic field of $B_\text{S}=\mu_0 i_\text{S} n$ along the axis of the solenoid.



The wire produces a field which is perpendicular to the radial vector of $B_\text{w}=\mu_0 i_\text{w}/\left(2\pi r\right).$ The magnitude of the field is then

$$B_\text{tot}=\sqrt{{B_\text{S}}^2+{B_\text{w}}^2}=\mu_0\sqrt{(i_\text{S}n)^2+(i_\text{w}/2\pi r)^2}$$

$$B_\text{tot}=\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\sqrt{\left(\left(0.250\text{ A}\right)\left(1000\text{ m}^{-1}\right)\right)^2+\left(\frac{\left(10.0\text{ A}\right)}{2\pi\left(0.0100\text{ m}\right)}\right)^2}=3.72\cdot 10^{-4}\text{ T}.$$

28.53. (a) The magnetic field produced by the wire is

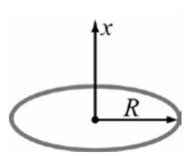
$$B=\mu_0 i/\left(2\pi r\right)=\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(2.5\text{ A}\right)/\left(2\pi\left(0.039\text{ m}\right)\right)=1.3\cdot 10^{-5}\text{ T}.$$

(b) The magnetic field of the solenoid is

$$B=\mu_0 in=\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(2.5\text{ A}\right)\left(\frac{32}{0.01\text{ m}}\right)=0.010\text{ T}=1.0\cdot 10^{-2}\text{ T}.$$

This field is much larger for the solenoid than the wire.

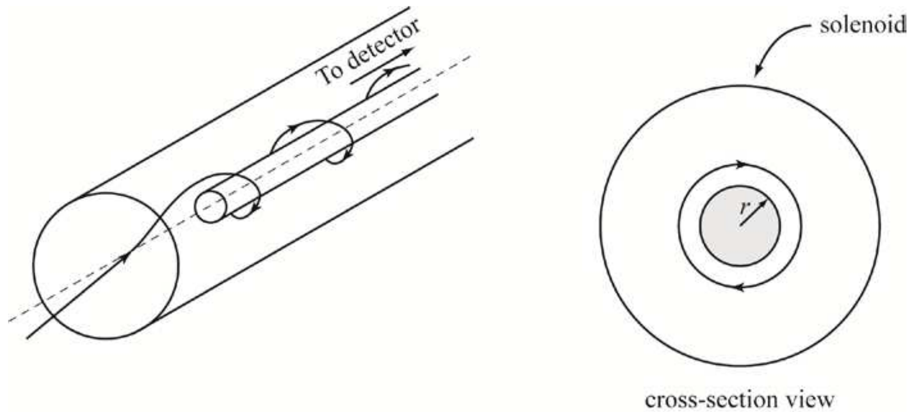
28.54. The magnetic field of a loop is $B=\dfrac{\mu_0 i}{2}\dfrac{R^2}{(x^2+R^2)^{3/2}}.$



Therefore a coil of N loops produces a field of $B = \frac{\mu_0 i N}{2}\frac{R^2}{(x^2+R^2)^{3/2}}$. Let $x = R/2$ gives $B = \frac{\mu_0 i N}{2}\frac{R^2}{(x^2+R^2)^{3/2}} = \frac{\mu_0 i N}{2R^3}\frac{R^2}{(5/4)^{3/2}} = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 i N}{2R}$. The field at the center of the coils is then $B_{\text{tot}} = 2B = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 i N}{R} = \left(\frac{4}{5}\right)^{3/2}\frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(0.123\text{ A})(15)}{(0.750\text{ m})} = 2.21\cdot 10^{-6}\text{ T}.$

28.55. **THINK:** If the perpendicular momentum of a particle is not large enough, its radius of motion will not be large enough to enter the detector. The minimum momentum perpendicular to the axis of the solenoid is determined by a condition such that the centripetal force is equal to the force due to the magnetic field.

SKETCH:



RESEARCH: Since the particle originates from the axis of the detector, the minimum radius of the circular motion of the particle must be equal to the radius of the detector as shown above. The magnetic force on the particle is $F = qvB$. Centripetal acceleration is $a_C = v^2/r$. The magnetic field due to the solenoid is $B = \mu_0 i n$.

SIMPLIFY: Using Newton's Second Law, the momentum is $qvB = mv^2/r \Rightarrow mv = p = qrB$. Therefore, the minimum momentum is $p = \mu_0 q r i n$.

CALCULATE: Substituting the numerical values yields.

$$p = \left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\left(1.602\cdot 10^{-19}\text{C}\right)\left(0.80\text{ m}\right)\left(22\text{ A}\right)\left(550\cdot 10^{2}\text{ m}^{-1}\right) = 1.949\cdot 10^{-19}\text{ kg m/s}$$

ROUND: Rounding the result to two significant figures gives $p = 1.9\cdot 10^{-19}$ kg m/s.

DOUBLE-CHECK: This is a reasonable value.

28.56. The magnetic potential energy of a magnetic dipole in an external magnetic field is given by $U = -\vec{\mu}\bullet\vec{B}$. Therefore, the magnitude of the difference in energy for an electron "spin up" and "spin down" is $\Delta U = \left|U_{\text{up}} - U_{\text{down}}\right| = 2\mu B$. This means the magnitude of the magnetic field is $B = \Delta U / 2\mu$.

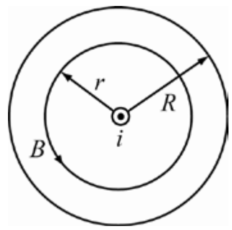
Putting in the numerical values gives $B = \frac{9.460\cdot 10^{-25}\text{ J}}{2\left(9.285\cdot 10^{-24}\text{ A m}^2\right)} = 0.05094\text{ T}.$

28.57. The energy of a dipole in a magnetic field is $U = -\vec{\mu}\bullet\vec{B}$. The dipole has its lowest energy $U_{\min} = -\vec{\mu}\bullet\vec{B} = -\mu B$, and its highest energy $U_{\max} = \mu B$. The energy required to rotate the dipole from its lowest energy to its highest energy is $\Delta U = 2\mu B$. This means that the thermal energy needed is ΔU which corresponds to a temperature $T = \Delta U / k_B = 2\mu B / k_B$.

Substituting the numerical values of the dipole moment of hydrogen atom and $B = 0.15$ T yields

$$T = \frac{2\left(9.27 \cdot 10^{-24} \text{ J/T}\right)\left(0.15 \text{ T}\right)}{\left(1.38 \cdot 10^{-23} \text{ J/K}\right)} = 0.20 \text{ K}.$$

28.58.



The magnetic permeability of aluminum is $\mu = \left(1 + \chi_{\text{Al}}\right)\mu_0$. Applying Ampere's Law around an Amperian loop of radius r gives

$$\oint \vec{B} \cdot d\vec{s} = B\oint ds = B(2\pi r) = \mu i_{\text{enc}}.$$

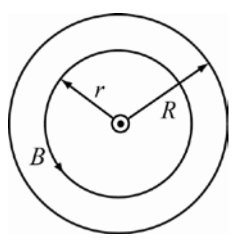
The current enclosed by the Amperian loop is $i_{\text{enc}} = i\dfrac{\pi r^2}{\pi R^2}$. Therefore, the magnetic field inside a wire is given by $B = \dfrac{\mu i r}{2\pi R^2}$. This means the maximum magnetic field is located at the surface of the wire where the magnitude is $B = \dfrac{\mu i}{2\pi R}$. Thus, the maximum current is

$$i_{\max} = \frac{2\pi R B_{\max}}{\left(1 + \chi_{\text{Al}}\right)\mu_0} = \frac{2\pi\left(1.0 \cdot 10^{-3} \text{ m}\right)\left(0.0105 \text{ T}\right)}{\left(1 + \left(2.2 \cdot 10^{-5}\right)\right)\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)} = 52 \text{ A}.$$

28.59. The magnitude of the magnetic field inside a solenoid is given by $B = \mu i n = \kappa_{\text{m}}\mu_0 i(N/L)$. Thus the relative magnetic permeability κ_{m} is given by the equation:

$$\kappa_{\text{m}} = \frac{BL}{\mu_0 i N} = \frac{\left(2.96 \text{ T}\right)\cdot\left(3.50 \cdot 10^{-2} \text{ m}\right)}{\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)\cdot\left(3.00 \text{ A}\right)\cdot\left(500.\right)} = 54.96 \approx 55.0.$$

28.60.



The magnetic permeability of tungsten is $\mu = \left(1 + \chi_{\text{W}}\right)\mu_0$. Applying Ampere's Law around an Amperian loop of radius r gives

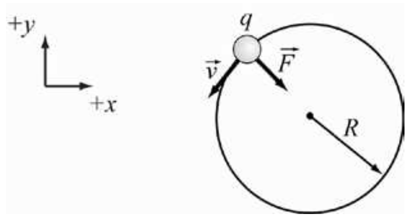
$$\oint \vec{B} \cdot d\vec{s} = B\oint ds = B(2\pi r) = \mu i_{\text{enc}}.$$

The current enclosed by the Amperian loop is $i_{\text{enc}} = i\dfrac{\pi r^2}{\pi R^2}$. Therefore, the magnetic field is

$$B = \left(\frac{\left(1 + \chi_{\text{W}}\right)\mu_0 i}{2\pi R^2}\right) r = \frac{\left(1 + 6.8 \cdot 10^{-5}\right)\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)\left(3.5 \text{ A}\right)\left(0.60 \cdot 10^{-3} \text{ m}\right)}{2\pi\left(1.2 \cdot 10^{-3} \text{ m}\right)^2} = 2.9 \cdot 10^{-3} \text{ T}.$$

28.61. **THINK:** To determine the magnetic moment, the effective current of the system is needed. This implies the speed of the ball is required.

SKETCH:



RESEARCH: The ball travels in a circular orbit and it travels a distance of $2\pi R$ in time T, where T is the time for one revolution. The effective current is given by $i = q/T$. Since $T = 2\pi R/v$, this becomes $i = qv/(2\pi R)$. The effective magnetic moment is $\mu = iA = qv\pi R^2/(2\pi R) = qvR/2$. From the centripetal force, it is found that the speed is $mv^2/R = F \Rightarrow v = \sqrt{FR/m}$.

SIMPLIFY: Combining the above results yields $\mu = \frac{1}{2}q\sqrt{\frac{FR}{m}}R$.

CALCULATE: Putting in the numerical values gives

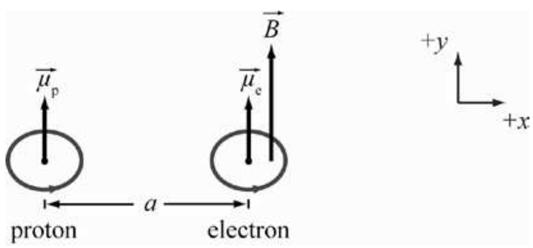
$$\mu = \frac{1}{2}\left(2.00\cdot 10^{-6}\ C\right)\sqrt{\frac{(25.0\text{ N})(1.00\text{ m})}{0.200\text{ kg}}}(1.00\text{ m}) = 1.118\cdot 10^{-5}\text{ A m}^2.$$

ROUND: Keeping 3 significant figures gives $\mu = 1.12\cdot 10^{-5}$ A m^2.

DOUBLE-CHECK: This magnetic moment is appropriately small for a small charge moving at a low velocity.

28.62. **THINK:** The magnetic field due to a proton is modeled as a dipole field. Using the value of the magnetic field, the potential energy of an electron spin in the magnetic field is $U = -\vec{\mu}\cdot\vec{B}$.

SKETCH:



RESEARCH: The electron field due to an electric dipole is given by $\vec{E} = \vec{P}/\left(2\pi\varepsilon_0 R^3\right)$. The corresponding magnetic field is obtained by replacing $1/(4\pi\varepsilon_0)$ with $\mu_0/(4\pi)$ and $\vec{P}$ with $\vec{\mu}$. Thus, $\vec{B} = \mu_0\vec{\mu}/\left(2\pi R^3\right)$.

SIMPLIFY: The energy difference between two electron-spin configurations is

$$\begin{aligned}\Delta U &= U_{\text{anti}} - U_{\text{parallel}} \\ &= -(-\vec{\mu}_e)\bullet\vec{B} - (-\vec{\mu}_e\bullet\vec{B}) \\ &= 2\vec{\mu}_e\bullet\vec{B} = 2\vec{\mu}_e\bullet\frac{\mu_0\vec{\mu}_P}{2\pi a_0^3} \\ &= \frac{\mu_0\mu_e\mu_P}{\pi a_0^3}\end{aligned}$$

CALCULATE: Inserting all the numerical values yields

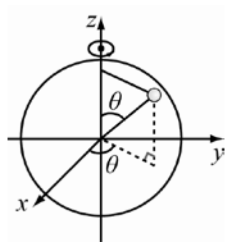
$$\Delta U = \frac{\left(4\pi \cdot 10^{-7}\text{ T m/A}\right)\left(9.27 \cdot 10^{-24}\text{ J/T}\right)\left(1.41 \cdot 10^{-26}\text{ J/T}\right)}{\pi\left(5.292 \cdot 10^{-11}\text{ m}\right)^3} = 3.528 \cdot 10^{-25}\text{ J} = 2.204 \cdot 10^{-6}\text{ eV}.$$

ROUND: Rounding the result to three significant digits produces $\Delta U = 2.20 \cdot 10^{-6}$ eV.

DOUBLE-CHECK: This is reasonable. A small difference in potential is expected for these small particles.

28.63. **THINK:** The classical angular momentum of rotating object is related to its moment of inertia. To get the magnetic dipole of a uniformly changed sphere, the spherical volume is divided into small elements. Each element produces a current and a magnetic dipole moment. The dipole moment of all elements is then added to get the net dipole moment.

SKETCH:



RESEARCH:

(a) The classical angular momentum of the sphere is given by $L = I\omega = \left(2/5\right)mR^2\omega.$

(b) The current produced by a small volume element dV is $i = \rho dV\omega/(2\pi)$. Thus the magnetic dipole moment of this element is $d\mu = \frac{\rho\omega dV}{2\pi}\pi(r\sin\theta)^2$. Integrating all the elements gives

$$\mu = \int_0^{2\pi}\int_0^{\pi}\int_0^{R}\frac{\rho\omega r^2}{2}\left(\sin^2\theta\right)\left(r^2\sin\theta\right)dr\,d\theta\,d\phi.$$

(c) The gyromagnetic ratio is simply the ratio of the results from parts (a) and (b): $\gamma_e = \mu/L$.

SIMPLIFY:

(b) $$\mu = \frac{\rho\omega}{2}\cdot 2\pi\int_0^{\pi}\int_0^{R} r^4\sin^3\theta drd\theta$$

$$= \rho\pi\omega\int_0^{\pi}\sin^3\theta d\theta \cdot \int_0^{R} r^4 dr$$

$$= \rho\pi\omega\left[\int_{\cos 0}^{\cos\pi} -(1-\cos^2\theta)d\cos\theta\right]\frac{R^5}{5} = \rho\pi\omega\left[-x+\frac{x^3}{3}\right]_1^{-1}\frac{R^5}{5} = \rho\pi\omega\left(\frac{4}{3}\right)\frac{R^5}{5}$$

Since $\rho\frac{4}{3}\pi R^3 = q$, the magnetic moment becomes $\mu = q\omega R^2/5$.

(c) Taking the ratio of the magnetic dipole moment and the angular momentum yields:

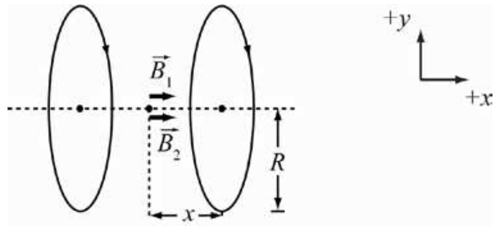
$$\gamma_e = \frac{\mu}{L} = \frac{\frac{q\omega R^2}{5}}{\frac{2}{5}mR^2\omega} = \frac{q}{2m}.$$ Substituting $q = -e$ gives: $\gamma_e = -e/\left(2m\right)$.

CALCULATE: Not required

ROUND: Not required

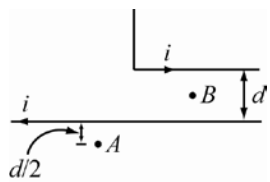
DOUBLE-CHECK: The magnetic dipole and the angular momentum should both be quadratic in *R*, so it is logical that the ratio of these two quantities is independent of *R*.

28.64.



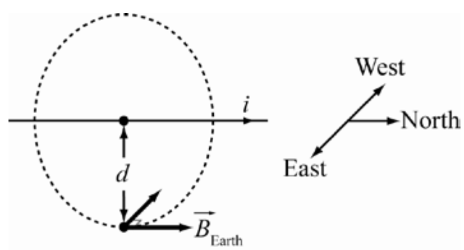
The magnitude of magnetic field due to one of the coils is $B_1 = \frac{\mu_0 i N_1}{2}\frac{R^2}{(x^2+R^2)^{3/2}}$. Since $B_1 = B_2$, the net magnetic field is $B = B_1 + B_2 = \frac{\mu_0 i N R^2}{(x^2+R^2)^{3/2}}$. Putting in $x = 0.500$ m, $R = 2.00$ m, $i = 7.00$ A and $N = 50$ yields $B = \frac{\left(4\pi \cdot 10^{-7}\text{ T m/A}\right)\left(7.00\text{ A}\right)\left(50\right)\left(2.00\text{ m}\right)^2}{\left[\left(0.500\text{ m}\right)^2 + \left(2.00\text{ m}\right)^2\right]^{3/2}} = 2.01 \cdot 10^{-4}$ T.

28.65.



Since the horizontal distance between points A and B is large compared to d, the magnetic field at point B can be approximated by two parallel wires carrying opposite currents. By the right hand rule, the magnetic field at point B is directed into the page from both currents. Since point B is a distance of $d/2$ away from each wire, the magnitude of magnetic field at point B is twice that at point A. So, the strength of the magnetic field at point B is $B = 2\left(2.00\text{ mT}\right) = 4.00$ mT.

28.66.



Applying the right hand rule gives the direction of the magnetic field due to the wire at the compass needle in the westward direction. The magnitude of B_{wire} is

$$B = \frac{\mu_0 I}{2\pi d} = \frac{\left(4\pi \cdot 10^{-7}\text{ T m/A}\right)\cdot 500.0\text{ A}}{2\pi \cdot 12.0\text{ m}} = 8.33\ \mu\text{T}.$$

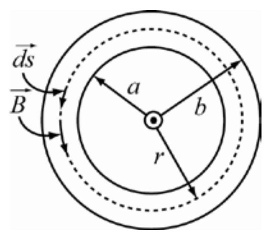
The deflection of the compass needle is $\delta\theta = \arctan\left(\frac{B_{\text{wire}}}{B_{\text{Earth}}}\right) = \arctan\left(\frac{8.33\ \mu\text{T}}{40.0\ \mu\text{T}}\right) = 11.8°$. The deflection is westward.

28.67. The magnetic dipole moment is defined as $\mu = iA = i\pi R^2$. This means the current that produces this magnetic dipole moment is $i = \mu / \left(\pi R^2\right)$. Substituting the numerical values gives the current of $i = \dfrac{8.0 \cdot 10^{22} \text{ A m}^2}{\pi (2.5 \cdot 10^6 \text{ m})^2} = 4.07 \cdot 10^9 \text{ A} \approx 4.1 \cdot 10^9 \text{ A}.$

28.68. The potential energy of a current loop in a magnetic field is given by $U = -\vec{\mu} \bullet \vec{B}$. The magnitude of the magnetic dipole moment is $\mu = iA = i\pi R^2$. The direction of the magnetic dipole moment can be determined using the right hand rule. In this case, the magnetic dipole is in the positive z-direction. Therefore, it follows that $\vec{\mu} = i\pi R^2 \hat{z} = 0.10 \text{ A} \cdot \pi \cdot \left(0.12 \text{ m}\right)^2 = 4.5 \cdot 10^{-3} \hat{z} \text{ A m}^2$. The energy is given by $U = -\vec{\mu} \bullet \vec{B} = \left(4.5 \cdot 10^{-3} \hat{z} \text{ A m}^2\right) \bullet \left(-1.5\hat{z} \text{ T}\right) = 6.8 \cdot 10^{-3} \text{ J}$. If the loop can move freely, the loop will rotate such that its magnetic dipole moment aligns with the direction of the magnetic field. This means the magnetic dipole moment is $\vec{\mu} = 4.5 \cdot 10^{-3} (-\hat{z}) \text{ A m}^2$. Thus the minimum energy is $U = -4.5 \cdot 10^{-3} \text{ A m}^2 (-\hat{z}) \bullet (-1.5\hat{z} \text{ T}) = -6.8 \cdot 10^{-3} \text{ J}.$

28.69. The magnitude of magnetic field inside a solenoid is given by $B = \mu_0 in = \mu_0 i\left(N / L\right)$. Simplifying this, the number of turns of the wire is $N = BL / \left(\mu_0 i\right)$. Putting in the numerical values, $i = 0.20$ A, $L = 0.90$ m and $B = 5.0 \cdot 10^{-3}$ T yields $N = \dfrac{\left(5.0 \cdot 10^{-3} \text{ T}\right)\left(0.90 \text{ m}\right)}{\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)\left(0.20 \text{ A}\right)} = 17904 \approx 18000$ turns.

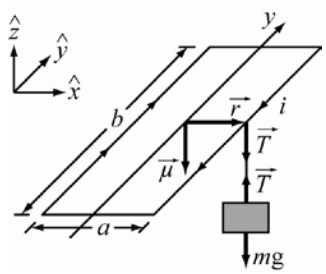
28.70.



Applying Ampere's Law around a loop as shown in the figure gives $\oint \vec{B} \bullet d\vec{s} = B \oint ds = \mu_0 i_{\text{enc}}$. Thus, the magnetic field is $B = \mu_0 i_{\text{enc}} / \left(2\pi r\right)$. The enclosed current is given by $i_{\text{enc}} = i - \dfrac{A_{\text{enc}} i}{A_{\text{total}}}$ when A_{enc} is cross sectional area of the shield that is enclosed by the loop and A_{total} is the cross sectional area shield. This means the areas are $A_{\text{enc}} = \pi(r^2 - a^2)$ and $A_{\text{total}} = \pi(b^2 - a^2)$. Thus the magnetic field inside the shield is $B = \dfrac{\mu_0 i}{2\pi r}\left(1 - \dfrac{r^2 - a^2}{b^2 - a^2}\right) = \dfrac{\mu_0 i}{2\pi r}\left(\dfrac{b^2 - r^2}{b^2 - a^2}\right).$

28.71. **THINK:** The torque due to the current in a loop of wire in a magnetic field must balance the torque due to weight.

SKETCH:



RESEARCH: The torque on a current loop in a uniform magnetic field is given by $\tau_{\text{B}} = \vec{\mu} \times \vec{B} = iN\vec{A} \times \vec{B} = iNA(-\vec{z}) \times \vec{B}$. Using Newton's Second Law, the torque due to the weight is found to be $\tau_{\text{W}} = \vec{r} \times \vec{T} = \left(\frac{1}{2}a\hat{x}\right) \times mg\left(-\hat{z}\right) = -\frac{1}{2}amg(\hat{x} \times \hat{z})$.

SIMPLIFY: Since the system is in equilibrium, the net torque must be zero: $\sum \tau = \tau_{\text{B}} + \tau_{\text{w}} = 0$.

Thus,

$$\tau_{\text{B}} = -\tau_{\text{w}}$$
$$-iNA\hat{z} \times \vec{B} = \frac{1}{2}amg(\hat{x} \times \hat{z}) = -\frac{1}{2}amg(\hat{z} \times \hat{x}).$$

This means that the magnetic field vector is in positive $\hat{x}$. Substituting $\vec{B} = B\hat{x}$ gives $iNAB = \frac{1}{2}amg$. After simplifying and using $A = ab$, $\vec{B} = \frac{1}{2}\frac{amg}{iNA}\hat{x} = \frac{1}{2}\frac{amg}{iN(ab)}\hat{x} = \frac{mg}{2iNb}\hat{x}$.

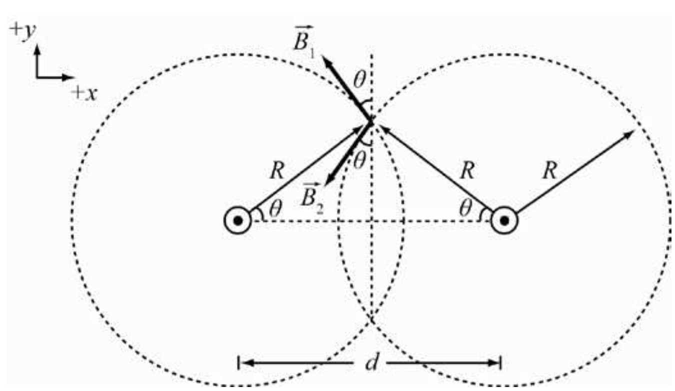
CALCULATE: Substituting the numerical values produces $\vec{B} = \frac{(0.0500 \text{ kg})(9.81 \text{ m/s}^2)}{2(1.00 \text{ A}) \cdot 50 \cdot (0.200 \text{ m})}\hat{x} = 0.02453 \text{ T}$.

ROUND: Three significant figures yields, $\vec{B} = 24.5 \text{ mT}$.

DOUBLE-CHECK: The magnetic force must be in the positive z-direction to balance gravity. By the right hand rule, it can be seen that the magnetic field must point in the positive x-direction for this to occur. This is consistent with the result calculated above. The result is reasonable.

28.72. **THINK:** In this problem, the net magnetic field due to two parallel wires is determined by adding the contributions from the wire.

SKETCH:



RESEARCH: The magnitude of the magnetic field of a long wire is given by $B = \mu_0 i / (2\pi R)$. The net magnetic field is $\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2$. Because of the symmetry of this problem, the y-component of the magnetic fields cancel out and only the x-component remains. Thus, the net magnetic field becomes

$$\vec{B} = -B\sin\theta\hat{x} - B\sin\theta\hat{x} = -2B\sin\theta\hat{x}$$

$$\vec{B} = \frac{-\mu_0 i}{\pi R}\sin\theta\hat{x}$$

SIMPLIFY: Since $\sin\theta = \dfrac{\sqrt{R^2 - (d/2)^2}}{R}$, the magnitude of the magnetic field simplifies to

$$B = \frac{\mu_0 i}{\pi R^2}\sqrt{R^2 - \frac{d^2}{4}}$$

CALCULATE: Inserting the numerical values of the parameters gives

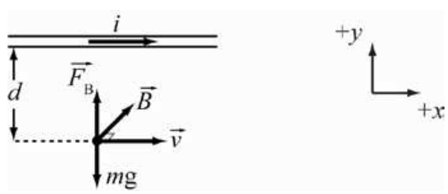
$$B = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(10.0\text{ A})}{\pi\left(12.0\cdot 10^{-2}\text{ m}\right)^2}\sqrt{\left(12.0\cdot 10^{-2}\text{ m}\right)^2 - \frac{\left(20.0\cdot 10^{-2}\text{ m}\right)^2}{4}} = 1.843\cdot 10^{-5}\text{ T}.$$

ROUND: Keeping three significant figures, $B = 18.4\ \mu\text{T}$.

DOUBLE-CHECK: The magnetic field due to one wire at the same position is 16.7μT. It is therefore reasonable that the answer for two wires is slightly larger than this, considering that the y-components cancel out.

28.73. **THINK:** In this problem the force on a particle due to a magnetic field must balance the force due to gravity.

SKETCH:



RESEARCH: The force acting on the particle due to the magnetic field is $F_B = qvB\sin\theta$. Since the angle between $\vec{v}$ and $\vec{B}$ is 90.0°, the force due to the magnetic field becomes $F_B = qvB$. This force must balance the gravitational force which is given by $F_g = mg$. Therefore $F_B = F_g$ or $qvB = mg$.

SIMPLIFY: The magnetic field due to the current in the wire is $B = \mu_0 i / (2\pi d)$. The change of the particle is then found to be $q = mg/(vB) = mg2\pi d/(v\mu_0 i)$.

CALCULATE: Inserting the numerical values gives a charge of

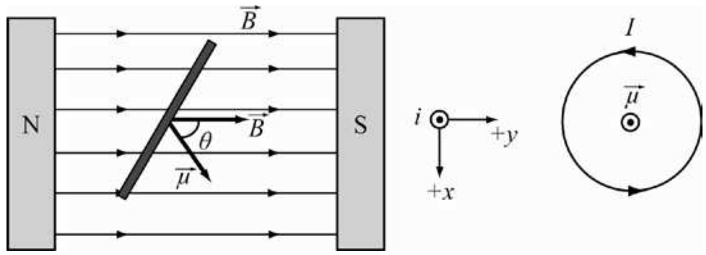
$$q = \frac{\left(1.00\cdot 10^{-6}\text{ kg}\right)\left(9.81\text{ m/s}^2\right)2\pi(0.100\text{ m})}{\left(1000.\text{ m/s}\right)\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(10.0\text{ A})} = 4.905\cdot 10^{-4}\text{ C}.$$

ROUND: Rounding the result to 3 significant figures gives $q = 4.91\cdot 10^{-4}$ C.

DOUBLE-CHECK: Dimensional analysis confirms the calculation provided the answer in the correct units: $q = \dfrac{[\text{kg}][\text{m/s}^2][\text{m}]}{[\text{m/s}][\text{T}][\text{m/A}][\text{A}]} = \dfrac{[\text{kg}][\text{m/s}^2]}{[\text{m/s}][\text{N}/(\text{A m})]} = \dfrac{[\text{A}][\text{m}]}{[\text{m/s}]} = [\text{A}][\text{s}] = [\text{C}].$

28.74. **THINK:** The torque on a loop of wire in a magnetic field is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the wire.

SKETCH:



RESEARCH:

(a) Using the right hand rule, the direction of current is counterclockwise as seen by an observer looking in the negative $\vec{\mu}$ direction as shown in the above figure.

(b) Using the magnetic dipole moment $\vec{\mu} = iNA\hat{n}$, the torque on the wire is $\vec{\tau} = iNA\hat{n} \times \vec{B}$, where $\hat{n}$ is a unit vector normal to the loop. Since $\left|\hat{n} \times \vec{B}\right| = B\sin\theta$ and $A = \pi R^2$, the magnitude of the torque is $\tau = iN\pi R^2 B\sin\theta$.

SIMPLIFY: From the equation, the number of turns needed to produce τ is $N = \dfrac{\tau}{\pi i R^2 B \sin\theta}$.

CALCULATE:

(b) Substituting the numerical values of the parameters yields

$$N_1 = \frac{(3.40 \text{ N m})}{\pi(5.00 \text{ A})(5.00 \cdot 10^{-2} \text{ m})^2 (2.00 \text{ T})\sin(60.0°)} = 49.98 = 50. \text{ turns.}$$

(c) Replacing the values of the above R with $R = 2.5 \cdot 10^{-2}$ m gives the number of turns

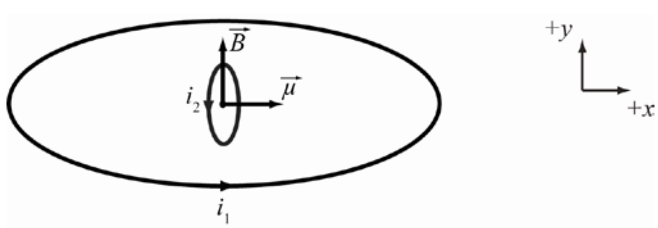
$$N_2 = \frac{(3.40 \text{ Nm})}{\pi(5.00 \text{ A})(2.50 \cdot 10^{-2} \text{ m})^2 (2.00 \text{ T})\sin(60.0°)} = 100. \text{ turns.}$$

ROUND: Not needed.

DOUBLE-CHECK: Since N is inversely proportional to R^2, the ratio of the results in (b) and (c) is $\dfrac{N_1}{N_2} = \dfrac{R_2^{\,2}}{R_1^{\,2}} = \dfrac{(R_1/2)^2}{R_1^{\,2}} = \dfrac{1}{4} = \dfrac{50}{200}$.

28.75. **THINK:** Assuming the inner loop is sufficiently small such that the magnetic field due to the larger loop is same across the surface of the smaller loop, the torque on the small loop can be determined by its magnetic moment.

SKETCH:



RESEARCH: The torque experienced by the small loop is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic field in the center of the loop is given by $\vec{B} = \dfrac{\mu_0 i_1}{2R}\hat{y}$. The magnetic dipole moment of the small loop is $\vec{\mu} = i_2 \vec{A}_2 = i_2 \pi r^2 \hat{x}$.

SIMPLIFY: Combining all the above expressions yields the torque.

$$\tau = |\vec{\tau}| = \left| \left(i_2 \pi r^2 \hat{x} \right) \times \left(\frac{\mu_0 i_1}{2R} \hat{y} \right) \right| = \frac{\pi \mu_0 i_1 i_2 r^2}{2R} |\hat{x} \times \hat{y}| = \frac{\pi \mu_0 i_1 i_2 r^2}{2R}$$

CALCULATE: Putting in all the numerical values gives

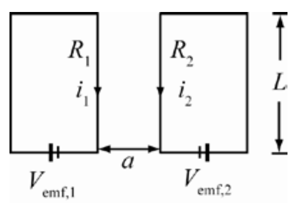
$$\tau = \frac{\pi \left(4\pi \cdot 10^{-7} \text{ T m/A}\right)(14.0 \text{ A})(14.0 \text{ A})(0.00900 \text{ m})^2}{2\left(0.250 \text{ m}\right)} = 1.254 \cdot 10^{-7} \text{ N m}.$$

ROUND: Rounding to 3 significant figures gives, $\tau = 1.25 \cdot 10^{-7}$ N m.

DOUBLE-CHECK: The units are correct: $\tau = \dfrac{\left[\text{T m/A}\right]\left[\text{A}\right]\left[\text{A}\right]\left[\text{m}^2\right]}{\left[\text{m}\right]} = \dfrac{\left[\text{N}\right]\left[\text{A}\right]\left[\text{m}^2\right]}{\left[\text{A}\right]\left[\text{m}\right]} = \left[\text{N m}\right]$.

28.76. **THINK:** Two parallel wires carrying currents in the same direction have an attractive force. Two parallel wires carrying currents in opposite directions have a repulsive force.

SKETCH:



RESEARCH: By considering the direction of *emf* potentials, the currents in the wires have the same direction. Therefore the force between the wires is attractive. The force between the two wires is given by

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi a}.$$

SIMPLIFY: The currents through the wires are given by $i_1 = \dfrac{V_{\text{emf},1}}{R_1}$ and $i_2 = \dfrac{V_{\text{emf},2}}{R_2}$. Thus, the force becomes $F = \dfrac{\mu_0 V_{\text{emf},1} V_{\text{emf},2} L}{2\pi a R_1 R_2}$. Solving for R_2 gives: $R_2 = \dfrac{\mu_0 V_{\text{emf},1} V_{\text{emf},2} L}{2\pi a R_1 F}$.

CALCULATE: Substituting the numerical values gives

$$R_2 = \frac{\left(4\pi \cdot 10^{-7} \text{ T m/A}\right)\left(9.00 \text{ V}\right)\left(9.00 \text{ V}\right)\left(0.250 \text{ m}\right)}{2\pi\left(0.00400 \text{ m}\right)\left(5.00\ \Omega\right)\left(4.00 \cdot 10^{-5} \text{ N}\right)} = 5.063\ \Omega.$$

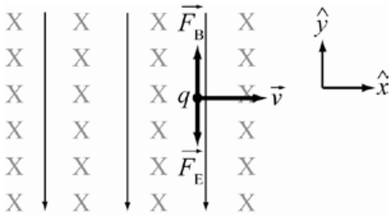
ROUND: Rounding the result to 3 significant figures gives $R_2 = 5.06\ \Omega$.

DOUBLE-CHECK: To 1 significant figure, the value of R_2 is the same as R_1. This is reasonable.

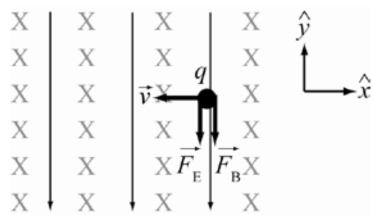
28.77. **THINK:** To solve this problem, the forces due to an electric field and a magnetic field are computed separately. The forces are added as vectors to get a net force.

SKETCH:

(a)



(b)



RESEARCH: Using the right hand rule and since the charge of proton is positive, the directions of forces are shown above. The magnitude of the electric force on the proton is $F_E = qE,$ and the magnitude of the magnetic force is $F_B = qvB.$

SIMPLIFY:

(a) The acceleration of the proton is $a = \frac{F_{\text{net}}}{m} = \frac{qvB - qE}{m} = \frac{q}{m}(vB - E).$

(b) The acceleration of the proton if the velocity is reversed is

$$a = \frac{F_{\text{net}}}{m} = -F_B - F_E = \frac{-qvB - qE}{m} = -\frac{q}{m}(vB + E).$$

CALCULATE: Substituting the numerical values yields the acceleration

(a) $a = \frac{1.60 \cdot 10^{-19} \text{ e}}{1.67 \cdot 10^{-27} \text{ kg}}\Big(\big(200. \text{ m/s}\big)\big(1.20 \text{ T}\big) - 1000. \text{ V/m}\Big) = -7.28 \cdot 10^{10} \text{ m/s}^2$

(b) $a = -\frac{1.60 \cdot 10^{-19} \text{ e}}{1.67 \cdot 10^{-27} \text{ kg}}\Big(\big(200. \text{ m/s}\big)\big(1.20 \text{ T}\big) + 1000. \text{ V/m}\Big) = -1.19 \cdot 10^{11} \text{ m/s}^2$

ROUND:

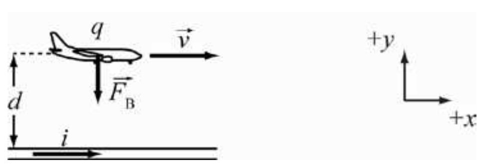
(a) $a = -7.28 \cdot 10^{10} \text{ m/s}^2$

(b) $a = -1.19 \cdot 10^{11} \text{ m/s}^2$

DOUBLE-CHECK: It is expected that the result in (b) is larger than in (a). This is consistent with the calculated values.

28.78. **THINK:** The net acceleration of a toy airplane is due to the gravitational acceleration and the magnetic field of a wire. However for this problem, the gravitational force is ignored.

SKETCH:



RESEARCH: Using a right hand rule, the magnetic force on the plane is directed toward the wire. The net acceleration of the plane due to the magnetic field is $a = F_B / m = qvB / m.$

SIMPLIFY: Substituting the magnetic field of the wire $B = \mu_0 i / (2\pi d)$ yields $a = \frac{qv\mu_0 i}{2\pi md}.$

CALCULATE: Putting in the numerical values gives the acceleration:

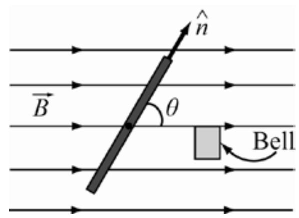
$$a = \frac{\big(36 \cdot 10^{-3} \text{ C}\big) \cdot (2.8 \text{ m/s})\big(4\pi \cdot 10^{-7} \text{ T m/A}\big) \cdot (25 \text{ A})}{2\pi\big(0.175 \text{ kg}\big)\big(0.172 \text{ m}\big)} = 1.674 \cdot 10^{-5} \text{ m/s}^2.$$

ROUND: Rounding the result to two significant figures gives $a = 1.7 \cdot 10^{-5} \text{ m/s}^2.$

DOUBLE-CHECK: It is expected that the result will be much less than the value of the gravitational acceleration.

28.79. **THINK:** To do this problem, the inertia of a long thin rod is required. The torque on a wire is also needed. The measure of the angle θ is 25.0°, and the current is $i = 2.00$ A. Let $A = 0.200 \cdot 10^{-4}$ m^2 and $B = 9.00 \cdot 10^{-2}$ T.

SKETCH:



RESEARCH: The magnetic dipole moment of the wire is given by $\vec{\mu} = NiA\hat{n}$.

(a) The torque on the wire is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnitude of this torque is $\tau = \mu B \sin\theta = NiAB\sin\theta$.

(b) The angular velocity of the rod when it strikes the bell is determined by using conservation of energy, that is, $E_{\text{i}} = E_{\text{f}}$ or $U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}}$.

SIMPLIFY:

(a) $\tau = \mu B \sin\theta = NiAB\sin\theta$.

(b) Since $K_{\text{i}} = 0$, the final kinetic energy is

$$K_{\text{f}} = U_{\text{i}} - U_{\text{f}}$$

$$\frac{1}{2}I\omega^2 = -\mu B\cos\theta + \mu B\cos(0°) = -\mu B\cos\theta + \mu B = \mu B(1-\cos\theta)$$

Thus the angular velocity is $\omega = \sqrt{\dfrac{2\mu B(1-\cos\theta)}{I}} = \sqrt{\dfrac{2NiAB(1-\cos\theta)}{(1/12)mL^2}}$, using $I = \dfrac{1}{12}mL^2$, the inertia of a thin rod.

CALCULATE: Putting in the numerical values gives the following values.

(a) $\tau = (70)(2.00 \text{ A})(0.200 \cdot 10^{-4} \text{ m}^2)(9.00 \cdot 10^{-2} \text{ T})\sin(25.0°) = 1.06 \cdot 10^{-4}$ N m

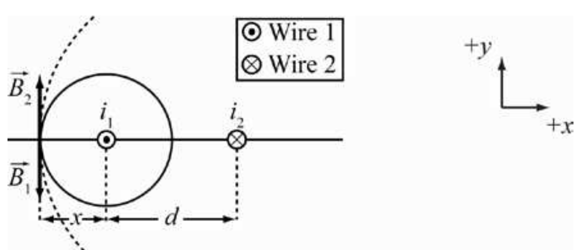
(b) $$\omega = \left[\frac{2(70)(2.00 \text{ A})(0.200 \cdot 10^{-4} \text{ m}^2)(9.00 \cdot 10^{-2} \text{ T})(1-\cos 25.0°)}{(1/12)(0.0300 \text{ kg})(0.0800 \text{ m})^2}\right]^{1/2} = 1.72 \text{ rad/s}$$

ROUND: Rounding to 3 significant figures yields $\tau = 1.06 \cdot 10^{-4}$ N m, $\omega = 1.72$ rad/s.

DOUBLE-CHECK: The torque should have units of Newton-meters, while the angular velocity should have units of radians per second.

28.80. **THINK:** Using a right hand rule, the sum of the magnetic fields of two parallel wires carrying opposite currents cannot be zero between the two wires.

SKETCH:



RESEARCH: The magnitude of the magnetic field of a long wire is $B=\mu_0 i/(2\pi R)$. Since $i_1 < i_2$ and i_1 is in an opposite direction to i_2, using the right hand rule, it is found that the location of the zero magnetic field must be to the left of the left-hand wire, as shown in the figure. Assuming the location is a distance x to the left of the left-hand wire, then the net magnetic field is $B_{\text{net}} = B_2 - B_1 = \dfrac{\mu_0 i_2}{2\pi(x+d)} - \dfrac{\mu_0 i_1}{2\pi x} = 0.$

SIMPLIFY: Solving for x yields

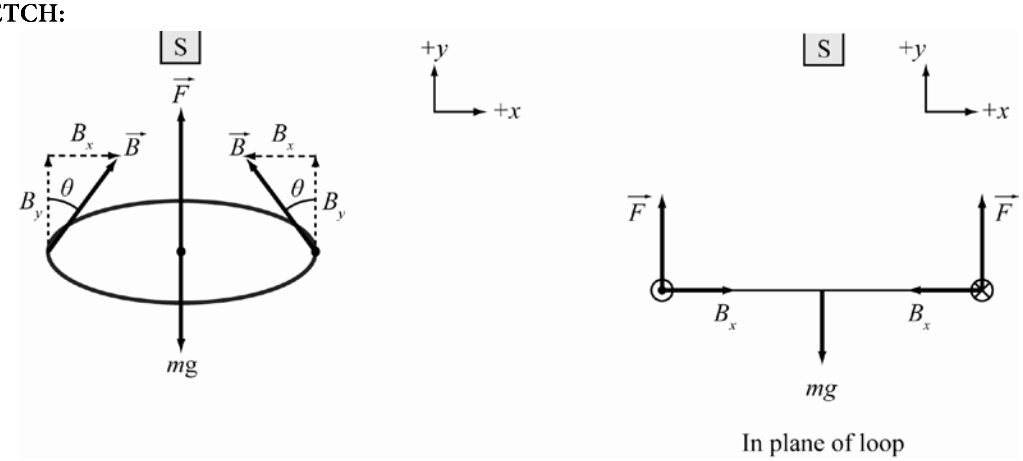
$$\frac{i_2}{x+d} = \frac{i_1}{x} \Rightarrow xi_2 = i_1 x + i_1 d \Rightarrow x = \frac{i_1 d}{i_2 - i_1}.$$

Since $i_2 = 2i_1$,

$$x = \frac{i_1 d}{2i_1 - i_1} = d.$$

CALCULATE: Not required.
ROUND: Not required.
DOUBLE-CHECK: This result is expected since the ratio of $i_2 / i_1 = 2.$ This means the ratio of distances is $\dfrac{d_2}{d_1} = \dfrac{2d}{d} = 2$ also.

28.81. **THINK:** In order for a coil to float in mid-air, the downward force of gravity must be balanced an upward force due to the current loop in the magnetic field.
SKETCH:



RESEARCH: By using right-hand rule 1, the direction of the forces can be determined. For the y-component B_y of the magnetic field the force due to the current is in the radial direction of the coil. Therefore, this component cannot be responsible for levitating the coil. For the x-component B_x of the magnetic field, with a counterclockwise current as viewed from the bar magnet, the resulting force is in the y-direction, towards the bar magnet (see figure on right). This is the correct direction for balancing the weight of the coil. The magnitude of the y-component of the force on an element dl is $dF_y = Ni\left|d\vec{l} \times \vec{B}_x\right| \sin\theta = NiB\sin\theta dl.$ Thus the total magnetic force on the current loop is $F_y = \int_0^{2\pi R} NiB\sin\theta dl.$ Newton's Second Law requires that $F_y = mg.$

SIMPLIFY: The integral simplifies to: $F_y = 2\pi RNiB\sin\theta.$ Therefore,

$$2\pi RNiB\sin\theta = mg \Rightarrow i = \frac{mg}{2\pi RNB\sin\theta}.$$

CALCULATE: Substituting in the numerical values yields

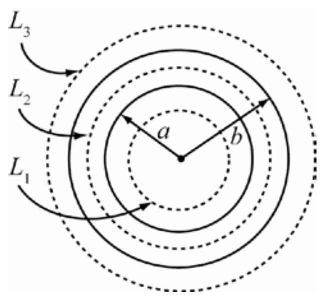
$$i = \frac{\left(10.0\cdot 10^{-3}\text{ kg}\right)(9.81\text{ m/s}^2)}{2\pi\left(5.00\cdot 10^{-2}\text{ m}\right)10.0(0.0100\text{ T})\sin(45.0°)} = 4.416\text{ A}.$$

ROUND: To 3 significant figures, the current is $i = 4.42$ A, counterclockwise as viewed from the bar magnet.

DOUBLE-CHECK: It takes large currents to generate strong magnetic forces. A current of 4 A is realistic to levitate a 10 g mass.

28.82. **THINK:** In this problem, Ampere's Law is applied on three different circular loops.

SKETCH:



RESEARCH: Loops L_1, L_2 and L_3 are Amperian loops.

(a) For distances $r < a$, applying Ampere's Law on the loop L_1, gives $\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 i_{\text{enc}}$. Since $i_{\text{enc}} = 0$, the field is also zero, $B = 0$.

(b) For distances r between a and b, applying Ampere's law on the loop L_2 yields $\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 i_{\text{enc}}$. The enclosed current is given by $i_{\text{enc}} = A_{\text{enc}} i / A$ or $i_{\text{enc}} = \dfrac{\pi\left(r^2 - a^2\right)}{\pi\left(b^2 - a^2\right)} i = \dfrac{r^2 - a^2}{b^2 - a^2} i.$

(c) For distances $r > b$, applying Ampere's Law on L_3 gives $B = \dfrac{\mu_0 i_{\text{enc}}}{2\pi r} = \dfrac{\mu_0 i}{2\pi r}$, since $i_{\text{enc}} = i.$

SIMPLIFY: Thus, the magnetic field is $B = \dfrac{\mu_0 i}{2\pi r}\dfrac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}.$

CALCULATE: Putting in the numerical values gives

(a) $B = 0$

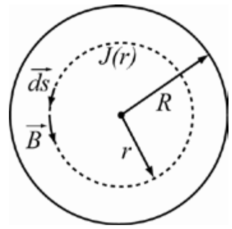
(b) $$B = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\cdot(0.100\text{ A})}{2\pi\left(6.50\cdot 10^{-2}\text{ m}\right)}\left[\frac{\left(6.50\text{ cm}\right)^2 - \left(5.00\text{ cm}\right)^2}{\left(7.00\text{ cm}\right)^2 - \left(5.00\text{ cm}\right)^2}\right] = 2.212\cdot 10^{-7}\text{ T}$$

(c) $$B = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)\cdot(0.100\text{ A})}{2\pi\left(9.00\cdot 10^{-2}\text{ m}\right)} = 2.222\cdot 10^{-7}\text{ T}$$

ROUND: Keeping 3 significant figures yields the following results for (b) and (c). Note that the value found in (a) is precise. (a) $B = 0$ (b) $B = 2.21\cdot 10^{-7}$ T (c) $B = 2.22\cdot 10^{-7}$ T

DOUBLE-CHECK: The units of the calculated values are T, which is appropriate for magnetic fields.

28.83. **THINK:** To solve this problem, the current enclosed by an Amperian loop must be determined.
SKETCH:



RESEARCH: Applying Ampere's Law on a loop as shown above gives $\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 i_{\text{enc}}$. i_{enc} is the current enclosed by the Amperian loop, that is $i_{\text{enc}} = \iint J(r)dA = \int_0^{2\pi}\int_0^r J(r')r'dr'd\theta.$

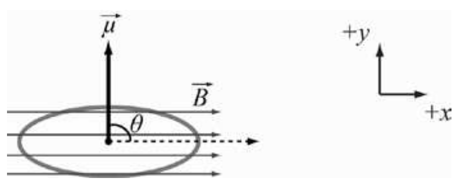
SIMPLIFY: Since $J(r)$ is a function of r only, the above integral becomes $i_{\text{enc}} = 2\pi\int_0^r J(r')r'dr'$. Substituting $J(r) = J_0(1 - r/R)$ yields

$$i_{\text{enc}} = 2\pi J_0 \int_0^r \left[r' - \frac{r'^2}{R}\right]dr' = 2\pi J_0\left[\frac{r'^2}{2} - \frac{r'^3}{3R}\right]_0^r = 2\pi J_0\left[\frac{r^2}{2} - \frac{r^3}{3R}\right].$$

Thus, the magnetic field is $B = \dfrac{\mu_0 2\pi J_0}{2\pi r}\left[\dfrac{r^2}{2} - \dfrac{r^3}{3R}\right] = \mu_0 J_0\left[\dfrac{r}{2} - \dfrac{r^2}{3R}\right].$

CALCULATE: Not required.
ROUND: Not required.
DOUBLE-CHECK: The form of the answer is reasonable.

28.84. **THINK:** The maximum torque on a circular wire in a magnetic field is when its magnetic moment is perpendicular to the magnetic field vector.
SKETCH:



RESEARCH: The torque on the circular wire is given by $\vec{\tau} = \vec{\mu}\times\vec{B}$. The magnitude of the torque is $\tau = \mu B\sin\theta$ where θ is the angle between $\vec{\mu}$ and $\vec{B}$.

SIMPLIFY:

(a) The maximum torque is when $\theta = 90°$, that is, $\tau = \mu B$. Using $\mu = iA = i\pi R^2$, the torque becomes $\tau = i\pi R^2 B$.

(b) The magnetic potential energy is given by $U = -\mu B\cos\theta$. The maximum and the minimum potential energies are when $\theta = 180°$ and $\theta = 0°$, that is, $U_{\text{max}} = +\mu B$ and $U_{\text{min}} = -\mu B$.

CALCULATE: (a) Inserting the numerical values gives the torque:

$$\tau = (3.0\text{ A})\pi\left(5.0\cdot 10^{-2}\text{ m}\right)^2\left(5.0\cdot 10^{-3}\text{ T}\right) = 1.18\cdot 10^{-4}\text{ N m}.$$

(b) Since the values of μB is the same as in (a), the range of the potential energy is
$\Delta U = U_{\text{max}} - U_{\text{min}} = 2\mu B = 2\cdot 1.2\cdot 10^{-4}\text{ J} = 2.4\cdot 10^{-4}\text{ J}.$

ROUND: Keeping only two significant figures yields $\tau = 1.2\cdot 10^{-4}$ N m and $\Delta U = 2.4\cdot 10^{-4}$ J.
DOUBLE-CHECK: The change in potential is a change in energy, so it is appropriate that the final answer have joules as units.

Multi-Version Exercises

Exercises 28.85–28.87 The magnetic field at the center of an arc of radius R subtended by an angle Φ is

$$B_\Phi = \int dB = \int_0^\Phi \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2} = \frac{\mu_0 i\Phi}{4\pi R}.$$

In this loop we have three sections:

$1:\ R = r,\ \Phi = \pi/2$

$2:\ R = 2r,\ \Phi = \pi/2$

$3:\ R = 3r,\ \Phi = \pi.$

The segments running directly toward/away from point *P* have no effect. So the magnetic field at *P* is

$$B = B_1 + B_2 + B_3 = \frac{\mu_0 i\left(\frac{\pi}{2}\right)}{4\pi(r)} + \frac{\mu_0 i\left(\frac{\pi}{2}\right)}{4\pi(2r)} + \frac{\mu_0 i(\pi)}{4\pi(3r)} = \frac{\mu_0 i}{8r} + \frac{\mu_0 i}{16r} + \frac{\mu_0 i}{12r} = \frac{6\mu_0 i}{48r} + \frac{3\mu_0 i}{48r} + \frac{4\mu_0 i}{48r} = \frac{13\mu_0 i}{48r}.$$

28.85. $$B = \frac{13\mu_0 i}{48r} = \frac{13\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(3.857\text{ A})}{48(1.411\text{ m})} = 9.303\cdot 10^{-7}\text{ T}$$

28.86. $$B = \frac{13\mu_0 i}{48r}$$

$$r = \frac{13\mu_0 i}{48B} = \frac{13\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(3.961\text{ A})}{48\left(7.213\cdot 10^{-7}\text{ T}\right)} = 1.869\text{ m}$$

28.87. $$B = \frac{13\mu_0 i}{48r}$$

$$i = \frac{48rB}{13\mu_0} = \frac{48(2.329\text{ m})\left(5.937\cdot 10^{-7}\text{ T}\right)}{13\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)} = 4.063\text{ A}$$

Exercises 28.88–28.90 The magnetic field inside a toroidal magnet is given by $B = \dfrac{\mu_0 Ni}{2\pi r}$.

28.88. $$B = \frac{\mu_0 Ni}{2\pi r}$$

$$N = \frac{2\pi rB}{\mu_0 i} = \frac{2\pi(1.985\text{ m})\left(66.78\cdot 10^{-3}\text{ T}\right)}{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(33.45\text{ A})} = 19{,}814$$

To four significant figures, the toroid has 19,810 turns.

28.89. $$B = \frac{\mu_0 Ni}{2\pi r}$$

$$i = \frac{2\pi rB}{\mu_0 N} = \frac{2\pi(1.216\text{ m})\left(78.30\cdot 10^{-3}\text{ T}\right)}{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(22{,}381)} = 21.27\text{ A}$$

28.90. $$B = \frac{\mu_0 Ni}{2\pi r} = \frac{\left(4\pi\cdot 10^{-7}\text{ T m/A}\right)(24{,}945)(49.13\text{ A})}{2\pi(1.446\text{ m})} = 0.1695\text{ T} = 169.5\text{ mT}$$