Emirates Schools Establishment

Aisha Bint Abi Baker School C3

Abu Dhabi

Grade 9 Advanced Physics

EOT EXAM SPECIFICATIONS

Term 3 (2023/2024)

Module 07: Gravitation

1.	Explain Kepler's Second Law which states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals.	Student Book	165

Kepler found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3.**

READING CHECK **Compare** the distances traveled from point 1 to point 2 and from point 6 to point 7 in **Figure 3.** Through which distance would Earth be traveling fastest?

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

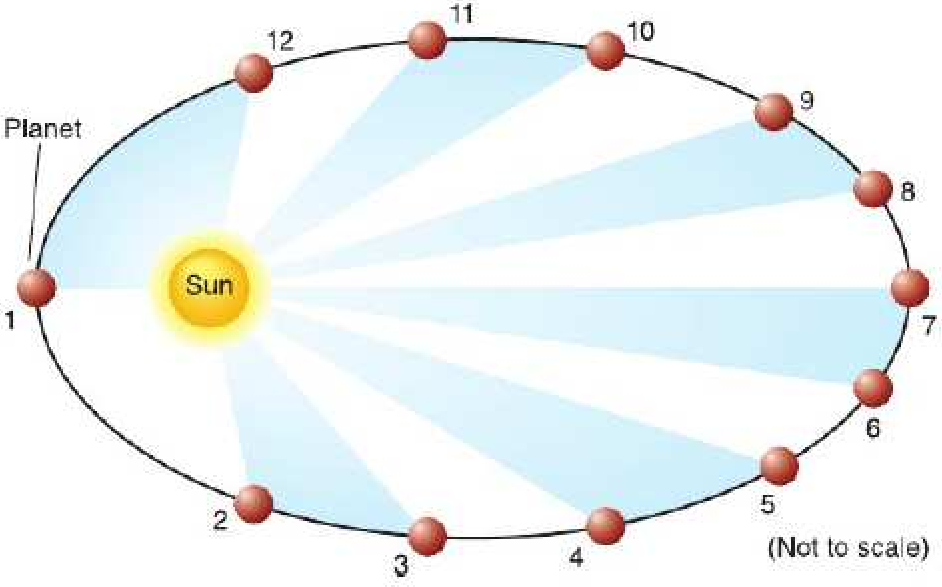


Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.

Explain why the equal time areas are shaped differently.

2.	Explain the law of universal gravitation and write it in equation form $\left[F_g = G\frac{m_1 m_2}{r^2}\right]$	Student Book	168
5.	Define gravitational force as the force of attraction between two objects of given mass.	Student Book	168

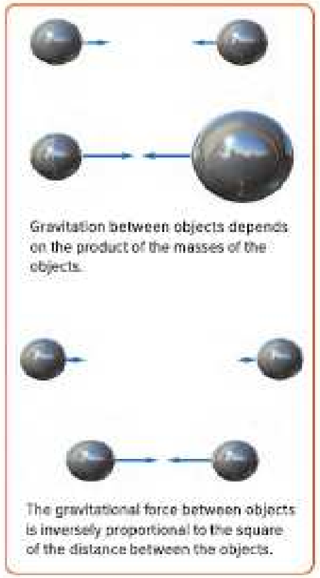


Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

Newton's Law of Universal Gravitation

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F_g) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F_g is proportional to $\frac{1}{r^2}$. The force (F_g) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them as shown below.

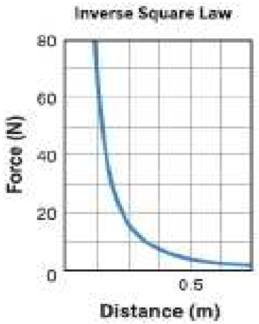
LAW OF UNIVERSAL GRAVITATION

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F_g = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature below to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed in the next sections.

Figure 6 This is a graphical representation of the inverse square relationship.



CONNECTING MATH TO PHYSICS

Direct and Inverse Relationships Newton's law of universal gravitation has both direct and inverse relationships.

$F_g \propto m_1m_2$		$F_g \propto \frac{1}{r^2}$	
Change	**Result**	**Change**	**Result**
$(2m_1)m_2$	$2F_g$	$2r$	$\frac{1}{4}F_g$
$(3m_1)m_2$	$3F_g$	$3r$	$\frac{1}{9}F_g$
$(2m_1)(3m_2)$	$6F_g$	$\frac{1}{2}r$	$4F_g$
$\left(\frac{1}{2}\right)m_1m_2$	$\frac{1}{2}F_g$	$\frac{1}{3}r$	$9F_g$

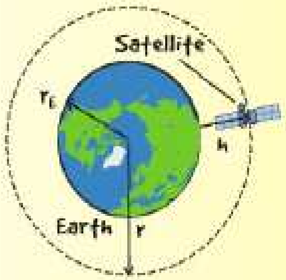
3.	Calculate the orbital period of a satellite.	Example Problem (2) Check your Progress Q.8	175 172

EXAMPLE 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.



KNOWN	UNKNOWN
$h = 2.25\times10^{5}$ m	$v = ?$
$r_E = 6.38\times10^{6}$ m	$T = ?$
$m_E = 5.97\times10^{24}$ kg	
$G = 6.67\times10^{-11}$ N·m²/kg²	

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$
$$= 2.25\times10^{5}\text{ m} + 6.38\times10^{6}\text{ m} = 6.60\times10^{6}\text{ m}$$

◄ Substitute $h = 2.25\times10^{5}$ m and $r_E = 6.38\times10^{6}$ m.

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{6.60\times10^{6}\text{ m}}}$$
$$= 7.77\times10^{3}\text{ m/s}$$

◄ Substitute $G = 6.67\times10^{-11}$ N·m²/kg², $m_E = 5.97\times10^{24}$ kg, and $r = 6.60\times10^{6}$ m.

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$
$$= 2\pi\sqrt{\frac{(6.60\times10^{6}\text{ m})^3}{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}$$
$$= 5.34\times10^{3}\text{ s}$$

◄ Substitute $r = 6.60\times10^{6}$ m, $G = 6.67\times10^{-11}$ N·m²/kg², and $m_E = 5.97\times10^{24}$ kg.

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

Are the units correct? The unit for speed is meters per second, and the unit for period is seconds.

3.	Calculate the orbital period of a satellite.	Example Problem (2) Check your Progress Q.8	175 172

Neptune's Orbital Period Neptune orbits the Sun at an average distance given in **Figure 10,** which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

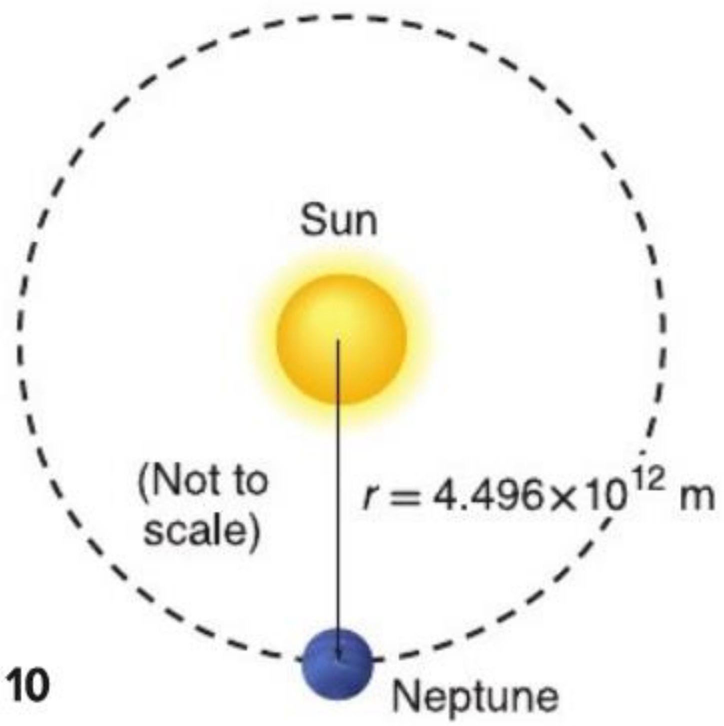


Figure 10

$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}} = 2\pi\sqrt{\frac{(4.495\times10^{12}\,\text{m})^3}{(6.67\times10^{-11}\,\text{N·m}^2/\text{kg}^2)(1.99\times10^{30}\,\text{kg})}}$$

$$= 5.20\times10^9\,\text{s}$$

$$= \frac{5.20\times10^9\,\text{s}}{1}\times\frac{1\,\text{day}}{86{,}400\,\text{s}} = 6.02\times10^4\,\text{days}$$

4.	Explain Kepler's First Law which states that the planets follow elliptical paths with the sun at one focus.	Student Book	165

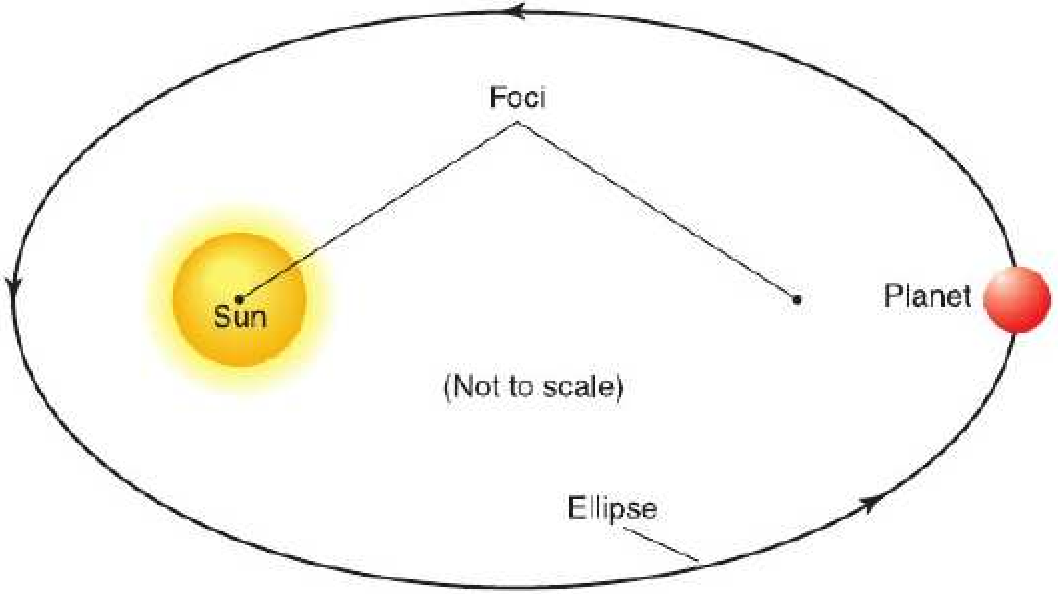


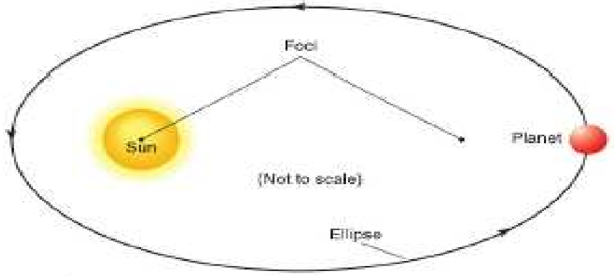
Figure 2 The orbit of each planet is an ellipse, with the Sun at one focus.

Kepler's Laws

In 1600 Tycho moved to Prague where Johannes Kepler, a 29-year-old German, became one of his assistants. Kepler analyzed Tycho's observations. After Tycho's death in 1601, Kepler continued to study Tycho's data and used geometry and mathematics to explain the motion of the planets. After seven years of careful analysis of Tycho's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite, natural or artificial. Here, the laws are presented in terms of planets.

Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 2.** Although exaggerated ellipses are used in the diagrams, Earth's actual orbit is very nearly circular. You would not be able to distinguish it from a circle visually.

18	(1) Explain Kepler's First Law which states that the planets follow elliptical paths with the sun at one focus. (2) Explain Kepler's Second Law which states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. (3) Explain Kepler's Third Law which states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun and write it in equation form $\left[\left(\frac{T_A}{T_B}\right)^2=\left(\frac{r_A}{r_B}\right)^3\right]$. (4) Explain the law of universal gravitation and write it in equation form $\left[F_g=G\frac{m_1 m_2}{r^2}\right]$	Student Book	165-168



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Kepler found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3**.

READING CHECK **Compare** the distances traveled from point 1 to point 2 and from point 6 to point 7 in **Figure 3**. Through which distance would Earth be traveling fastest?

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

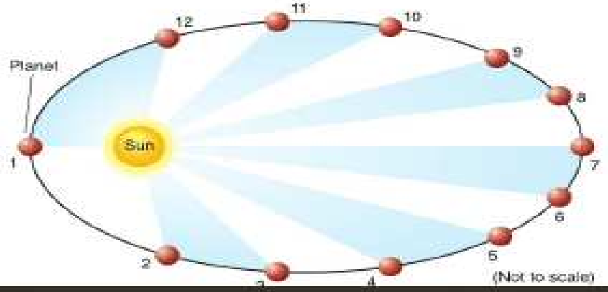

Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.

Explain why the equal time areas are shaped differently.

Table 1 Solar System Data

Name	Average Radius (m)	Mass (kg)	Average Distance from the Sun (m)
Sun	6.96×10^{8}	1.99×10^{30}	—
Mercury	2.44×10^{6}	3.30×10^{23}	5.79×10^{10}
Venus	6.05×10^{6}	4.87×10^{24}	1.08×10^{11}
Earth	6.38×10^{6}	5.97×10^{24}	1.50×10^{11}
Mars	3.40×10^{6}	6.42×10^{23}	2.28×10^{11}
Jupiter	7.15×10^{7}	1.90×10^{27}	7.78×10^{11}
Saturn	6.03×10^{7}	5.69×10^{26}	1.43×10^{12}
Uranus	2.56×10^{7}	8.68×10^{25}	2.87×10^{12}
Neptune	2.48×10^{7}	1.02×10^{26}	4.50×10^{12}

Kepler's third law states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun. Thus, if the periods of the planets are T_A and T_B and their average distances from the Sun are r_A and r_B, Kepler's third law can be expressed as follows.

KEPLER'S THIRD LAW

The square of the ratio of the period of planet A to the period of planet B is equal to the cube of the ratio of the distance between the centers of planet A and the Sun to the distance between the centers of planet B and the Sun.

$$\left(\frac{T_A}{T_B}\right)^2=\left(\frac{r_A}{r_B}\right)^3$$

Note that Kepler's first two laws apply to each planet, moon, and satellite individually. The third law, however, relates the motion of two objects around a single body. For example, it can be used to compare the planets' distances from the Sun, shown in **Table 1**, to their periods around the Sun. It also can be used to compare distances and periods of the Moon and artificial satellites orbiting Earth.

PhysicsLAB

MODELING ORBITS

What is the shape of the orbits of planets and satellites in the solar system?

Comet periods Comets are classified as long-period comets or short-period comets based on orbital periods. Long-period comets have orbital periods longer than 200 years and short-period comets have orbital periods shorter than 200 years. Comet Hale-Bopp, shown in **Figure 4**, with a period of approximately 2400 years, is an example of a long-period comet. Comet Halley, with a period of 76 years, is an example of a short-period comet. Comets also obey Kepler's laws. Unlike planets, however, comets have highly elliptical orbits.

Figure 4 Hale-Bopp is a long-period comet, with a period of 2400 years. This photo was taken in 1997, when Hale-Bopp was highly visible.

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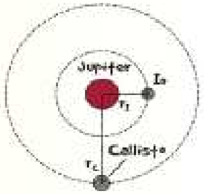
EXAMPLE 1

CALLISTO'S DISTANCE FROM JUPITER Galileo measured the orbital radii of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, has a period of 1.8 days and is 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, has a period of 16.7 days. Using the same units that Galileo used, predict Callisto's distance from Jupiter.

ANALYZE AND SKETCH THE PROBLEM

- Sketch the orbits of Io and Callisto.
- Label the radii.



KNOWN	UNKNOWN
$T_C = 16.7$ days	$r_C = ?$
$T_I = 1.8$ days	
$r_I = 4.2$ units	

SOLVE FOR CALLISTO'S DISTANCE FROM JUPITER

Solve Kepler's third law for r_C.

$$\left(\frac{T_C}{T_I}\right)^2 = \left(\frac{r_C}{r_I}\right)^3$$

$$r_C^3 = r_I^3\left(\frac{T_C}{T_I}\right)^2$$

$$r_C = \sqrt[3]{r_I^3\left(\frac{T_C}{T_I}\right)^2}$$ ◄ Substitute $r_I = 4.2$ units, $T_C = 16.7$ days, $T_I = 1.8$ days

$$= \sqrt[3]{(4.2\text{ units})^3\left(\frac{16.7\text{ days}}{1.8\text{ days}}\right)^2}$$

$$= \sqrt[3]{6.4\times10^3\text{ units}^3}$$

$$= 19\text{ units}$$

EVALUATE THE ANSWER

- **Are the units correct?** r_C should be in Galileo's units, like r_I.
- **Is the magnitude realistic?** The period is larger, so the radius should be larger.

APPLICATION

1. If Ganymede, one of Jupiter's moons, has a period of 32 days, how many units is its orbital radius? Use the information given in Example 1.
2. An asteroid revolves around the Sun with a mean orbital radius twice that of Earth's. Predict the period of the asteroid in Earth years.
3. Venus has a period of revolution of 225 Earth days. Find the distance between the Sun and Venus as a multiple of Earth's average distance from the Sun.
4. Uranus requires 84 years to circle the Sun. Find Uranus's average distance from the Sun as a multiple of Earth's average distance from the Sun.
5. From **Table 1** you can find that, on average, Mars is 1.52 times as far from the Sun as Earth is. Predict the time required for Mars to orbit the Sun in Earth days.
6. The Moon has a period of 27.3 days and a mean distance of 3.9×10^5 km from its center to the center of Earth.
   a. Use Kepler's laws to find the period of a satellite in orbit 6.70×10^3 km from the center of Earth.
   b. How far above Earth's surface is this satellite?
7. **CHALLENGE** Using the data in the previous problem for the period and radius of revolution of the Moon, predict what the mean distance from Earth's center would be for an artificial satellite that has a period of exactly 1.00 day.

1. If Ganymede, one of Jupiter's moons, has a period of 7.15 days, how many units is its orbital radius? Use the information given in Example Problem 1.

$$\left(\frac{T_G}{T_I}\right)^2 = \left(\frac{r_G}{r_I}\right)^3$$

$$r_G = \sqrt[3]{(4.2\text{ units})^3\left(\frac{7.15\text{ days}}{1.8\text{ days}}\right)^2}$$

$$= \sqrt[3]{1.17\times10^3\text{ units}^3} = 11\text{ units}$$

2. An asteroid revolves around the Sun with a mean orbital radius twice that of Earth's. Predict the period of the asteroid in Earth years.

$$\left(\frac{T_a}{T_E}\right)^2 = \left(\frac{r_a}{r_E}\right)^3 \text{ with } r_a = 2r_E$$

$$T_a = \sqrt{\left(\frac{r_a}{r_E}\right)^3 T_E^2}$$

$$= \sqrt{\left(\frac{2r_E}{r_E}\right)^3(1.0\text{ y})^2} = 2.8\text{ y}$$

3. Venus has a period of revolution of 225 Earth days. Find the distance between the Sun and Venus as a multiple of Earth's average distance from the Sun.

$$\left(\frac{T_V}{T_E}\right)^2 = \left(\frac{r_V}{r_E}\right)^3$$

$$\frac{r_V}{r_E} = \sqrt[3]{\left(\frac{T_V}{T_E}\right)^2}$$

$$= \sqrt[3]{\left(\frac{225\text{ days}}{365\text{ days}}\right)^2}$$

$$= 0.724$$

So $r_V = 0.724 r_E$

4. Uranus requires 84 years to circle the Sun. Find Uranus's average distance from the Sun as a multiple of Earth's average distance from the Sun.

$$\left(\frac{T_U}{T_E}\right)^2 = \left(\frac{r_U}{r_E}\right)^3$$

$$\frac{r_U}{r_E} = \sqrt[3]{\left(\frac{T_U}{T_E}\right)^2}$$

$$= \sqrt[3]{\left(\frac{84\text{ y}}{1.0\text{ y}}\right)^2} = 19$$

So $r_U = 19 r_E$

5. From **Table 1** you can find that, on average, Mars is 1.52 times as far from the Sun as Earth is. Predict the time required for Mars to orbit the Sun in Earth days.

$$\left(\frac{T_M}{T_E}\right)^2 = \left(\frac{r_M}{r_E}\right)^3 \text{ with } r_M = 1.52 r_E$$

Thus, $T_M = \sqrt{\left(\frac{r_M}{r_E}\right)^3 T_E^2}$

$$= \sqrt{\left(\frac{1.52 r_E}{r_E}\right)^3(365\text{ days})^2}$$

$$= \sqrt{4.68\times10^5\text{ days}^2}$$

$$= 684\text{ days}$$

6. The Moon has a period of 27.3 days and a mean distance of 3.9×10^5 km from the center of Earth.

   a. Use Kepler's laws to find the period of a satellite in orbit 6.70×10^3 km from the center of Earth.

$$\left(\frac{T_S}{T_M}\right)^2 = \left(\frac{r_S}{r_M}\right)^3$$

$$T_S = \sqrt{\left(\frac{r_S}{r_M}\right)^3 T_M^2}$$

$$= \sqrt{\left(\frac{6.70\times10^3\text{ km}}{3.9\times10^5\text{ km}}\right)^3(27.3\text{ days})^2}$$

$$= \sqrt{3.78\times10^{-3}\text{ days}^2}$$

$$= 6.15\times10^{-2}\text{ days} = 89\text{ min}$$

   b. How far above Earth's surface is this satellite?

$$h = r_S - r_E$$

$$= 6.70\times10^6\text{ m} - 6.38\times10^6\text{ m}$$

$$= 3.2\times10^5\text{ m}$$

$$= 3.2\times10^2\text{ km}$$

7. **CHALLENGE** Using the data in the previous problem for the period and radius of revolution of the Moon, predict what the mean distance from Earth's center would be for an artificial satellite that has a period of exactly 1.00 day.

$$\left(\frac{T_S}{T_M}\right)^2 = \left(\frac{r_S}{r_M}\right)^3$$

$$r_S = \sqrt[3]{r_M^3\left(\frac{T_S}{T_M}\right)^2} = \sqrt[3]{(3.90\times10^5\text{ km})^3\left(\frac{1.00\text{ days}}{27.3\text{ days}}\right)^2}$$

$$= \sqrt[3]{7.96\times10^{13}\text{ km}^3}$$

$$= 4.3\times10^4\text{ km}$$

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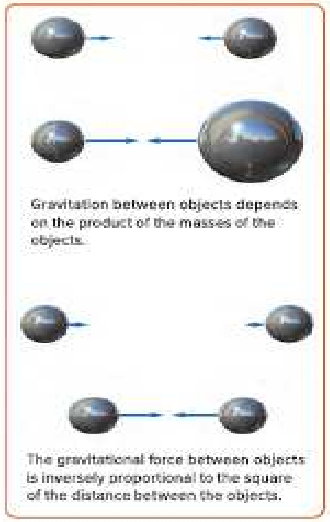

Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

Newton's Law of Universal Gravitation

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F_g) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F_g is proportional to $\frac{1}{r^2}$. The force ($\boldsymbol{F_g}$) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them as shown below.

LAW OF UNIVERSAL GRAVITATION
The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F_g = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature below to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed in the next sections.

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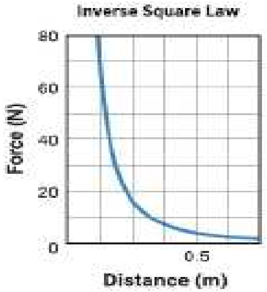


CONNECTING MATH TO PHYSICS

Direct and Inverse Relationships Newton's law of universal gravitation has both direct and inverse relationships.

$F_g \propto m_1m_2$		$F_g \propto \frac{1}{r^2}$	
Change	**Result**	**Change**	**Result**
$(2m_1)m_2$	$2F_g$	$2r$	$\frac{1}{4}F_g$
$(3m_1)m_2$	$3F_g$	$3r$	$\frac{1}{9}F_g$
$(2m_1)(3m_2)$	$6F_g$	$\frac{1}{2}r$	$4F_g$
$\left(\frac{1}{2}\right)m_1m_2$	$\frac{1}{2}F_g$	$\frac{1}{3}r$	$9F_g$

Module 10: Energy and Its Conservation

6.	Identify work as a scalar quantity measured in N.m or Joule (J).	Student Book	188

Work

Consider a force exerted on an object while the object moves a certain distance, such as the book bag in **Figure 1.** There is a net force, so the object is accelerated, $a = \frac{F}{m}$, and its velocity changes. Recall from your study of motion that acceleration, velocity, and distance are related by the equation $v_f^2 = v_i^2 + 2ad$. This can be rewritten as $2ad = v_f^2 - v_i^2$. Replace a with the term $\frac{F}{m}$ to get $2\left(\frac{F}{m}\right)d = v_f^2 - v_i^2$. Multiplying both sides by $\frac{m}{2}$ gives $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

The left side of the equation describes an action that was done to the system by the external world. Recall that a system is the object or objects of interest and the external world is everything else. A force (F) was exerted on a system while the point of contact moved. When a force is applied through a displacement, **work** (W) is done on the system.

The SI unit of work is called a **joule** (J). One joule is equal to 1 N·m. One joule of work is done when a force of 1 N acts on a system over a displacement of 1 m. An apple weighs about 1 N, so it takes roughly 1 N of force to lift the apple at a constant velocity. Thus, when you lift an apple 1 m at a constant velocity, you do 1 J of work on it.

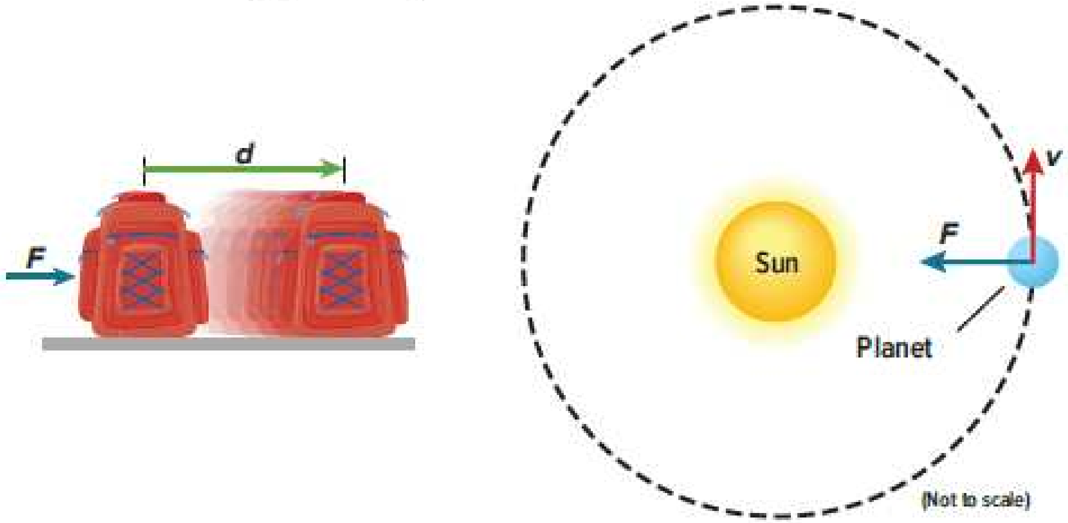


Figure 1 Work is done when a force is applied through a displacement.

Identify *another example of when a force does work on an object.*

7.	Illustrate when work is positive, negative or zero with suitable examples.	Student Book Practice Problem (9)	189 193

Work done by a constant force In the book bag example, F is a constant force exerted in the direction in which the object is moving. In this case, work (W) is the product of the force and the system's displacement. That is,

$$W = Fd.$$

What happens if the exerted force is perpendicular to the direction of motion? For example, for a planet in a circular orbit, the force is always perpendicular to the direction of motion, as shown in **Figure 1.** Recall that a perpendicular force only changes the direction. The speed of the planet doesn't change, so the right side of the equation, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, is zero. Therefore, the work done is also zero.

Constant force exerted at an angle What work does a force exerted at an angle do? For example, what work does the person pushing the car in **Figure 2** do? Recall that any force can be replaced by its components. If you use the coordinate system shown in **Figure 2**, the 125-N force (F) exerted in the direction of the person's arm has two components.

The magnitude of the horizontal component (F_x) is related to the magnitude of the applied force (F) by $\cos 25.0° = \frac{F_x}{F}$. By solving for F_x, you obtain

$$F_x = F\cos 25.0° = (125\ \text{N})(\cos 25.0°) = 113\ \text{N}.$$

Using the same method, the vertical component is

$$F_y = -F\sin 25.0°$$

$$F_y = -(125\ \text{N})(\sin 25.0°) = -52.8\ \text{N}.$$

The negative sign shows that the force is downward. Because the displacement is in the x direction, only the x-component does work. The y-component does no work. The work you do when you exert a force on a system at an angle to the direction of motion is equal to the component of the force in the direction of the displacement multiplied by the displacement.

The magnitude of the component (F_x) force acting in the direction of displacement is found by multiplying the magnitude of force (F) by the cosine of the angle (θ) between the force and the direction of the displacement:

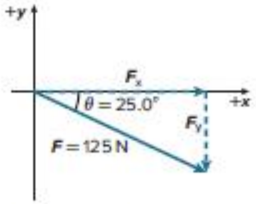


Figure 2 Only the horizontal component of the force that the man exerts on the car does work because the car's displacement is horizontal.

$F_x = F\cos\theta$. Thus, the work done is represented by the following equation.

Work

Work is equal to the product of the magnitude of the force and magnitude of displacement times the cosine of the angle between them.

$$W = Fd\cos\theta$$

Get It?

Determine the work you do when you exert a force of 3 N at an angle of 45° from the direction of motion for 1 m.

The equation above agrees with our expectations for constant forces exerted in the direction of displacement and for constant forces perpendicular to the displacement. In the book bag example, $\theta = 0°$ and $\cos 0° = 1$. Thus, $W = Fd(1) = Fd$, just as we found before. In the case of the orbiting planet, $\theta = 90°$ and $\cos 90° = 0$. Thus, $W = Fd(0) = 0$. This agrees with our previous conclusions.

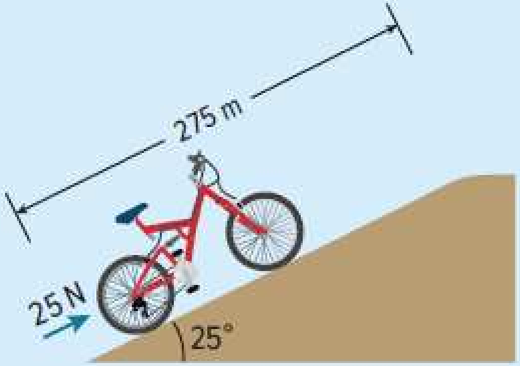


Figure 5

9. CHALLENGE A bicycle rider pushes a 13-kg bicycle up a steep hill. The incline is 25° and the hill is 275 m long, as shown in **Figure 5.** The rider pushes the bike parallel to the road with a force of 25 N.

a. How much work does the rider do on the bike?

b. How much work is done by the force of gravity on the bike?

a. How much work does the rider do on the bike?

Force and displacement are in the same direction.

$$W = Fd$$
$$= (25\ \text{N})(275\ \text{m}) = 6.9\times10^3\ \text{J}$$

b. How much work is done by the force of gravity on the bike?

The force is downward (–90°), and the displacement is 25° above the horizontal or 115° from the force.

$$W = Fd\cos\theta = mgd\cos\theta$$
$$= (13\ \text{kg})(9.8\ \text{N/kg})(275\ \text{m})(\cos 115°)$$
$$= -1.5\times10^4\ \text{J}$$

8.	Determine graphically the work done by a force from the area of force versus displacement graph.	Student Book	191

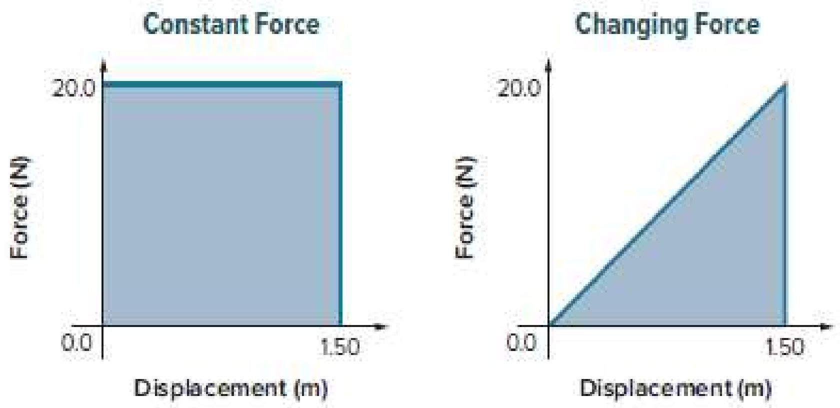


Figure 4 The area under a force-displacement graph is equal to the work.

Finding work done when forces change In the last example, the force changed but we could determine the work done in each segment. But what if the force changes in a more complicated way?

A graph of force versus displacement lets you determine the work done by a force. This graphical method can be used to solve problems in which the force is changing. The left graph in **Figure 4** shows the work done by a constant force of 20.0 N that is exerted to lift an object 1.50 m. The work done by this force is represented by

$$W = Fd = (20.0\ \text{N})(1.50\ \text{m}) = 30.0\ \text{J}.$$

Note that the shaded area under the left graph is also equal to (20.0 N) (1.50 m), or 30.0 J. The area under a force-displacement graph is equal to the work done by that force.

This is true even if the force changes. The right graph in **Figure 4** shows the force exerted by a spring that varies linearly from 0.0 to 20.0 N as it is compressed 1.50 m. The work done by the force that compressed the spring is the area under the graph, which is the area of a triangle, $\left(\frac{1}{2}\right)$(base)(altitude), or

$$W = \left(\frac{1}{2}\right)(20.0\ \text{N})(1.50\ \text{m}) = 15.0\ \text{J}.$$

Use the problem-solving strategy below when you solve problems related to work.

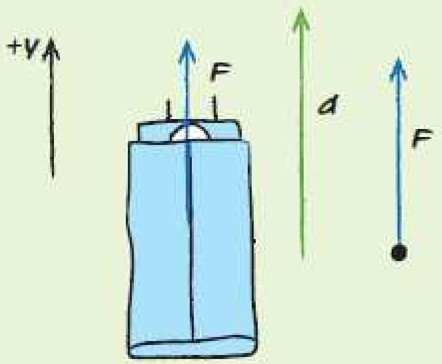
9.	Apply the relationship between power, the work done by a force, and the time interval in which that work is done ($P = \frac{W}{\Delta t}$)	Example Problem (3) Practice Problem (17)	197 197

EXAMPLE Problem 3

POWER An electric motor lifts an elevator 9.00 m in 15.0 s by exerting an upward force of 1.20×10^4 N. What power does the motor produce in kW?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation showing the system as the elevator with its initial conditions.
- Establish a coordinate system with up as positive.
- Draw a vector diagram for the force and displacement.



Known	Unknown
$d = 9.00$ m	$P = ?$
$t = 15.0$ s	
$F = 1.20\times10^4$ N	

2 SOLVE FOR THE UNKNOWN

Use the definition of power.

$$P = \frac{W}{t}$$

$$= \frac{Fd}{t}$$ Substitute $W = Fd\cos 0° = Fd$.

$$= \frac{(1.20\times10^4\ \text{N})(9.00\ \text{m})}{(15.0\ \text{s})}$$ Substitute $F = 1.20\times10^4$ N, $d = 9.00$ m, $t = 15.0$ s.

$$= 7.20\ \text{kW}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Power is measured in joules per second, or watts.
- **Does the sign make sense?** The positive sign agrees with the upward direction of the force.

17. An electric motor develops 65 kW of power as it lifts a loaded elevator 17.5 m in 35 s. How much force does the motor exert?

$$P = \frac{W}{t} = \frac{Fd}{t}$$

$$F = \frac{Pt}{d} = \frac{(65\times10^3\ \text{W})(35\ \text{s})}{17.5\ \text{m}}$$

$$= 1.3\times10^5\ \text{N}$$

10.	Relate the rotational kinetic energy to the object's moment of inertia and it's angular velocity: ($K_{rotational} = \frac{1}{2} I\omega^2$)	Student Book	201

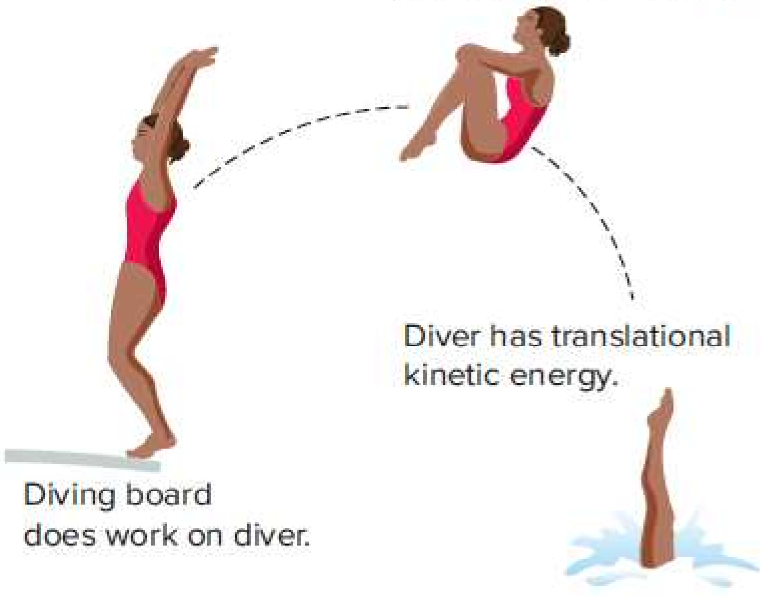


Figure 14 The diving board does work on the diver. This work increases the diver's kinetic energy.

Rotational kinetic energy If you spin a toy top in one spot, you might say that it does not have kinetic energy because the top does not change its position. However, to make the top rotate, you had to do work on it. The top has **rotational kinetic energy,** which is energy due to rotational motion. Rotational kinetic energy can be calculated using $KE_{rot} = \frac{1}{2} I\omega^2$, where I is the object's moment of inertia and ω is the object's angular velocity.

In **Figure 14,** the diving board does work on a diver, transferring translational and rotational kinetic energy to the diver. Her center of mass moves as she leaps, so she has translational kinetic energy. She rotates about her center of mass, so she has rotational kinetic energy. When she slices into the water, she has mostly translational kinetic energy.

11.	Relate the gravitational potential energy to the mass of the object and its height above or below a reference level ($GPE=mgh$)	Example Problem (4)	204

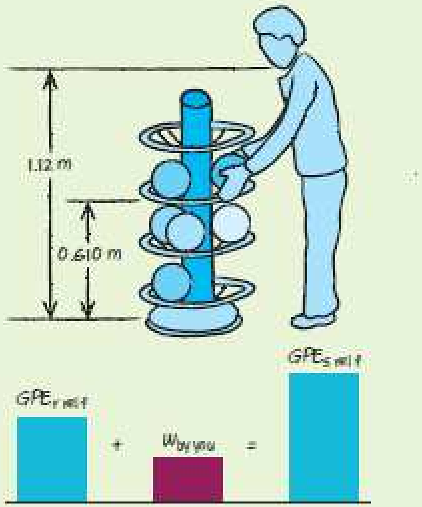
EXAMPLE Problem 4

GRAVITATIONAL POTENTIAL ENERGY You lift a 7.30-kg bowling ball from the storage rack and hold it up to your shoulder. The storage rack is 0.610 m above the floor and your shoulder is 1.12 m above the floor.

a. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the floor?

b. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the rack?

c. How much work was done by gravity as you lifted the ball from the rack to shoulder level?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose a reference level.
- Draw an energy bar diagram showing the gravitational potential energy with the floor as the reference level.

Known

$m = 7.30$ kg $\quad g = 9.8$ N/kg

$h_r = 0.610$ m (rack relative to the floor)

$h_s = 1.12$ m (shoulder relative to the floor)

Unknown

$GPE_{s\,rel\,f} = ?$

$GPE_{s\,rel\,r} = ?$

$W = ?$

2 SOLVE FOR THE UNKNOWN

a. Set the reference level to be at the floor.

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$$GPE_{s\,rel\,f} = mgh_s = (7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m}) = 8.0\times10^{1}\text{ J}$$

Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m

b. Set the reference level to be at the rack height.

Determine the height of your shoulder relative to the rack.

$$h = h_s - h_r$$

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$$GPE_{s\,rel\,r} = mgh = mg(h_s - h_r)$$

Substitute $h = h_s - h_r$

$$= (7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m} - 0.610\text{ m})$$

Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$$= 36\text{ J}$$

This also is equal to the work done by you as you lifted the ball.

c. The work done by gravity is the weight of the ball times the distance the ball was lifted.

$$W = Fd = -(mg)h = -(mg)(h_s - h_r)$$

The weight opposes the motion of lifting, so the work is negative.

$$= -(7.30\text{ kg})(9.8\text{ N/kg})(1.12\text{ m} - 0.610\text{ m})$$

Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$$= -36\text{ J}$$

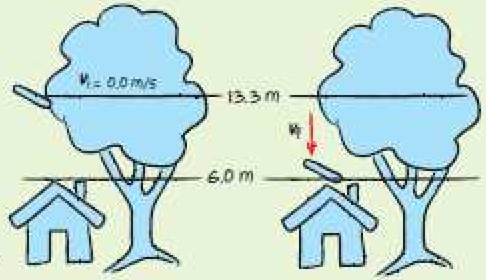
12.	Apply the law of conservation of mechanical energy to solve problems ($KE_i + PE_i = KE_f + PE_f$).	Example Problem (5) Check your Progress Q.51	212 217

EXAMPLE Problem 5

CONSERVATION OF MECHANICAL ENERGY A 22.0-kg tree limb is 13.3 m above the ground. During a tropical storm, it falls on a roof that is 6.0 m above the ground.

a. Find the kinetic energy of the limb when it reaches the roof. Assume that the air does no work on the tree limb.

b. What is the limb's speed when it reaches the roof?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw an energy bar diagram.

Known		Unknown	
$m = 22.0$ kg	$g = 9.8$ N/kg	$GPE_i = ?$	$KE_f = ?$
$h_{limb} = 13.3$ m	$v_i = 0.0$ m/s	$GPE_f = ?$	$v_f = ?$
$h_{roof} = 6.0$ m	$KE_i = 0.0$ J		

2 SOLVE FOR THE UNKNOWN

a. Set the reference level as the height of the roof.

Find the initial height of the limb relative to the roof.

$h = h_{limb} - h_{roof} = 13.3\text{ m} - 6.0\text{ m} = 7.3\text{ m}$ — Substitute $h_{limb} = 13.3$ m, $h_{roof} = 6.0$ m

Determine the initial gravitational potential energy of the limb-Earth system.

$GPE_i = mgh = (22.0\text{ kg})(9.8\text{ N/kg})(7.3\text{ m})$ — Substitute $m = 22.0$ kg, $g = 9.8$ N/kg, $h = 7.3$ m

$= 1.6\times10^3$ J

Identify the initial kinetic energy of the system.

$KE_i = 0.0$ J — The tree limb is initially at rest.

Identify the final potential energy of the system.

$GPE_f = 0.0$ J — $h = 0.0$ m at the roof.

Use the law of conservation of energy to find KE_f.

$KE_f + GPE_f = KE_i + GPE_i$

$KE_f = KE_i + GPE_i - GPE_f$

$= 0.0\text{ J} + 1.6\times10^3\text{ J} - 0.0\text{ J}$ — Substitute $KE_i = 0.0$ J, $GPE_i = 1.6\times10^3$ J, and $GPE_f = 0.0$ J.

$= 1.6\times10^3$ J

b. Determine the speed of the limb.

$KE_f = \frac{1}{2}mv_f^2$

$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(1.6\times10^3\text{ J})}{22.0\text{ kg}}}$ — Substitute $KE_f = 1.6\times10^3$ J, $m = 22.0$ kg

$= 12$ m/s

51. **Energy** In **Figure 27,** a child slides down a playground slide. At the bottom, she is moving at 3.0 m/s. How much energy was transformed by friction as she slid down the slide?

Figure 27

$$E_i = mgh$$
$$= (36.0\text{ kg})(9.8\text{ N/kg})(2.5\text{ m})$$
$$= 880\text{ J}$$
$$E_f = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(36.0\text{ kg})(3.0\text{ m/s})^2$$
$$= 160\text{ J}$$
$$W = \Delta KE = 880\text{ J} - 160\text{ J}$$
$$= 720\text{ J}$$

13.	Define kinetic energy and apply the relationship between a particle's kinetic energy, mass, and speed ($KE = \frac{1}{2}mv^2$).	Student Book	200

Translational and Rotational Kinetic Energy

In the examples we have considered so far, moving objects that were changing position had kinetic energy $\left(\frac{1}{2}mv^2\right)$ due to their motion. What about energy due to an object's changing position?

Translational kinetic energy Energy due to changing position is called **translational kinetic energy** and can be represented by the following equation.

Translational Kinetic Energy

A system's translational kinetic energy is equal to one-half times the system's mass multiplied by the system's speed squared.

$$KE_{trans} = \frac{1}{2}mv^2$$

Translational kinetic energy is proportional to the object's mass. For example, a 7.26-kg bowling ball thrown through the air has more translational kinetic energy than a 0.148-kg baseball, like the one shown in **Figure 13**, moving with the same speed.

An object's translational kinetic energy is also proportional to the square of the object's speed. A car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.

Figure 13 A baseball has less translational kinetic energy than a bowling ball moving with the same speed.

Get It?

Explain Using the equation for translational kinetic energy, show why a car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.

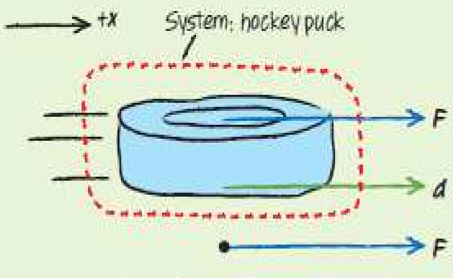
16	(1) Apply the relationship between a force $\vec{F}$ and the work done on a system by the force when the system undergoes a displacement $\vec{d}$, $W = \lvert\vec{F}\rvert \times \lvert\vec{d}\rvert \cos(\theta)$ where θ is the angle between the direction of the force and the direction of displacement. (2) Apply the work–energy theorem to relate the net work done on a system and the resulting change in kinetic energy	Example Problem (1) Example Problem (2) Practice Problem (11)	192 193 196

EXAMPLE Problem 1

WORK A hockey player uses a stick to apply a constant 4.50-N force forward to a 105-g puck sliding on ice over a displacement of 0.150 m forward. How much work does the stick do on the puck? Assume friction is negligible.

1 ANALYZE AND SKETCH THE PROBLEM

- Identify the system and the force doing work on it.
- Sketch the situation showing initial conditions.
- Establish a coordinate system with $+x$ to the right.
- Draw a vector diagram.



Player does work on hockey puck.

Known	Unknown
$m = 105$ g	$W = ?$
$F = 4.50$ N	
$d = 0.150$ m	
$\theta = 0°$	

2 SOLVE FOR THE UNKNOWN

Use the definition for work.

$W = Fd\cos\theta$

$= (4.50\ \text{N})(0.150\ \text{m})(\cos\theta)$ — Substitute $F = 4.50$ N, $d = 0.150$ m, $\cos\theta = \cos 0° = 1$.

$= 0.675\ \text{N·m}$

$= 0.675\ \text{J}$ — $1\ \text{J} = 1\ \text{N·m}$

3 EVALUATE THE ANSWER

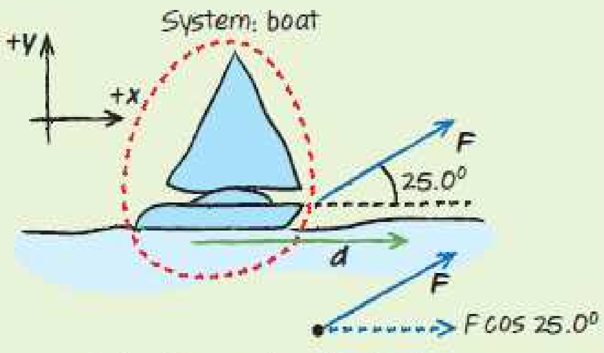
- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The stick (external world) does work on the puck (the system), so the sign of work should be positive.

EXAMPLE Problem 2

FORCE AND DISPLACEMENT AT AN ANGLE A sailor pulls a boat a distance of 30.0 m along a dock using a rope that makes a 25.0° angle with the horizontal. How much work does the rope do on the boat if its tension is 255 N?

1 ANALYZE AND SKETCH THE PROBLEM

- Identify the system and the force doing work on it.
- Establish coordinate axes.
- Sketch the situation showing the boat with initial conditions.
- Draw vectors showing the displacement, the force, and its component in the direction of the displacement.



Sailor does work on the boat.

Known		Unknown
$F = 255$ N	$\theta = 25.0°$	$W = ?$
$d = 30.0$ m		

2 SOLVE FOR THE UNKNOWN

Use the definition of work.

$W = Fd\cos\theta$

$= (255\ \text{N})(30.0\ \text{m})(\cos 25.0°)$ — Substitute $F = 255$ N, $d = 30.0$ m, $\theta = 25.0°$.

$= 6.93\times10^{3}\ \text{J}$

16	(1) Apply the relationship between a force $\vec{F}$ and the work done on a system by the force when the system undergoes a displacement $\vec{d}$, $W = \lvert\vec{F}\rvert \times \lvert\vec{d}\rvert \cos(\theta)$ where θ is the angle between the direction of the force and the direction of displacement. (2) Apply the work–energy theorem to relate the net work done on a system and the resulting change in kinetic energy	Example Problem (1) Example Problem (2) Practice Problem (11)	192 193 196

11. A 52.0-kg skater moves at 2.5 m/s and stops over a distance of 24.0 m. Find the skater's initial kinetic energy. How much of her kinetic energy is transformed into other forms of energy by friction as she stops? How much work must she do to speed up to 2.5 m/s again?

While stopping:

$$E_{\text{transformed}} = E_i - E_f = KE_i - 0$$
$$= \frac{1}{2} m v_i^2$$
$$= \frac{1}{2}(52.0\text{ kg})(2.5\text{ m/s})^2$$
$$= 160\text{ J}$$

All of her kinetic energy is transformed to other forms of energy when she stops.

To Speed up again:

$$W = \Delta E = KE_f - KE_i$$
$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$
$$= \frac{1}{2}(52.0\text{ kg})(2.5\text{ m/s})^2 - \frac{1}{2}(52.0\text{ kg})(0.00\text{ m/s})^2$$
$$= +160\text{ J}$$

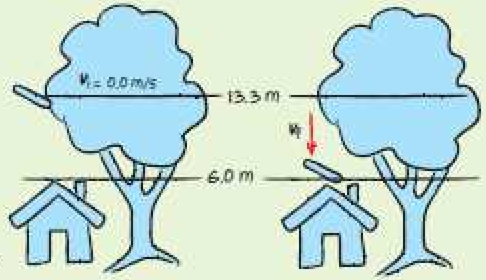
17	(1) Apply the law of conservation of mechanical energy to solve problems ($KE_i + PE_i = KE_f + PE_f$). (2) Apply the law of conservation of energy to examples like roller coaster rides, ski slopes, inclined planes/ hills, and pendulums	Example Problem (5) Practice Problem (41)	212 213

EXAMPLE Problem 5

CONSERVATION OF MECHANICAL ENERGY A 22.0-kg tree limb is 13.3 m above the ground. During a tropical storm, it falls on a roof that is 6.0 m above the ground.

a. Find the kinetic energy of the limb when it reaches the roof. Assume that the air does no work on the tree limb.

b. What is the limb's speed when it reaches the roof?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw an energy bar diagram.

Known		**Unknown**	
$m = 22.0$ kg	$g = 9.8$ N/kg	$GPE_i = ?$	$KE_f = ?$
$h_{limb} = 13.3$ m	$v_i = 0.0$ m/s	$GPE_f = ?$	$v_f = ?$
$h_{roof} = 6.0$ m	$KE_i = 0.0$ J		

2 SOLVE FOR THE UNKNOWN

a. Set the reference level as the height of the roof.

Find the initial height of the limb relative to the roof.

$h = h_{limb} - h_{roof} = 13.3\text{ m} - 6.0\text{ m} = 7.3\text{ m}$ — Substitute $h_{limb} = 13.3$ m, $h_{roof} = 6.0$ m

Determine the initial gravitational potential energy of the limb-Earth system.

$GPE_i = mgh = (22.0\text{ kg})(9.8\text{ N/kg})(7.3\text{ m})$ — Substitute $m = 22.0$ kg, $g = 9.8$ N/kg, $h = 7.3$ m

$= 1.6\times10^3$ J

Identify the initial kinetic energy of the system.

$KE_i = 0.0$ J — The tree limb is initially at rest.

Identify the final potential energy of the system.

$GPE_f = 0.0$ J — $h = 0.0$ m at the roof.

Use the law of conservation of energy to find KE_f.

$KE_i + GPE_i = KE_f + GPE_f$

$KE_f = KE_i + GPE_i - GPE_f$

$= 0.0\text{ J} + 1.6\times10^3\text{ J} - 0.0\text{ J}$ — Substitute $KE_i = 0.0$ J, $GPE_i = 1.6\times10^3$ J, and $GPE_f = 0.0$ J.

$= 1.6\times10^3$ J

b. Determine the speed of the limb.

$KE_f = \frac{1}{2}mv_f^2$

$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(1.6\times10^3\text{ J})}{22.0\text{ kg}}}$ — Substitute $KE_f = 1.6\times10^3$ J, $m = 22.0$ kg

$= 12$ m/s

41. A skier starts from rest at the top of a hill that is 45.0 m high, skis down a 30° incline into a valley, and continues up a hill that is 40.0 m high. The heights of both hills are measured from the valley floor. Assume that friction is negligible and ignore the effect of the ski poles.

a. How fast is the skier moving at the bottom of the valley?

b. What is the skier's speed at the top of the second hill?

a.

Mechanical energy is conserved,

so $KE_i + PE_i + KE_f + PE_f$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{(2)(9.8\text{ N/kg})(45.0\text{ m})}$$

$$= 3.0\times10^1\text{ m/s}$$

b.

$$KE_i + PE_i = KE_f + PE_f$$

$$0 + mgh_i = \frac{1}{2}mv^2 + mgh_f$$

$$v^2 = 2g(h_i - h_f)$$

$$= \sqrt{2g(h_i - h_f)}$$

$$= \sqrt{(2)(9.8\text{ N/kg})(45.0\text{ m} - 40.0\text{ m})}$$

$$= 9.9\text{ m/s}$$

c. No, the angles do not have any impact.

Module 12:
States of Matter

14.	Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle	Student Book Check your Progress Q.38	249-250 251

Bernoulli's Principle

Study the flow of water from the hose in **Figure 14.** In the top photo, the water flows from the hose unobstructed. In the bottom photo, the hose opening has been narrowed by a person's thumb. Notice that the streams of water look different. The velocity of the water stream in the bottom photo is greater compared to the velocity of the stream in the top photo. What you can't see is that pressure exerted by the water in the bottom photo decreased. The relationship between the velocity and pressure exerted by a moving fluid is named for Swiss scientist Daniel Bernoulli. **Bernoulli's principle** states that as the velocity of a fluid increases, the pressure exerted by that fluid decreases. This principle is a statement of work and energy conservation as applied to fluids.

Figure 14 You can demonstrate Bernoulli's principle by narrowing the opening of the hose as water flows out. As the velocity of the water increases, the pressure it exerts decreases. Richard Hutchings/Digital Light Source

Another instance in which the velocity of water can change is in a stream. You might have seen the water in a stream speed up as it passed through narrowed sections of the stream bed. As the opening of the hose and the stream channel become wider or narrower, the velocity of the fluid changes to maintain the overall flow of water. The pressure of blood in our circulatory systems depends partly on Bernoulli's principle. Bernoulli's principle also helps explain how smoke is pulled up a fireplace chimney.

Consider a horizontal pipe completely filled with a smoothly flowing ideal fluid. If a certain mass of the fluid enters one end of the pipe, then an equal mass must come out the other end. What happens if the cross section becomes narrower, as shown in **Figure 15?** To keep the same mass of fluid moving through the narrow section in a fixed amount of time, the velocity of the fluid in the tube must increase. As the fluid's velocity increases, so does its kinetic energy. This means that net work has been done on the swifter fluid. This net work comes from the difference between the work that was done to move the mass of fluid into the pipe and the work that was done by the fluid pushing the same mass out of the pipe. The work is proportional to the force on the fluid, which, in turn, depends on the pressure. If the net work is positive, the pressure at the input end of the section, where the velocity is lower, must be larger than the pressure at the output end, where the velocity is higher.

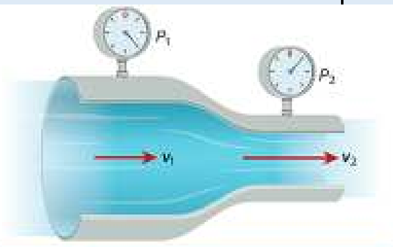


Figure 15 The fluid flowing through this pipe also demonstrates Bernoulli's principle. As the velocity of the fluid increases (v_2 is greater than v_1), the pressure it exerts decreases (P_2 is less than P_1).

Get It?

Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle.

Applications of Bernoulli's principle

There are many common applications of Bernoulli's principle, such as paint sprayers and sprayers attached to garden hoses to apply fertilizers and pesticides to lawns and gardens. In a hose-end sprayer, a strawlike tube is sunk into the chemical solution in the sprayer. The sprayer is attached to a hose. A trigger on the sprayer allows water from the hose to flow at a high speed, producing an area of low pressure above the tube. The solution is then sucked up through the tube and into the stream of water.

A gasoline engine's carburetor, which is where air and gasoline are mixed, is another common application of Bernoulli's principle. Part of the carburetor is a tube with a constriction, as shown in the diagram in **Figure 16.** The pressure on the gasoline in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be varied. Carburetors are used in motorcycles, stock car race cars, and the motors of small gasoline-powered machines, such as lawn mowers.

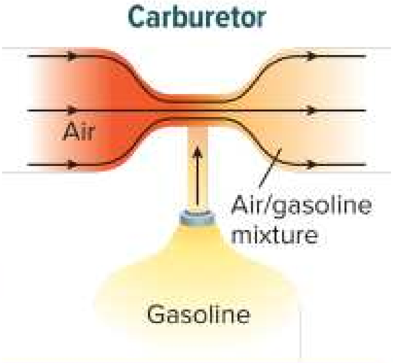


Figure 16 In a carburetor, low pressure in the narrow part of the tube draws fuel into the air flow.

14.	Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle	Student Book Check your Progress Q.38	249-250 251

38. **Critical Thinking** A tornado passing over a house sometimes makes the house explode from the inside out. How might Bernoulli's principle explain this phenomenon? What could be done to reduce the danger of a door or window exploding outward?

The fast-moving air of the tornado has a lower pressure than the still air inside the house. Therefore, the air inside the house is at a higher pressure and produces an enormous force on the windows, doors, and walls of the house. This pressure difference is reduced by opening doors and windows to let the air flow freely to the outside.

15.	(1) Recall Pascal's principle. (2) Apply Pascal's principle to hydraulic systems to solve problems.	Student Book	244

Fluids at Rest

If you have ever dived deep into a swimming pool or a lake, you likely felt pressure on your ears. Blaise Pascal, a French physician, found that the pressure at a point in a fluid depends on its depth in the fluid and is unrelated to the shape of the fluid's container. He also noted that any change in pressure applied at any point on a confined fluid is transferred undiminished throughout the fluid, a fact that is now known as Pascal's principle. One application of Pascal's principle is using fluids in machines to multiply forces. In the hydraulic system shown in **Figure 10,** a fluid is confined to two connecting chambers. Each chamber has a piston that is free to move, and the pistons have different surface areas. Recall that if a force (F_1) is exerted on the first piston with a surface area of A_1, the pressure (P_1) exerted on the fluid is $P_1 = \frac{F_1}{A_1}$. The pressure exerted by the fluid on the second piston, with a surface area A_2, is $P_2 = \frac{F_2}{A_2}$.

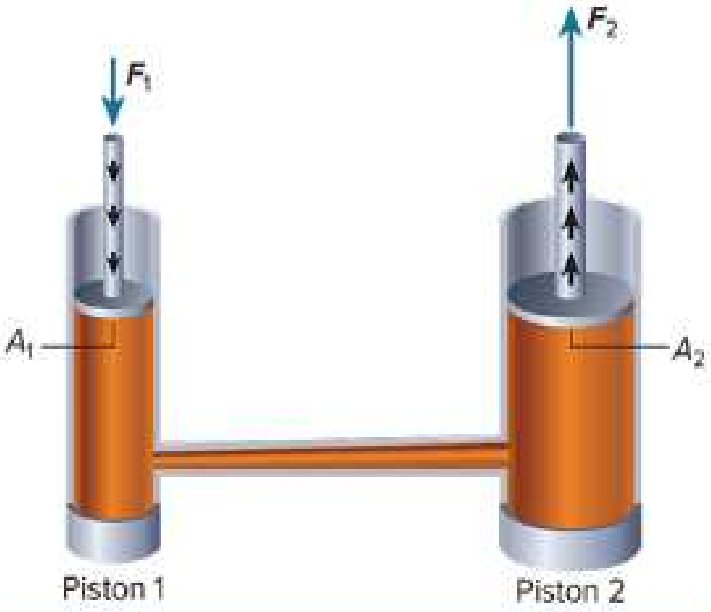


Figure 10 As F_1 exerts pressure on the smaller piston (piston 1), the pressure is transmitted throughout the fluid. As a result, a multiplied force (F_2) is exerted on the larger piston (piston 2).

Infer *How would F_2 change if F_1 increased? Explain why.*

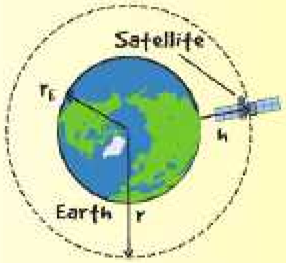
19	(1) Calculate the orbital period of a satellite. (2) Define pressure as the perpendicular component of a force on a surface divided by the area of the surface: $\left(P = \frac{F}{A}\right)$	Example Problem (2) Check your Progress Q.8 Example Problem (1) Practice Problem (3)	175 172 234 235

EXAMPLE 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.



KNOWN	UNKNOWN
$h = 2.25\times10^{5}$ m	$v = ?$
$r_E = 6.38\times10^{6}$ m	$T = ?$
$m_E = 5.97\times10^{24}$ kg	
$G = 6.67\times10^{-11}$ N·m²/kg²	

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$
$$= 2.25\times10^{5}\text{ m} + 6.38\times10^{6}\text{ m} = 6.60\times10^{6}\text{ m}$$

◄ Substitute $h = 2.25\times10^{5}$ m and $r_E = 6.38\times10^{6}$ m.

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{6.60\times10^{6}\text{ m}}}$$
$$= 7.77\times10^{3}\text{ m/s}$$

◄ Substitute $G = 6.67\times10^{-11}$ N·m²/kg², $m_E = 5.97\times10^{24}$ kg, and $r = 6.60\times10^{6}$ m.

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$
$$= 2\pi\sqrt{\frac{(6.60\times10^{6}\text{ m})^3}{(6.67\times10^{-11}\text{ N·m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}$$
$$= 5.34\times10^{3}\text{ s}$$

◄ Substitute $r = 6.60\times10^{6}$ m, $G = 6.67\times10^{-11}$ N·m²/kg², and $m_E = 5.97\times10^{24}$ kg.

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

Are the units correct? The unit for speed is meters per second, and the unit for period is seconds.

Neptune's Orbital Period Neptune orbits the Sun at an average distance given in **Figure 10,** which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

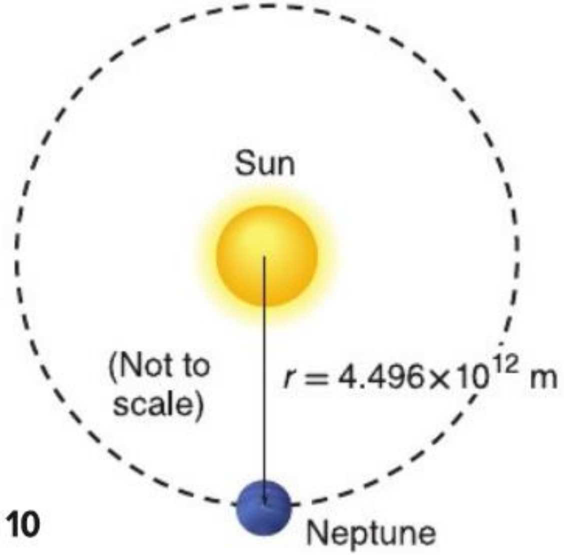


Figure 10

$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}} = 2\pi\sqrt{\frac{(4.495\times10^{12}\,\text{m})^3}{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\,\text{kg})}}$$
$$= 5.20\times10^{9}\,\text{s}$$
$$= \frac{5.20\times10^{9}\,\text{s}}{1}\times\frac{1\text{ day}}{86{,}400\text{ s}} = 6.02\times10^{4}\text{ days}$$

19	(1) Calculate the orbital period of a satellite. (2) Define pressure as the perpendicular component of a force on a surface divided by the area of the surface: $\left(P = \frac{F}{A}\right)$	Example Problem (2) Check your Progress Q.8 Example Problem (1) Practice Problem (3)	175 172 234 235

EXAMPLE Problem 1

CALCULATING PRESSURE A child weighs 364 N and sits on a three-legged stool, which weighs 41 N. The bottoms of the stool's legs touch the ground over a total area of 19.3 cm^2.

a. What is the average pressure that the child and the stool exert on the ground?

b. How does the pressure change when the child leans over so that only two legs of the stool touch the floor?

1. ANALYZE AND SKETCH THE PROBLEM

- Sketch the child and the stool, labeling the total force that they exert on the ground.
- List the variables, including the force that the child and the stool exert on the ground and the areas for parts **a** and **b**.

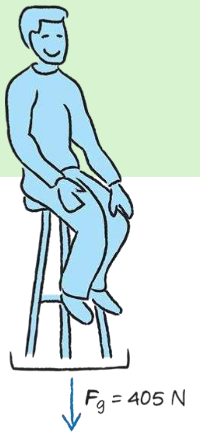
Known

$$F_{g\ \text{child}} = 364\ \text{N}$$
$$F_{g\ \text{stool}} = 41\ \text{N}$$
$$F_{g\ \text{total}} = F_{g\ \text{child}} + F_{g\ \text{stool}} = 364\ \text{N} + 41\ \text{N} = 405\ \text{N}$$

$$A_a = 19.3\ \text{cm}^2$$
$$A_b = \frac{2}{3} \times 19.3\ \text{cm}^2 = 12.9\ \text{cm}^2$$

Unknown

$$P_a = ?$$
$$P_b = ?$$



2. SOLVE FOR THE UNKNOWN

Find each pressure.

$$P = \frac{F}{A}$$

a. $$P_a = \left(\frac{405\ \text{N}}{19.3\ \text{cm}^2}\right)\left(\frac{(100\ \text{cm})^2}{(1\ \text{m})^2}\right) = 2.10 \times 10^2\ \text{kPa}$$

Substitute $F = F_{g\ \text{total}} = 405$ N, $A = A_a = 19.3\ \text{cm}^2$.

b. $$P_b = \left(\frac{405\ \text{N}}{12.9\ \text{cm}^2}\right)\left(\frac{(100\ \text{cm})^2}{(1\ \text{m})^2}\right) = 3.14 \times 10^2\ \text{kPa}$$

Substitute $F = F_{g\ \text{total}} = 405$ N, $A = A_b = 12.9\ \text{cm}^2$.

3. EVALUATE THE ANSWER

- **Are the units correct?** The units for pressure should be Pa, and 1 N/m^2 = 1 Pa.

19	(1) Calculate the orbital period of a satellite. (2) Define pressure as the perpendicular component of a force on a surface divided by the area of the surface: $\left(P = \frac{F}{A}\right)$	Example Problem (2) Check your Progress Q.8 Example Problem (1) Practice Problem (3)	175 172 234 235

3. A lead brick, 5.0 cm × 10.0 cm × 20.0 cm, rests on the ground on its smallest face. Lead has a density of 11.8 g/cm^3. What pressure does the brick exert on the ground?

$$m_{\text{brick}} = \rho V = \rho lwh$$
$$= (11.8\ \text{g/cm}^3)(5.0\ \text{cm})(10.0\ \text{cm})(20.0\ \text{cm})$$
$$= 1.18\times10^4\ \text{g} = 11.8\ \text{kg}$$
$$P = \frac{F_{\text{g, brick}}}{A} = \frac{m_{\text{brick}}g}{lw}$$
$$= \frac{\rho Vg}{lw} = \frac{\rho lwhg}{lw} = \rho hg$$
$$= (11.8\ \text{g/cm}^3)(20.0\ \text{cm})(9.80\ \text{m/s}^2)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\frac{(100\ \text{cm})^2}{(1\ \text{m})^2}\right)$$
$$= 23\ \text{kPa}$$