حل تمارين الوحدة ٧

[1] $\int 15x^2(x^3+1)^4\,dx$ $\quad u = x^3+1$

$\int 15x^2 (u)^4 \frac{du}{3x^2}$ $\quad \frac{du}{dx} = 3x^2$

$\boxed{\frac{5}{5}u^5 + C = (x^3+1)^5 + C}$ $\quad dx = \frac{du}{3x^2}$

(c)

[2] $\int \frac{x}{\sqrt{3x^2+5}}\,dx$ $\quad u = 3x^2+5$

$\int \frac{x}{\sqrt{u}} \frac{du}{6x}$ $\quad \frac{du}{dx} = 6x$

$\int \frac{1}{6} u^{-\frac{1}{2}}\,du$ $\quad dx = \frac{du}{6x}$

$\boxed{\frac{2}{6} u^{\frac{1}{2}} + C = \frac{1}{3}(3x^2+5)^{\frac{1}{2}} + C}$

(c)

[3] $\int \sin^2\alpha \cos^3\alpha\,d\alpha$ $\quad u = \sin\alpha$

$\int \sin^2\alpha \cos^2\alpha \cos\alpha$ $\quad \frac{du}{d\alpha} = \cos\alpha$

$\int \sin^2\alpha(1-\sin^2\alpha)\cos\alpha$ $\quad d\alpha = \frac{du}{\cos\alpha}$

$\int \sin^2\alpha(1-\sin^2\alpha)\,du$

$\int u^2(1-u^2)\,du$

$\int u^2 - u^4\,du$

$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C = \boxed{\frac{1}{3}\sin^3 - \frac{1}{5}\sin^5 x + C}$

(d)

[4] $\int 3^{x^2} x\,dx = 3^{x^2}\frac{du}{2}$ $\quad u = x^2$

$\frac{1}{2}\int 3^u\,du = \boxed{\frac{1}{2}\frac{3^u}{\ln 3} = \frac{3^{x^2}}{\ln 9} + C}$ $\quad \frac{du}{dx} = 2x$ $\quad dx = \frac{du}{2x}$

(c)

[5] $\int_0^1 x e^{-x}\,dx$

u		dv
x	+	e^{-x}
1	−	$-e^{-x}$
0		e^{-x}

$-xe^{-x}\Big]_0^1 - e^{-x}\Big]_0^1 = \boxed{1-2e^{-1}}$ (b)

[6] $\int x\sec^2\alpha\,d\alpha$

U		dV
x	+	$\sec^2\alpha$
1	−	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
0		$-\ln\cos\alpha$

$\boxed{x\tan x + \ln|\cos\alpha| + C}$

(b)

[7] $\int \frac{\sqrt{4-x^2}}{x}$ $\quad x = 2\sin\theta$ $\quad dx = 2\cos\theta\,d\theta$

$\int \frac{\sqrt{4-4\sin^2\theta}}{2\sin\theta} 2\cos\theta\,d\theta$

$\int \frac{4\cos^2\theta}{2\sin\theta}\,d\theta = \boxed{2\int \frac{\cos^2\theta}{\sin\theta}\,d\theta}$

(c) ✓

[8] $\int \frac{\sin^2 x}{1-\cos x}\,dx \times \frac{1+\cos x}{1+\cos x}$

$\frac{\cancel{\sin^2 x}}{\cancel{1-\cos^2 x}} \cdot 1+\cos x\,dx$

$\int 1+\cos x\,d\alpha$

$\boxed{x + \sin x + C}$ (d)

[9] $\int_0^1 \frac{x^2}{x^2+1}\,dx$ $\qquad$ $x^2+1 \,\big)\, x^2$; $-x^2-1$; -1

$\int_0^1 1 + \frac{-1}{x^2+1}\,dx$

$x - \tan^{-1}x\Big]_0^1$

$[1-\tan^{-1}(1)] - [0-\tan^{-1}(0)]$

$\boxed{1 - \frac{\pi}{4} = \frac{4-\pi}{4}}$ (d)

10 $\int_0^{\frac{\pi}{2}} \cos^2\alpha \sin\alpha \, d\alpha$

$U = \cos\alpha$

$\frac{du}{d\alpha} = -\sin\alpha$

$d\alpha = \frac{du}{-\sin\alpha}$

$\int_{\frac{\pi}{2} \to \cos\frac{\pi}{2} = 0}^{} \quad 0 \to \cos(0) = 1$

$\int -u^2 \, du$

$-\frac{1}{3}u^3 \Big]_1^0$

$\boxed{-\frac{1}{3}(0) - -\frac{1}{3}(1)^3 = \frac{1}{3} \quad (c)}$ ✓

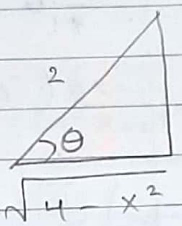
11 $\int_0^1 (4-x^2)^{-\frac{3}{2}} \, dx$

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$\int_0^1 \frac{1}{\sqrt{(4-x^2)^3}} \, dx$

$\int \frac{1}{\sqrt{(4-4\sin^2\theta)^3}} \, 2\cos\theta \, d\theta$



$\frac{1}{\sqrt{(4(1-\sin^2\theta))^3}}$

$\frac{1}{\sqrt{4^3(\cos^2\theta)^3}}$

$\int \frac{1}{\cancel{\sqrt{}}(2^3)^{\cancel{2}}(\cos^3\theta)^{\cancel{2}}}$

$\int \frac{1}{8\cos^{\cancel{3}}\theta} \, 2\cancel{\cos\theta}$

$\frac{1}{4}\int \frac{1}{\cos^2\theta}$

$\frac{1}{4}\int \sec^2\theta \, d\theta$

$\frac{1}{4}\tan\theta \Big]$

$\frac{1}{4}\frac{x}{\sqrt{4-x^2}}\Big]_0^1$

$\frac{1}{4}\frac{1}{\sqrt{4-1}} - 0$

$\frac{1}{4} \quad \frac{1}{\sqrt{3}} = \boxed{\frac{1}{4\sqrt{3}} \quad (a)}$

12 $\int 8x^3 \ln x \, dx$

U		dV
$\ln x$	+	$8x^3$
$\frac{1}{x}$	−	$2x^4$

$2x^4 \ln x - \int 2x^3 \, dx$

$2x^4 \ln x - \frac{1}{2}x^4 + c$

(c) ✓

13 $\int 4e^{x^2 + \ln x} \, dx$

$4\int e^{x^2} \cdot e^{\ln x}$

$\frac{1}{2} 4 \int 2x e^{x^2}$

$\boxed{2e^{x^2} + C \quad (c)}$ ✓

14 $\int \frac{1}{\sqrt{x^4 - x^2}} \, dx = \int \frac{1}{\sqrt{x^2(x^2-1)}}$

$\int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + C \quad (a)$ ✓

15 $\int 3x e^{x^2+1} \, dx$

$\frac{1}{2} 3 \int 2x e^{x^2+1} \, dx$

$\boxed{\frac{3}{2} e^{x^2+1} + C \quad (d)}$

16 $\int_0^1 x\sqrt{8x^2+1} \, dx$

$U = 8x^2 + 1$

$\frac{du}{dx} = 16x$

$dx = \frac{du}{16x}$

$\int x \sqrt{U} \, \frac{du}{16x}$

$\frac{1}{16}\int \sqrt{U} \, du$

$\frac{1}{16}\int U^{\frac{1}{2}} \, du$

$\frac{1}{16} \frac{2}{3} U^{\frac{3}{2}} + C =$

$\frac{1}{24}(8x^2+1)^{\frac{3}{2}}\Big]_0^1$

$\frac{9}{8} - \frac{1}{24} = \boxed{\frac{13}{12}}$

17 $\int \frac{1}{x^2+4}\,dx$ $\quad u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$dx = 2\,du$

$\int \frac{1}{4\left(\left(\frac{x}{2}\right)^2+1\right)}\,dx$

$\int \frac{1}{4(u^2+1)}\,2\,du$

$\boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c \quad (c)}$

18 $\int \frac{(\tan^{-1}x)^2}{x^2+1}\,dx$ $\quad u = \tan^{-1}x$

$\frac{du}{dx} = \frac{1}{x^2+1}$

$dx = x^2+1\,du$

$\int \frac{u^2}{\cancel{x^2+1}}\,\cancel{x^2+1}\,du$

$\frac{1}{3}u^3 + C = \boxed{\frac{1}{3}(\tan^{-1}x)^3 + C}$

19 $\int \sec^2 x\sqrt{\tan x}\,dx$ $\quad u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int \sec^2 x\sqrt{u}\,\frac{du}{\cancel{\sec^2 x}}$

$\int \sqrt{u}\,du$

$\int u^{\frac{1}{2}}\,du = \boxed{\frac{2}{3}(\tan x)^{\frac{3}{2}} + c}$

20 $\int \sin x\cos^6 x$ $\quad u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int \cancel{\sin x}\,u^6\,\frac{du}{-\cancel{\sin x}}$

$\int -u^6\,du$

$-\frac{1}{7}u^7 + C$

$\boxed{-\frac{1}{7}(\cos x)^7 + C}$

21 $\int \sqrt[3]{x^5 - x^3}\,dx$ $\quad u = x^2 - 1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \sqrt[3]{x^3(x^2-1)}\,dx$

$\int x\sqrt[3]{x^2-1}$

$\int \cancel{x}\sqrt[3]{u}\,\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^{\frac{1}{3}}\,du$

$\frac{1}{2}\cdot\frac{3}{4}\,u^{\frac{4}{3}} + C = \boxed{\frac{3}{8}(x^2-1)^{\frac{4}{3}} + c}$ ✓

22 $\int \sin^2 2x\,dx$

$\int \frac{1}{2}(1 - \cos 4x)\,dx$

$\frac{1}{2}\int 1 - \cos 4x\,dx$

$\frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right] + C$

$\frac{1}{2}x - \frac{1}{8}\sin 4x + C$

$\boxed{\frac{1}{8}4x - \frac{1}{8}\sin 4x + c}$ ✓ (d)

23 $\int x^2\cos x^3\,dx$ $\quad u = x^3$

$\frac{du}{dx} = 3x^2$

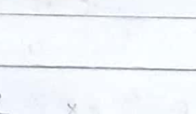
$dx = \frac{du}{3x^2}$

$\frac{1}{3}\int \cos u\,du$

$\boxed{\frac{1}{3}\sin x^3 + C}$

(a)

24 $\int_0^1 \sqrt{1-x^2}\,dx$



$A = \frac{\pi r^2}{4} = \boxed{\frac{\pi}{4}}$

[25] $\int \sin^{-1} x$

u		dv
$\sin^{-1} x$	$+$	dx
$\frac{1}{\sqrt{1-x^2}}$	$-$	x

$\boxed{x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx}$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

<b>

[26] $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{e^u}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}}\,du$

$\int_{1 \to 1}^{4 \to 2}$

$\boxed{2\int_1^2 e^u\,du}$

<c>

[27] $\int_0^1 \frac{x}{x^2+1}$

$\frac{1}{2}\ln|x^2+1|\Big]_0^1$

$\frac{1}{2}\ln|1^2+1| - \frac{1}{2}\ln|0+1|$

$= \frac{1}{2}\ln|2|$

$= \ln 2^{\frac{1}{2}}$

$= \boxed{\ln\sqrt{2}}$

<b>

[28] $\int \frac{1}{(x-1)(x+2)}\,dx = \frac{A}{(x-1)} + \frac{B}{(x+2)}$

$A(x+2) + B(x-1) = 1$

$x = 1 \Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}$

$x = -2 \Rightarrow -3B = 1 \Rightarrow B = \frac{-1}{3}$

$\int \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}}{x+2}\,dx$

$\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + c = \boxed{\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + c}$ <A>

[29] $\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x(x+1)}\,dx$

$\int \frac{A}{x} + \frac{B}{x+1}\,dx$

$A(x+1) + Bx = 1$

$x = 0 \Rightarrow A = 1 \mid x = -1 \Rightarrow B = -1$

$\int \frac{1}{x} + \frac{-1}{x+1}\,dx$

$\ln|x| - \ln|x+1| + c = \boxed{\ln\left|\frac{x}{x+1}\right| + c}$

[30] $\int_2^3 \frac{x+1}{x^2+2x-3}\,dx$

$\int_2^3 \frac{x+1}{(x-1)(x+3)}\,dx = \int \frac{A}{x-1} + \frac{B}{x+3}\,dx$

$A(x+3) + B(x-1) = x+1$

$x = 1 \Rightarrow 4A = 2 \quad A = \frac{1}{2}$

$x = -3 \Rightarrow -4B = -2 \quad A = \frac{1}{2}$

$\int \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+3}\,dx$

$\frac{1}{2}\ln|x-1| + \frac{1}{2}|x+3|\Big]_2^3$

$\frac{1}{2}\ln 2 + \frac{1}{2}\ln|6| - \frac{1}{2}\ln 5$

$\boxed{\frac{1}{2}\left[\ln\frac{12}{5}\right]}$

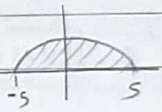
[31] $\int \frac{1}{(x^2+x)(x+1)} = \frac{1}{(x)(x+1)(x+1)}$

$\int \frac{1}{x(x+1)^2}\,dx = \int \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}\,dx$

$\boxed{A\ln|x| + B\ln|x+1| + \frac{C}{x+1} + C}$

<a>

[32] $\int_{-5}^{5} \sqrt{25-x^2}\,dx$



$A = \frac{\pi r^2}{2} = \boxed{\frac{25\pi}{2}}$

<b>

[33] $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$\int_0^{\frac{1}{2}} \frac{\sin^2\theta}{\cancel{\sqrt{1-\sin^2\theta}}} \cancel{\cos\theta}\,d\theta$

$\int \sin^2\theta\,d\theta$

$\theta = \sin^{-1} x$

$\int_{0 \to 0}^{0 \to \pi/6}$

$\boxed{\int_0^{\pi/6} \sin^2\theta\,d\theta}$

<D>

[34] $\int_{\pi/4}^{\pi/2} \sin^3 x \cos x\,dx$

$= \boxed{\frac{3}{16}}$ ✓

[35] $\int \frac{x}{x+2}$

$x \div (x+2) = 1$, remainder -2: $\quad x - (x+2) = -2$

$\int 1 + \frac{-2}{x+2}\,dx$

$\boxed{x - 2\ln|x+2| + C}$

<C>

36) $\int_0^{\pi/2} \frac{\cos x}{\sqrt{1+\sin x}} dx$

$U = \sin x + 1$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \frac{1}{\sqrt{U}} du$

$2U^{\frac{1}{2}} \Big]_1^2$

$2(\sqrt{2}-1)$

37) $\int_0^1 \frac{e^x}{(3-e^x)^2} dx = \boxed{3.049}$

(c)

38) $\int \frac{1}{x^2 - 6x + 10} dx$

$\int \frac{1}{x^2-6x+9+10-9} dx$

$\int \frac{1}{(x-3)^2+1}$

$U = x-3$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{U^2+1} du$

$\boxed{\tan^{-1}(x-3) + C}$

39) $\int \frac{x^3}{x^8+1} dx = \int \frac{x^3}{(x^4)^2+1} dx$

$U = x^4$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$\int \frac{\cancel{x^3}}{U^2+1} \frac{du}{4\cancel{x^3}}$

$\boxed{\frac{1}{4}\tan^{-1} x^4 + C}$

(d)

40) $\int \sin 3x \cos 2x \, dx$

$\int \frac{1}{2}[\sin(3-2)x + \sin(5)x]$

$\int \frac{1}{2}[\sin(x) + \sin 5x]$

$\frac{1}{2}[-\cos x - \frac{1}{5}\sin 5x] + C$

$\boxed{-\frac{1}{2}[\cos x + \frac{1}{5}\sin 5x] + C}$

(a)

41) $\int_3^5 x\sqrt{2x-1}\, dx = K\int_a^b (U+1)\sqrt{U}$

$U = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\to 2x = U+1$

$x = \frac{U+1}{2}$

$\int \frac{U+1}{2} \sqrt{U} \frac{du}{2}$

$\frac{1}{4}\int (U+1)\sqrt{U}\, du$

$\boxed{(d)}$ $(\frac{1}{4})$

42) $\int_3^5 x\sqrt{2x-1}\, dx = K\int_a^b (U+1)\sqrt{U} du$

$U = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\to 2x = U+1$

$x = \frac{U+1}{2}$

$\int_3^5 x\sqrt{U} \frac{du}{2}$

$\frac{1}{2}\int \frac{U+1}{2} \sqrt{U}\, du$

$\square \to 2(5)-1 = 9$

$\square \to 2(3)-1 = 5$

43) $\int_0^K \frac{\sec^2 x}{1+\tan x} = \ln 2$

$\ln|1+\tan x| \Big]_0^K = \ln 2$

$\ln|1+\tan K| - \ln|1+\tan 0|$

$= \ln 2$

$\ln_e|1+\tan K| = \ln_e 2$

$1+\tan K = 2$

$\tan K = 2-1$

$\tan K = 1$

$K = \tan^{-1} 1 = \boxed{\frac{\pi}{4}}$

44) $\int x(2x+1)^5 dx$

$U = 2x+1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\to 2x = U-1$

$x = \frac{U-1}{2}$

$\int x(U)^5 \frac{du}{2}$

$\frac{1}{2}\int \frac{U-1}{2} (U)^5 du$

$\frac{1}{4}\int (U-1)U^5 du$

$\frac{1}{4}\int U^6 - U^5 du$

$\frac{1}{4}\left[\frac{1}{7}U^7 - \frac{1}{6}U^6\right] + C$

$\frac{1}{4}\left[\frac{1}{7}(2x+1)^7 - \frac{1}{6}(2x+1)^6\right] + C$

$\frac{1}{28}(2x+1)^7 - \frac{1}{24}(2x+1)^6 + C$ $\boxed{(b)}$

45) $\int \sin\sqrt{x}\, dx$

$U = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\, du$

$\int \sin U \, 2\sqrt{x}\, du$

$\int 2U \sin U \, du$

$2U \;\; + \;\; \sin U$

$2 \;\; - \;\; -\cos U$

$0 \;\; \;\; -\sin U$

$-2U\cos U + 2\sin U + C$

$\boxed{-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C}$

(b)

46 $\int e^{\sqrt{x}}\,dx$ $u=\sqrt{x}$

$\int e^{u}\,du\;2\sqrt{x}$ $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ $dx=2\sqrt{x}\,du$

$\int 2ue^{u}\,du$

$2u \xrightarrow{+} e^{u}$
$2 \xrightarrow{-} e^{u}$
$0 \quad e^{u}$

$2ue^{u}-2e^{u}+C$

$\boxed{2\sqrt{x}\,e^{\sqrt{x}}-2e^{\sqrt{x}}+C}$

⟨D⟩

47 $\int e^{2x}\sin e^{x}\,dx$ $u=e^{x}$

$\int e^{2x}\sin u\,\frac{du}{e^{x}}$ $\frac{du}{dx}=e^{x}$ $dx=\frac{du}{e^{x}}$

$\int u\sin u\,du$

$u \xrightarrow{+} \sin u$
$1 \xrightarrow{-} -\cos u$
$0 \quad -\sin u$

$-u\cos u+\sin u+C$

$\boxed{-e^{x}\cos e^{x}+\sin e^{x}+C}$

48 $\sec^{3}x\,dx$

$\sec^{2}x\,\sec x\,dx$

$\sec x \xrightarrow{+} \sec^{2}x$
$\sec x\tan x \xleftarrow{-} \tan x$

$\sec x\tan x-\int\sec x\tan^{2}x\,dx$

$\sec x\tan x-\int\sec x(\sec^{2}x-1)$

$\sec x\tan x-\int\sec^{3}x-\sec x$

$\boxed{\frac{\sec x\tan x+\ln|\sec x+\tan x|}{2}}$

⟨A⟩

49 $g(5)=2$

$\int_{2}^{5} f'(x)g(x)+f(x)g'(x)\,dx$

$=g(x)f(x)\Big]_{2}^{5}$

$=g(5)f(5)-g(2)f(2)$

$=(2)(7)-(3)(4)$

$\boxed{14-12=2}$

(d)

50 $\int_{1}^{4} f(x)\,dx=12$

$\int_{1}^{4}(2x+3)f'(x)\,dx$

$2x+3 \xrightarrow{+} f'(x)$
$2 \longleftarrow f(x)$

$(2x+3)f(x)\Big]_{1}^{4}-\int_{1}^{4}2f(x)$

$(2\times4+3)f(4)-(2\times1+3)f(1)-\int_{1}^{4}2f(x$

$(11)(-8)-(5)(3)-2(12)$

$=\boxed{-127}$ ⟨C⟩

51 $\int\tan^{3}x\sec x\,dx$

$\int\tan^{2}x\,du$ $u=\sec x$

$\int(\sec^{2}x-1)\,du$ $\frac{du}{dx}=\sec x\tan x$

$\int u^{2}-1\,du$ $dx=\frac{du}{\sec x\tan x}$

$\frac{1}{3}u^{3}-u+C$

$\boxed{\frac{1}{3}(\sec^{3}x)-\sec x+C}$

⟨b⟩

52 $\int\sin^{3}x\,dx$ $u=\cos x$

$\int\sin^{2}x\cdot\sin x$ $\frac{du}{dx}=-\sin x$

$\int(1-\cos^{2}x)\sin x$ $dx=\frac{du}{-\sin x}$

$-\int(1-u^{2})\,du$

$=-u+\frac{1}{3}u^{3}+C$

$=\boxed{-\cos x+\frac{1}{3}\cos^{3}x+C}$

⟨C⟩

53 $\int\sec^{4}x\,dx$ $u=\tan x$

$\int\sec^{2}x\sec^{2}x\,dx$ $\frac{du}{dx}=\sec^{2}x$

$\int(1+\tan^{2}x)\sec^{2}x$ $dx=\frac{du}{\sec^{2}x}$

$\int(1+u^{2})\,du$

$u+\frac{1}{3}u^{3}+C$

$\boxed{\tan x+\frac{1}{3}\tan^{3}x+C}$

54 $\int\sec^{4}x\sqrt{\tan x}\,dx$

$\int\sec^{2}x\sqrt{u}\,du$ $u=\tan x$

$\int(1+\tan^{2}x)\sqrt{u}\,du$ $\frac{du}{dx}=\sec^{2}x$

$\int(1+u^{2}\sqrt{u})\,du$ $dx=\frac{du}{\sec^{2}x}$

$\int u^{1/2}+u^{5/2}\,du$

$\frac{2}{3}u^{3/2}+\frac{2}{7}u^{7/2}+C$

$\boxed{\frac{2}{3}(\tan x)^{3/2}+\frac{2}{7}(\tan x)^{7/2}}$

⟨b⟩

$\cot^{2}x+1=\csc^{2}x$

55 $\int\frac{2\cos x}{\sin^{2}x-4}$ $u=\sin x$ $\frac{du}{dx}=\cos x$ $dx=\frac{du}{\cos x}$

$\int\frac{2}{u^{2}-4}\,du \longrightarrow$ قابلة للتحليل
درجة البسط <

$\int\frac{2}{(u-2)(u+2)}\,du$ $\frac{A}{u-2}+\frac{B}{u+2}$

$A(u+2)+B(u-2)=2$

$u=-2 \Rightarrow -4B=2$ $B=-\frac{1}{2}$

$u=2 \Rightarrow 4A=2$ $A=\frac{1}{2}$

$\int\frac{\frac{1}{2}}{u-2}+\frac{-\frac{1}{2}}{u+2}\,du$ ⟨a⟩

$\frac{1}{2}\ln|u-2|-\frac{1}{2}\ln|u+2|+C$

$\boxed{\frac{1}{2}\ln|\sin x-2|-\frac{1}{2}\ln|\sin x+2|+C}$

56 $\int\cot^{2}x\csc^{2}x\,dx$ $u=\cot x$

$\int u^{2}\,du=\frac{1}{3}u^{3}+C$ $\frac{du}{dx}=-\csc^{2}x$

$\boxed{-\frac{1}{3}\cot^{3}x+C}$ $dx=\frac{du}{-\csc^{2}x}$

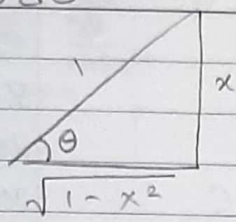
⟨C⟩

57 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx \qquad x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\int \frac{1}{\sin^2\theta\sqrt{1-\sin^2\theta}} \cos\theta\, d\theta$

$\int \frac{1}{\sin^2\theta}\, d\theta$



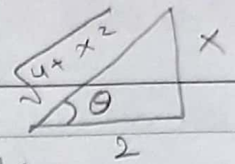
$\int \csc^2\theta\, d\theta$

$-\cot\theta + C$

$\boxed{-\frac{\sqrt{1-x^2}}{x} + C}$ (d)

58 $\int \frac{1}{\sqrt{4+x^2}}\,dx \qquad x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$



$\int \frac{1}{\sqrt{4+4\tan^2\theta}}$

$\int \frac{1}{2\sqrt{1+\tan^2\theta}}\, 2\sec^2\theta\, d\theta$

$\int \frac{1}{\cancel{2}\,\cancel{\sec\theta}}\, \cancel{2}\sec^{\cancel{2}}\theta$

$\cos\theta$

$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

$= \boxed{\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right| + C}$

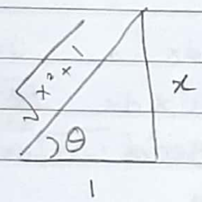
59 $\int \frac{1}{(x^2+1)^{3/2}}\,dx$

$\int \frac{1}{\sqrt{(x^2+1)^3}}\,dx \qquad x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\int \frac{1}{\sqrt{(\tan^2\theta+1)^3}} \sec^2\theta\, d\theta$

$\int \frac{1}{\sqrt{(\sec^2\theta)^3}} \sec^2\theta\, d\theta$



$\int \frac{1}{\sqrt{(\sec^3\theta)^2}} \sec^2\theta$

$\int \frac{1}{\sec^{\cancel{3}}\theta} \cancel{\sec^2\theta}$

$\int \cos\theta\, d\theta = \sin\theta + C$

$\boxed{\frac{x}{\sqrt{x^2+1}} + C}$ (c)

62 $\int \csc x\, dx = \boxed{\ln|\csc x - \cot x| + C}$

63 $\int \tan x \sec^3 x\, dx \qquad u = \sec x$

$\int \sec^2 x\, du \qquad \frac{du}{dx} = \sec x \tan x$

$\int u^2\, du = \frac{1}{3}u^3 + C$

$\boxed{\frac{1}{3}(\sec^3 x) + C} \qquad dx = \frac{du}{\sec x \tan x}$

✓(b)

60 $\int \frac{\sqrt{x^2-1}}{x}\,dx$

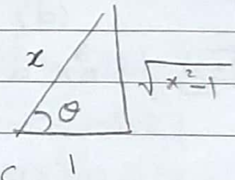
المشتقة موجودة
لكن ليس في البسط

$\int \frac{\sqrt{\sec^2\theta-1}}{\cancel{\sec\theta}}\, \cancel{\sec\theta}\tan\theta$

$x = \sec\theta$ (cos)

$\int \tan^2\theta\, d\theta \qquad dx = \sec\theta\tan\theta\, d\theta$

$\int \sec^2\theta - 1$



$\tan\theta - \theta + C$

$\sqrt{x^2-1} - \sec^{-1}x + C$

(c)

61 $\int \sec x = \boxed{\ln|\sec x + \tan x| + C}$

(a)

67 (a) $\frac{dy}{dx} = 3x(x+y)$

(b) $\frac{dy}{dx} = e^x \cdot e^{\ln y} = e^x y \qquad \frac{dy}{y} = e^x dx$ ✓

$\boxed{(b)}$

68 $y' = \frac{x}{x^2+1} \qquad \frac{dy}{dx} = \frac{x}{x^2+1} \qquad dy = \frac{x}{x^2+1}\,dx$

$y = \frac{1}{2}\ln|x^2+1| + C$

$2 = \frac{1}{2}\ln|0^2+1| + C$

$C = 2$

$\boxed{y = \frac{1}{2}\ln|x^2+1| + 2}$

(b)

69 $y' = x\sin x$

$\frac{dy}{dx} = x\sin x$

$x \quad + \sin x$
$1 \quad - \cos x$
$0 \quad - \sin x$

$-x\cos x + \sin x$

$y = -x\cos x + \sin x$

$1 = 0 + 0 + C$

$\boxed{y = -x\cos x + \sin x + 1}$ (d)

70 $y' = y^2 + 1$

$\frac{dy}{dx} = y^2 + 1$

$\frac{dy}{y^2+1} = dx$

$\tan^{-1} y = x + C$

$\boxed{y = \tan(x + c)}$

(c)

71 $y' = \frac{2xy}{x^2+1}$

$\frac{dy}{dx} = \frac{2xy}{x^2+1}$

$\frac{dy}{y} = \frac{2x}{x^2+1}\,dx$

$\ln y = \ln|x^2+1| + C$

$y = e^{\ln(x^2+1)} \cdot e^{c}$

$\boxed{y = A(x^2+1)}$ (a)

72 $y' = e^{x-y}$

$\frac{dy}{dx} = e^{x-y}$

$= e^{x} \cdot e^{-y}$

$\frac{dy}{e^{-y}} = e^{x}\,dx$

$e^{y}\,dy = e^{x}\,dx$

$e^{y} = e^{x} + C$

$\boxed{y = \ln(e^{x} + c)}$

(b)

73 $y' = \frac{\sin x}{y\cos y}$

$\frac{dy}{dx} = \frac{\sin x}{y\cos y}$

$y\cos y\,dy = \sin x\,dx$

$y \quad + \cos y$
$1 \quad \sin y$
$0 \quad -\cos y$

$y\sin y + \cos y = -\cos x + C$

$\pi\sin\pi + \cos\pi = -\cos(0) + C$

$\cancel{-1} = \cancel{-1} + C$

$C = 2$

$\boxed{y\sin y + \cos y = -\cos x}$ (d)

74 $y' = \sqrt{1-y^2}$

$\frac{dy}{dx} = \sqrt{1-y^2}$

$\frac{dy}{\sqrt{1-y^2}} = dx$

$\sin^{-1} y = x + C$

$\boxed{y = \sin(x + c)}$

(d) ✓

75 $y' = \frac{-x}{ye^{x^2}}$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$

$y\,dy = \frac{-x}{e^{x^2}}\,dx$

$y\,dy = -\frac{1}{2}\frac{1}{e^{u}}\,du$

$\frac{1}{2}y^2 = +\frac{1}{2}e^{-u}$

$y^2 = e^{-x^2} + C$

$1 = e^{-0} + C$

$1 = 1 + C$

$C = 0$

$\boxed{y^2 = e^{-x^2}}$ (d)

76 $y' = \frac{8}{x^2+1} + \sec^2 x$

$\frac{dy}{dx} = \frac{8}{x^2+1} + \sec^2 x$

$dy = \left(\frac{8}{x^2+1} + \sec^2 x\right)dx$

$\boxed{y = 8\tan^{-1} x + \tan x + C}$

(d)

77 $y' = \frac{x\sin x^2}{y}$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$y'y = x\sin x^2$

$\frac{dy}{dx}y = x\sin x^2$

$y\,dy = x\sin x^2\,dx$

$y\,dy = \frac{1}{2}\sin u\,du$

$\frac{1}{2}y^2 = -\frac{1}{2}\cos x^2 + C$

$\boxed{y = \pm\sqrt{2 - \cos x^2}}$

(a) ✓

$y^2 = -\cos x^2 + C$

$1^2 = -\cos(0) + C$

$1 = -1 + C$

$C = 2$

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78 $a(x) = 12t^2 + 4$ $\quad V(0) = 4$ $\quad S(0) = 1$ $\quad S(2) = ?$

$\int a(x) = \int 12t^2 + 4$

$V(x) = 4t^3 + 4t + C$

$4 = 0 + C$

$C = 4$

$V(x) = 4t^3 + 4t + 4$

$\int V(x) = \int 4t^3 + 4t + 4$

$S(x) = t^4 + 2t^2 + 4t + C$

$1 = C$

$S(x) = t^4 + 2t^2 + 4t + 1$

$S(2) = \boxed{33 \ (b)}$

79 $P = 2500$

$y'(t) = \frac{-2500}{(t+1)^2}$ $\quad u = t+1$ $\quad \frac{du}{dt} = 1$ $\quad dt = \frac{du}{1}$

$\frac{dy}{dt} = \frac{-2500}{(t+1)^2}$

$dy = \frac{-2500}{(t+1)^2} dt$

$\int dy = \int \frac{-2500}{u^2} du$

$\int dy = \int -2500 u^{-2} du$

$y = \frac{2500}{u} + C$

$y = \frac{2500}{t+1} + C$

$2500 = \frac{2500}{0+1} + C$

$C = 0$

$y(4) = \boxed{\frac{2500}{4+1} = 500}$ (c) ✓

80 $y(0) = 300$ $\quad y(30) = 900$

$y = Ae^{kt}$ $\quad y = 300e^{kt}$

$300 = Ae^0$ $\quad 900 = 300e^{30k}$

$300 = A$ $\quad 3 = e^{30k}$

$\ln 3 = \ln e^{30k}$

$\ln 3 = 30k$

$k = \frac{\ln 3}{30}$

$y = 300e^{0.0366t}$

$y(3) = 300e^{0.0366(180)} = \boxed{217898}$

(a)

81 $y(0) = 300$ $\quad y(30) = 900$

$T_b = \frac{\ln 2}{k}$ $\quad y = Ae^{kt}$

$300 = Ae^0$

$A = 300$

$T_b = \frac{\ln 2}{\frac{\ln 3}{30}}$ $\quad 900 = 300e^{30k}$

$3 = e^{30k}$

$\ln 3 = 30k$

$\boxed{T_b = 19 min}$ $\quad k = \frac{\ln 3}{30}$

82 $y(0) = 100$ $\quad T_b = 4$ $\quad k = \frac{\ln 2}{T_d}$

$y = Ae^{kt}$ $\quad k = \frac{\ln 2}{4}$

$100 = Ae^0$

$A = 100$

$y = 100e^{\ln 2/4 (6)} = \boxed{336.3 \ (b)}$

83 $y(0) = 50$ $\quad T_d = 6000$

$y = Ae^{kt}$ $\quad k = \frac{\ln 1/2}{6000} = 1.15 \times 10^{-4}$

$50 = A$

$y(t) = 50e^{kt}$

$y(t) = 50e^{\frac{\ln 1/2}{6000} t}$

$y(8000) = \boxed{20g}$

84 $y(0) = 0.4$ $\quad T_h = 3$ $\quad t = 24$

$y = Ae^{kt}$ $\quad k = \frac{\ln \frac{1}{2}}{3} =$

$0.4 = A$

$y = 0.4e^{\frac{\ln \frac{1}{2}}{3} t} =$

$y(24) = \boxed{1.5625 \times 10^{-3}}$

(a)

85 $y(0) = 0.8g$ $\quad k = \frac{\ln \frac{1}{2}}{3} =$

$y = Ae^{kt}$

$0.8 = Ae^0$ $\quad t = \frac{\ln \frac{1}{8}}{\frac{1}{3}\ln \frac{1}{2}}$

$A = 0.8$

$y = 0.8e^{\frac{1}{3}\ln \frac{1}{2} t}$ $\quad \boxed{t = 9 \ (b)}$ ✓

$0.1 = 0.8e^{\frac{1}{3}\ln \frac{1}{2} t}$

$\frac{1}{8} = e^{\frac{1}{3}\ln \frac{1}{2} t}$

$\ln \frac{1}{8} = \ln e$

$\boxed{= \frac{1}{3}\ln \frac{1}{2} (t)}$

86) $T_a = 20$

$y(0) = 80 \quad y(2) = 75$

$y = Ae^{kt} + T_a$

$80 = Ae^{0} + 20$

$60 = A$

$y = 60e^{kt} + 20$

$75 = 60e^{2k} + 20$

$55 = 60e^{2k}$

$\frac{55}{60} = e^{2k}$

$\ln\frac{55}{60} = \ln e^{2k}$

$k = \frac{\ln\frac{55}{60}}{2}$

$y = 60e^{\frac{1}{2}\ln\frac{55}{60}t} + 20$

$\boxed{y = 68.27}$

87) $y = Ae^{kt}$

$y = Pe^{rt}$

$r = 0.03 \quad P = 100\,000$

$\boxed{y = 100\,000e^{0.03t}}$

(a)

88) $P = 60\,000 \quad r = 0.10$

$y = Ae^{kt}$

$y = 60\,000e^{0.10t}$

$y(5) = 60\,000e^{-0.10(5)} = \boxed{36391.8}$

89) $y = \frac{AMe^{kMt}}{1 + Ae^{kMt}}$ $\quad M = 100 \quad K = 0.007 \quad t = 5 \quad y(0) = 20$

$20 = \frac{A(100)e^{0.7(0)}}{1 + Ae^{0.7(0)}}$ $\quad KM = 0.7$

$20 = \frac{100A}{1 + A}$

$20 + 20A = 100A$

$20 = 80A$

$A = 1/4$

$y(5) = \frac{1/4(100)e^{0.7(5)}}{1 + 1/4e^{0.7(5)}}$

$\boxed{= 89.2 \text{ million}}$

(c)

90) $y' = e^{x-y}$

$\frac{dy}{dx} = e^{x} \cdot e^{-y}$

$dy\, e^{y} = e^{x}\, dx$

$e^{y} = e^{x} + C$

$\ln e^{y} = \ln e^{x} + C$

$y = \ln(e^{x} + C)$

$0 = \ln(1 + C)$

$e^{0} = \cancel{e}^{\ln(1+c)}$

$1 = 1 + C$

$C = 0$

$y(2) = \ln(e^{2})$

$\boxed{y(2) = 2 \text{ (d)}}$

91) $y' = \sin x \cos^2 x$ $\quad U = \cos x$

$\frac{dy}{dx} = \sin x \cos^2 x$ $\quad \frac{du}{dx} = -\sin x$ $\quad dx = \frac{du}{-\sin x}$

$dy = \sin x \cos^2 x\, dx$

$= \cancel{\sin x}\, U^2 \frac{du}{-\cancel{\sin x}}$

$= \int -U^2 du$

$= -\frac{1}{3}U^3 + C$

$y = -1/3 \cos^3 x + C$

$0 = -\frac{1}{3}\cos^3(\pi/2) + C$

$0 = C$

$y = -\frac{1}{3}\cos^3 x$

$\boxed{y(0) = -1/3 \text{ (d)}}$ ✓

92) $K = \frac{\ln 2}{T_d} = \frac{\ln 2}{10} = 0.0693$

(a) ✓

93) $y = P\left(1 + \frac{r}{n}\right)^{nt}$

$y = 100\,000\left(1 + \frac{0.03}{12}\right)^{12(5)}$

$= \boxed{100\,000\left(1 + \frac{0.03}{12}\right)^{60}}$

94) $v(t) = 10t + 2$

$\int v(t) = \int 10t + 2$

$s(t) = \frac{10}{2}t^2 + 2t + C$

$10 = C$

$s(t) = \boxed{5t^2 + 2t + 10}$

(c)

[95] $\int \frac{\sqrt{x^2-9}}{x}\,dx$

$\boxed{x = 3\sec\theta}$

(c) ✓

[96] $\int \sqrt{x^2-4x}$

$\int \sqrt{x^2-4x+4-4}\,dx$

$\int \sqrt{(x-2)^2-4}\,dx$

$x-2 = 2\sec\theta$

$\boxed{x = 2\sec\theta+2}$

(c)

[97] $\int_0^{\pi} \cos x\, f'(\sin x)\,dx$

$U = \sin x$

$\frac{dU}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int_{\square\to 0}^{\square\to 0}$

$\int \cos x\, f'(U)\,\frac{du}{\cancel{\cos x}}$

$\boxed{\int_0^0 f'(U)\,du = 0}$ ✓

[98] $\int \tan x \sec^n x$

$U = \sec x$

$\frac{du}{dx} = \sec x \tan x$

$dx = \frac{du}{\sec x \tan x}$

$\int \cancel{\tan x}\, \sec^n x\, \frac{du}{\sec x \cancel{\tan x}}$

$\int \frac{\sec^n x}{\sec x}\,du$

$\int \sec^{n-1} x\,du$

$\int U^{n-1}\,du$

$\frac{U^{n-1+1}}{n} + C = \frac{\sec^n}{n} + C$

$\boxed{n = 3 \quad (e)}$

[99] $\int \sec^2 x \sqrt{\tan x}\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int \cancel{\sec^2 x} \sqrt{U}\, \frac{du}{\cancel{\sec x}}$

$\int U^{\frac{1}{2}}\,du$

$\frac{2}{3}U^{3/2} + C = \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$ ✓

(a)

[100] $\int \frac{e^{\sin^2 x}}{e^{-\cos^2 x}}\,dx$

$\int e^{\sin^2 x} e^{\cos^2 x}\,dx$

$\int e^{\sin^2 x + \cos^2 x}\,dx$

$\int e^1\,dx$

$\boxed{ex + C}$

(c)

[101] $y' = 2x\cos^2 y$

$\frac{dy}{dx} = 2x\cos^2 y$

$\frac{dy}{\cos^2 y} = 2x\,dx$

$\sec^2 y\,dy = 2x\,dx$

$\tan y = x^2 + C$

$y = \boxed{\tan^{-1}(x^2 + C)}$

(d) ✓

[102] $\int f(x)\sin x\,dx$

$U \qquad dV$

$f(x) \quad + \quad \sin x$

$f'(x) \quad - \quad -\cos x$

$-f(x)\cos x + \int f'(x)\cos x\,dx$

$f'(x) = 3x^2$

$\boxed{f(x) = x^3 \quad (b)}$ ✓

[103] $\int_{-3}^{5} f'(x)g(x)\,dx$

$U \qquad dV$

$g(x) \quad + \quad f'(x)$

$g'(x) \quad - \quad f(x)$

$\boxed{g(x)f(x)\Big]_{-3}^{5} - \int_{-3}^{5} f(x)g'(x)\,dx}$

$g(5)f(5) - g(-3)f(-3) - 9$

$(4)(3) - (-1)(2) - 9 = \boxed{5 \quad (b)}$ ✓

104

$$\frac{f'(x)}{f(x)} = \frac{f(x)}{f(x)}$$

$$\frac{f'(x)}{f(x)} = 1$$

$$\ln|f(x)| = x + c$$

$$e^{\ln|f(x)|} = e^{x+c}$$

$$f(x) = e^{x+c}$$

$$1 = e^{1+c}$$

$$\ln 1 = \ln e^{1+c}$$

$$0 = 1 + c$$

$$c = -1$$

$$\boxed{f(x) = e^{x-1}}$$

<d>

105

$$f'(x) = 3x^2 + 2x$$

$$\int f'(x) = \int 3x^2 + 2x$$

$$f(x) = x^3 + x^2 + C$$

$$3 = 2^3 + 2^2 + C$$

$$3 = 12 + C$$

$$C = -9$$

$$f(1) = x^3 + x^2 - 9$$

$$\boxed{f(1) = -7 \quad \langle b \rangle}$$

108

$$\frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Dx + C}{x^2 + 1}$$

$\boxed{\langle c \rangle}$

109

$$\left[\begin{array}{cccc} & 2 & & 2x \\ 0 & x & 1 & x \end{array} \right.$$

$$H(x) = \int_0^x f(t)\,dt$$

$$H(x) = \int_0^x 2\,dx = (x-0)2 = 2x$$

$$H(x) = \int_0^x 2x\,dx = \int_0^1 2\,dx + \int_1^x 2x\,dx$$

$$= 2 + x^2 \Big]_1^x$$

$$2 + x^2 - 1 = x^2 + 1$$

$$\boxed{f(x) = \begin{cases} 2x & , 0 \le x < 1 \\ x^2 + 1 & , x \ge 1 \end{cases}}$$

107

$$y' = \sin(x+y)$$

$$\frac{dy}{dx} = \sin(x+y)$$

$$\frac{dy}{\sin(x+y)} = dx$$

$\boxed{\langle a \rangle}$

110

$$\int_0^1 e^x f(x) + e^x f'(x)\,dx$$

$$e^x f(x) \Big]_0^1$$

$$e^1 f(1) - e^0 f(0) = \boxed{2e + 1}$$

$\langle a \rangle$

111 $y = Ae^{kt}$ $\quad T_h = \frac{\ln\frac{1}{2}}{K}$

$y = Ae^{-0.024t}$ $\quad 28 = \frac{\ln\frac{1}{2}}{K}$

$y = 100e^{-0.024t}$ $\quad K = \frac{\ln\frac{1}{2}}{28}$

$y = 100e^{-0.024(84)}$ $\quad K = -0.024$

$\boxed{y = 12.5\%}$ (a)

112 $y'(t) = -0.14\, y(t)$

$\frac{dy}{dt} = -0.14\, y(t)$

$\int \frac{dy}{y} = \int -0.14\, dt$

$e^{\ln y} = e^{-0.14t + C}$

$y = Ae^{-0.14t}$

$\frac{1}{2}\cancel{A} = \cancel{A}e^{-0.14t}$

$\frac{1}{2} = e^{-0.14t}$

$\ln\frac{1}{2} = -0.14t$

$t = \frac{\ln\frac{1}{2}}{-0.14} \approx \boxed{5}$ (a)

113 $y'(t) = Ky(M-y)$ $\quad \boxed{y'(t) = 0.025y(5000-y)}$ (d)

114 $y = e^{2x}$

$y' = 2e^{2x}$

$y'' = 4e^{2x}$

نشتق الخيارات ثم نعوض بالمعادلة التفاضلية

$4e^{2x} - 4(e^{2x}) = 0$

(a) $\boxed{y = e^{2x}}$

115 $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx$ $\quad U = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$= \int \frac{\cancel{e^x}}{1+U^2}\,\frac{dU}{\cancel{e^x}}$

$= \boxed{\tan^{-1} e^x + C}$ (b)

116 $\int \frac{1}{\sqrt{2x - x^2}}$

$U = x - 1$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{\sqrt{-(x^2 - 2x + 1 - 1)}}$

$\int \frac{1}{\sqrt{-(x-1)^2 + 1}}$

$\int \frac{1}{\sqrt{1 - U^2}} du = \boxed{\sin^{-1}(x-1) + C}$ <b>

117 $\int \frac{1}{2}\left[\sin(x) + \sin(3+2)x\right] dx$

$\int \frac{1}{2}\left[\sin x + \sin 5x\right] dx$

$\frac{1}{2}\left[-\cos x - \frac{1}{5}\cos 5x\right] + C$

$\boxed{-\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C}$ <a>

118 $\int \sec^3 \alpha \, d\alpha$

$\sec\alpha \quad + \quad \sec^2\alpha$

$\sec\alpha\tan\alpha \quad - \quad \tan\alpha$

$\sec\alpha\tan\alpha - \int \sec\alpha \tan^2\alpha$

<d> $\boxed{g(x) = \sec\alpha\tan\alpha}$ [illegible]

119 $\int \frac{x+1}{\sqrt{3-2x-x^2}} dx = \int \frac{\cancel{x+1}}{\sqrt{U}} \frac{du}{-2\cancel{(x+1)}}$

$U = 3 - 2x - x^2$

$\frac{du}{dx} = -2x - 2$

$dx = \frac{du}{-2(x+1)}$

$f'(x) = -2x - 2$

$f'(x) = -2(x+1)$

$= \int \frac{du}{-2\sqrt{U}}$

$= \frac{-1}{2}\int U^{-\frac{1}{2}} du$

$= -\frac{1}{2}\cdot 2 U^{\frac{1}{2}} + C$

$= -\sqrt{U} + C$

$= \boxed{-\sqrt{3 - 2\alpha - \alpha^2} + C}$ <b>

120 $\int \frac{1}{x^2+a^2}\,dx$ $\quad U = \frac{x}{a}$

$\int \frac{1}{a^2\left(\left(\frac{x}{a}\right)^2+1\right)}$ $\quad \frac{du}{dx} = \frac{1}{a}$

$dx = a\,du$

$\int \frac{1}{a^2(U^2+1)} a\,du$

$= \boxed{\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C}$ <c>

121 $\frac{A}{x-1} + \frac{B}{x+2}$

$A(x+2) + B(x-1) = 3x$

$x = -2 \Rightarrow B(-3) = 3(-2) \Rightarrow B = \frac{6}{3} = 2$

$x = 1 \Rightarrow 3A = 3 \Rightarrow A = 1$ $\quad \boxed{A=1 \quad B=2}$ <a>

122 $T_h = \frac{\ln\frac{1}{2}}{K} = 5730$ $\quad K = -1.2\times10^{-4}$

$y = Ae^{-Kt}$

$0.20A = Ae^{-1.2\times10^{-4}t}$

$\ln(0.20) = \ln e^{-1.2\times10^{-4}t}$

$\ln(0.20) = -1.2\times10^{-4}t$

$t \approx 13411.98 \approx \boxed{13304}$ <a>

123 $y' = K(y - T_a) = \boxed{K(y-30)}$ <c>

124 $\int 2(\tan x + \tan^3 \alpha)\, dx$

$U = \tan \alpha$

$\frac{du}{d\alpha} = \sec^2 \alpha$

$d\alpha = \frac{du}{\sec^2 \alpha}$

$2 \int U + U^3 \frac{du}{1 + U^2}$

$\int U(\cancel{1+U^2}) \frac{du}{\cancel{(1+U^2)}}$

$2 \int U\, du = U^2 + C = \boxed{\tan^2 \alpha + C}$ (A)

125 $y' = ky \quad y(0) = A$

$\frac{dy}{dt} = ky$

$\frac{dy}{y} = k\,dt$

$\ln y = kt + c$

$y = e^{kt} + e^{c}$

$\boxed{y = Ae^{kt}}$ (a)

126 $\int (\ln \alpha)^2\, d\alpha$

$U = \ln \alpha$

$\frac{du}{d\alpha} = \frac{1}{\alpha}$

$d\alpha = du\, \alpha$

$e^{u} = x$

$\int u^2\, du\, x$

$\boxed{\int u^2 e^{u}\, du}$ (b)