United Arab Emirates
دولة الإمارات العربية المتحدة
Second School Branch\ Cluster2
الفرع المدرسي الثاني / النطاق2
مؤسسة الإمارات للتعليم المدرسي
Al-Khalil Bin Ahmed Boys' School - Cycle 3
Emirates School Establishment
مدرسة الخليل بن أحمد - الحلقة الثالثة بنين

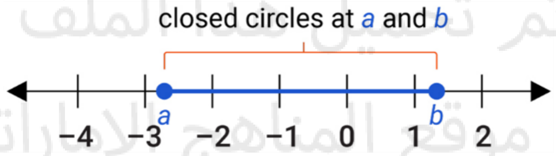
Chapter 1	Preliminaries to Calculus	
		Alef Platform
		Lesson: 1 to 17

Key Terms

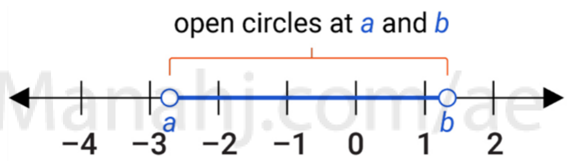
Types of Intervals

Let a and b be any two real numbers with $a < b$.

closed interval $[a, b] = \{x \in R \mid a \leq x \leq b\}$; set of all real numbers between a and b including the endpoints a and b.



open interval $(a, b) = \{x \in R \mid a < x < b\}$; set of all real numbers between a and b, not including the endpoints a and b.



$(a, \infty) = \{x \in R \mid x > a\}$; set of all real numbers that are greater than a.

$(-\infty, b) = \{x \in R \mid x < b\}$; set of all real numbers that are less than b.

$-\infty$ and ∞ are not real numbers

Types of Inequality

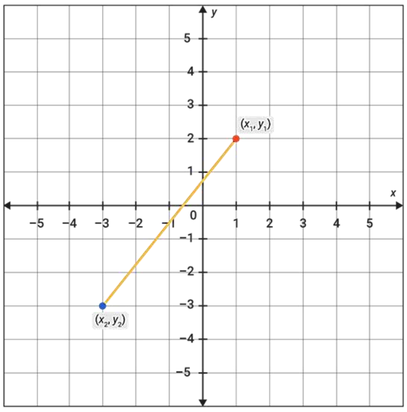
Linear Inequality an unequal relation between an unknown and constant numbers

Quadratic Inequality a second-degree polynomial which may be greater than (or equal) or less than (or equal) to zero

Absolute Value Inequality The absolute value of a real number is $|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$

Distance Formula.

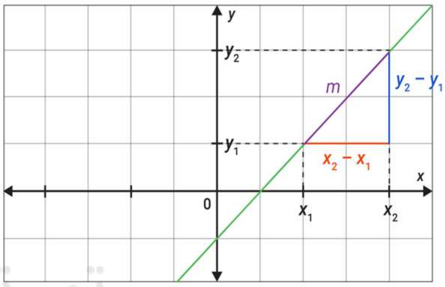
Distance Between any Two Points $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



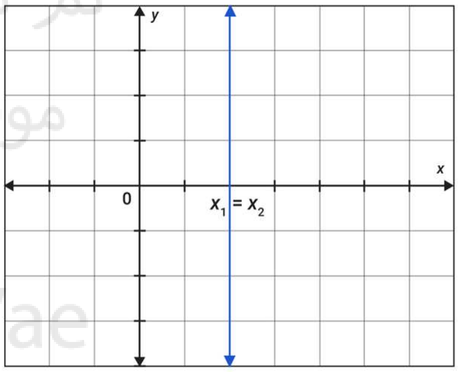
slope of the line

The slope of the line passing through the two points $(x_1, y_1), (x_2, y_2)$

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$



If $x_1 = x_2$ then the line passing through the two points is vertical and the slope is undefined.



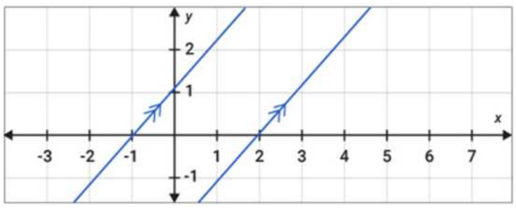
Equation of a Line

slope m

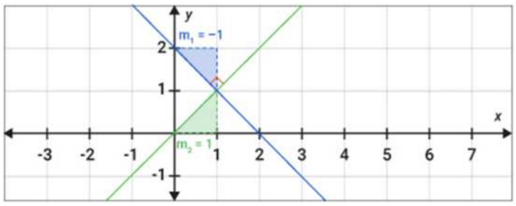
$y = m(x - x_0) + y_0$
point slope form

$y = mx + b$
slope-intercept form

Parallel Lines: Two lines having same slope with no point in common

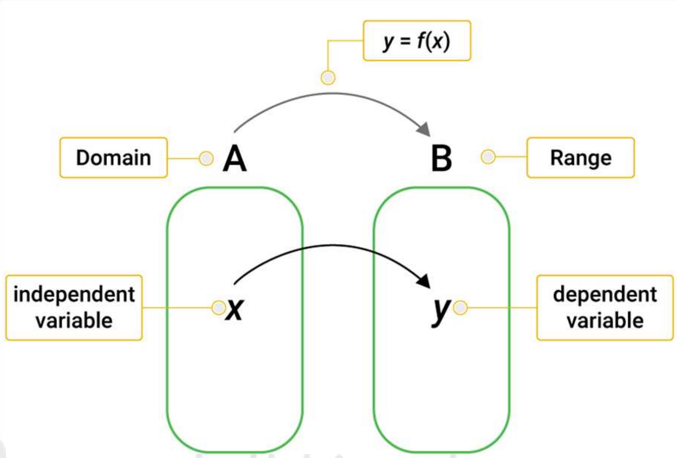


Perpendicular Lines: Two lines having the product of their slopes equal to -1



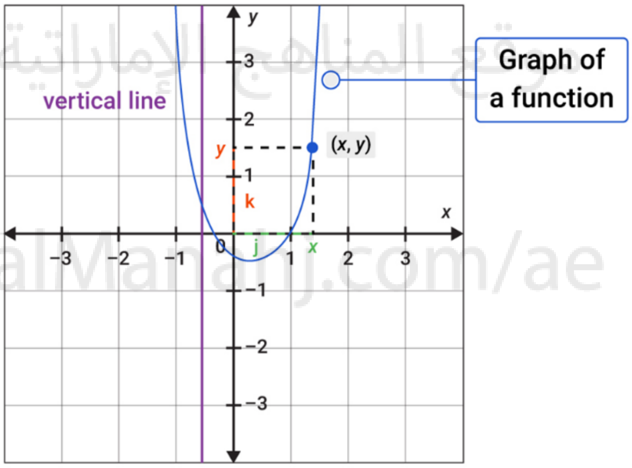
Function

A function, f, is a relationship between an independent variable, denoted in general by x, and a dependent variable, denoted in general by y, such that for every x there exists one and only one y where $y = f(x)$.



Vertical Line Test

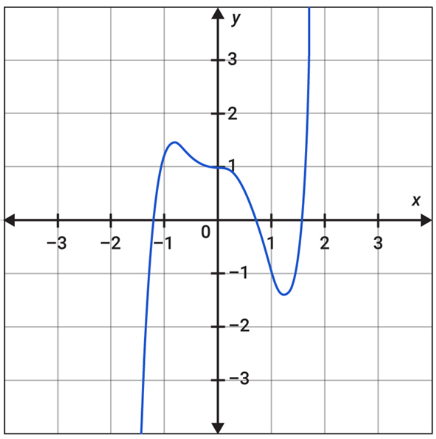
If any vertical line cuts the graph in more than one point, then the curve is not a graph of a function



Polynomial Function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots\dots + a_1 x + a_0$ where $a_0, a_1, \dots, a_n$ are its coefficients with $a_n \neq 0$. n is a whole number and represents the degree of the polynomial.

Polynomial Function



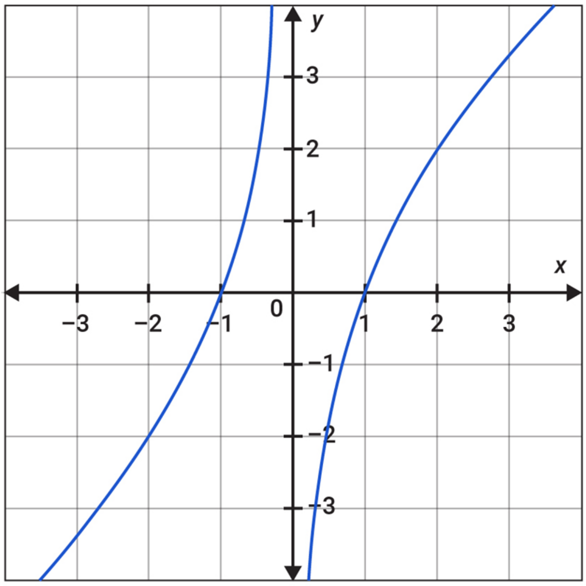
Example:

* Constant polynomial: $f(x) = 5$
* Linear polynomial: $f(x) = -3x + 6$
* Quadratic polynomial: $f(x) = x^2 + 2x - 1$
* Cubic polynomial: $f(x) = -3x^3 + 2x^2 + x - 1$
* Quartic polynomial: $f(x) = x^4 - 2x^3 - x^2 + x - 4$

The **domain** of any polynomial function is the set of all real numbers.

Rational Function

Rational Function

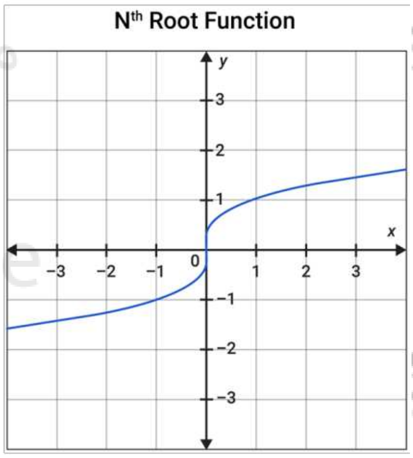


$f(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials.

The **domain** of a rational function is the set of all values of x such that $Q(x) \neq 0$. $D = \{x \mid x \neq \dots,\ x \in \mathbb{R}\}$

The values of x for which the denominator is zero

N^{th} Root Function



$f(x) = \sqrt[n]{p(x)}$ where $p(x)$ is a polynomial, and $n \geq 2$.

Example: $f(x) = \sqrt[3]{x}$ and $f(x) = \sqrt{x^2 - 5}$

The **domain** of this function is:

Square-Root Function •all real values of x such that $p(x) \geq 0$, if n is **even.**

Cubic Root Function •all real numbers if n is **odd**.

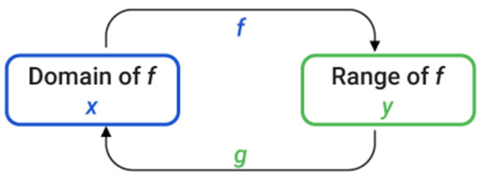
Inverse Functions

The idea of the **inverse function** is to find an input that produces a given output.

Given a $y \in$ **Range** of f , find the x ∈**Domain** of f such that, $y = f(x)$

If g is the inverse of f , then $g(y) = x$.

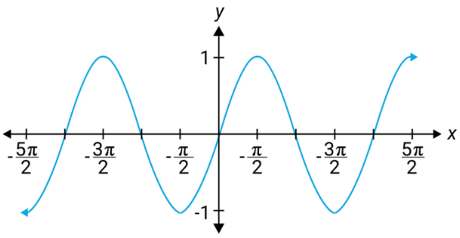
Two functions f and g are inverses if and only if: $f(g(x)) = x$ for all values of x in the domain of $g(x)$ and $g(f(x)) = x$ for all values of x in the domain of $f(x)$.



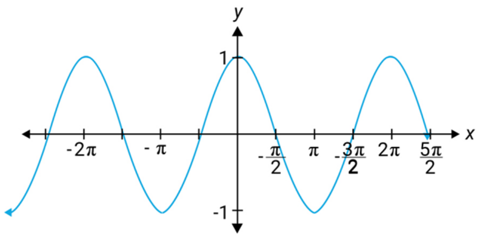
Trigonometric Functions

Sine and Cosine Functions

The domain of $\boldsymbol{\theta}$ is given as $(-\infty, +\infty)$ and the range of these functions is the interval $[-\mathbf{1}, \mathbf{1}]$.



$y = \sin x$



$y = \cos x$

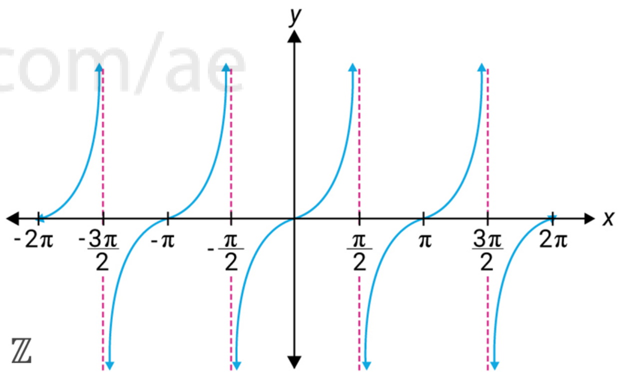
The functions $\boldsymbol{f(\theta) = sin\theta}$ and $\boldsymbol{g(\theta) = cos\theta}$ are periodic functions, of period $\mathbf{2\pi}$.

- Domain: All real numbers $(-\infty, +\infty)$
- Range: $-\mathbf{1} \leq \boldsymbol{x} \leq \mathbf{1}$

The tangent function

$$\boldsymbol{tanx = \frac{sinx}{cosx}}$$

The tangent function is a periodic function of period $\boldsymbol{\pi}$.



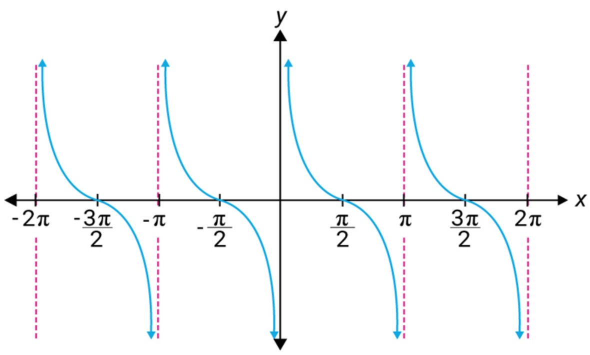
$y = \tan x$

- Domain: All real numbers such that $\boldsymbol{x \neq \frac{\pi}{2} + n\pi \text{ , } n} \in \mathbb{Z}$
- Range: All real numbers

The cotangent function

$$\boldsymbol{cotx = \frac{cosx}{sinx}}$$

The cotangent function is a periodic function of period $\boldsymbol{\pi}$.



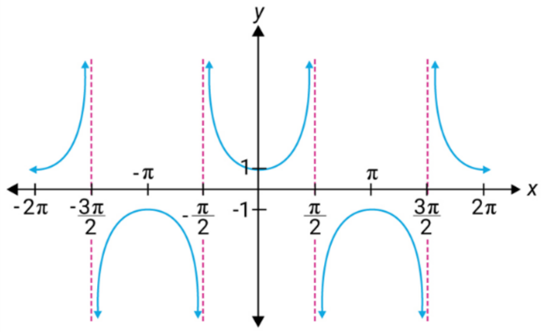
$y = \cot x$

- Domain: All real numbers such that $\boldsymbol{x \neq n\pi \text{ , } n} \in \mathbb{Z}$
- Range: All real numbers

The secant function

$$sec x = \frac{1}{cos x}$$

The secant function is a periodic function of period 2π.



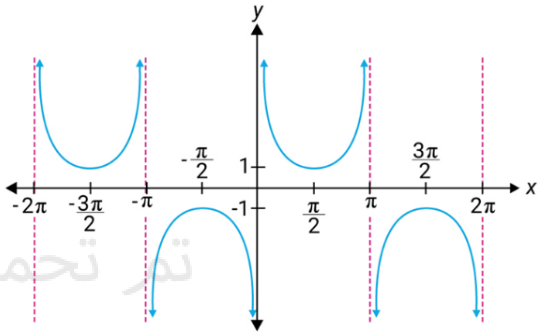
y = sec x

- Domain: All real numbers such that $x \neq \frac{\pi}{2} + n\pi\ , n \in \mathbb{Z}$
- Range: $(-\infty\ , -1] \cup [1\ , \infty)$

The cosecant function

$$csc x = \frac{1}{sin x}$$

The cosecant function is a periodic function of period 2π.



y = csc x

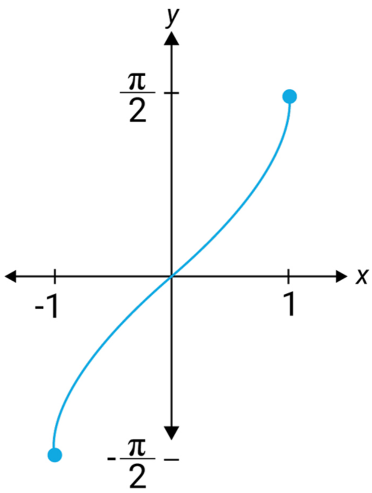
- Domain: All real numbers such that $x \neq n\pi\ , n \in \mathbb{Z}$
- Range: $(-\infty\ , -1] \cup [1\ , \infty)$

Inverse Trigonometric Functions

Arc sine function

The inverse sine function or Arcsine

$$y = arc\ sin x$$
$$y = sin^{-1} x$$

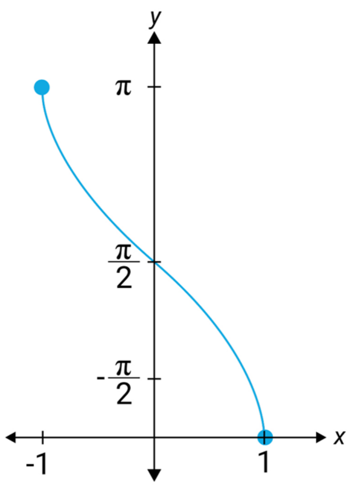


- Domain: $-1 \leq x \leq 1$
- Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
- $y = sin^{-1} x$ if and only if $sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Arc cosine function

The inverse cosine function or Arccosine

$$y = arc\ cos x$$
$$y = cos^{-1} x$$



- Domain: $-1 \leq x \leq 1$
- Range: $0 \leq y \leq \pi$
- $y = cos^{-1} x$ if and only if $cos y = x$ and $0 \leq y \leq \pi$.

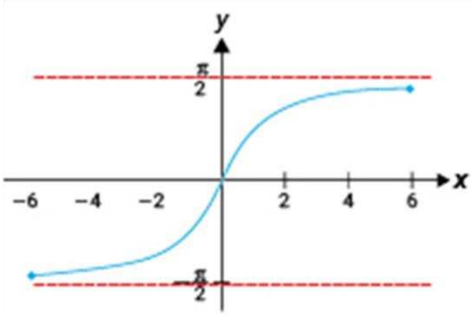
Arc tangent function

The inverse tangent function or Arctangent

$$y = arc\ tanx$$

$$y = tan^{-1}x$$

- Domain: All real numbers $= -\infty \le x \le \infty$
- Range: $0 < y < \pi$
- $y = cot^{-1}x$ if and only if $coty = x$ and $0 < y < \pi$.



$y = \tan^{-1} x$

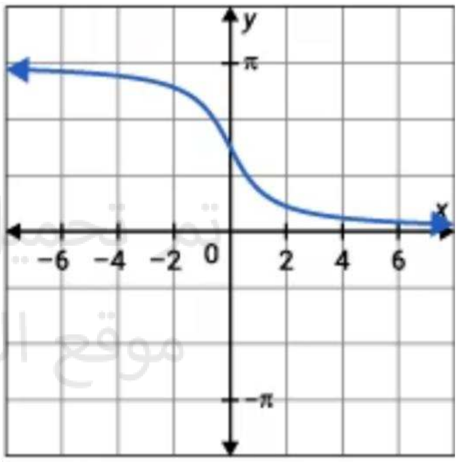
Arc cotangent function

The inverse cotangent function or Arc cotangent

$$y = arc\ cotx$$

$$y = cot^{-1}x = \frac{\pi}{2} - tan^{-1}x$$

- Domain: All real numbers $= -\infty \le x \le \infty$
- Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$
- $y = tan^{-1}x$ if and only if $tany = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.



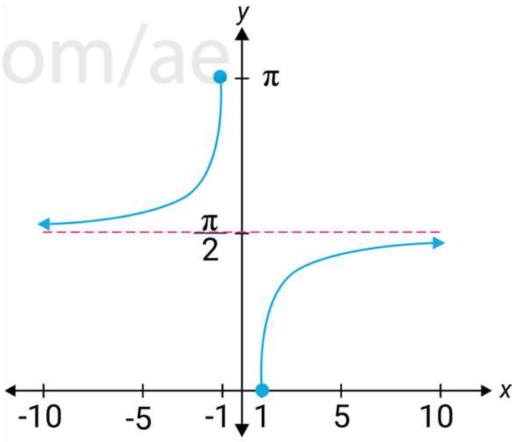
Arc secant function

The inverse secant function or Arc secant

$$y = arc\ secx$$

$$y = sec^{-1}x = cos^{-1}\left(\frac{1}{x}\right)$$

- Domain: $(-\infty, -1] \cup [1, \infty)$
- Range: $0 \le y \le \pi$ such that $y \ne \frac{\pi}{2}$.
- $y = sec^{-1}x$ if and only if $secy = x$, $0 \le y \le \pi$ such that $y \ne \frac{\pi}{2}$.



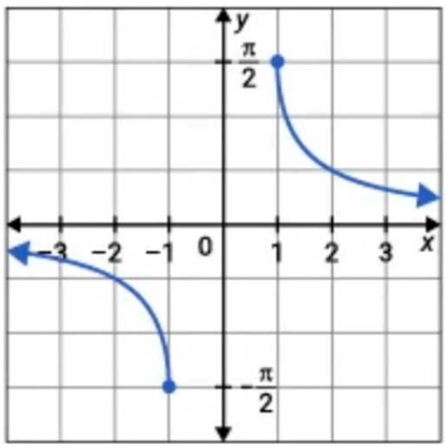
Arc cosecant function

The inverse cosecant function or Arc cosecant

$$y = arc\ cscx$$

$$y = csc^{-1}x = sin^{-1}\left(\frac{1}{x}\right)$$

- Domain: $(-\infty, -1] \cup [1, \infty)$
- Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ such that $y \ne 0$.
- $y = csc^{-1}x$ if and only if $cscy = x$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ such that $y \ne 0$.



Rules of Exponents

The rules of exponents that are needed to change between forms of expressions

Product Rule Given $x > 0$; for any real numbers p and q we have:

$$x^p . x^q = x^{p+q}$$

Quotient Rule Given $x > 0$; for any real numbers p and q we have:

$$\frac{x^p}{x^q} = x^{p-q}$$

Power of a Power Rule Given $x > 0$; for any real numbers p and q we have:

$$(x^p)^q = x^{p.q}$$

Negative Exponent Given $x > 0$; for any real numbers p we have:

$$x^{-p} = \frac{1}{x^p}$$

Power of a Product Rule Given $x > 0$ and $y > 0$; for any real numbers p we have:

$$(x.y)^p = x^p . y^p$$

Power of a Quotient Rule Given $x > 0$ and $y > 0$; for any real numbers p we have:

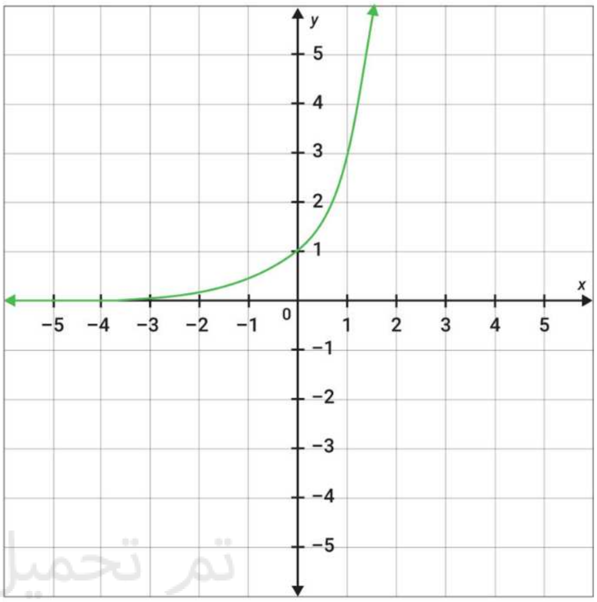
$$\left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}$$

Fractional Exponent Given $x > 0$; for any integers m and n with $n \geq 2$ we have:

$$(x^m)^n = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Exponential Functions

An **exponential function** is any function of the form $f(x) = a.b^x$ where a is a nonzero constant and $1 \neq b > 0$ is the base and x is the exponent.



$f(x) = a.b^x$, ($b > 0$ and $b \neq 1$)

- Domain: All real numbers $(-\infty, +\infty)$.

  Range: $(0, \infty)$.

Euler's Number

The irrational number e, or Euler's number, is defined by:

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$$

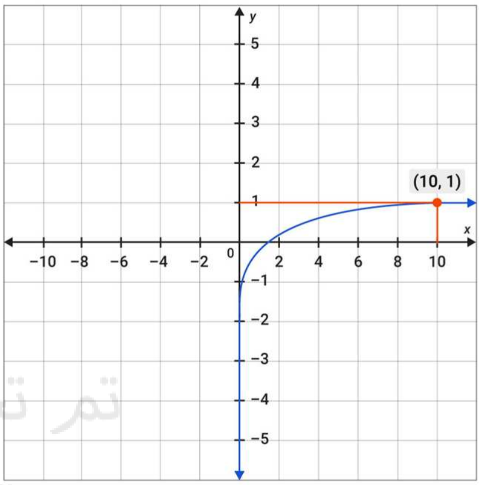
As the irrational number π is approximated to be used as 3.14, similarly, e is approximated to 2.718.

Logarithmic functions

Logarithmic functions are the functions defined to be the inverses of exponential functions.

The logarithmic function with base $\mathbf{1} \neq \boldsymbol{b} > \mathbf{0}$, written $\mathbf{log}_b\, \boldsymbol{x}$; $\boldsymbol{b} \neq \mathbf{1}$ is defined by:

$$\boldsymbol{y} = \mathbf{log}_b\, \boldsymbol{x} \text{ if and only if } \boldsymbol{x} = \boldsymbol{b}^{\boldsymbol{y}}$$

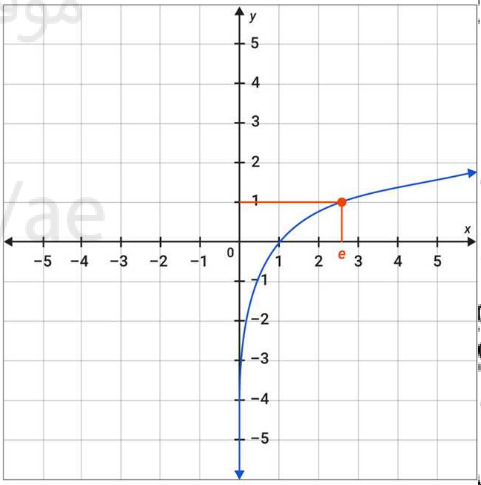


$\boldsymbol{f}(\boldsymbol{x}) = \mathbf{log}_b\, \boldsymbol{x}$, ($\boldsymbol{b} > \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{1}$)

- Domain: $(\mathbf{0}, \infty)$
- Range: All real numbers $(-\infty, +\infty)$.

Natural Logarithm.

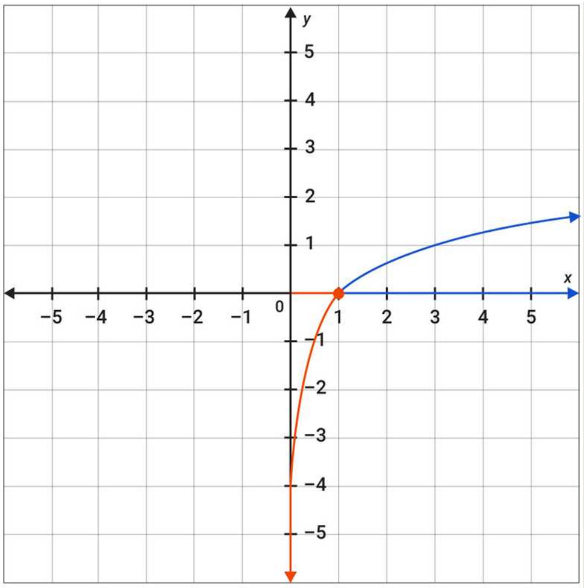
is the logarithm of a number with base $\boldsymbol{e}$



$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{ln}\, \boldsymbol{x}$

- Domain: $(\mathbf{0}, \infty)$
- Range: All real numbers $(-\infty, +\infty)$.

Theorem



For any base $\boldsymbol{b} > \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{1}$:

•$\mathbf{log}_b\, \mathbf{1} = \mathbf{0}$

•If $\boldsymbol{b} > \mathbf{1}$, then $\boldsymbol{log x} < \mathbf{0}$ (graph is below the x-axis)

for $\mathbf{0} < \boldsymbol{x} < \mathbf{1}1$,

and $\boldsymbol{log x} > \mathbf{0}$ (graph is above the x-axis) for $\boldsymbol{x} > \mathbf{1}$.

For any base $b > 0$ and $b \neq 1$, $x > 0$ and $y > 0$ the following applies:

Product Rule $\log_b(x.y) = \log_b(x) + \log_b(y)$

Quotient Rule $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$

Power Rule $\log_b(x^y) = y.\log_b(x)$

$\ln(e^x) = x. \qquad e^{\ln(x)} = x. \qquad x > 0$

You can rewrite any exponential as an exponential with base e.

$$a^x = e^{\ln(a^x)} = e^{x\ln(a)} \qquad a > 0$$

Also, when having logarithms of base different from e or 10 the following rule can be used.

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}, b > 0, b \neq 1, x > 0$$

Function operations

Given functions f and g, their sum $f + g$, difference $f - g$, product $f \cdot g$ and quotient $\frac{f}{g}$ are described by:

Operation	Definition	Domain
Sum	$(f+g)(x) = f(x) + g(x)$	intersection of the domains of f and g
Difference	$(f-g)(x) = f(x) - g(x)$	intersection of the domains of f and g
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	intersection of the domains of f and g
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	domain is defined as the intersection of the domains of f and g with excluding values of x where $g(x) = 0$

Composition of Functions

If f and g are functions, then the composite function $f \circ g$ is defined by:

$$(f \circ g)(x) = f(g(x))$$

We read the left-hand side as f composed with g or the composition of functions f and g at x and the right-hand side as f of g of x.

And the domain of $f \circ g = \{x | x \in$ **domain of** g **and** $g(x) \in$ **domain of** $f\}$.

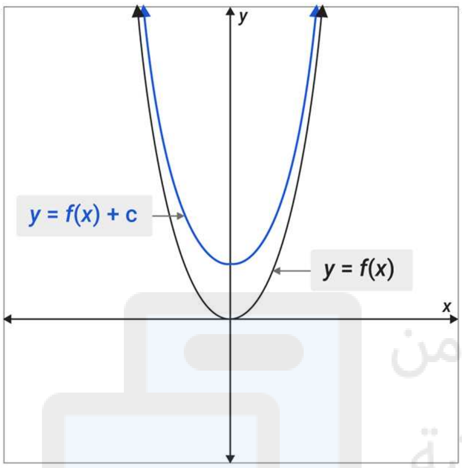
Translation

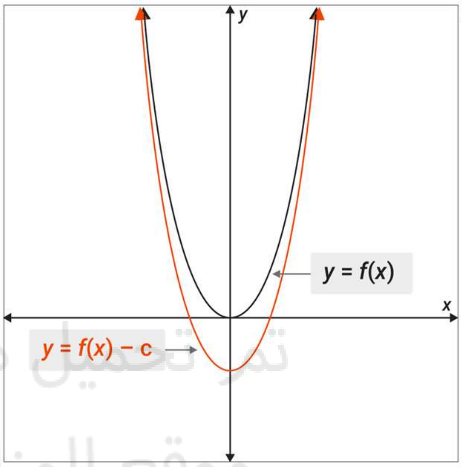
Vertical Translation

Let $y = f(x)$ and $c > 0$.

To obtain the graph of $y = f(x) + c$, shift the graph of $y = f(x)$ up c units.

To obtain the graph of $y = f(x) - c$, shift the graph of $y = f(x)$ down c Units.



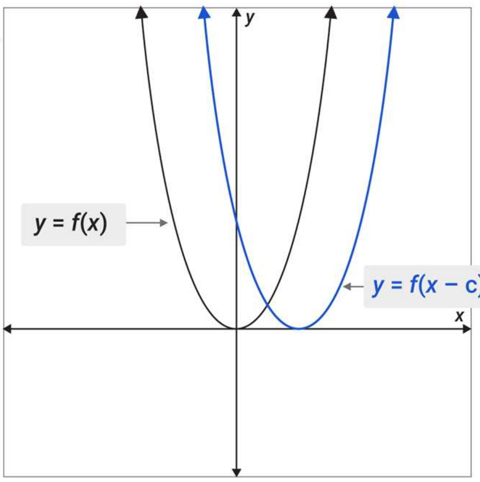


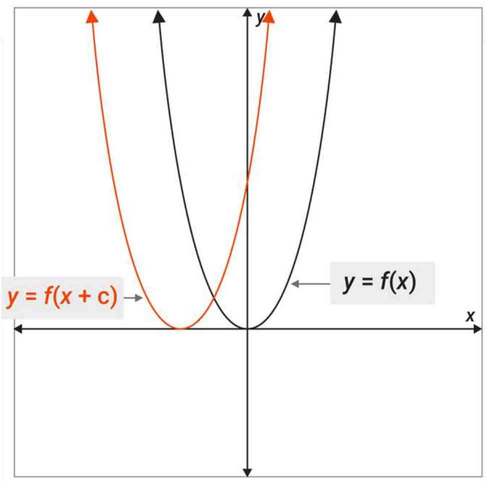
Horizontal Translation

Let $y = f(x)$ and $c > 0$.

To obtain the graph of $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.

To obtain the graph of $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c Units.





To graph the function $y = cf(x), c > 0$, copy the graph of $f(x)$ and multiply the scale of the y -axis by c.

To graph the function $y = f(cx), c > 0$, copy the graph of $f(x)$ and divide the scale of the x -axis by c.

12G Advanced Lessons: 1.1, 1.2, 1.3

12G Advanced Lessons: 1.1, 1.2, 1.3

12G Advanced Lessons: 1.4, 1.5

12G Advanced Lessons: 1.4, 1.5

The End

Good Luck to you

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