المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الثالث	
حل أسئلة الدرس الرابع حركة المقذوفات من الوحدة السادسة وفق الهيكل الوزاري	3
حل أسئلة الدرس الثالث طول القوس ومساحة السطح من الوحدة السادسة وفق الهيكل الوزاري	4
حل نموذج اختبار تجريبي يحاكي الهيكل الوزاري	5

Part 7

الجزء السابع - هيكل 12 متقدم

الفصل الدراسي الثالث 2023 / 2024

الدرس 3-7 | طرائق تكامل الدوال المثلثية

Lesson 7-3 | Trigonometric Techniques of Integration

13	Integrate functions of the form $\sin^n x \cdot \cos^m x$ إيجاد تكاملات دوال بصيغة $\sin^n x \cdot \cos^m x$	Exercises (1-6)	P507

Evaluate the integral. $\frac{\pi}{3} \rightarrow \frac{9}{32}$ **جــد قيمة التكامل**

1) $\int \cos^1 x \sin^4 x \; dx$

A) $\frac{1}{5}\cos^5 x + c$

B) $\frac{1}{5}\sin^5 x + c$

C) $\frac{1}{5}\sin^5 x + \cos x + c$

D) $-\frac{1}{5}\cos^5 x + c$

$= \int \sin^4 x \; \cos x\, dx$ $\qquad u = \sin x$, $du = \cos x\, dx$

$= \int u^4 \, du$

$= \frac{1}{5} u^5 + C$

$= \frac{1}{5} \sin^5 x + C$

$$\boxed{\int [f(x)]^n \cdot f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C}$$

$\int [\sin x]^4 \cos x \; dx$

$= \frac{1}{5} \sin^5 x + C$

Evaluate the integral. $\cos^2 x = 1 - \sin^2 x$ $\boxed{x = \frac{\pi}{3}}$ جـــد قيمة التكامل

2) $\int \cos^3 x \sin^4 x \; dx$

$= \int \cos^2 x \;\; \sin^4 x \;\;\; \boxed{\cos x \, dx}$

$= \int (1 - \sin^2 x) \sin^4 x \; \boxed{\cos x \, dx}$

$= \int (1 - u^2) u^4 \; du$

$u = \sin x$
$du = \cos x \, dx$

$= \int u^4 - u^6 \, du$

$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C$

$= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$

A) $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

B) $-\frac{1}{5}\sin^5 x + \frac{1}{7}\sin^7 x + c$

C) $\frac{1}{5}\sin^5 x + \frac{1}{7}\sin^7 x + c$

D) $\frac{1}{5}\cos^5 x - \frac{1}{7}\cos^7 x + c$

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Evaluate the integral. جـــد قيمة التكامل

3) $\int_0^{\pi/4} \cos 2x \sin^3 2x \; dx$

Select 2 answers

$= \frac{1}{2} \int_0^{\frac{\pi}{4}} [\sin 2x]^3 \, 2\cos 2x \quad dx$

$= \frac{1}{2} \; \frac{\sin^4 2x}{4} \Big|_0^{\frac{\pi}{4}} = \frac{1}{8} \sin^4 2x \Big|_0^{\frac{\pi}{4}}$

$\int_0^{\frac{\pi}{4}} \sin^3 2x \quad \cos 2x \, dx$

$u = \sin 2x$
$du = 2\cos 2x \, dx$
$\frac{1}{2} du = \cos 2x \, dx$

$x = 0 \Rightarrow u = 0$
$x = \frac{\pi}{4} \Rightarrow u = 1$

$\int_0^1 u^3 \cdot \frac{1}{2} du = \frac{1}{2} \cdot \frac{1}{4} u^4 \Big|_0^1$

$= \frac{1}{8} u^4 \Big|_0^1 = \boxed{\frac{1}{8}}$

A) $\frac{1}{8}$

B) $\frac{3}{8}$

C) $\frac{1}{8}\sin^4 2x \Big|_0^{\pi/4}$

D) $\frac{1}{4}\sin^4 2x \Big|_0^{\pi/4}$

Mr. Ali Abdalla

Evaluate the integral. جــــد قيمة التكامل

4) $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \; dx$

$\cos^2 3x = 1 - \sin^2 3x$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2 3x \; \sin^3 3x \; \cos 3x \; dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \sin^2 3x) \sin^3 3x \; \cos 3x \, dx$

$u = \sin 3x$

$du = 3\cos 3x \, dx$

$\frac{1}{3} du = \cos 3x \, dx$

$x = \frac{\pi}{4} \qquad u = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{3} \qquad u = \sin \pi = 0$

$= \int_{\frac{\sqrt{2}}{2}}^{0} (1 - u^2) u^3 \cdot \frac{1}{3} du$

$= \frac{1}{3} \int_{0}^{\frac{\sqrt{2}}{2}} u^5 - u^3 \, du = \frac{1}{3} \left[\frac{1}{6} u^6 - \frac{1}{4} u^4 \right] \Big|_{0}^{\frac{\sqrt{2}}{2}}$

$= -\frac{1}{72}$

A) $\frac{1}{3}$

B) $\frac{1}{72}$

C) $-\frac{1}{72}$

D) -1

Evaluate the integral. جــــد قيمة التكامل

5) $\int_{0}^{\pi/2} \cos^2 x \; \underset{(-\sin x)}{\sin x} \; dx$

$\int [f(x)]^n f'(x) \, dx = \frac{1}{n+1} [f(x)]^{n+1} + C$

$= \frac{1}{3} \cos^3 x \Big|_{0}^{\frac{\pi}{2}}$

$= \frac{1}{3} \left[\cos^3 \frac{\pi}{2} - \cos^3 0 \right]$

$= \frac{1}{3} [0 - 1]$

$= -\frac{1}{3}$

A) $\frac{1}{3}$

B) $\frac{1}{27}$

C) $-\frac{1}{27}$

D) $-\frac{1}{3}$

Evaluate the integral.

جـــد قيمة التكامل

6) $-\int_{-\pi/2}^{0} \cos^3 x \sin x \ dx$ $-\sin$

$\frac{1}{4} \cos^4 x \Big|_{-\frac{\pi}{2}}^{0}$

$=$

A) 1

B) $\frac{1}{4}$

C) $-\frac{1}{4}$

D) -1

14	Integrate functions of the form $\sec^n x \tan^m x$ إيجاد تكاملات دوال بصيغة $\sec^n x \tan^m x$	Exercises (9,11,12,15,16)	P507

Evaluate the integral.

جـــد قيمة التكامل

9) $\int \tan x \sec^3 x \ dx$

اس ال tan المعزول
نأخذ $\tan x \sec x$ مع dx

$= \int \sec^2 x \ \sec x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \int u^2 \, du$

$= \frac{1}{3} u^3 + C$

$= \frac{1}{3} \sec^3 x + C$

A) $\frac{1}{3}\sec^3 x + c$

B) $\frac{1}{2}\tan^2 x + c$

C) $\frac{2}{3}\sec^3 x + c$

D) $\sec x \tan x + c$

Evaluate the integral.

جـــد قيمة التكامل

15

11) $\int x \tan^3(x^2+1)\sec(x^2+1)\,dx$

$x^2+1=t \Rightarrow 2x\,dx = dt$

$\Rightarrow dx = \frac{dt}{2x}$

$\int \cancel{x} \tan^3 t \sec t \cdot \frac{dt}{2\cancel{x}}$

$= \frac{1}{2}\int \tan^3 t \sec t \, dt$

$= \frac{1}{2}\int \tan^2 t \sec t \tan t \, dt$

$\tan^2 t = \sec^2 t - 1$

$= \frac{1}{2}\int (\sec^2 t - 1) \sec t \tan t \, dt$

$u = \sec t$

$du = \sec t \tan t \, dt$

$= \frac{1}{2}\int u^2 - 1 \, du = \frac{1}{2}\left[\frac{1}{3}u^3 - u\right] + C$

$\frac{1}{6}\sec^3 t - \frac{1}{2}\sec t + C$

$= \frac{1}{6}\sec^3(x^2+1) - \frac{1}{2}\sec(x^2+1) + C$

A) $\frac{1}{6}\sec^3(x^2+1)+\frac{1}{2}\sec(x^2+1)+c$

B) $\frac{1}{2}\sec^3(x^2+1)+\frac{1}{6}\sec(x^2+1)+c$

C) $\frac{1}{6}\sec^3(x^2+1)-\frac{1}{2}\sec(x^2+1)+c$

D) $-\frac{1}{6}\sec^3(x^2+1)-\frac{1}{2}\sec(x^2+1)+c$

12) $\int \boxed{\tan(2x+1)\sec^3(2x+1)}\,dx$

$t = 2x+1$

$dt = 2dx \Rightarrow dx = \frac{1}{2}dt$

$\frac{1}{2}\int \tan t \sec^3 t \, dt$

$= \frac{1}{2}\int \sec^2 t \sec t \tan t \, dt$

$u = \sec t$

$du = \sec t \tan t \, dt$

$= \frac{1}{2}\int u^2 \, du$

$= \frac{1}{2}\cdot\frac{1}{3}u^3 + C$

$= \frac{1}{6}\sec^3 t + C$

$= \frac{1}{6}\sec^3(2x+1) + C$

A) $\frac{1}{6}\sec^3(2x+1)+c$

B) $\frac{1}{2}\sec^3(2x+1)+c$

C) $\frac{1}{3}\sec^3(x^2+1)+c$

D) $6\sec^3(x^2+1)+c$

Evaluate the integral. جـــد قيمة التكامل

$1+\tan^2 x = \sec^2 x$

15) $\int_0^{\pi/4} \tan^4 x \sec^4 x \; dx$

Select 2 answers.

A) $\left(\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x\right)\Big|_0^{\pi/4}$

B) $\left(\frac{1}{5}\tan^5 x - \frac{1}{7}\tan^7 x\right)\Big|_0^{\pi/4}$

C) $\frac{12}{35}$

D) $\frac{11}{35}$

$= \int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x \sec^2 x \, dx$

$= \int_0^{\frac{\pi}{4}} \tan^4 x (1+\tan^2 x) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{4} \Rightarrow u = 1$

$= \int_0^1 u^4 (1+u^2) \, du$

$= \int_0^1 u^4 + u^6 \, du$

$= \frac{1}{5}u^5 + \frac{1}{7}u^7 \Big|_0^1 = \frac{12}{35}$

Evaluate the integral. جـــد قيمة التكامل

16) $\int_{-\pi/4}^{\pi/4} \tan^4 x \sec^2 x \; dx$

Select 2 answers.

A) $\left(\frac{1}{5}\tan^5 x\right)\Big|_{-\pi/4}^{\pi/4}$

B) $\left(\frac{1}{5}\tan^5 x - \frac{1}{7}\tan^7 x\right)\Big|_{-\pi/4}^{\pi/4}$

C) $\frac{2}{5}$

D) $\frac{1}{5}$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^4 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = -\frac{\pi}{4} \Rightarrow u = -1$

$x = \frac{\pi}{4} \Rightarrow u = 1$

$= \int_{-1}^{1} u^4 \, du$

$= \frac{1}{5} u^5 \Big|_{-1}^{1} = \frac{1}{5} \tan^5 x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

Determine the value of m if: $\int \tan x \sec^m x \, dx = \frac{1}{3}\sec^3 x + c$

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) 2

D) 3

$\int \sec^{m-1} x \sec x \tan x \, dx$ $u = \sec x$

$du = \sec x \tan x \, dx$

$= \int u^{m-1} \, du$

$= \frac{u^m}{m} + c = \frac{1}{m}\sec^m x + c$ $\boxed{m = 3}$

حل آخر

استخدام الاشتقاق (العكس)

$\frac{d}{dx}\left(\frac{1}{3}\sec^3 x + c\right) = \frac{1}{3} \cdot 3 \sec^2 x \cdot \sec x \tan x$

$= \sec^3 x \tan x$ $m = 3$

20) A certain substitution gives $\int \sin^3 x \cos^n x \, dx = \int u^9 - u^7 \, du$

What is the value of n?

A) 3

B) 5

C) 7

D) 9

$\int \sin^2 x \cos^n x \sin x \, dx$

$= \int (1 - \cos^2 x) \cos^n x \sin x \, dx$

$= \int (1 - u^2) u^n (-du)$ $u = \cos x$

$du = -\sin x \, dx$

$= \int u^{n+2} - u^n \, du$

$= \int u^9 - u^7 \, du$

$n = 7$

If : $\int \tan^3 x \sec x \, dx = \frac{1}{n}\sec^n x - \sec x + c$ then the value of n is

A) $\frac{1}{3}$

B) 3

C) $\frac{1}{4}$

D) 4

$\int \tan^2 x \sec x \tan x \, dx$ $\qquad \tan^2 x = \sec^2 x - 1$

$= \int (\sec^2 x - 1) \sec x \tan x \, dx$ $\qquad u = \sec x$

$du = \sec x \tan x \, dx$

$= \int u^2 - 1 \, du$

$= \frac{1}{3}u^3 - u + C$

$= \frac{1}{3}\sec^3 x - \sec x + C$

Garde 12 Advanced

Easy math – Mr. Ali Abdalla

If : $\int \sec^4 x \, dx = \frac{1}{3} f(x) + c$ then

A) $f(x) = \tan x + 3\tan^3 x$

B) $f(x) = 3\tan x + \tan^3 x$

C) $f(x) = 3\tan x - \tan^3 x$

D) $f(x) = \tan x - 3\tan^3 x$

$\int \boxed{\sec^2 x} \sec^2 x \, dx$

$= \int 1 + \tan^2 x \sec^2 x \, dx$

$= \int (1 + u^2) \, du$

$u = \tan x$

$du = \sec^2 x \, dx$

$= u + \frac{1}{3}u^3 + C$

$= 1\tan x + \frac{1}{3}\tan^3 x + C$

$= \frac{1}{3}\left[3\tan x + \tan^3 x\right] + C$

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15	Integrate trigonometric functions using the substitution: $x = a\tan(\theta)$ إيجاد تكاملات دوال مثلثية باستخدام التعويض بـ $x = a\tan(\theta)$	Exercises (33-36,38-40)	P507

Evaluate the integral. **جـــد قيمة التكامل**

33) $\int \frac{x^2}{\sqrt{9+x^2}}\boxed{dx}$

$$\int \sec^n x \; dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x \; dx$$

A) $\frac{x\sqrt{9+x^2}}{2} - \frac{9}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

B) $\frac{x\sqrt{9+x^2}}{2} + \frac{9}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

C) $\frac{x\sqrt{9+x^2}}{2} + \frac{5}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

D) $\frac{x\sqrt{9+x^2}}{2} - \frac{5}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

$a^2 = 9 \Rightarrow \boxed{a = 3}$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\sqrt{9+x^2} = 3\sec\theta$

$\int \frac{(3\tan\theta)^2}{3\sec\theta} \cdot 3\sec^2\theta\, d\theta = \int \frac{9\tan^2\theta}{\cancel{3}\sec\theta} \cdot \cancel{3}\sec^2\theta\, d\theta$

$= 9\int \tan^2\theta \sec\theta\, d\theta$

$\boxed{\tan^2\theta = \sec^2\theta - 1}$

$= 9\int (\sec^2\theta - 1)\sec\theta\, d\theta = 9\int (\sec^3\theta - \sec\theta)\, d\theta$

The appropriate change of variables for the integral $\int \frac{x^2}{\sqrt{9+x^2}}dx$ is

A) $x = \cancel{25}\,9 \tan\theta$

B) $x = \cancel{5}\,3 \sin\theta$

C) $x = \cancel{5}\,3 \sec\theta$

D) $x = \cancel{5}\,3 \tan\theta$

$a^2 = 9 \Rightarrow a = 3$

$x = 3\tan\theta$

Evaluate the integral. جـــد قيمة التكامل

34) $\int x^3\sqrt{8+x^2}\,dx$

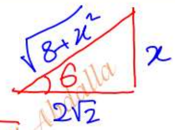
$a^2 = 8$

$a = 2\sqrt{2}$

$\boxed{x = 2\sqrt{2}\tan\theta}$

$dx = 2\sqrt{2}\sec^2\theta\, d\theta$

$\sqrt{8+x^2} = 2\sqrt{2}\sec\theta$



A) $\frac{2}{3}(8+x^2)^{\frac{3}{2}} - 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

B) $\frac{2\sqrt{2}}{3}(8+x^2)^{\frac{3}{2}} + 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

C) $\frac{4}{3}(8+x^2)^{\frac{3}{2}} - 32(8+x^2)^{\frac{1}{2}} + c$

D) $\frac{2\sqrt{2}}{3}(8+x^2)^{\frac{3}{2}} - 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

$= \int (2\sqrt{2}\tan\theta)^3 \cdot 2\sqrt{2}\sec\theta \cdot 2\sqrt{2}\sec^2\theta\, d\theta$

$= 128\sqrt{2}\int \tan^3\theta\,\sec^3\theta\, d\theta$

$= 128\sqrt{2}\int \tan^2\theta\,\sec^2\theta\,\sec\theta\tan\theta\, d\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= 128\sqrt{2}\int (\sec^2\theta - 1)\sec^2\theta\,\sec\theta\tan\theta\, d\theta$

$u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta$

$= 128\sqrt{2}\int (u^2-1)u^2\, du = 128\sqrt{2}\int u^4 - u^2\, d\theta$

$= 128\sqrt{2}\left[\frac{1}{5}u^5 - \frac{1}{3}u^3\right] + C$

$= 128\sqrt{2}\left[\frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta\right] + C$

$= 128\sqrt{2}\left[\frac{1}{5}\cdot\frac{(8+x^2)^{5/2}}{(2\sqrt{2})^5} - \frac{1}{3}\cdot\frac{(8+x^2)^{3/2}}{(2\sqrt{2})^3}\right] + C$

$= \frac{1}{5}(8+x^2)^{5/2} - \frac{8}{3}(8+x^2)^{3/2} + C$

Evaluate the integral. جـــد قيمة التكامل

35) $\int \sqrt{16+x^2}\,dx$

$\boxed{x = 4\tan\theta}$ $\tan\theta = \frac{x}{4}$

$dx = 4\sec^2\theta$

$\sqrt{16+x^2} = 4\sec\theta$

A) $x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

B) $2x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

C) $\frac{1}{2}x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

D) $\frac{1}{2}x\sqrt{16+x^2} - 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

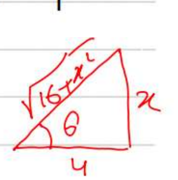
$= \int 4\sec\theta \cdot 4\sec^2\theta\, d\theta$

$= 16\int \sec^3\theta\, d\theta$

$= 16\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta\, d\theta\right]$

$\tan\theta = \frac{x}{4}$

$\sec\theta = \frac{\sqrt{16+x^2}}{4}$



$= 8\sec\theta\tan\theta + 8\ln|\sec\theta + \tan\theta| + C$

$= 8\left(\frac{\sqrt{16+x^2}}{4}\cdot\frac{x}{4}\right) + 8\ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$

$= \frac{x\sqrt{16+x^2}}{2} + 8\ln$

$$\int \sec^n x\, dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x\, dx$$

The appropriate change of variables for the integral $\int \sqrt{16 + x^2}\, dx$ is

A) $x = 16 \tan\theta$

B) $x = 4 \sin\theta$

C) $x = 4 \sec\theta$

D) $x = 4 \tan\theta$

توقعات الاختبار

Evaluate the integral. جـــد قيمة التكامل

36) $\int \frac{1}{\sqrt{4 + x^2}}\, dx$

$\tan\theta = \frac{x}{2}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta \quad \Big| \quad \sqrt{4+x^2} = 2\sec\theta$

A) $\ln\left|x + \sqrt{4 + x^2}\right| + c$

B) $\ln\left|x - \sqrt{4 + x^2}\right| + c$

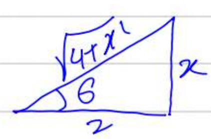
C) $\frac{1}{2}\ln\left|x + \sqrt{4 + x^2}\right| + c$

D) $\frac{1}{3}\ln\left|x + \sqrt{4 + x^2}\right| + c$

$= \int \frac{1}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$

$= \int \sec\theta\, d\theta = \int \frac{\sec\theta}{1} \cdot \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\, d\theta$

$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta}\, d\theta = \ln|\sec\theta + \tan\theta| + C$



$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C = \ln\left|x + \sqrt{4+x^2}\right| + C$

Evaluate the integral. جـــد قيمة التكامل

38) $\int_0^2 x^2\sqrt{x^2+9}\,dx$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta \quad , \quad \sqrt{x^2+9} = 3\sec\theta$

$\int (3\tan\theta)^2 \cdot 3\sec\theta \cdot 3\sec^2\theta\,d\theta$

$= \int 81\tan^2\theta\sec^3\theta\,d\theta \qquad \tan^2\theta = \sec^2\theta - 1$

$= \int 81(\sec^2\theta - 1)\sec^3\theta\,d\theta$

$= \int 81\sec^5\theta\,d\theta - 81\int\sec^3\theta\,d\theta$

$n=5$

$$\boxed{\int \sec^n\theta\,d\theta = \frac{1}{n-1}\sec^{n-2}\theta\tan\theta + \frac{n-2}{n-1}\int\sec^{n-2}\theta\,d\theta}$$

$n=3$

A) 8.99399

B) 9.99399

C) $2-\sqrt{13}$

D) $2+\sqrt{13}$

Evaluate the integral. جـــد قيمة التكامل

39) $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

$x = \tan\theta \Rightarrow \boxed{\theta = \tan^{-1}x}$

$dx = \sec^2\theta\,d\theta$

$\sqrt{1+x^2} = \sec\theta$

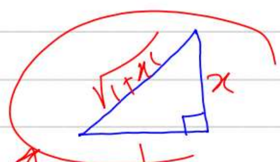
A) $\frac{1}{2}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + c$

B) $\sec^3(\tan^{-1}x) - \frac{1}{3}\sec(\tan^{-1}x) + c$

C) $\frac{1}{3}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + c$

D) $\frac{1}{3}\sec^3(\tan^{-1}x) + \sec(\tan^{-1}x) + c$

$= \int \frac{\tan^3\theta}{\sec\theta}\cdot\sec^2\theta\,d\theta$



$= \int \tan^3\theta\sec\theta\,d\theta \qquad \tan^2\theta = \sec^2\theta - 1 \qquad u = \sec\theta, \; du = \sec\theta\tan\theta\,d\theta$

$= \int \tan^2\theta\sec\theta\tan\theta\,d\theta = \int(\sec^2\theta - 1)\sec\theta\tan\theta\,d\theta$

$= \int u^2 - 1\,du = \frac{1}{3}u^3 - u + C = \frac{1}{3}\sec^3\theta - \sec\theta + C$

$\boxed{= \frac{1}{3}\left(\sqrt{1+x^2}\right)^3 - \sqrt{1+x^2} + c} \qquad = \frac{1}{3}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + c$

The appropriate change of variables for the integral $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$ is

A) $x = \tan\theta$

B) $x = \sin\theta$

C) $x = \sec\theta$

D) $x = \tan\theta$ ✓

Evaluate the integral. جـــد قيمة التكامل

40) $\int \frac{x+1}{\sqrt{4+x^2}}\,dx$

$\boxed{x = 2\tan\theta} \Longrightarrow \tan\theta = \frac{x}{2}$ or $\theta = \tan^{-1}\left(\frac{x}{2}\right)$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{4+x^2} = 2\sec\theta$

A) $2\sec\left[\tan^{-1}\left(\frac{x}{2}\right)\right] - \ln\left|\sec\left[\tan^{-1}\left(\frac{x}{2}\right)\right] + \left(\frac{x}{2}\right)\right| + c$

B) ✓ $2\sec\left[\tan^{-1}\left(\frac{x}{2}\right)\right] + \ln\left|\sec\left[\tan^{-1}\left(\frac{x}{2}\right)\right] + \left(\frac{x}{2}\right)\right| + c$

$= \int \frac{2\tan\theta + 1}{\cancel{2\sec\theta}} \cdot \cancel{2}\sec^{\cancel{2}}\theta \; d\theta$

$= \int 2\tan\theta\sec\theta + \sec\theta \; d\theta$

$= 2\sec\theta + \ln|\sec\theta + \tan\theta| + C$

$= 2\sec\left(\tan^{-1}\left(\frac{x}{2}\right)\right) + \ln\left|\sec\left(\tan^{-1}\frac{x}{2}\right) + \tan\left(\tan^{-1}\frac{x}{2}\right)\right| + C$

The appropriate change of variables for the integral $\int \frac{x+1}{\sqrt{4+x^2}}\,dx$ is

A) $x = 2\tan\theta$

B) $x = 2\sin\theta$

C) $x = 2\sec\theta$

D) $x = 4\tan\theta$