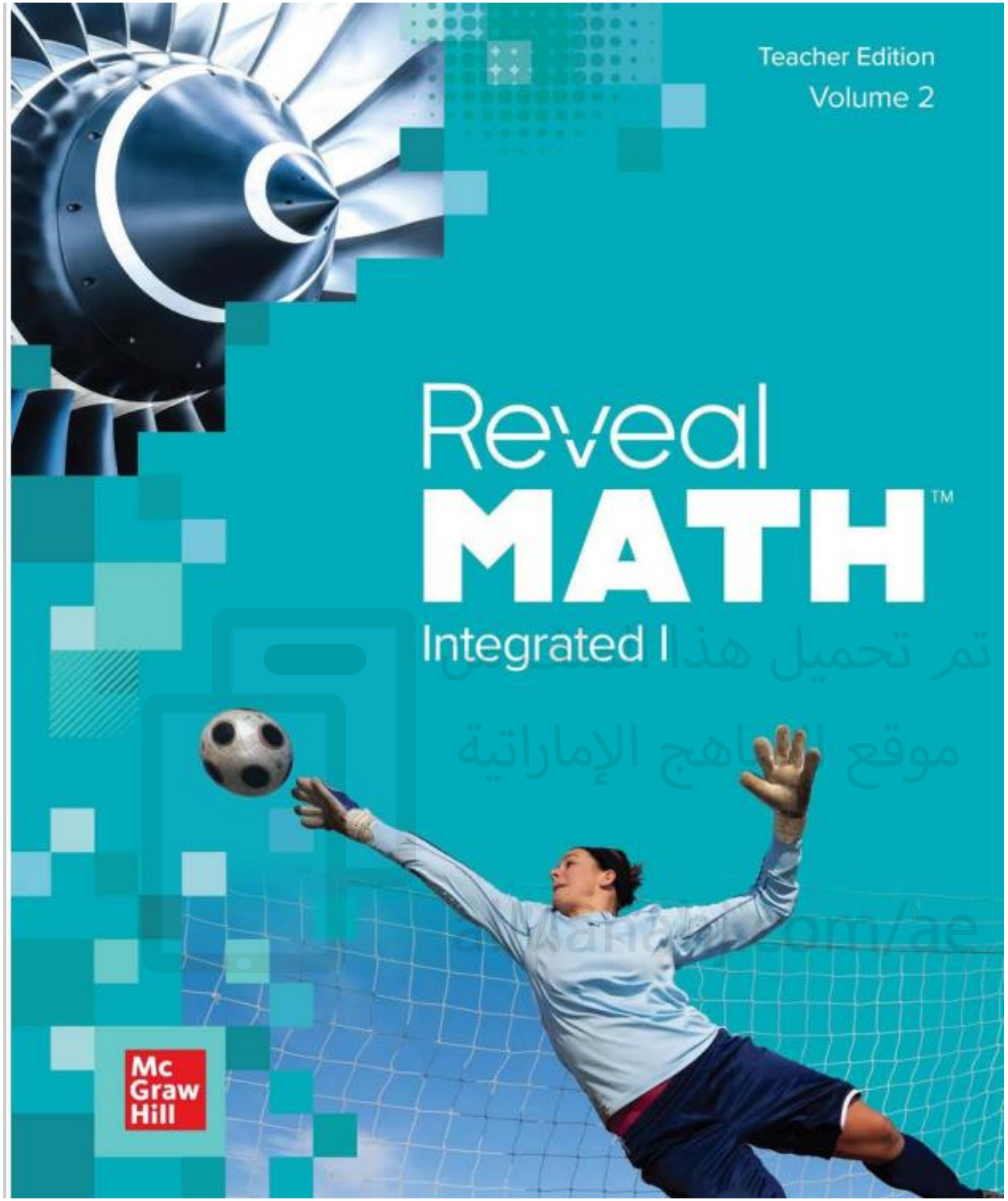
مراجعة أسئلة وفق الهيكل الوزاري ريفيل



EOT Coverage – 9General

Semester 3

Saif abdulaziz alsabhi

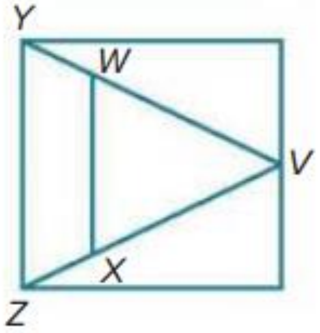
Part	Question** السؤال**	Learning Outcome*** ناتج التعلم***	Reference(s) in the Student Book المرجع في كتاب الطالب: Example/Exercise مثال/تمرين	Page الصفحة
Part 1	1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744
Part 1	2	Prove theorems about angles by using the Angle Addition Postulate.	1 to 5	753
Part 1	3	Identify special angle pairs, parallel and skew lines, and transversals.	1 to 7	761
Part 1	4	Apply angle relationship theorems to identify parallel lines and find missing values.	1 to 6	779
Part 1	5	Use symmetry to describe the reflections that carry a figure onto itself.	1 to 6	829
Part 1	6	Use rotational symmetry to describe the rotations that carry a figure onto itself.	11 to 14	830
Part 1	7	Prove the Triangle Angle-Sum Theorem and apply the theorem to solve problems	1 to 4	843
Part 1	8	Use the SAS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	11 to 15	860
Part 1	9	Use the right triangle congruence theorems to prove relationships in geometric figures	4 to 9	873
Part 1	10	Use the AAS congruence criterion for triangles to prove relationships in geometric figures	12 to 15	869
Part 2	11	Prove theorems about angles by using properties and theorems of angle congruence.	11 to 18	754
Part 2	12	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines.	1 to 9	771
Part 2	13	Classify lines as parallel, perpendicular, or neither by comparing the equations of the lines	10 to 15	771
Part 2	14	Find the distance between parallel lines by using perpendicular distance.	9 to 17	790
Part 2	15	Determine the translation vector	1 to 13	805, 806
Part 2	16	Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.	6 to 13	851 , 852
Part 2	17	Use the SSS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	1 to 6	859
Part 2	18	Use the ASA congruence criterion for triangles to solve problems and prove relationships in geometric figures.	18 to 21	870
Part 2	19	Solve problems involving isosceles triangles	5 to 15	879, 880
Part 2	20	Solve problems involving equilateral triangles	5 to 15	879, 880
Part 3	21	Find values by applying theorems about parallel lines and transversals	29 to 37	762
Part 3	22	Use rigid motions to reflect figures on the coordinate plane and describe the effects of the reflections	1 to 6	801
Part 3	23	Prove the Exterior Angle Theorem and apply the theorem to solve problems	5 to 8	843
Part 3	24	A learning outcome from the SoW	Undisclosed	Undisclosed
Part 3	25	A learning outcome from the SoW	Undisclosed	Undisclosed

1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

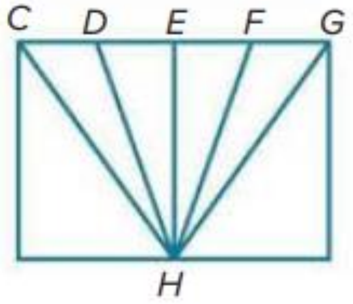
Example 2

PROOF Write a two-column proof to prove each geometric relationship.

3. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



4. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.



1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

5. FAMILY Maria is 11 inches shorter than her sister Clara. Luna is 11 inches shorter than her brother Chad. If Maria is shorter than Luna, how do the heights of Clara and Chad compare? What else can be concluded if Maria and Luna are the same height?

1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

6. LUMBER Byron works in a lumberyard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know that plank 7 and plank 10 are the same length even though they were never directly compared to each other.

1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

7. NEIGHBORHOODS Karla, Lola, and Mandy live in three houses that are on the same line. Lola lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for Lola's house to be a mile from both Karla's and Mandy's houses?

1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

8. PROOF Five lights, A, B, C, D, and E, are aligned in a row. The middle light is the midpoint of the segment between the second and fourth lights and also the midpoint of the segment between the first and last lights.

a. Draw a figure to illustrate the situation.

b. Complete this proof.

Given: C is the midpoint of $\overline{BD}$ and $\overline{AE}$.

Prove: $AB = DE$

Statement	Reason
1. C is the midpoint of $\overline{BD}$ and $\overline{AE}$.	**1.** Given
2. $BC = CD$ and ________	**2.** ________
3. $AC = AB + BC$, $CE = CD + DE$	**3.** ________
4. $AC - BC = AB$	**4.** ________
5. ________	**5.** Substitution Property
6. $CE - CD = DE$	**6.** ________
7. $AB = CE - CD$	**7.** Symmetric Property of Equality
8. ________	**8.** ________

1	Prove theorems about line segments by using properties of segment congruence	3 to 8	744

Example 3

Solution

5. FAMIL Y Maria is 11 inches shorter than her sister Clara. Luna is 11 inches shorter than her brother Chad. If Maria is shorter than Luna, how do the heights of Clara and Chad compare? What else can be concluded if Maria and Luna are the same height? Clara is shorter than Chad when Maria is shorter than Luna; Clara and Chad are the same height when Maria is the same height as Luna.

6. LUMBER Byron works in a lumberyard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know that plank 7 and plank 10 are the same length even though they were never directly compared to each other. Plank 7 is the same length as plank 1, and plank 1 is the same length as plank 10. By the Transitive Property, plank 7 must be the same length as plank 10.

7. NEIGHBORHOODS Karla, Lola, and Mandy live in three houses that are on the same line. Lola lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for Lola's house to be a mile from both Karla's and Mandy's houses? No, it's not possible. Lola's house must be less than a mile from each house because she lives between them.

Mixed Exercises

8. PROOF Five lights, A, B, C, D, and E, are aligned in a row. The middle light is the midpoint of the segment between the second and fourth lights and also the midpoint of the segment between the first and last lights.

a. Draw a figure to illustrate the situation. See margin.

b. Complete this proof.

Given: C is the midpoint of $\overline{BD}$ and $\overline{AE}$.

Prove: $AB = DE$

Statement	Reason
1. C is the midpoint of $\overline{BD}$ and $\overline{AE}$.	1. Given
2. $BC = CD$ and ? $AC = CE$	2. ? Def. of mdpt.
3. $AC = AB + BC$, $CE = CD + DE$	3. ? Seg. Add. Post.
4. $AC - BC = AB$	4. ? Subtr. Prop.
5. ? $CE - CD = AB$	5. Substitution Property
6. $CE - CD = DE$	6. ? Subtr. Prop.
7. $AB = CE - CD$	7. Symmetric Property of Equality
8. ? $AB = DE$	8. ? Trans. Prop.

solution method

Example 2 Prove Segment Congruence

Write the correct statement and reasons to complete the two-column proof.

Given: R is the midpoint of $\overline{QS}$.
T is the midpoint of $\overline{VS}$.
$\overline{QR} \cong \overline{VT}$

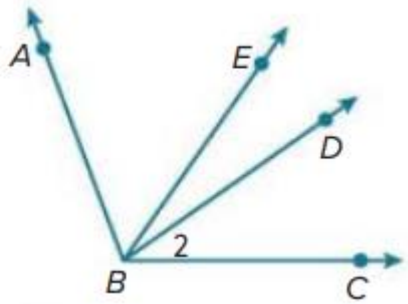
Prove: $\overline{RS} \cong \overline{TS}$

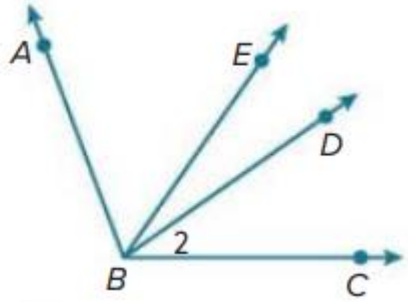
Proof:

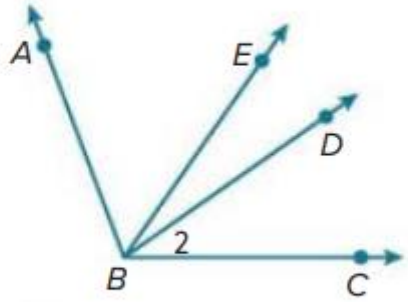
Statements	Reasons
1. R is the midpoint of $\overline{QS}$. T is the midpoint of $\overline{VS}$.	1. Given
2. $\overline{QR} \cong \overline{RS}$; $\overline{VT} \cong \overline{TS}$	2. Midpoint Theorem
3. $\overline{QR} \cong \overline{VT}$	3. Given
4. $\overline{QR} \cong \overline{TS}$	4. Transitive Property of Congruence
5. $\overline{RS} \cong \overline{QR}$	5. Symmetric Property of Congruence
6. $\overline{RS} \cong \overline{TS}$	6. Transitive Property of Congruence

2	Prove theorems about angles by using the Angle Addition Postulate.	1 to 5	753

Find the measure of each angle.







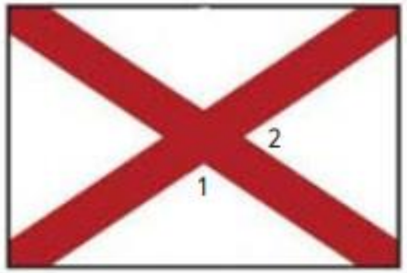
1. Find $m\angle ABC$ if $m\angle ABD = 70^\circ$ and $m\angle DBC = 43^\circ$.

2. If $m\angle EBC = 55^\circ$ and $m\angle EBD = 20^\circ$, find $m\angle 2$.

3. Find $m\angle ABD$ if $m\angle ABC = 110^\circ$ and $m\angle 2 = 36^\circ$.

2	Prove theorems about angles by using the Angle Addition Postulate.	1 to 5	753

4. FLAGS The Alabama state flag is white and has two diagonal red stripes. If the $m\angle 1 = 112°$, what is $m\angle 2$?



5. CONSTRUCTION Aaron has installed a new window above the entrance of an office building. If $m\angle 2 = 44°$, what is $m\angle 1$?

2	Prove theorems about angles by using the Angle Addition Postulate.	1 to 5	753

solution method

Solution

Learn Angle Addition

A protractor is used to measure angles. The Protractor Postulate illustrates the relationship between angle measures and real numbers. You will use these theorems and postulates to find angle measures.

Postulate 12.10: Protractor Postulate
The measure of any angle has a measure that is between 0 and 180.

Postulate 12.11: Angle Addition Postulate
D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.

Theorem 12.3: Supplement Theorem
If two angles form a linear pair, then they are supplementary angles.

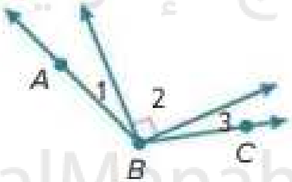
Theorem 12.4: Complement Theorem
If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

You will prove Theorems 12.3 and 12.4 in Exercises 19–20.

Example 1 Angle Addition Postulate

What is $m\angle 3$ if $m\angle 1 = 23°$ and $m\angle ABC = 131°$?

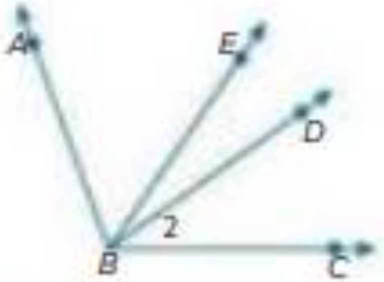
Choose from the reasons provided to justify each step.



$m\angle 1 + m\angle 2 + m\angle 3 = m\angle ABC$	Angle Addition Postulate
$23° + 90° + m\angle 3 = 131°$	Substitution Property
$113° + m\angle 3 = 131°$	Substitution Property
$113° + m\angle 3 - 113° = 131° - 113°$	Subtraction Property
$m\angle 3 = 18°$	Substitution Property

Example 1

Find the measure of each angle.



1. Find $m\angle ABC$ if $m\angle ABD = 70°$ and $m\angle DBC = 43°$. $113°$
2. If $m\angle EBC = 55°$ and $m\angle EBD = 20°$, find $m\angle 2$. $35°$
3. Find $m\angle ABD$ if $m\angle ABC = 110°$ and $m\angle 2 = 36°$. $74°$

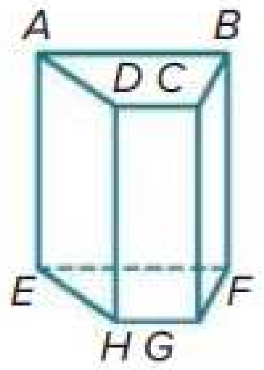
Example 2

4. **FLAGS** The Alabama state flag is white and has two diagonal red stripes. If the $m\angle 1 = 112°$, what is $m\angle 2$? $68°$

5. **CONSTRUCTION** Aaron has installed a new window above the entrance of an office building. If $m\angle 2 = 44°$, what is $m\angle 1$? $46°$

3	Identify special angle pairs, parallel and skew lines, and transversals.	1 to 7	761

Identify each of the following using the figure shown. Assume lines and planes that appear to be parallel or perpendicular are parallel or perpendicular, respectively.



1. three segments parallel to $\overline{AE}$

2. a segment skew to $\overline{AB}$

3. a pair of parallel planes

4. a segment parallel to $\overline{AD}$

5. three segments parallel to $\overline{HG}$

6. five segments skew to $\overline{BC}$

7. How could you characterize the relationship between faces *ABCD* and *DCGH*? Explain.

3	Identify special angle pairs, parallel and skew lines, and transversals.	1 to 7	761

solution method

Solution

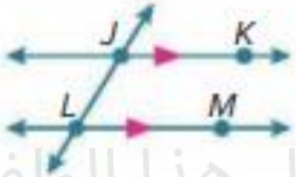
Learn Parallel Lines and Transversals

If two lines do not intersect, then they are either parallel or skew.

Parallel and Skew

Parallel Lines

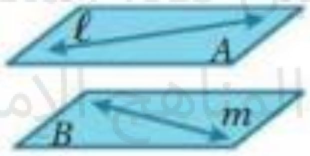
Parallel lines are coplanar lines that do not intersect.



Example $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$

Skew Lines

Skew lines are lines that do not intersect and are not coplanar.



Example Lines ℓ and m are skew.

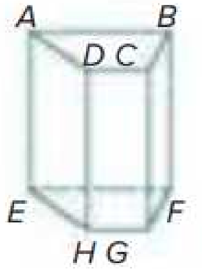
Parallel Planes

Parallel planes are planes that do not intersect.

Example Planes A and B are parallel.

If segments or rays are contained within lines that are parallel or skew, then the segments or rays are parallel or skew.

Identify each of the following using the figure shown. Assume lines and planes that appear to be parallel or perpendicular are parallel or perpendicular, respectively.



1. three segments parallel to $\overline{AE}$
$\overline{BF}$, $\overline{CG}$, and $\overline{DH}$

2. a segment skew to $\overline{AB}$
Sample answer: $\overline{EH}$

3. a pair of parallel planes
ABCD and *EFGH* or *ABFE* and *CDHG*

4. a segment parallel to $\overline{AD}$
$\overline{EH}$

5. three segments parallel to $\overline{HG}$
$\overline{EF}$, $\overline{AB}$,and $\overline{DC}$

6. five segments skew to $\overline{BC}$
$\overline{AE}$, $\overline{EH}$, $\overline{HG}$,$\overline{EF}$, and $\overline{DH}$

7. How could you characterize the relationship between faces *ABCD* and *DCGH*? Explain. Sample answer: *ABCD* and *DCGH* could be characterized as perpendiculars, because *DCGH* contains segment $\overline{CG}$, which is perpendicular to *ABCD*.

4	Apply angle relationship theorems to identify parallel lines and find missing values.	1 to 6	779

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. $\angle 3 \cong \angle 7$

____

____

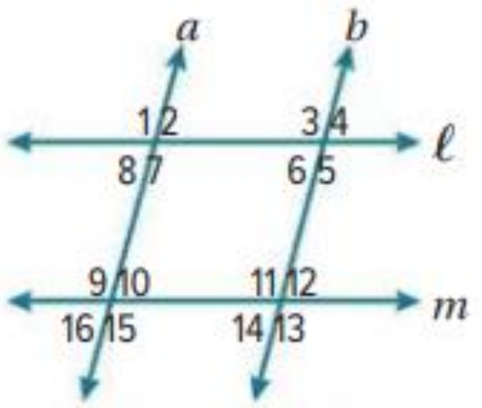
____

2. $\angle 9 \cong \angle 11$

____

____

____



3. $\angle 2 \cong \angle 16$

____

____

____

4. $m\angle 5 + m\angle 12 = 180°$

____

____

____

Given the following information, determine which lines, if any, are parallel. State the theorem that justifies your answer.

5. $\angle 1 \cong \angle 6$

____

____

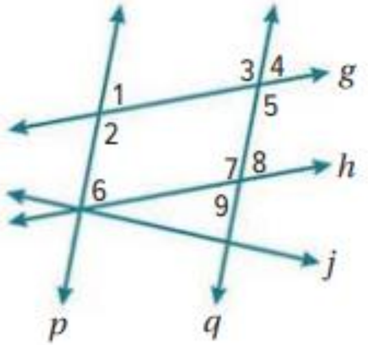
____

6. $m\angle 7 + m\angle 6 = 180°$

____

____

____



4	Apply angle relationship theorems to identify parallel lines and find missing values.	1 to 6	779

solution method

Learn Identifying Parallel Lines

Corresponding angles are congruent when the lines cut by the transversal are parallel. The converse of this relationship is also true.

Theorem 12.19: Converse of Corresponding Angles Theorem

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Postulate 12.13: Parallel Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Parallel lines that are cut by a transversal create several pairs of congruent angles. These special angle pairs can be used to prove that a pair of lines is parallel.

Theorem 12.20: Alternate Exterior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the lines are parallel.	If $\angle 1 \cong \angle 5$, then $a \parallel b$.

Theorem 12.21: Consecutive Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.	If $m\angle 7 + m\angle 6 = 180°$, then $a \parallel b$.

Theorem 12.22: Alternate Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.	If $\angle 7 \cong \angle 3$, then $a \parallel b$.

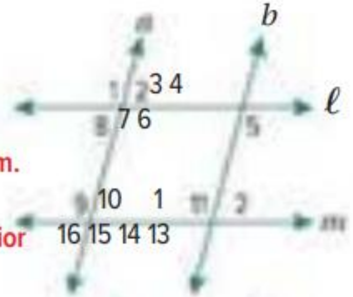
Theorem 12.23: Perpendicular Transversal Converse

If two lines in a plane are perpendicular to the same line, then the lines are parallel.

Solution

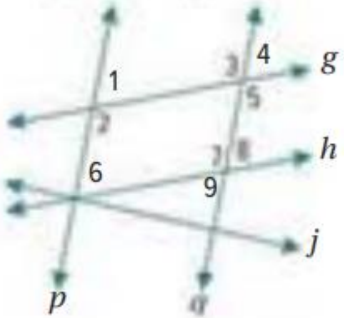
Example 1

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



1. $\angle 3 \cong \angle 7$
   $a \parallel b$; Alternate Interior Angles Converse
2. $\angle 9 \cong \angle 11$
   $a \parallel b$ Converse of Corresponding Angles Thm.
3. $\angle 2 \cong \angle 16$
   $\ell \parallel m$; Alternate Exterior Angles Converse
4. $m\angle 11 + m\angle 12 = 180°$
   $\ell \parallel m$; Consecutive Interior Angles Converse

Given the following information, determine which lines, if any, are parallel. State the theorem that justifies your answer.



5. $\angle 1 \cong \angle 6$
   $g \parallel h$; Converse of Corresponding Angles Thm.
6. $m\angle 7 + m\angle 6 = 180°$
   $p \parallel q$; Consecutive Interior Angles Converse

Example 2

5	Use symmetry to describe the reflections that carry a figure onto itself.	1 to 6	829

Determine whether each figure has a line of symmetry. If so, draw the lines of symmetry and state how many lines of symmetry it has.

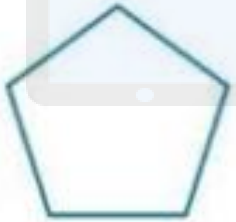
1.

2.

3.

4.

5.



6.

5	Use symmetry to describe the reflections that carry a figure onto itself.	1 to 6	829

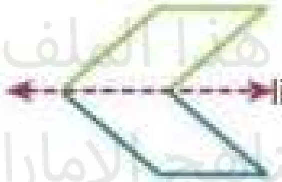
solution method

Solution

Learn Line Symmetry

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself. Figures that have symmetry are self-congruent. One type of symmetry is *line symmetry*.

A figure in the plane has **line symmetry** (or *reflectional symmetry*) if each half of the figure matches the other side exactly. When a figure has line symmetry, the figure can be mapped onto itself by a reflection in a line, called the **line of symmetry** (or *axis of symmetry*).



Determine whether each figure has a line of symmetry. If so, draw the lines of symmetry and state how many lines of symmetry it has. 1–2, 5–6. See margin for

1. yes; 3

2. yes; 2

3. no

4. no

5. yes; 5

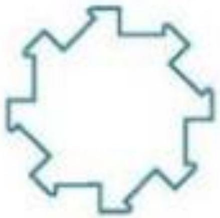
6. yes; 8

6	Use rotational symmetry to describe the rotations that carry a figure onto itself.	11 to 14	830

Determine whether each figure has rotational symmetry. If so, locate the center of symmetry, and state the order and magnitude of symmetry.

11.

12.



13.

14.

6	Use rotational symmetry to describe the rotations that carry a figure onto itself.	11 to 14	830

solution method

Learn Rotational Symmetry

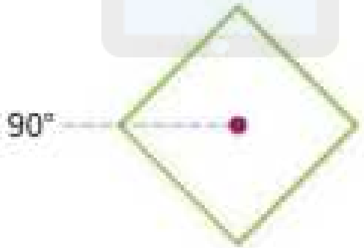
A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by being rotated less than 360° about the center of the figure so the image and the preimage are indistinguishable. The point in which a figure can be rotated onto itself is called the **center of symmetry** (or *point of symmetry*).

This figure has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.

The number of times that a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry.** The **magnitude of symmetry** (or *angle of rotation*) is the smallest angle through which a figure can be rotated so it maps onto itself. The order and magnitude of a rotation are related by the following equation.

$$\text{magnitude} = 360° \div \text{order}$$

This figure has order 4 and magnitude 90°.



Solution

Determine whether each figure has rotational symmetry. If so, locate the center of symmetry, and state the order and magnitude of symmetry.

11. yes; 3; 120°

12. yes; 8; 45°

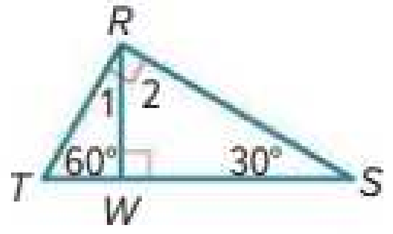
13. yes; 2; 180°

14. no

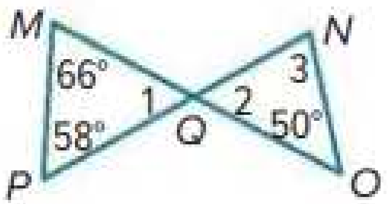
7	Prove the Triangle Angle-Sum Theorem and apply the theorem to solve problems	1 to 4	843

Find the measure of each numbered angle.

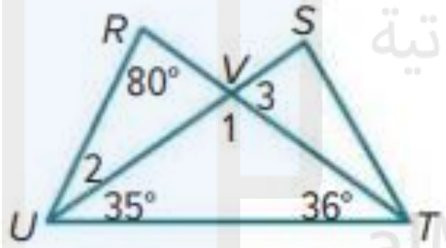
1.



2.



3.



4.

7	Prove the Triangle Angle-Sum Theorem and apply the theorem to solve problems	1 to 4	843

solution method

Learn Interior Angles of Triangles

An **interior angle of a triangle** is the angle at a vertex of a triangle. Because a triangle has three vertices, it also has three interior angles. The Triangle Angle-Sum Theorem describes the relationships among the interior angle measures of any triangle.

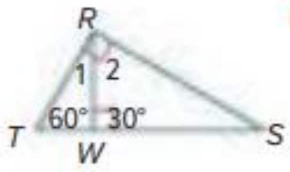
Theorem 14.1: Triangle Angle-Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Solution

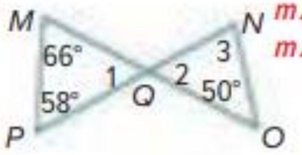
Find the measure of each numbered angle.

1.



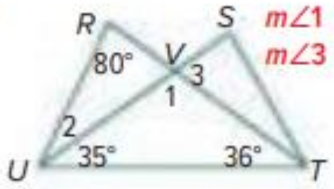
$m\angle 1 = 30°, m\angle 2 = 60°$

2.



$m\angle 1 = 56°, m\angle 2 = 56$
$m\angle 3 = 74°$

3.



$m\angle 1 = 109°, m\angle 2 = 29°,$
$m\angle 3 = 71°$

4.

$m\angle 1 = m\angle 2 = 17°$

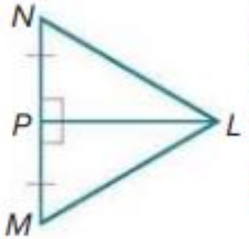
8	Use the SAS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	11 to 15	860

PROOF **Write the specified type of proof.**

11. two-column proof

Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$

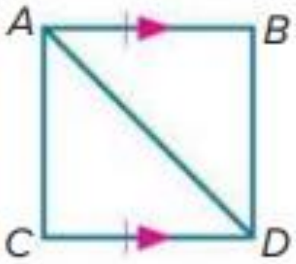
Prove: $\triangle NPL \cong \triangle MPL$



12. two-column proof

Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$

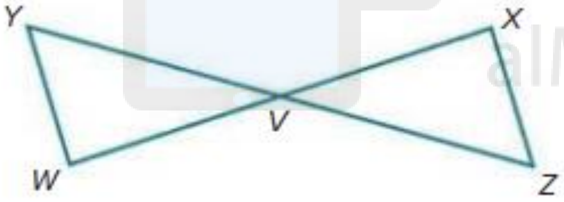
Prove: $\triangle ACD \cong \triangle DBA$



13. paragraph proof

Given: V is the midpoint of $\overline{WX}$ and $\overline{YZ}$.

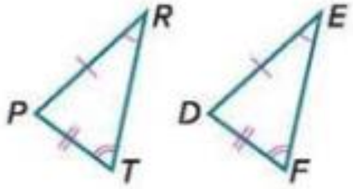
Prove: $\triangle XVZ \cong \triangle WVY$



14. flow proof

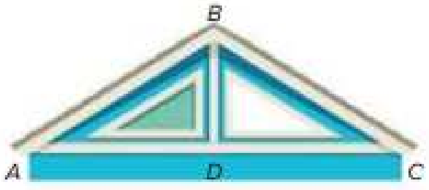
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$, $\angle R \cong \angle E$, $\angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$



8	Use the SAS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	11 to 15	860

15. GAMING Devontae is building a house in a simulation video game. He wants the roof of the house and the main support beam to create congruent triangles. If $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ bisects $\overline{AC}$,write a two-column proof to prove $\triangle ABD \cong \triangle CBD$.



8	Use the SAS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	11 to 15	860

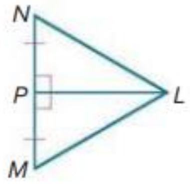
Example 3

PROOF **Write the specified type of proof.**

11. two-column proof

Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$

Prove: $\triangle NPL \cong \triangle MPL$



Proof:

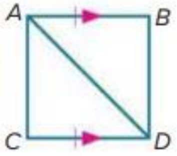
Statements (Reasons)

1. $NP = PM$, $\overline{NP} \perp \overline{PL}$ (Given)
2. $\overline{NP} \cong \overline{MP}$ (Def. of congruence)
3. $\angle MPL$ and $\angle NPL$ are rt. angles. ($\perp$ lines form rt. angles.)
4. $\angle MPL \cong \angle NPL$ (All right angles are congruent.)
5. $\overline{PL} \cong \overline{PL}$ (Reflexive Property of $\cong$)
6. $\triangle NPL \cong \triangle MPL$ (SAS)

12. two-column proof

Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle DBA$



Proof:

Statements (Reasons)

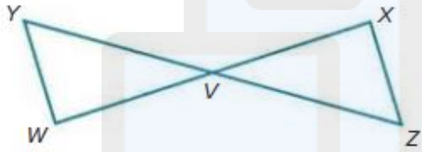
1. $AB = CD$, $\overline{AB} \parallel \overline{CD}$ (Given)
2. $\angle BAD \cong \angle CDA$ (Alternate Interior Angles Thm.)
3. $\overline{AD} \cong \overline{AD}$ (Reflexive Property)
4. $\overline{AB} \cong \overline{CD}$ (Def. of congruent segments)
5. $\triangle ACD \cong \triangle DBA$ (SAS)

Solution

13. paragraph proof

Given: V is the midpoint of $\overline{WX}$ and $\overline{YZ}$.

Prove: $\triangle XVZ \cong \triangle WVY$

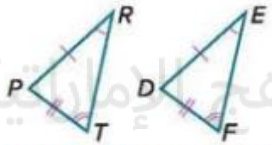


See Mod. 14 Answer Appendix.

14. flow proof

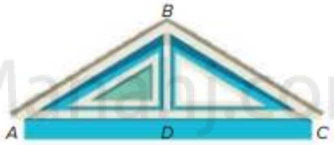
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$, $\angle R \cong \angle E$, $\angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$



See Mod. 14 Answer Appendix.

15. GAMING Devontae is building a house in a simulation video game. He wants the roof of the house and the main support beam to create congruent triangles. If $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ bisects $\overline{AC}$,write a two-column proof to prove $\triangle ABD \cong \triangle CBD$.



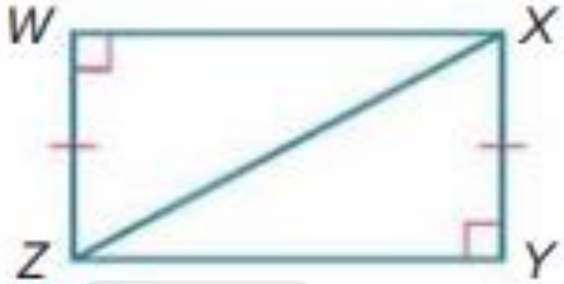
Proof:

Statements (Reasons)

1. $\overline{BD} \perp \overline{AC}$; $\overline{BD}$ bisects $\overline{AC}$.(Given)
2. $\angle BDA$ and $\angle BDC$ are rt. angles. ($\perp$ lines form rt. angles.)
3. $\angle BDA \cong \angle BDC$ (All right angles are congruent.)
4. $\overline{AD} \cong \overline{DC}$ (Def. of segment bisector)
5. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of $\cong$)
6. $\triangle ABD \cong \triangle CBD$ (SAS)

9	Use the right triangle congruence theorems to prove relationships in geometric figures	4 to 9	873

Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.

4.


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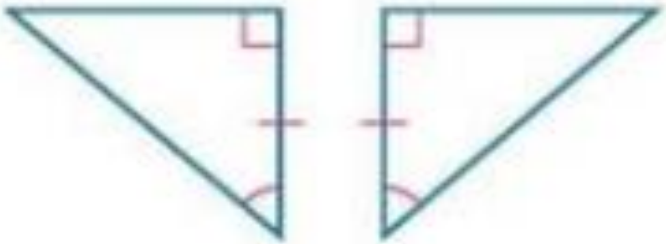

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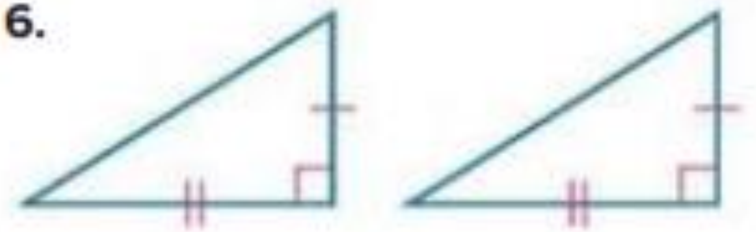

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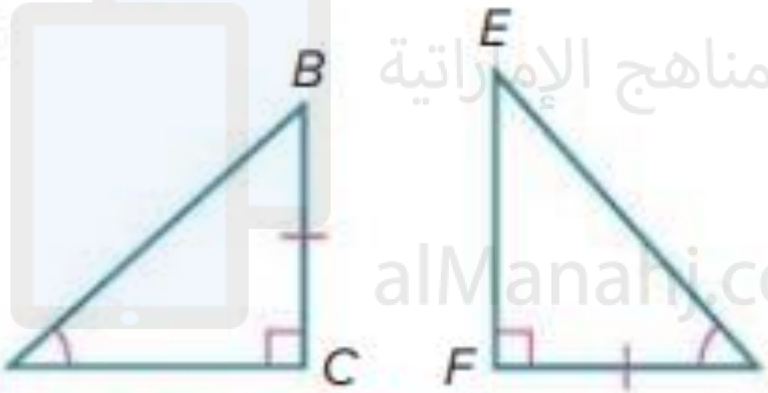
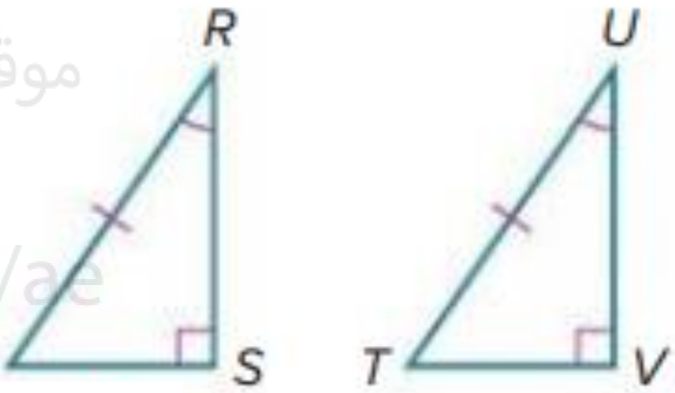
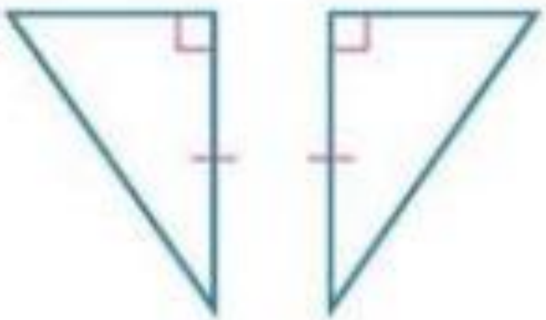


8.


9.


9	Use the right triangle congruence theorems to prove relationships in geometric figures	4 to 9	873

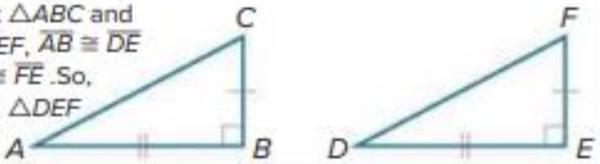
solution method

Learn Right Triangle Congruence

Theorem 14.6: Leg-Leg (LL) Congruence

Words If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

Example Given right $\triangle ABC$ and right $\triangle DEF$, $\overline{AB} \cong \overline{DE}$ and $\overline{CB} \cong \overline{FE}$. So, $\triangle ABC \cong \triangle DEF$ by LL.



Theorem 14.7: Hypotenuse-Angle (HA) Congruence

Words If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and the corresponding acute angle of another right triangle, then the triangles are congruent.

Example Given right $\triangle ABC$ and right $\triangle DEF$, $\overline{AC} \cong \overline{DF}$ and $\angle C \cong \angle F$. So, $\triangle ABC \cong \triangle DEF$ by HA.

Theorem 14.8: Leg-Angle (LA) Congruence

If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

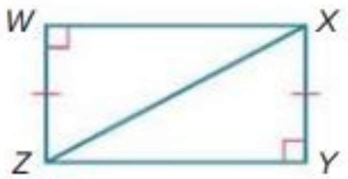
Theorem 14.9: Hypotenuse-Leg (HL) Congruence

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent.

Solution

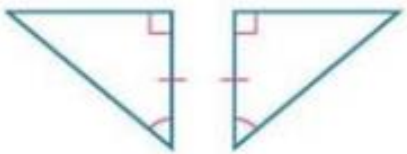
Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.

4.



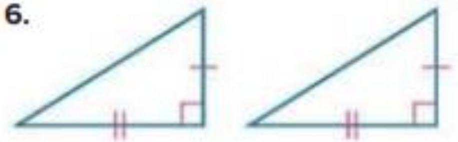
Yes; HL

5.



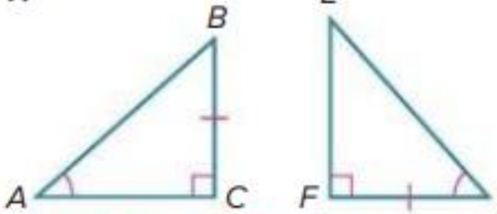
Yes; LA

6.



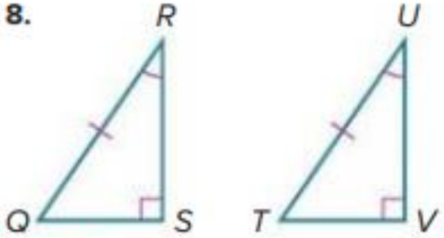
Yes; LL

7.



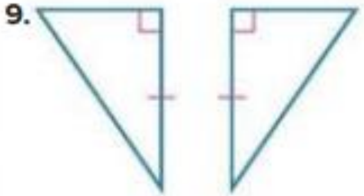
No; not enough information

8.



Yes; HA

9.



No; not enough information

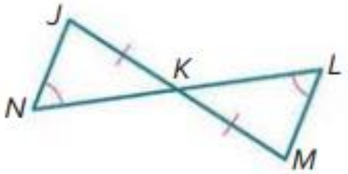
10	Use the AAS congruence criterion for triangles to prove relationships in geometric figures	12 to 15	869

Write the specified type of proof.

12. flow proof

Given: $\overline{JK} \cong \overline{MK}$, $\angle N \cong \angle L$

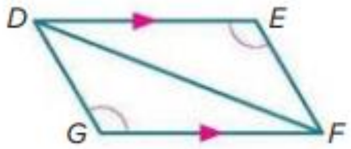
Prove: $\triangle JKN \cong \triangle MKL$



13. paragraph proof

Given: $\overline{DE} \parallel \overline{FG}$, $\angle E \cong \angle G$

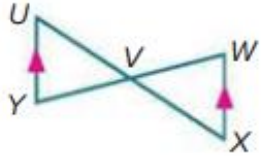
Prove: $\triangle DFG \cong \triangle FDE$



14. two-column proof

Given: V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$,

Prove: $\triangle UVY \cong \triangle XVW$

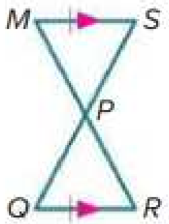


15. two-column proof

Given: $\overline{MS} \cong \overline{RQ}$,

$\overline{MS} \parallel \overline{RQ}$,

Prove: $\triangle MSP \cong \triangle RQP$

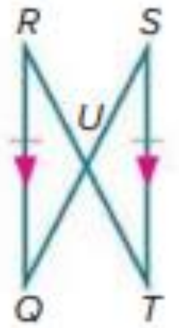


10	Use the AAS congruence criterion for triangles to prove relationships in geometric figures	12 to 15	869

solution method

Example 3 Use AAS to Prove Triangles Congruent

Choose the correct statements and reasons to complete the flow proof.



Given: $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

Prove: $\triangle RUQ \cong \triangle TUS$

$\overline{RQ} \parallel \overline{ST}$ — Given → $\angle RQS \cong \angle TSQ$ — Alternate Interior Angles Theorem

$\overline{RQ} \cong \overline{ST}$ — Given

$\angle RUQ \cong \angle TUS$ — Vertical Angles Theorem

→ $\triangle RUQ \cong \triangle TUS$ — AAS

Solution

Example 3

Write the specified type of proof.

12. flow proof

Given: $\overline{JK} \cong \overline{MK}$, $\angle N \cong \angle L$

Prove: $\triangle JKN \cong \triangle MKL$

Proof:

$\angle N \cong \angle L$ — Given

$\overline{JK} \cong \overline{MK}$ — Given

$\angle JKN \cong \angle MKL$ — Vertical $\angle$s are $\cong$.

→ $\triangle JKN \cong \triangle MKL$ — AAS

13. paragraph proof

Given: $\overline{DE} \parallel \overline{FG}$, $\angle E \cong \angle G$

Prove: $\triangle DFG \cong \triangle FDE$

Proof:

It is given that $\angle E \cong \angle G$ and $\overline{DE} \parallel \overline{FG}$. By the Alternate Interior Angles Theorem, $\angle DFG \cong \angle FDE$. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence. Therefore, $\triangle DFG \cong \triangle FDE$ by AAS.

14. two-column proof

Given: V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.

Prove: $\triangle UVY \cong \triangle XVW$

Proof:

Statements (Reasons)

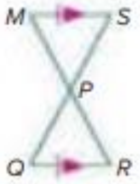
1. V is the midpoint of $\overline{YW}$, and $\overline{UY} \parallel \overline{XW}$. (Given)
2. $\overline{YV} \cong \overline{VW}$ (Midpoint Theorem)
3. $\angle VWX \cong \angle VYU$ (Alternate Interior Angles Thm.)
4. $\angle VUY \cong \angle VXW$ (Alternate Interior Angles Thm.)
5. $\triangle UVY \cong \triangle XVW$ (AAS)

15. two-column proof

Given: $\overline{MS} \cong \overline{RQ}$,

$\overline{MS} \parallel \overline{RQ}$.

Prove: $\triangle MSP \cong \triangle RQP$



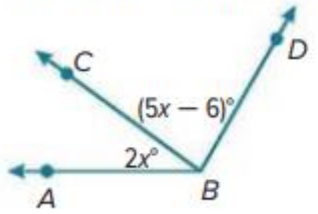
Proof:

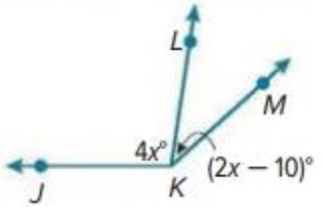
Statements (Reasons)

1. $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$ (Given)
2. $\angle SPM \cong \angle QPR$ (Vertical Angles Thm.)
3. $\angle SMP \cong \angle QRP$ (Alternate Interior Angles Thm.)
4. $\triangle MSP \cong \triangle RQP$ (AAS)

11	**Prove theorems about angles by using properties and theorems of angle congruence.**	**11 to 18**	**754**

11. Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120°$. **12.** Find $m\angle JKL$ and $m\angle LKM$ if $m\angle JKM = 140°$.

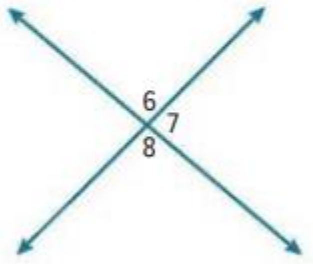




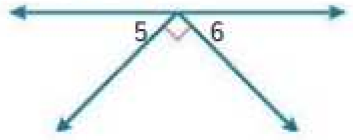
11	Prove theorems about angles by using properties and theorems of angle congruence.	11 to 18	754

Find the measure of each numbered angle and name the theorems that you used to justify your work.

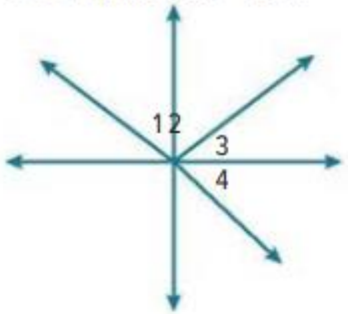
13. $m\angle 6 = (2x - 21)°$
$m\angle 7 = (3x - 34)°$



14. $m\angle 5 = m\angle 6$



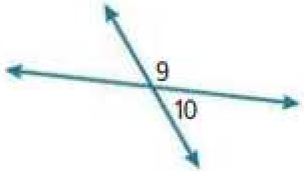
15. $\angle 2$ and $\angle 3$ are complementary.
$\angle 1 \cong \angle 4$ and $m\angle 2 = 28°$.



11	**Prove theorems about angles by using properties and theorems of angle congruence.**	**11 to 18**	**754**

16. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary.
$m\angle 4 = 105°$.

17. $m\angle 9 = (3x + 12)°$
$m\angle 10 = (x - 24)°$



18. $m\angle 3 = (2x + 23)°$
$m\angle 4 = (5x - 112)°$

11	Prove theorems about angles by using properties and theorems of angle congruence.	11 to 18	754

solution method

Solution

Learn Congruent Angles

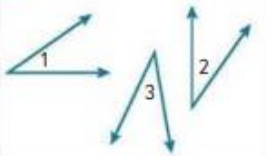
The properties of algebra that apply to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Theorem 12.5: Properties of Angle Congruence

Reflexive Property of Congruence
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence
If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.



Proof: Symmetric Property of Congruence

Given: $\angle J \cong \angle K$

Prove: $\angle K \cong \angle J$

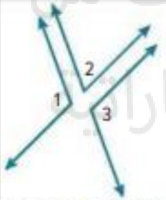
Paragraph Proof:
We are given that $\angle J \cong \angle K$. By the definition of congruent angles, $m\angle J = m\angle K$. Using the Symmetric Property of Equality, $m\angle K = m\angle J$. Thus, $\angle K \cong \angle J$ by the definition of congruent angles.

Theorems

Theorem 12.6: Congruent Supplements Theorem

Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

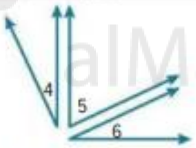


If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$.

Theorem 12.7: Congruent Complements Theorem

Angles complementary to the same angle or to congruent angles are congruent.

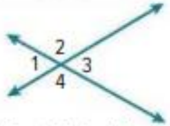
Abbreviation $\angle$s compl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.



If $m\angle 4 + m\angle 5 = 90°$ and $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.

Theorem 12.8: Vertical Angles Theorem

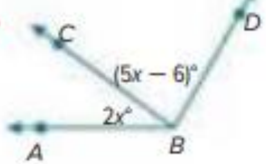
If two angles are vertical angles, then they are congruent.



$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

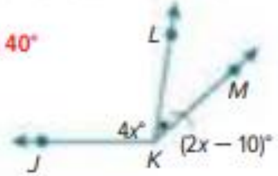
11. Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120°$.

36°; 84°



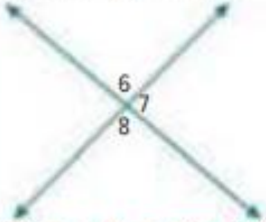
12. Find $m\angle JKL$ and $m\angle LKM$ if $m\angle JKM = 140°$.

100°; 40°



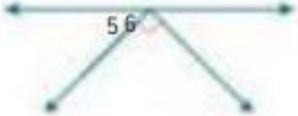
Find the measure of each numbered angle and name the theorems that you used to justify your work.

13. $m\angle 6 = (2x - 21)°$
$m\angle 7 = (3x - 34)°$



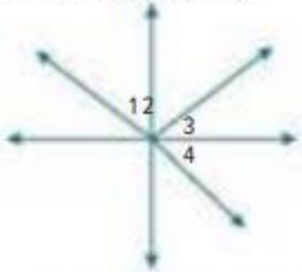
$m\angle 6 = m\angle 8 = 73°$, $m\angle 7 = 107°$
($\cong$ Supp. Thm. and Vert. $\angle$s Thm.)

14. $m\angle 5 = m\angle 6$



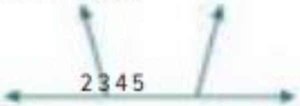
$m\angle 5 = m\angle 6 = 45°$
($\cong$ Supp. Thm.)

15. $\angle 2$ and $\angle 3$ are complementary.
$\angle 1 \cong \angle 4$ and $m\angle 2 = 28°$.



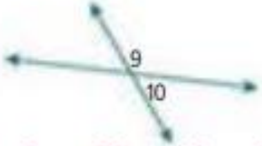
$m\angle 3 = 62°$, $m\angle 1 = m\angle 4 = 45°$
($\cong$ Comp. and Supp. Thm.)

16. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary.
$m\angle 4 = 105°$.



$m\angle 2 = 75°$, $m\angle 3 = 105°$, $m\angle 5 = 75°$ ($\cong$ Supp. Thm.)

17. $m\angle 9 = (3x + 12)°$
$m\angle 10 = (x - 24)°$



$m\angle 9 = 156°$, $m\angle 10 = 24°$
($\cong$ Supp. Thm.)

18. $m\angle 3 = (2x + 23)°$
$m\angle 4 = (5x - 112)°$

$m\angle 3 = m\angle 4 = 113°$
(Vert. $\angle$s Thm.)

12	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines.	1 to 9	771

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

1. $A(1, 5)$, $B(4, 4)$, $C(9, -10)$, $D(-6, -5)$

2. $A(-6, -9)$, $B(8, 19)$, $C(0, -4)$, $D(2, 0)$

3. $A(4, 2)$, $B(-3, 1)$, $C(6, 0)$, $D(-10, 8)$

4. $A(8, -2)$, $B(4, -1)$, $C(3, 11)$, $D(-2, -9)$

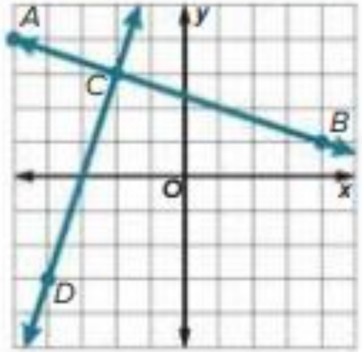
5. $A(8, 4)$, $B(4, 3)$, $C(4, -9)$, $D(2, -1)$

6. $A(4, -2)$, $B(-2, -8)$, $C(4, 6)$, $D(8, 5)$

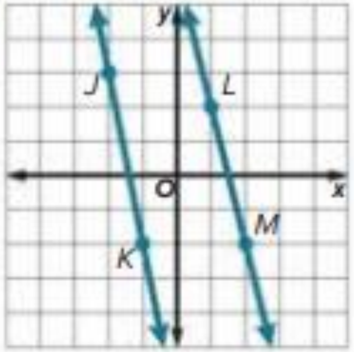
12	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines.	1 to 9	771

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

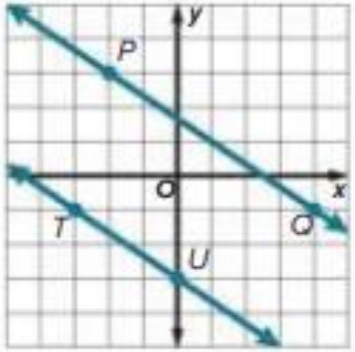
7.



8.



9.



12	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines.	1 to 9	771

solution method

Solution

Postulate 12.12: Slope Criteria for Parallel and Perpendicular Lines

Slopes of Parallel Lines
Two distinct nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

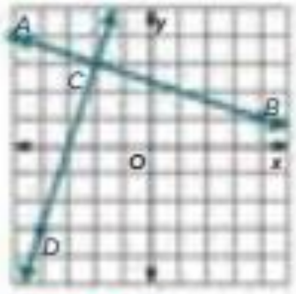
Slopes of Perpendicular Lines
Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. Vertical and horizontal lines are perpendicular.

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel, perpendicular,* or *neither*. Graph each line to verify your answer. See margin for graphs.

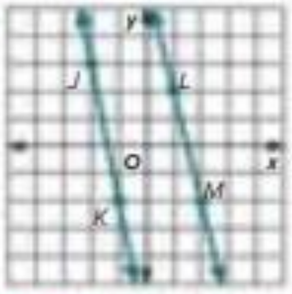
1. $A(1, 5)$, $B(4, 4)$, $C(9, -10)$, $D(-6, -5)$ parallel
2. $A(-6, -9)$, $B(8, 19)$, $C(0, -4)$, $D(2, 0)$ parallel
3. $A(4, 2)$, $B(-3, 1)$, $C(6, 0)$, $D(-10, 8)$ neither
4. $A(8, -2)$, $B(4, -1)$, $C(3, 11)$, $D(-2, -9)$ perpendicular
5. $A(8, 4)$, $B(4, 3)$, $C(4, -9)$, $D(2, -1)$ perpendicular
6. $A(4, -2)$, $B(-2, -8)$, $C(4, 6)$, $D(8, 5)$ neither

Example 2

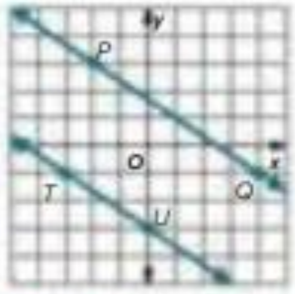
Determine whether each pair of lines is *parallel, perpendicular,* or *neither*.

7. 

perpendicular

8. 

neither

9. 

parallel

13	Classify lines as parallel, perpendicular, or neither by comparing the equations of the lines	10 to 15	771

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

10. $y = 2x + 4, y = 2x - 10$

11. $y = -\frac{1}{2}x - 12, y - 3 = 2(x + 2)$

12. $y - 4 = 3(x + 5), y + 3 = -\frac{1}{3}(x + 1)$

13. $y - 3 = 6(x + 2), y + 3 = -\frac{1}{3}(x - 4)$

14. $x = -2, y = 10$

15. $y = 5, y = -3$

13	Classify lines as parallel, perpendicular, or neither by comparing the equations of the lines	10 to 15	771

solution method

Example 3 Determine Line Relationships When Given Equations

Determine whether each pair of lines is *parallel, perpendicular,* or *neither*.

a. $y = 3x - 2$; $y - 0 = -\frac{1}{3}(x - 2)$

slope-intercept form   point-slope form

$y = 3x - 2 \quad y - 0 = -\frac{1}{3}(x - 2)$

slope

The two lines do not have the same slope, so the lines are not parallel. To determine whether the lines are perpendicular, find the product of the slopes.

$3\left(-\frac{1}{3}\right) = -1$ Product of slopes

Because the product of their slopes is -1, the two lines are perpendicular.

b. $y = 3$; $x = 1$

$y = 3$	$x = 1$
horizontal line	vertical line
slope of 0	undefined slope

Vertical and horizontal lines are always perpendicular.

c. $y - 5 = -\frac{3}{4}(x + 2)$; $y = -\frac{3}{4}x + 2$

point-slope form   slope-intercept form

$y - 5 = -\frac{3}{4}(x + 2) \quad y = -\frac{3}{4}x + 2$

slope

Because the slopes of both lines are $-\frac{3}{4}$ the lines are parallel.

d. $y = 2x + 3$; $y - 1 = \frac{1}{2}(x + 2)$

slope-intercept form   point-slope form

$y = 2x + 3 \quad y - 1 = \frac{1}{2}(x + 2)$

slope

The two lines do not have the same slope, so the lines are not parallel. To determine whether the lines are perpendicular, find the product of the slopes.

$2\left(\frac{1}{2}\right) = 1$ Product of slopes

Because the product of the slopes is not -1, the two lines are not perpendicular. So, the two lines are neither parallel nor perpendicular.

e. $x = -2$; $x = 4$

Both lines are vertical with undefined slope. Vertical lines are always parallel.

Solution

Example 3

Determine whether each pair of lines is *parallel, perpendicular,* or *neither*.

10. $y = 2x + 4$, $y = 2x - 10$

parallel

11. $y = -\frac{1}{2}x - 12$, $y - 3 = 2(x + 2)$

perpendicular

12. $y - 4 = 3(x + 5)$, $y + 3 = -\frac{1}{3}(x + 1)$

perpendicular

13. $y - 3 = 6(x + 2)$, $y + 3 = -\frac{1}{3}(x - 4)$

neither

14. $x = -2$, $y = 10$

perpendicular

15. $y = 5$, $y = -3$

parallel

14	Find the distance between parallel lines by using perpendicular distance.	9 to 17	790

Find the distance between each pair of parallel lines with the given equations.

9. $y = 7$
$y = -1$

10. $x = -6$
$x = 5$

11. $y = 3x$
$y = 3x + 10$

12. $y = -5x$
$y = -5x + 26$

13. $y = x + 9$
$y = x + 3$

14. $y = -2x + 5$
$y = -2x - 5$

14	Find the distance between parallel lines by using perpendicular distance.	9 to 17	790

15. $y = \frac{1}{4}x + 2$
$4y - x = -60$

16. $3x + y = 3$
$y + 17 = -3x$

17. $y = -\frac{5}{4}x + 3.5$
$4y + 10.6 = -5x$

14	Find the distance between parallel lines by using perpendicular distance.	9 to 17	790

solution method

Solution

Example 3 Distance Between Parallel Lines

Find the distance between the parallel lines r and t with equations $y = -3x - 5$ and $y = -3x + 6$, respectively.

You need to solve a system of equations to find the endpoints of a segment perpendicular to lines r and t. Lines r and t have slope -3.

Step 1 Write an equation of line q.

The slope of q is the opposite reciprocal of -3, or $\frac{1}{3}$. Use the y-intercept of line r, $(0, -5)$, as a point through which line q will pass.

$(y - y_1) = m(x - x_1)$	Point-slope form
$[y - (-5)] = \frac{1}{3}(x - 0)$	$x_1 = 0$, $y_1 = -5$, and $m = \frac{1}{3}$
$y = \frac{1}{3}x - 5$	Solve

Step 2 Solve the system of equations.

Determine the point of intersection of lines t and q.

t: $y = -3x + 6$	q: $y = \frac{1}{3}x - 5$
$-3x + 6 = \frac{1}{3}x - 5$	Substitute.
$6 + 5 = \frac{1}{3}x + 3x$	Group like terms.
$\frac{33}{10} = x$	Solve.

Solve for y when $x = \frac{33}{10}$.

$y = \frac{1}{3}\left(\frac{33}{10}\right) - 5$	Substitute $\frac{33}{10}$ for x in the equation for q.
$y = -\frac{39}{10}$	Simplify.

The point of intersection is $\left(\frac{33}{10}, -\frac{39}{10}\right)$ or $(3.3, -3.9)$.

Step 3 Calculate the distance between lines r and t.

Use the Distance Formula to determine the distance between $(0, -5)$ and $(3.3, -3.9)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(3.3 - 0)^2 + [-3.9 - (-5)]^2}$	$x_2 = 3.3$, $x_1 = 0$, $y_2 = -3.9$, and $y_1 = -5$
≈ 3.5	Use a calculator.

The distance between the lines is about 3.5 units.

Example 3

Find the distance between each pair of parallel lines with the given equations.

9. $y = 7$
$y = -1$
8 units

10. $x = -6$
$x = 5$
11 units

11. $y = 3x$
$y = 3x + 10$
$\sqrt{10}$ or about 3.16 units

12. $y = -5x$
$y = -5x + 26$
$\sqrt{26}$ or about 5.10 units

13. $y = x + 9$
$y = x + 3$
$3\sqrt{2}$ or about 4.24 units

14. $y = -2x + 5$
$y = -2x - 5$
$2\sqrt{5}$ or about 4.47 units

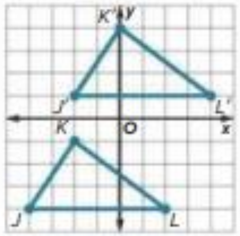
15. $y = \frac{1}{4}x + 2$
$4y - x = -60$
$4\sqrt{17}$ or about 16.49 units

16. $3x + y = 3$
$y + 17 = -3x$
$2\sqrt{10}$ or about 6.32 units

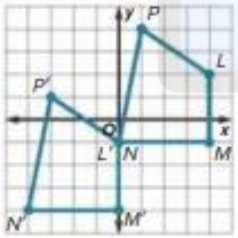
17. $y = -\frac{5}{4}x + 3.5$
$4y + 10.6 = -5x$
$\sqrt{14.76}$ or about 3.84 units

15	Determine the translation vector	1 to 13	805, 806

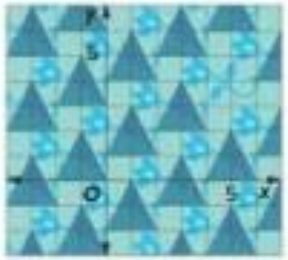
1. Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector. If not, explain why.



2. Determine whether a translation maps quadrilateral $LMNP$ onto quadrilateral $L'M'N'P'$. If so, find the translation vector. If not, explain why.



3. **WALLPAPER** A wallpaper design consists of repeated translations of a single isosceles triangle. The pattern is shown overlaid on a coordinate plane. The space above the triangle around the coordinate (5, 1) should be filled with a missing triangle. What are the coordinates of the vertices of the triangle that fill this space consistently with the rest of the pattern?

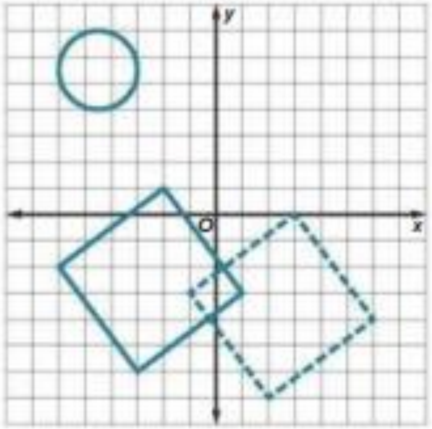


15	Determine the translation vector	1 to 13	805, 806

4. FURNITURE Alejandro plotted the location of a reclining chair and an end table on a coordinate plane. The end table is represented by the circle, and the chair is represented by the square with solid sides. The image of the chair along a translation is represented by the square with dashed sides.

a. Describe this translation of the chair.

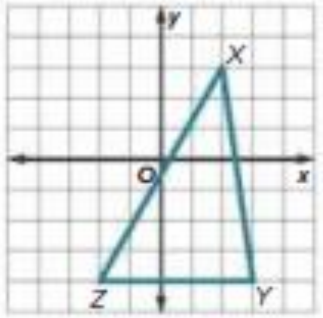
b. Draw the image of the end table under the same translation that you described in part **a**.



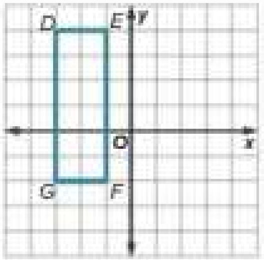
15	Determine the translation vector	1 to 13	805, 806

Copy the graph. Draw and label the image of each figure after the given translation.

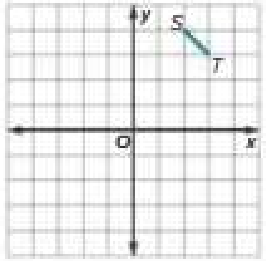
5. 3 units to the left



6. translation vector ⟨1, −2.5⟩



7. translation vector ⟨−5, −7⟩



15	Determine the translation vector	1 to 13	805, 806

Name the image of each point after the given translation vector.

8. $F(-3, 1)$; $\langle 5, -1 \rangle$

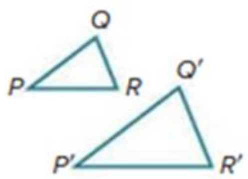
9. $Q(4, -2)$; $\langle -2, -5 \rangle$

10. $P(9, 1.5)$; $\langle 3, -0.5 \rangle$

11. The image of $A(-3, -5)$ under a translation is $A'(6, -1)$. Find the image of $B(3, -2)$ under the same translation.

12. CONSTRUCT ARGUMENTS Explain why $\triangle A'B'C'$ with vertices $A'(-1, -2)$, $B'(0, 0)$, and $C'(-6, 0)$ is not a translation image of $\triangle ABC$ with vertices $A(1, 2)$, $B(0, 0)$, and $C(6, 0)$.

13. Determine whether $\triangle P'Q'R'$ is a translation image of $\triangle PQR$. Explain.



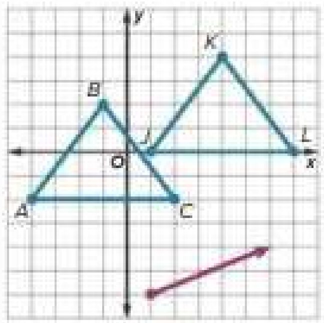
15	Determine the translation vector	1 to 13	805, 806

solution method

Learn Translations

You've learned that a translation is a function in which all of the points of a figure move the same distance in the same direction as described by a translation vector.

When a translation has been applied to a figure:



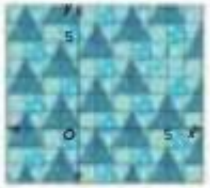
1. The distance between each pair of corresponding vertices is the same.
2. The segments that connect each pair of corresponding vertices are parallel.

Recall that a translation vector describes the magnitude and direction of the translation. The **magnitude** of a vector is its length from the initial point to the terminal point.

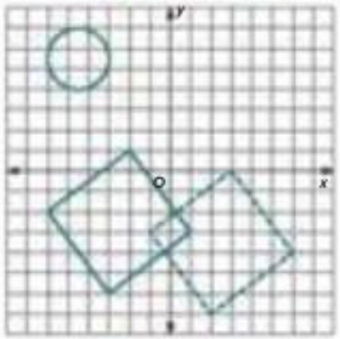
15	Determine the translation vector	1 to 13	805, 806

Solution

3. WALLPAPER A wallpaper design consists of repeated translations of a single isosceles triangle. The pattern is shown overlaid on a coordinate plane. The space above the triangle around the coordinate (5, 1) should be filled with a missing triangle. What are the coordinates of the vertices of the triangle that fill this space consistently with the rest of the pattern? (4, 3), (5, 5), (6, 3)



4. FURNITURE Alejandro plotted the location of a reclining chair and an end table on a coordinate plane. The end table is represented by the circle, and the chair is represented by the square with solid sides. The image of the chair along a translation is represented by the square with dashed sides.



a. Describe this translation of the chair.
$(x, y) \rightarrow (x + 5, y - 1)$

b. Draw the image of the end table under the same translation that you described in part a. See margin.

Name the image of each point after the given translation vector.

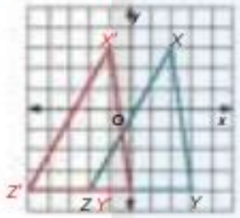
8. $F(-3, 1)$; $\langle 5, -1\rangle$ $F'(2, 0)$
9. $Q(4, -2)$; $\langle -2, -5\rangle$ $Q'(2, -7)$
10. $P(9, 1.5)$; $\langle 3, -0.5\rangle$ $P'(12, 1)$

11. The image of $A(-3, -5)$ under a translation is $A'(6, -1)$. Find the image of $B(3, -2)$ under the same translation. $B'(12, 2)$

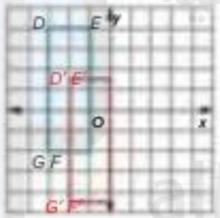
Mixed Exercises

Copy the graph. Draw and label the image of each figure after the given translation.

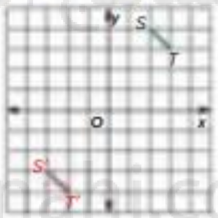
5. 3 units to the left



6. translation vector $\langle 1, -2.5\rangle$

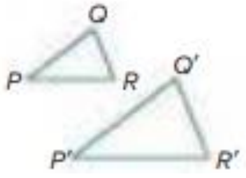


7. translation vector $\langle -5, -7\rangle$



12. CONSTRUCT ARGUMENTS Explain why $\triangle A'B'C'$ with vertices $A'(-1, -2)$, $B'(0, 0)$, and $C'(-6, 0)$ is not a translation image of $\triangle ABC$ with vertices $A(1, 2)$, $B(0, 0)$, and $C(6, 0)$.
Sample answer: All the points are not all moved the same distance or in the same direction.

13. Determine whether $\triangle P'Q'R'$ is a translation image of $\triangle PQR$. Explain.
No; sample answer: The size has been changed.



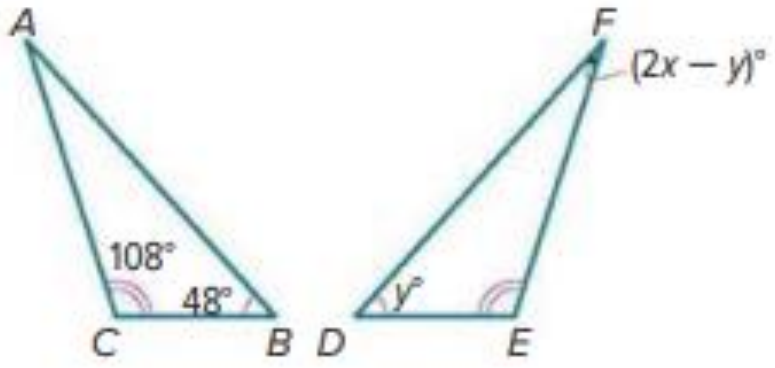
16	Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.	6 to 13	851 , 852

Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x.

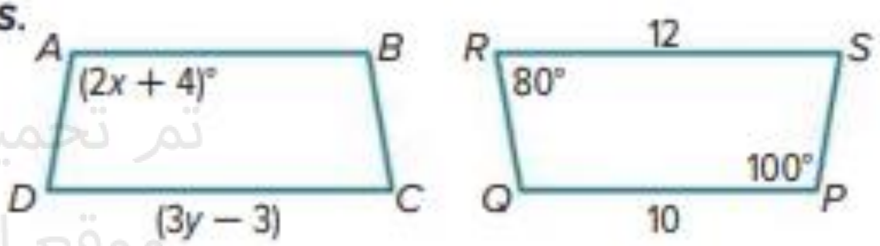
7. Find the value of y.



In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x.

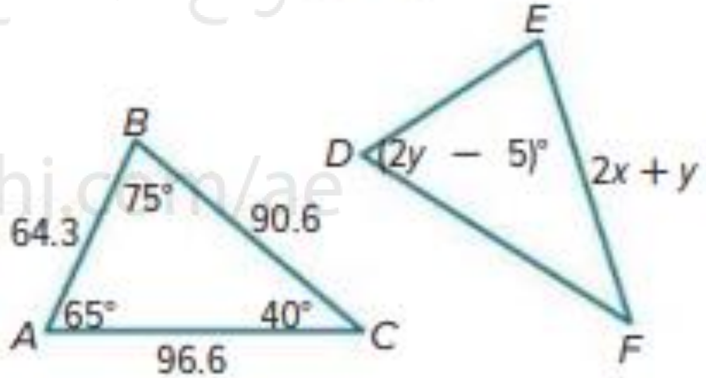
9. Find the value of y.



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x.

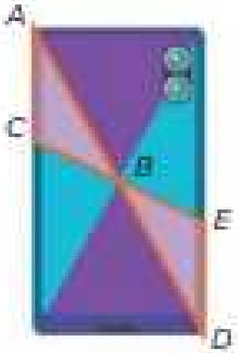
11. Find the value of y.



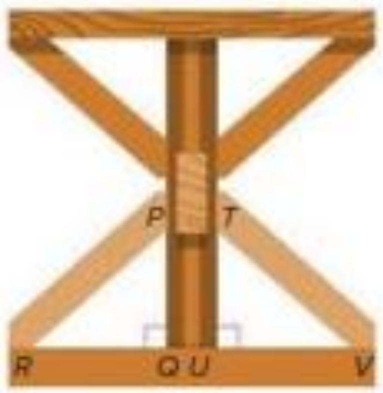
16	**Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.**	6 to 13	851 , 852

Example 3

12. DESIGN Camila is designing a new image for her cell phone case. If $m\angle ABC = 35°$, $m\angle BAC = 29°$, and $\angle ACB \cong \angle DEB$, what is $m\angle DEB$?



13. CARPENTRY Mr. Lewis is building a rustic dining table. Instead of having four legs, the table has a set of supports at each end. If $\angle PRQ \cong \angle TVU$ and $m\angle RPQ = 49°$, what is $m\angle TVU$?



16	Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.	6 to 13	851 , 852

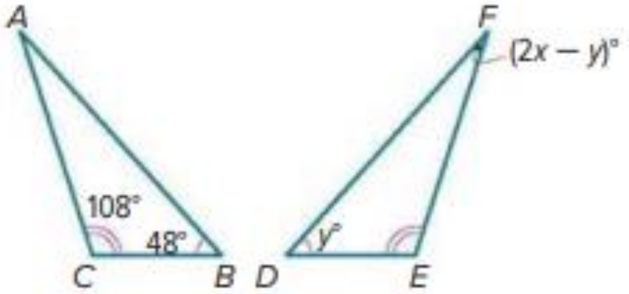
Solution

Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x. 36

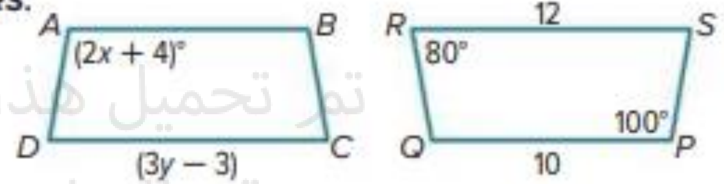
7. Find the value of y. 48



In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x. 48

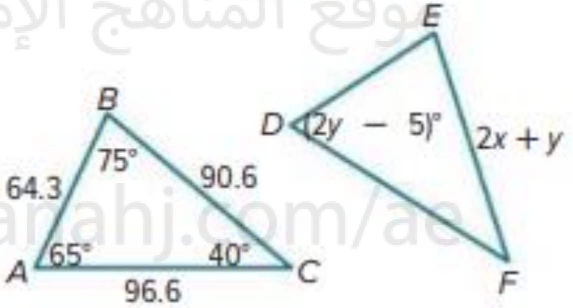
9. Find the value of y. 5



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x. 27.8

11. Find the value of y. 35

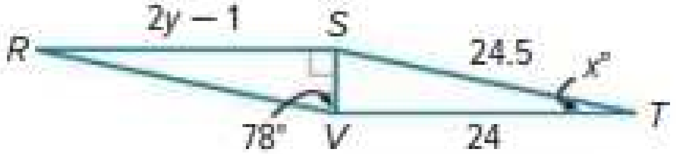


16	Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.	6 to 13	851 , 852

solution method

Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y.



Part A Find the value of x.

$\angle T \cong \angle R$	CPCTC
$m\angle T = m\angle R$	Definition of congruence
$= 180° - 90° - 78°$	Triangle Angle-Sum Theorem
$= 12°$	Solve.

The value of x is 12.

Part B Find the value of y.

$\overline{RS} \cong \overline{TV}$	CPCTC
$RS = TV$	Definition of congruence
$2y - 1 = 24$	Substitution
$y = 12.5$	Solve.

The value of y is 12.5.

Theorem 14.4: Properties of Triangle Congruence

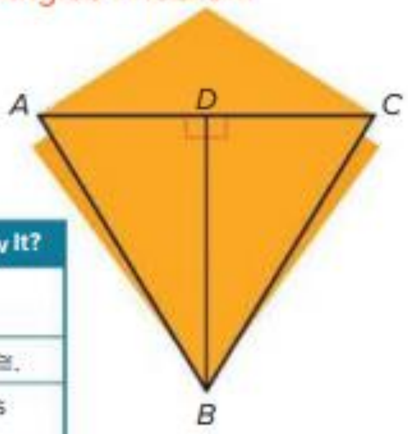
Reflexive Property of Triangle Congruence
$\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence
If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence
If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Example 3 Use the Third Angles Theorem

ORIGAMI Aika is folding origami dragons for a party she is hosting. If $\angle ABD \cong \angle CBD$ and $m\angle BAD = 58°$, find $m\angle CBD$.



What Do You Know?	How Do You Know It?
$\angle ABD \cong \angle CBD$, $m\angle BAD = 58°$	Given
$\angle BDC \cong \angle BDA$	All rt. $\angle$s are $\cong$.
$\angle BAD \cong \angle BCD$	Third Angles Theorem
$m\angle CBD + m\angle BCD = 90°$	The acute $\angle$s of a rt. $\triangle$ are compl.
$m\angle BCD = m\angle BAD$	Def. of congruence

$m\angle BCD = 58°$.	Transitive Property
$m\angle CBD + 58° = 90°$	Substitute.
$m\angle CBD = 32°$.	Solve.

The measure of $\angle CBD$ is 32°.

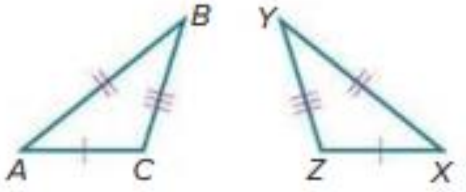
17	Use the SSS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	1 to 6	859

PROOF **Write the specified type of proof.**

1. two-column proof

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

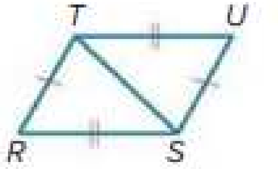
Prove: $\triangle ABC \cong \triangle XYZ$



2. flow proof

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

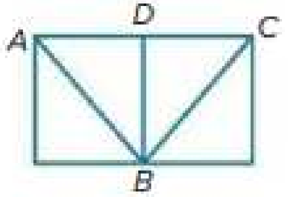
Prove: $\triangle RST \cong \triangle UTS$



3. two-column proof

Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.

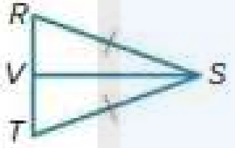
Prove: $\triangle ABD \cong \triangle CBD$



4. flow proof

Given: $\overline{RS} \cong \overline{TS}$, V is the midpoint of $\overline{RT}$.

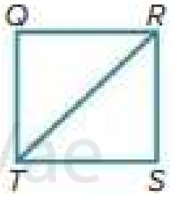
Prove: $\triangle RSV \cong \triangle TSV$



5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$

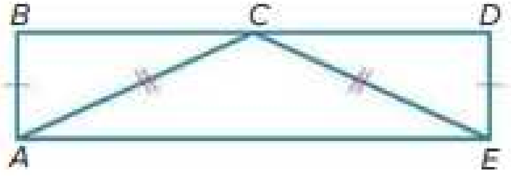
Prove: $\triangle QRT \cong \triangle SRT$



6. two-column proof

Given: $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$, $\overline{AC}$ bisects $\overline{BD}$

Prove: $\triangle ABC \cong \triangle EDC$



17	Use the SSS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	1 to 6	859

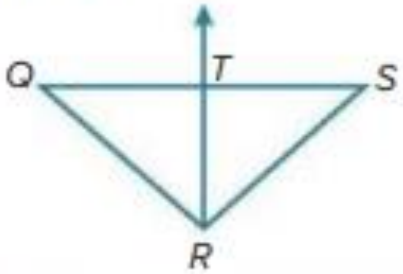
Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof to show that $\triangle QRT \cong \triangle SRT$.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overrightarrow{RT}$ bisects $\overline{QS}$ at point T.

Prove: $\triangle QRT \cong \triangle SRT$

Proof:



$\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$.
Given

$\overrightarrow{RT}$ bisects $\overline{QS}$ at point T.
Given

$\overline{RT} \cong \overline{RT}$
Reflexive Property of Congruence

$\overline{QT} \cong \overline{ST}$
Definition of segment bisector

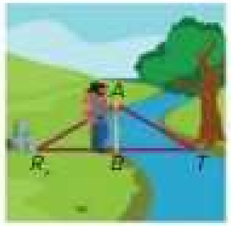
$\triangle QRT \cong \triangle SRT$
SSS

Go Online You can complete an Extra Example online

solution method

18	**Use the ASA congruence criterion for triangles to solve problems and prove relationships in geometric figures.**	**18 to 21**	**870**

18. USE ESTIMATION Delma came to a river during a hike, and she wanted to estimate the distance across it. She held her walking stick $\overline{BA}$ vertically on the ground at the edge of the river and sighted along the top of the stick across the river to the base of a tree T. Then she turned without changing the angle of her head and sighted along the top of the stick to a rock R, located on her side of the river.



a. Explain why $\triangle ABT \cong \triangle ABR$.

b. Delma finds that it takes 27 paces to walk from her current location to the rock. She also knows that each of her paces is 14 inches long. Explain how she can use this information to estimate the distance across the river.

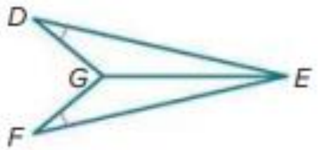
18	Use the ASA congruence criterion for triangles to solve problems and prove relationships in geometric figures.	18 to 21	870

19. PROOF Write a paragraph proof.

Given: $\angle D \cong \angle F$

$\overline{GE}$ bisects $\angle DEF$.

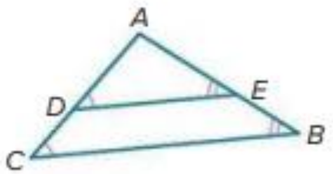
Prove: $\overline{DG} \cong \overline{FG}$



20. ANALYZE Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

18	Use the ASA congruence criterion for triangles to solve problems and prove relationships in geometric figures.	18 to 21	870

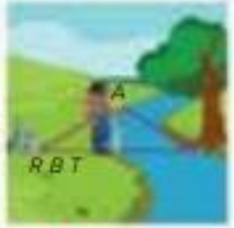
21. FIND THE ERROR Tyrone says that it is not possible to show that $\triangle ADE \cong \triangle ACB$. Lorenzo disagrees, explaining that because $\angle ADE \cong \angle ACB$, $\angle AED \cong \angle ABC$, and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ADE \cong \triangle ACB$. Who is correct? Explain your reasoning.



18	Use the ASA congruence criterion for triangles to solve problems and prove relationships in geometric figures.	18 to 21	870

Solution

18. USE ESTIMATION Delma came to a river during a hike, and she wanted to estimate the distance across it. She held her walking stick $\overline{AB}$ vertically on the ground at the edge of the river and sighted along the top of the stick across the river to the base of a tree T. Then she turned without changing the angle of her head and sighted along the top of the stick to a rock R, located on her side of the river.



a. Explain why $\triangle ABT \cong \triangle ABR$. Because Delma did not change the angle of her head, $\angle BAT \cong \angle BAR$. $\overline{AB} \cong \overline{AB}$ by the Reflexive Property of $\cong$. Because the walking stick is vertical, $\angle ABT$ and $\angle ABR$ are right angles, so $\angle ABT \cong \angle ABR$. Therefore, $\triangle ABT \cong \triangle ABR$ by ASA.

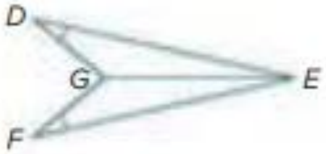
b. Delma finds that it takes 27 paces to walk from her current location to the rock. She also knows that each of her paces is 14 inches long. Explain how she can use this information to estimate the distance across the river. $BR = 27 \cdot 14 = 378$ in. or 31.5 ft. $\triangle ABT \cong \triangle ABR$, so $\overline{BT} \cong \overline{BR}$, because they are corresponding parts of congruent triangles. Therefore, the approximate distance across the river is 31.5 ft.

19. PROOF Write a paragraph proof. See margin.

Given: $\angle D \cong \angle F$

$\overline{GE}$ bisects $\angle DEF$.

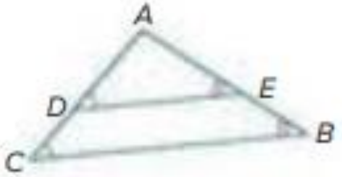
Prove: $\overline{DG} \cong \overline{FG}$



Higher-Order Thinking Skills

20. ANALYZE Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles. See margin.

21. FIND THE ERROR Tyrone says that it is not possible to show that $\triangle ADE \cong \triangle ACB$. Lorenzo disagrees, explaining that because $\angle ADE \cong \angle ACB$, $\angle AED \cong \angle ABC$, and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ADE \cong \triangle ACB$. Who is correct? Explain your reasoning.



Tyrone; Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

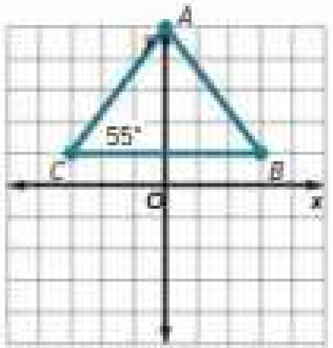
19	Solve problems involving isosceles triangles	5 to 15	879, 880
20	Solve problems involving equilateral triangles	5 to 15	879, 880

Example 2

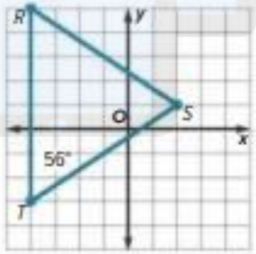
5. Refer to the figure.

a. Find the measures of the sides of $\triangle ABC$. Show your work.

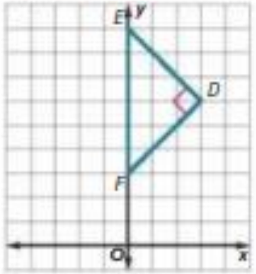
b. Find $m\angle A$. Show your work.



6. Find SR, ST, RT, $m\angle TRS$, and $m\angle RST$. Round to the nearest tenth, if necessary.



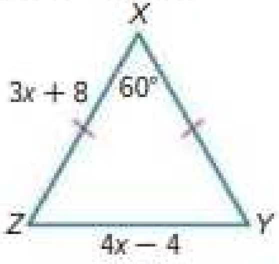
7. Find the measures of $\angle DEF$ and $\angle EFD$. Round to the nearest tenth, if necessary.



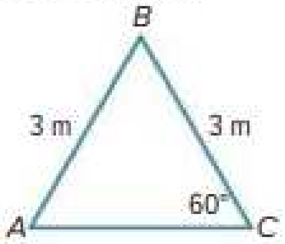
19	Solve problems involving isosceles triangles	5 to 15	879, 880
20	Solve problems involving equilateral triangles	5 to 15	879, 880

Examples 3 and 4

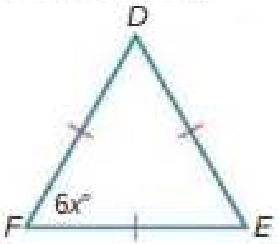
8. Find the value of x.



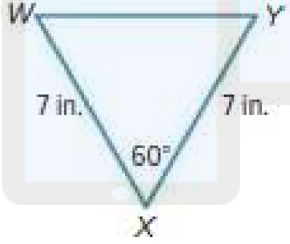
9. Find $m\angle B$ and AC.



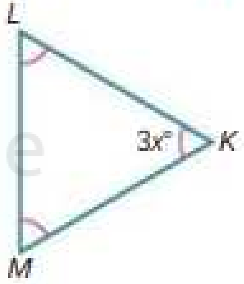
10. Find the value of x.



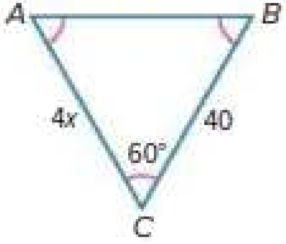
11. Find $m\angle Y$ and WY.



12. Find the value of x.



13. Find the value of x.



19	Solve problems involving isosceles triangles	5 to 15	879, 880
20	Solve problems involving equilateral triangles	5 to 15	879, 880

Find the value of each variable.

14. CHIPS Some tortilla chips can be modeled by a triangle.

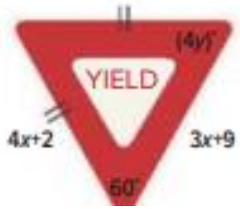
a. Solve for x.

b. Solve for y.

15. SIGNS Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first.

a. Solve for x.

b. Solve for y.



19	Solve problems involving isosceles triangles	5 to 15	879, 880
20	Solve problems involving equilateral triangles	5 to 15	879, 880

Solution

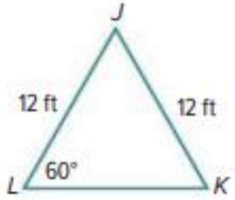
solution method

Example 3 Find Missing Measures in Equilateral Triangles

Find $m\angle J$.

Because $JL = JK$, $\overline{JL} \cong \overline{JK}$. By the Isosceles Triangle Theorem, base angles L and K are congruent, so $m\angle L = m\angle K$.

Use the Triangle Angle-Sum Theorem to write and solve an equation to find $m\angle J$.



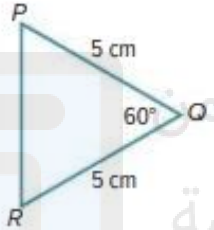
$m\angle J + m\angle K + m\angle L = 180°$	Triangle Angle-Sum Theorem
$m\angle J + 60° + 60° = 180°$	Isosceles Triangle Theorem
$m\angle J = 60°$	Solve.

Check

Find $m\angle R$ and PR.

$m\angle R =$ __?__

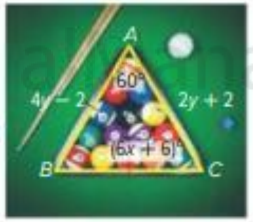
$PR =$ __?__ cm



Example 4 Find Missing Values

BILLIARDS Find the value of each variable.

Because $\overline{AB} \cong \overline{BC}$, $\angle ACB \cong \angle BAC$ by the Isosceles Triangle Theorem.

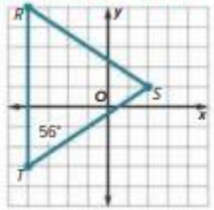


$(6x + 6)° = 60°$	Isosceles Tiangle Theorem
$x = 9$	Solve.

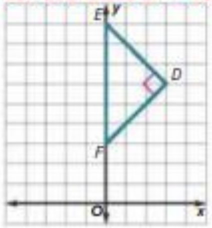
Because each angle of the triangle measures 60° by the Triangle Angle-Sum Theorem, the triangle is an equilateral triangle by Corollary 5.3.

$4y - 2 = 2y + 2$	Corollary 14.3; definition of equilateral △
$y = 2$	Solve.

6. Find SR, ST, RT, $m\angle TRS$, and $m\angle RST$. Round to the nearest tenth, if necessary. $SR = 7.2$ units; $ST = 7.2$ units; $RT = 8$ units; $m\angle TRS = 56°$; $m\angle RST = 68°$

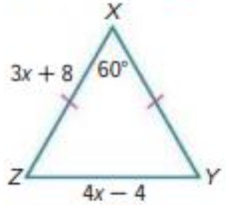


7. Find the measures of $\angle DEF$ and $\angle EFD$. Round to the nearest tenth, if necessary. $m\angle DEF = 45°$ and $m\angle EFD = 45°$

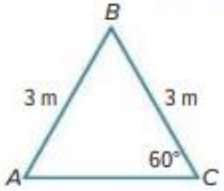


Examples 3 and 4

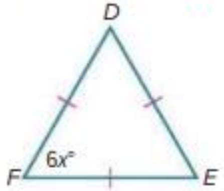
8. Find the value of x. 12



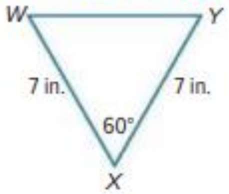
9. Find $m\angle B$ and AC. 60°; 3 m



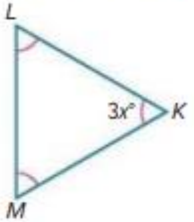
10. Find the value of x. 10



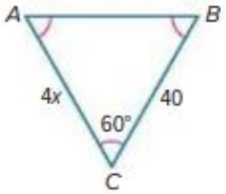
11. Find $m\angle Y$ and WY. 60°; 7 in.



12. Find the value of x. 20



13. Find the value of x. 10

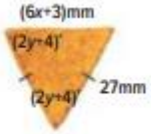


Find the value of each variable.

14. CHIPS Some tortilla chips can be modeled by a triangle.

a. Solve for x. 4

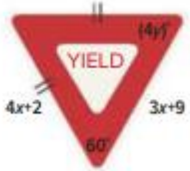
b. Solve for y. 28



15. SIGNS Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first.

a. Solve for x. 7

b. Solve for y. 15

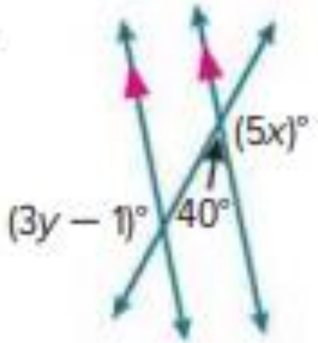


21	Find values by applying theorems about parallel lines and transversals	29 to 37	762

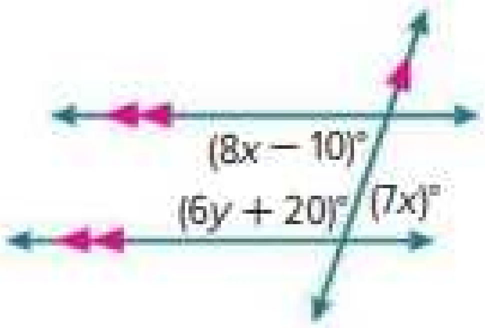
Example 5

Find the value of the variables in each figure. Explain your reasoning.

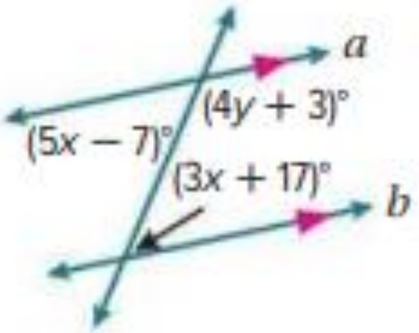
29.



30.



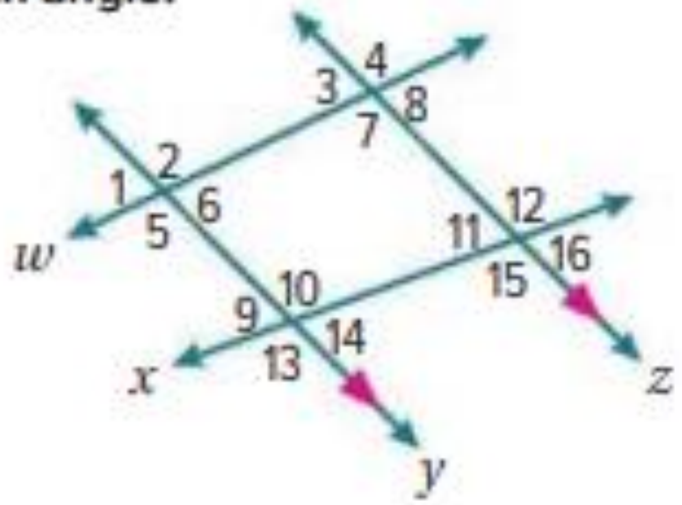
31. Find the value of the variables in the figure.



21	Find values by applying theorems about parallel lines and transversals	29 to 37	762

Mixed Exercises

In the figure, $m\angle 3 = 75$ and $m\angle 10 = 105°$. Find the measure of each angle.



32. $\angle 2$

33. $\angle 5$

34. $\angle 7$

35. $\angle 15$

36. $\angle 14$

37. $\angle 9$

21	Find values by applying theorems about parallel lines and transversals	29 to 37	762

Solution

solution method

Example 5 Find Values of Variables

Use the figure to find the value of the indicated variable. Justify your reasoning.

a. If $m\angle 3 = (4x + 7)^\circ$ and $m\angle 6 = (5x - 13)^\circ$, find x.

$\angle 3 \cong \angle 6$	Alternate Interior Angles Theorem
$m\angle 3 = m\angle 6$	Definition of congruent angles
$4x + 7 = 5x - 13$	Substitution
$x = 20$	Simplify.

b. Find y if $m\angle 8 = 68^\circ$ and $m\angle 3 = (3y - 2)^\circ$.

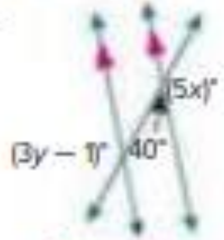
$\angle 5 \cong \angle 8$	Vertical Angles Theorem
$m\angle 5 = m\angle 8$	Definition of congruent angles
$m\angle 5 = 68^\circ$	Substitution

Because lines j and k are parallel, $\angle 5$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 3 + m\angle 5 = 180^\circ$	Definition of supplementary angles
$3y - 2 + 68 = 180$	Substitution
$3y + 66 = 180$	Simplify.
$y = 38$	Simplify.

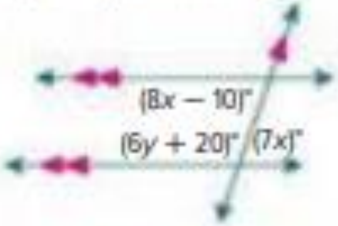
Find the value of the variables in each figure. Explain your reasoning.

29.



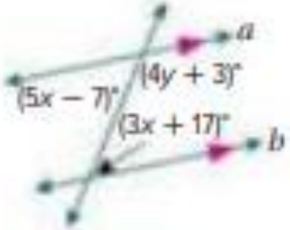
See margin.

30.



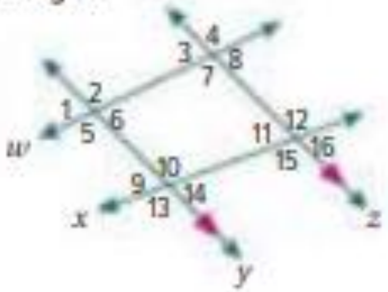
See margin.

31. Find the value of the variables in the figure. $x = 12$, $y = 31$



Mixed Exercises

In the figure, $m\angle 3 = 75$ and $m\angle 10 = 105^\circ$. Find the measure of each angle.



32. $\angle 2$ 105°

33. $\angle 5$ 105°

34. $\angle 7$ 105°

35. $\angle 15$ 105°

36. $\angle 14$ 75°

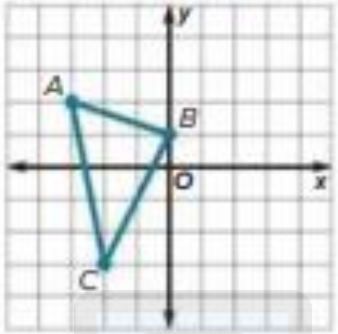
37. $\angle 9$ 75°

22	Use rigid motions to reflect figures on the coordinate plane and describe the effects of the reflections	1 to 6	801

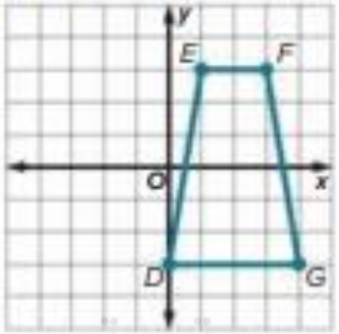
Examples 1 and 2

Graph the image of each figure under the given reflection. Determine the coordinates of the image.

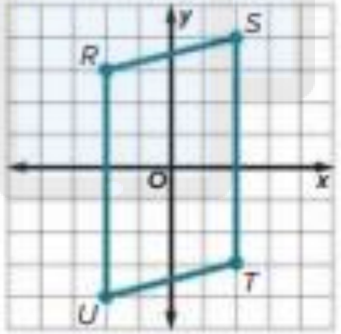
1. $\triangle ABC$ in the line $y = x$



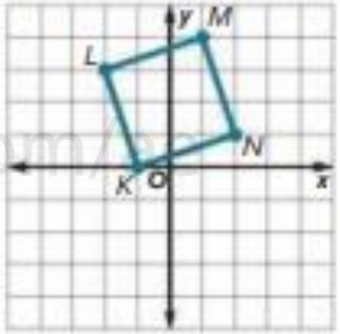
2. trapezoid $DEFG$ in the line $x = -1$



3. parallelogram $RSTU$ in the line $y = x$



4. square $KLMN$ in the line $y = -2$



22	Use rigid motions to reflect figures on the coordinate plane and describe the effects of the reflections	1 to 6	801

5. Determine the coordinates of $S(-7, 1)$ after a reflection in the line $y = 3$.

6. Determine the coordinates of $Q(6, -4)$ after a reflection in the line $x = 2$.

22	Use rigid motions to reflect figures on the coordinate plane and describe the effects of the reflections	1 to 6	801

solution method

Solution

Key Concept • Reflection

Reflection in a Vertical Line

When a figure is reflected in a vertical line that is not the y-axis, the y-coordinates of the image remain the same as the preimage. The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

Reflection in a Horizontal Line

When a figure is reflected in a horizontal line that is not the x-axis, the x-coordinates of the image remain the same as the preimage. The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

Reflection in $y = x$

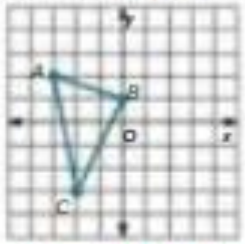
To reflect a point in the line $y = x$ interchange the x- and y-coordinates; $(x, y) \rightarrow (y, x)$.

Practice

Examples 1 and 2 1–4. See margin for graphs.

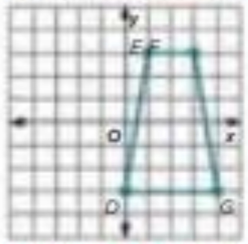
Graph the image of each figure under the given reflection. Determine the coordinates of the image.

1. $\triangle ABC$ in the line $y = x$



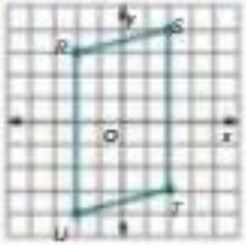
$A'(2, -3), B'(1, 0), C'(-3, -2)$

2. trapezoid $DEFG$ in the line $x = -1$



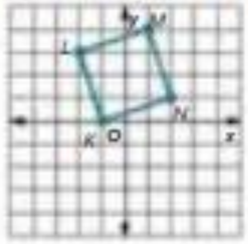
$D'(-2, -3), E'(-3, 3), F'(-5, 3), G'(-6, -3)$

3. parallelogram $RSTU$ in the line $y = x$



$R'(3, -2), S'(4, 2), T'(-3, 2), U'(-4, -2)$

4. square $KLMN$ in the line $y = -2$



$K'(-1, -4), L'(-2, -7), M'(1, -8), N'(2, -5)$

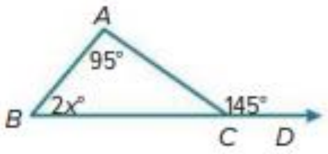
5. Determine the coordinates of $S(-7, 1)$ after a reflection in the line $y = 3$. $S'(-7, 5)$

6. Determine the coordinates of $Q(6, -4)$ after a reflection in the line $x = 2$. $Q'(-2, -4)$

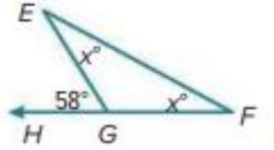
23	Prove the Exterior Angle Theorem and apply the theorem to solve problems	5 to 8	843

Find each measure.

5. $m\angle ABC$



6. $m\angle F$



7. TOWERS A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower. If $m\angle 1 = (7x - 7)°$, $m\angle 2 = (4x + 2)°$, and $m\angle 3 = (2x + 6)°$, what is $m\angle 1$?

8. GARDENING A gardener uses a grow light to grow vegetables indoors. If $m\angle 1 = (8x)°$ and $m\angle 2 = (7x - 4)°$, what is $m\angle 1$?

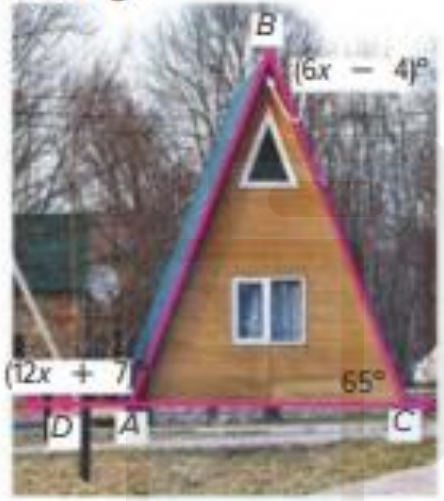
22	Use rigid motions to reflect figures on the coordinate plane and describe the effects of the reflections	1 to 6	801

solution method

Solution

Example 2 Use the Exterior Angle Theorem

ARCHITECTURE Find the measure of $\angle DAB$ in the front face of the building.

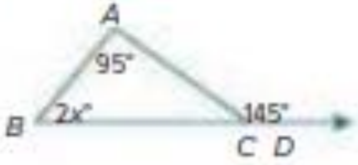


$m\angle DAB = m\angle ABC + m\angle BCA$	Exterior Angle Theorem
$12x + 7 = 6x - 4 + 65$	Substitution
$x = 9$	Solve.
$m\angle DAB = 12(9) + 7$ or $115°$	

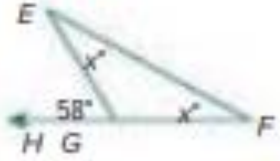
Example 2

Find each measure.

5. $m\angle ABC$ 50°



6. $m\angle F$ 29°



7. TOWERS A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower. If $m\angle 1 = (7x - 7)°$, $m\angle 2 = (4x + 2)°$, and $m\angle 3 = (2x + 6)°$, what 98° $m\angle 1$?

8. GARDENING A gardener uses a grow light to grow vegetables indoors. If $m\angle 1 = (8x)°$ and $m\angle 2 = (7x - 4)°$, what is $m\angle 1$? 64°


عن أنس رضي الله عنه
أن رسول الله ﷺ قال :
اللهم لا سهل
إلا ما جعلته سهلا
وأنت تجعل الحزن إذا شئت
سهلا
bedayatv
+tvbedaya
bedayatv
tvbedaya
BEDAYA