حل اختبار الفصل 1

(1) $\frac{dy}{dz} = z - (-z^{-2}) = z + z^{-2}$

$\frac{dz}{dx} = 7$

$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$

$= (z + z^{-2})(7)$

$= 7z + 7z^{-2}$

(2) $\frac{dz}{dx} = 24x^2$

$\frac{dy}{dx} = (3z^2 - 7)(24x^2)$

$= 72z^2x^2 - 168x^2$

(3) $y = (z+6)^{-1}$

$\frac{dy}{dz} = -(z+6)^{-2}$

$\frac{dz}{dx} = 6x$

$\frac{dy}{dx} = -(z+6)^{-2}(6x)$

$= -6x(z+6)^{-2}$

$\left(\frac{dy}{dx}\right)_{x=-1} = -6(-1)(z+6)^{-2}$

$= 6(z+6)^{-2}$

(4) ~~$(fog)'(x) =$~~

$f'(x) = 2x - 5$, $g'(x) = 6$

$(fog)'(x) = [2(6) - 5](6) = 42$

(5) $f'(x) = 2x^{-3}$, $g'(x) = 4$

$(fog)'(x) = 2(4)^{-3}(4) = \frac{1}{4}$

(6) $y = 9(x^2+16)^{-\frac{1}{2}}$

$y' = -4.5(x^2+16)(2x)$

(7) $y' = -x^5(x^2+3)^{-2}(2x)$

(8) $y' = 8\left(\frac{1+x}{x-3}\right)^7\left(\frac{\cancel{x-3} + \cancel{1-x}}{(x-3)^2}\right)$

$= 8\left(\frac{1+x}{x-3}\right)^7\left(\frac{-2}{(x-3)^2}\right)$

(9) $3(y+2)^2\left(\frac{dy}{dx}\right) = 2(5x-3)(5)$

$\frac{dy}{dx} = \frac{10(5x-3)}{3(y+2)^2}$

$\left(\frac{dy}{dx}\right)^2 = \left(\frac{10(5x-3)}{3(y+2)^2}\right)^2 = \frac{100(5x-3)^2}{9(y+2)}$

L.H.S $= \cancel{3}(y+2)\left(\frac{100(5x-3)^2}{\cancel{9}(y+2)^{\cancel{4}3}}\right)$

$= 100 =$ R.H.S

(10) $2\left(\frac{x}{25}\right) - 2\left(\frac{\frac{dy}{dx} \cdot y}{9}\right) = 0$

$\frac{dy}{dx} = \frac{2x}{25} \cdot \frac{9}{2}y = \frac{9x}{25y}$

$=$ RHS $=$ LHS

(13)

~~$[f \circ g](x)$~~

$f'(x) = 2\cos x$

$g'(x) = x^{-\frac{1}{2}}$

$[f \circ g]'(x) = 2\cos x^{-\frac{1}{2}} \left(x^{-\frac{1}{2}}\right)$

$= 2x^{-\frac{1}{2}} \cos x^{-\frac{1}{2}}$

$[f \circ g]'\left(\frac{\pi^2}{4}\right) = 2\left(\frac{\pi^2}{4}\right)^{-\frac{1}{2}} \cos\left(\frac{\pi^2}{4}\right)^{-\frac{1}{2}}$

$= 1.02$

(11)

$2x - 5\left(\frac{dy}{dx}\right) - 2y\left(\frac{dy}{dx}\right) = 0$

$2x = 5\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right)$

$2x = (5 + 2y)\left(\frac{dy}{dx}\right)$

$\frac{2x}{5+2y} = \left(\frac{dy}{dx}\right)$

$\left(\frac{dy}{dx}\right)_{(1,-2)} = \frac{2(1)}{5+2(-2)} = 2$

(14)

$f'(x) = 2\sec^2 x^2 \left(\sec x^2 \tan x^2\right)(2x)$

$= 4x\sec^3 x^2 \tan x^2$

$f'(\sqrt{\pi}) = 4\sqrt{\pi}\left(\frac{1}{\cos(\sqrt{\pi})^2}\right)^3 \tan(\sqrt{\pi})^2$

$= 0$

(12)

$\frac{dz}{dy} = 3y^2$

$\frac{dy}{dx} = 2\csc x\left(-\csc x \cot x\right)$

$= -2\csc^2 x \cot x$

$\frac{dz}{dx} = 3y^2\left(-2\csc^2 x \cot x\right)$

$= -2y^2 \csc^2 x \cot x$

$\left(\frac{dz}{dx}\right)_{x=\frac{\pi}{4}} =$

(15)

$\frac{dy}{dx} = 4\sin^3 x(\cos x) - 4\cos^3 x(-\sin x)$

$= 4\sin x \cos x\left(\sin^2 x + \cos^2 x\right)$

$= 4\sin x \cos x$

RHS = LHS

(12)

$\left(\frac{dz}{dx}\right) = (z \circ y)'(x)$

$(z \circ y)(x) = (\csc^2 x)^3$

$= \csc^6 x$

$(z \circ y)'(x) = 6\csc^5 x\left(-\csc x \cot x\right)$

$= -6\csc^6 x \cot x$

$\left(\frac{dz}{dy}\right)_{x=\frac{\pi}{4}} = -6\csc^6 \frac{\pi}{4} \cot \frac{\pi}{4}$

$= -6\sqrt{2}$

(16)

$$f'(x) = (\csc x - 1)\,3(\csc x + 1)^2(-\csc x \cot x) + (\csc x + 1)^3(-\csc x \cot x)$$

$$= (-\csc x \cot x)(\csc x + 1)^2\left[(3\csc x - 1) + (\csc x + 1)\right]$$

$$f'\left(\frac{\pi}{4}\right) = \left(-\frac{1}{\sin\frac{\pi}{4}} \cdot \frac{1}{\tan\frac{\pi}{4}}\right)\left(\frac{1}{\sin\frac{\pi}{4}} + 1\right)^2\left[\left(3 \cdot \frac{1}{\sin\frac{\pi}{4}} - 1\right) + \left(\frac{1}{\sin\frac{\pi}{4}} + 1\right)\right]$$

$$= -4 - 3\sqrt{2} + 4\sqrt{2} = -4\sqrt{2}$$

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(17)

$$2x + 2y\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx}(2y+1) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y+1}$$

$$\frac{d^2y}{dx^2} = \frac{-2(2y+1) - (-2x)\left(2\left(\frac{dy}{dx}\right)\right)}{2y+1}$$

$$= \frac{-4y - 2 + 4x\frac{dy}{dx}}{2y+1}$$

$$2 + 2\left(\frac{d^2y}{dx^2}\right) = 0 \quad \Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2}{2} = 1$$

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(19)

$$g'(x) = 2x$$

$$(f \circ g)'(x) = (\sec x^2)(2x) = 2x \sec x^2$$

$$(f \circ g)''(x) = 2x(\sec x^2 \tan x^2)(2x) + \sec x^2(2) = 4x^2 \sec x^2 \tan x^2 + 2\sec x^2$$

$$= 2\sec x^2(2x^2 \tan x^2 + \sec x^2)$$

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(20)

$$f'(x) = -4\sin 4x \quad , \quad g'(x) = 3x^2$$

$$(g \circ f)'(x) = 3(\cos 4x)(3x^2)$$

$$= 9x^2 \cos 4x$$

$$(g \circ f)''(x) = 9x^2(-4\sin 4x) + \cos 4x\,(18x)$$

$$= -36x^2 \sin 4x + 18x \cos 4x$$

$$= 18x(-2x\sin 4x + \cos 4x)$$

(11)

$$2x - 5\left[x \cdot \frac{dy}{dx} + y\right] - 2y\left(\frac{dy}{dx}\right) = 0$$

$$2x - 5x \cdot \frac{dy}{dx} - 5y - 2y \cdot \frac{dy}{dx} = 0$$

$$2x - 5y = 2y\left(\frac{dy}{dx}\right) + 5x\left(\frac{dy}{dx}\right)$$

$$\frac{2x + 5y}{2y + 5x} = \frac{(2y + 5x)\left(\frac{dy}{dx}\right)}{2y + 5x}$$

$$\frac{2(1) - 5(-2)}{2(-2) + 5(1)} = \frac{dy}{dx}(1, -2)$$

$$12 = 8$$

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(17)

$$2x + 2y\left(\frac{dy}{dx}\right) + \left[x \cdot \frac{dy}{dx} + y\right] = 0$$

$$2x + 2y\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) + y = 0$$

$$2y\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) = -2x - y$$

$$\frac{(2y + x)\left(\frac{dy}{dx}\right)}{2y + x} = \frac{-2x - y}{2y + x}$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-2(1) - (1)}{2(1) + (1)} = \frac{-3}{3} = -1$$

$$2 + 2\left(\frac{d^2y}{dx^2}\right) + 1 + \left(\frac{d^2y}{dx^2}\right) = 0$$

$$\frac{d^2y}{dx^2} = \frac{-3}{3} = 1$$