تجميعة أسئلة وفق الهيكل الوزاري الجديد

موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الثاني عشر المتقدم ⇦ رياضيات ⇦ الفصل الثالث ⇦ الملف

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المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الثالث	
5	مراجعة أوراق عمل الوحدة السادسة تطبيقات على التكامل المحدود

تعليم
مؤسسة الإمارات للتعليم المدرسي
EMIRATES SCHOOLS ESTABLISHMENT

المعرفة
بالمعرفة تبنى العقول

EOT Math 12 Advanced 2023-2024 – term 3

1	Find the area between two curves using definite integration إيجاد مساحة المنطقة المحصورة بين منحنيين باستخدام التكامل المحدود	(1-18)	414

In exercises 1–4, find the area between the curves on the given interval.

1. $y = x^3, y = x^2 - 1, 1 \le x \le 3$
2. $y = \cos x, y = x^2 + 2, 0 \le x \le 2$
3. $y = e^x, y = x - 1, -2 \le x \le 0$
4. $y = e^{-x}, y = x^2, 1 \le x \le 4$

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In exercises 5–12, sketch and find the area of the region determined by the intersections of the curves.

5. $y = x^2 - 1,\ y = 7 - x^2$
6. $y = x^2 - 1,\ y = \frac{1}{2}x^2$
7. $y = x^3,\ y = 3x + 2$
8. $y = \sqrt{x},\ y = x^2$
9. $y = 4xe^{-x^2},\ y = |x|$
10. $y = \dfrac{2}{x^2 + 1},\ y = |x|$

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11. $y = \dfrac{5x}{x^2+1}, y = x$

12. $y = \sin x \, (0 \le x \le 2\pi), y = \cos x$

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In exercises 13–18, sketch and estimate the area determined by the intersections of the curves.

13. $y = e^x, y = 1 - x^2$

14. $y = x^4, y = 1 - x$

15. $y = \sin x, y = x^2$

16. $y = \cos x, y = x^4$

17. $y = x^4, y = 2 + x$

18. $y = \ln x, y = x^2 - 2$

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2	Compute volume by means of definite integration using areas of cross sections حساب الحجم بالتكامل المحدود مع استخدام مساحات المقاطع العرضية	Ex.(1-4)	429

In exercises 1–4, find the volume of the solid with cross-sectional area $A(x)$.

1. $A(x) = x + 2, -1 \le x \le 3$
2. $A(x) = 10e^{0.01x}, 0 \le x \le 10$

2	Compute volume by means of definite integration using areas of cross sections حساب الحجم بالتكامل المحدود مع استخدام مساحات المقاطع العرضية	Ex.(1-4)	429

3. $A(x) = \pi(4 - x)^2, 0 \le x \le 2$

4. $A(x) = 2(x + 1)^2, 1 \le x \le 4$

3	Find the volume of a solid of revolution using the method of disks	(17,19,25)-(disks parts)	430
	إيجاد حجم مجسم باستخدام طريقة الأقراص	(27,28)-(disks parts)	431

In exercises 17–20, compute the volume of the solid formed by revolving the given region about the given line.

17. Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about (a) the x-axis; (b) $y = 3$

19. Region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about (a) the y-axis; (b) $x = 4$

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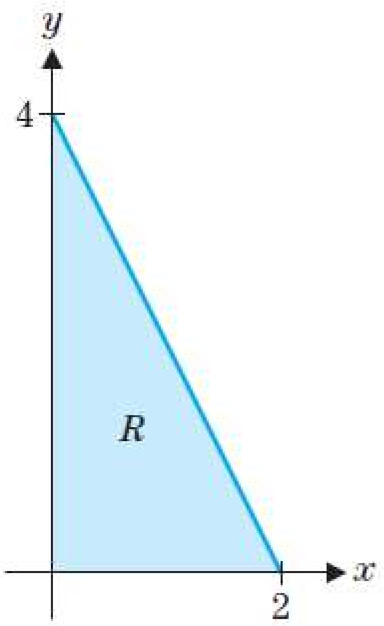
25. Let R be the region bounded by $y = 4 - 2x$, the x-axis and the y-axis. Compute the volume of the solid formed by revolving R about the given line.

(a) the y-axis (b) the x-axis (c) $y = 4$

(d) $y = -4$ (e) $x = 2$ (f) $x = -2$



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27. Let R be the region bounded by $y = x^2$, $y = 0$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.

(a) the y-axis (b) the x-axis (c) $x = 1$

(d) $y = 1$ (e) $x = -1$ (f) $y = -1$

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		(27,28)-(disks parts)	431

28. Let R be the region bounded by $y = x$, $y = -x$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.

(a) the x-axis (b) the y-axis

(c) $y = 1$ (d) $y = -1$

4	Find arc length in a given interval using definite integration إيجاد طول قوس من منحنى دالة في فترة معطاة باستخدام التكامل المحدود	(5-14)	446

In exercises 5–14, compute the arc length exactly.

5. $y = 2x + 1, 0 \leq x \leq 2$
6. $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$
7. $y = 4x^{3/2} + 1, 1 \leq x \leq 2$
8. $y = \frac{1}{4}(e^{2x} + e^{-2x}), 0 \leq x \leq 1$
9. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, 1 \leq x \leq 2$
10. $y = \frac{1}{6}x^3 + \frac{1}{2x}, 1 \leq x \leq 3$
11. $x = \frac{1}{8}y^4 + \frac{1}{4y^2}, -2 \leq y \leq -1$
12. $x = e^{y/2} + e^{-y/2}, -1 \leq y \leq 1$
13. $y = \frac{1}{3}x^{3/2} - x^{1/2}, 1 \leq x \leq 4$
14. $y = 2\ln(4 - x^2), 0 \leq x \leq 1$

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5	Find surface area of a solid of revolution using definite integration حساب مساحة السطح الناتج عن دوران منطقة معينة باستخدام التكامل المحدود	(29-36)	447

In exercises 29–36, set up the integral for the surface area of the surface of revolution and approximate the integral with a numerical method.

29. $y = x^2$, $0 \le x \le 1$, revolved about the x-axis

30. $y = \sin x$, $0 \le x \le \pi$, revolved about the x-axis

31. $y = 2x - x^2$, $0 \le x \le 2$, revolved about the x-axis

32. $y = x^3 - 4x$, $-2 \le x \le 0$, revolved about the x-axis

33. $y = e^x$, $0 \le x \le 1$, revolved about the x-axis

34. $y = \ln x$, $1 \le x \le 2$, revolved about the x-axis

35. $y = \cos x$, $0 \le x \le \pi/2$, revolved about the x-axis

36. $y = \sqrt{x}$, $1 \le x \le 2$, revolved about the x-axis

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36. $y = \sqrt{x}, 1 \le x \le 2$, revolved about the x-axis

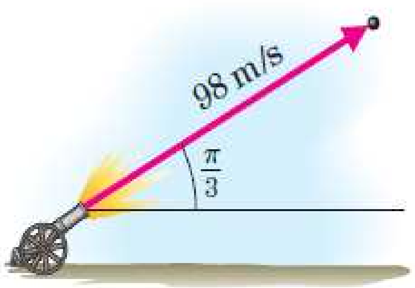
6	Solve physical problems involving velocity حل مسائل تطبيقات فيزيائية على السرعة المتجهة	Ex.(1-4)	455

In exercises 1–4, identify the initial conditions $y(0)$ and $y'(0)$.

1. An object is dropped from a height of 80 ft.
2. An object is dropped from a height of 100 ft.
3. An object is released from a height of 60 ft with an upward velocity of 10 ft/s.
4. An object is released from a height of 20 ft with a downward velocity of 4 ft/s.

7	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	(17-22)	456

17. An object is launched at angle $\theta = \pi/3$ radians from the horizontal with an initial speed of 98 m/s. Determine the time of flight and the horizontal range. Compare to example 5.4.



18. Find the time of flight and horizontal range of an object launched at angle 30° with initial speed 40 m/s. Repeat with an angle of 60°.

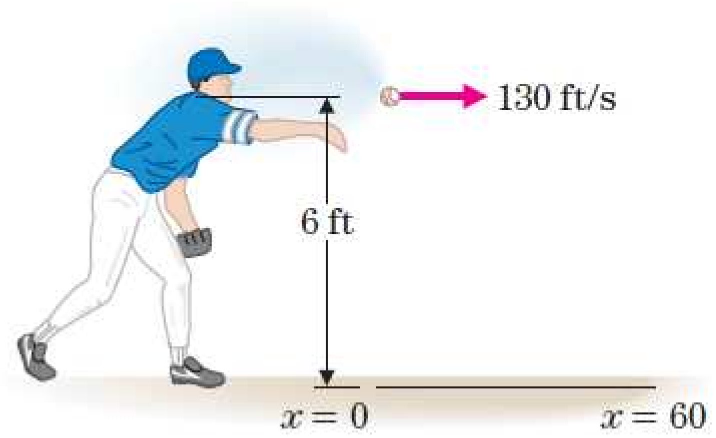
7	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	(17-22)	456

19. Repeat example 5.5 with an initial angle of 6°. By trial and error, find the smallest and largest angles for which the serve will be in.

20. Repeat example 5.5 with an initial speed of 170 ft/s. By trial and error, find the smallest and largest initial speeds for which the serve will be in.

7	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	(17-22)	456

21. A baseball pitcher releases the ball horizontally from a height of 60 ft with an initial speed of 130 ft/s. Find the height of the ball when it reaches home plate 60 ft away. (Hint: Determine the time of flight from the *x*-equation, then use the *y*-equation to determine the height.)



7	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	(17-22)	456

22. Repeat exercise 21 with an initial speed of 80 ft/s. (Hint: Carefully interpret the negative answer.)

8	Compute integrals using direct computation and rules إيجاد تكاملات دوال متنوعة بصيغة مباشرة باستخدام الصيغ	Ex.(1,2,5-10)	489

In exercises 1–40, evaluate the integral.

1. $\int e^{ax}\, dx,\ a \neq 0$

2. $\int \cos(ax)\, dx,\ a \neq 0$

5. $\int \sin 6t\, dt$

6. $\int \sec 2t \tan 2t\, dt$

7. $\int (x^2+4)^2\, dx$

8. $\int x(x^2+4)^2\, dx$

9. $\int \frac{3}{16+x^2}\, dx$

10. $\int \frac{2}{4+4x^2}\, dx$

9	Compute integrals using direct computation and rules إيجاد تكاملات دوال متنوعة بصيغة مباشرة باستخدام الصيغ	(17,18,25,26,33,36,38,39)	489

17. $\int e^{3-2x}\,dx$

18. $\int \frac{3}{e^{6x}}\,dx$

25. $\int_{-\pi/4}^{0} \frac{\sin t}{\cos^2 t}\,dt$

26. $\int_{\pi/4}^{\pi/2} \frac{1}{\sin^2 t}\,dt$

9	Compute integrals using direct computation and rules إيجاد تكاملات دوال متنوعة بصيغة مباشرة باستخدام الصيغ	(17,18,25,26,33,36,38,39)	489

33. $\int \frac{1+x}{1+x^2}\,dx$

36. $\int_1^3 e^{2\ln x}\,dx$

38. $\int_0^1 x(x-3)^2\,dx$

39. $\int_1^4 \frac{x^2+1}{\sqrt{x}}\,dx$

10	Compute various integrals using integration by substitution إيجاد تكاملات دوال متنوعة باستخدام طريقة التكامل بالتعويض	(27,30,31,32)	489

27. $\displaystyle\int \frac{x^2}{1+x^6}\,dx$

30. $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

31. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

32. $\displaystyle\int \frac{2x^3}{\sqrt{1-x^4}}\,dx$

11	Learn the notion of integration by parts التعرف على التكامل بطريقة التكامل بالأجزاء	(53,54)	497

In exercises 53 and 54, name the method by identifying whether substitution or integration by parts can be used to evaluate the integral.

53. (a) $\int x \sin x^2\, dx$ (b) $\int x^2 \sin x\, dx$

(c) $\int x \ln x\, dx$ (d) $\int \frac{\ln x}{x}\, dx$

54. (a) $\int x^3 e^{4x}\, dx$ (b) $\int x^3 e^{x^4}\, dx$

(c) $\int x^{-2} e^{4/x}\, dx$ (d) $\int x^2 e^{-4x}\, dx$

12	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء	(56-61)	497

In exercises 56–61, use the method of exercise 55 to evaluate the integral.

56. $\int x^4 \sin x \, dx$

57. $\int x^4 \cos x \, dx$

58. $\int x^4 e^x \, dx$

59. $\int x^4 e^{2x} \, dx$

12	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء	(56-61)	497

In exercises 56–61, use the method of exercise 55 to evaluate the integral.

60. $\int x^5 \cos 2x \, dx$

61. $\int x^3 e^{-3x} \, dx$

13	Integrate functions of the form sin^n(x) · cos^m(x) sin^n(x) .cos^m(x) إيجاد تكاملات دوال بصيغة	(1-6)	507

In exercises 1–44, evaluate the integrals.

1. $\int \cos x \sin^4 x \, dx$

2. $\int \cos^3 x \sin^4 x \, dx$

3. $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$

4. $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \, dx$

5. $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

6. $\int_{-\pi/2}^{0} \cos^3 x \sin x \, dx$

13	Integrate functions of the form sin^n(x) · cos^m(x) sin^n(x) .cos^m(x) إيجاد تكاملات دوال بصيغة	(1-6)	507

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2. $\int \cos^3 x \sin^4 x \, dx$

3. $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$

4. $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \, dx$

5. $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

6. $\int_{-\pi/2}^{0} \cos^3 x \sin x \, dx$

14	Integrate functions of the form sec^n(x) · tan^m(x) sec^n(x). tan^m(x) إيجاد تكاملات دوال بصيغة	(9,11,12,15,16)	507

9. $$\int \tan x \sec^3 x \, dx$$

11. $$\int x \tan^3 (x^2+1) \sec (x^2+1) \, dx$$

12. $$\int \tan (2x+1) \sec^3 (2x+1) \, dx$$

14	Integrate functions of the form sec^n(x) · tan^m(x) sec^n(x). tan^m(x) إيجاد تكاملات دوال بصيغة	(9,11,12,15,16)	507

15. $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$

16. $\int_{-\pi/4}^{\pi/4} \tan^4 x \sec^2 x \, dx$

15	Integrate trigonometric functions using the substitution x = a·tan(y) x = a·tan(y) إيجاد تكاملات دوال مثلثية باستخدام التعويض بـ	(33-36,38-40)	507

33. $\int \frac{x^2}{\sqrt{9+x^2}}\,dx$

34. $\int x^3\sqrt{8+x^2}\,dx$

35. $\int \sqrt{16+x^2}\,dx$

36. $\int \frac{1}{\sqrt{4+x^2}}\,dx$

15	Integrate trigonometric functions using the substitution x = a·tan(y) x = a·tan(y) إيجاد تكاملات دوال مثلثية باستخدام التعويض بـــ	(33-36,38-40)	507

38. $\int_0^2 x^2\sqrt{x^2+9}\,dx$

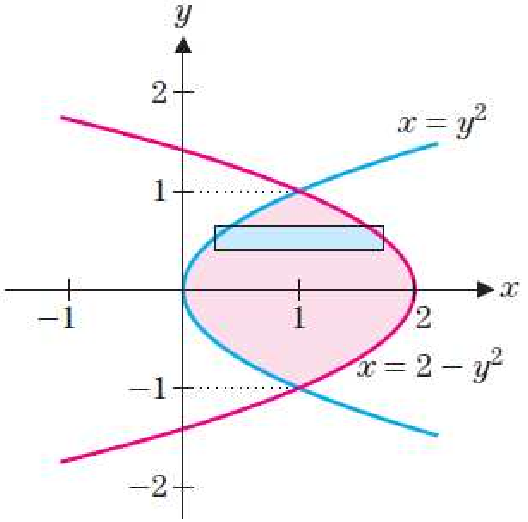
39. $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

40. $\int \frac{x+1}{\sqrt{4+x^2}}\,dx$

16	Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة كتكامل محدود بمعلومية y عوضا عن x	Example 1.6 (19,20,22,24)	413 414

EXAMPLE 1.6 The Area of a Region Bounded by Functions of *y*

Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.



16	Compute the area of a region using definite integration with y as a variable x	Example 1.6	413
	إيجاد مساحة منطقة كتكامل محدود بمعلومية y عوضا عن x	(19,20,22,24)	414

In exercises 19–26, sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral. Verify your answers to exercises 19–21 with a basic geometric area formula.

19. $y = x, y = 2 - x, y = 0$

20. $y = x, y = 2, y = 6 - x, y = 0$

16	Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة كتكامل محدود بمعلومية y عوضا عن x	Example 1.6 (19,20,22,24)	413 414

22. $x = 3y, x = 2 + y^2$

24. $x = y^2, x = 4$

17	Find the volume of a solid of revolution by using the method of washers	(17,19,25)-(washers parts)	430
	إيجاد حجم مجسم باستخدام طريقة الحلقات	(27,28)-(washers parts)	431

In exercises 17–20, compute the volume of the solid formed by revolving the given region about the given line.

17. Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about (a) the x-axis; (b) $y = 3$

19. Region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about (a) the y-axis; (b) $x = 4$

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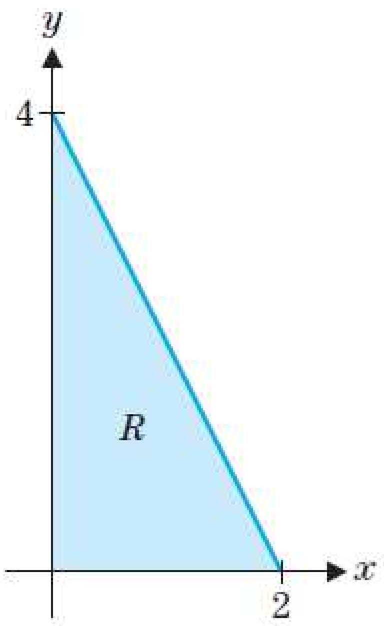
(d) $y = -4$ (e) $x = 2$ (f) $x = -2$



17	Find the volume of a solid of revolution by using the method of washers	(17,19,25)-(washers parts)	430
	إيجاد حجم مجسم باستخدام طريقة الحلقات	(27,28)-(washers parts)	431

27. Let R be the region bounded by $y = x^2$, $y = 0$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.

(a) the y-axis (b) the x-axis (c) $x = 1$

(d) $y = 1$ (e) $x = -1$ (f) $y = -1$

17	Find the volume of a solid of revolution by using the method of washers	(17,19,25)-(washers parts)	430
	إيجاد حجم مجسم باستخدام طريقة الحلقات	(27,28)-(washers parts)	431

28. Let R be the region bounded by $y = x$, $y = -x$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.

(a) the x-axis (b) the y-axis

(c) $y = 1$ (d) $y = -1$

18	Solve mathematical problems involving applications on arc length or surface area حل مسائل رياضية كتطبيقات على طول القوس أو مساحة السطح	(23-25)	447

23. A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 10(e^{x/20} + e^{-x/20})$, $-20 \le x \le 20$, compute the length of the rope.

24. A rope is to be hung between two poles 60 meters apart. If the rope assumes the shape of the catenary $y = 15(e^{x/30} + e^{-x/30})$, $-30 \le x \le 30$, compute the length of the rope.

25. In example 4.4, compute the "sag" in the cable—that is, the difference between the y-values in the middle ($x = 0$) and at the poles ($x = 10$). Given this, is the arc length calculation surprising?

19	**Use integration by parts to compute definite and indefinite integrals** إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء	(9,10,11,14)	496

9. $\int e^x \sin 4x\, dx$

10. $\int e^{2x} \cos x\, dx$

19	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء	(9,10,11,14)	496

11. $\int \cos x \cos 2x \, dx$

14. $\int (\ln x)^2 \, dx$

20	Integrate trigonometric functions using the substitution x = a·sin(y) x = a·sin(y) إيجاد تكاملات دوال مثلثية باستخدام التعويض بـــ	(21-24)	507

21. $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

22. $\displaystyle\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

23. $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

24. $\displaystyle\int \frac{x^3}{\sqrt{9-x^2}}\,dx$

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