مدرسة سار الثانوية للبنات
قسم الرياضيات

نموذج إجابة

بطاقة إثرائية في التكامل (1)

أوجدي كلاً من التكاملات الآتية:

(1) $\int (4x^3 - 5x^2 + 4x - 9)\,dx$

$= \frac{4x^4}{4} - \frac{5}{3}x^3 + \frac{4}{2}x^2 - 9x + c.$

$= x^4 - \frac{5}{3}x^3 + 2x^2 - 9x + c.$

(2) $\int \sqrt{x}\,(x+2)\,dx$

$= \int x\sqrt{x} + 2\sqrt{x}\,dx$

$= \int x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\,dx$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C.$

(3) $\int \frac{1}{x^6}(x^2 + x^6)\,dx$

$= \int x^{2-6} + x^{6-6}\,dx = \int x^{-4} + 1\,dx$

$= \frac{-1}{3}x^{-3} + x + C$

(4) $\int (x^2+3)(2x)\,dx$

$= \frac{1}{2}(x^2+3)^2 + C$

(5) $\int \sqrt{x+2}\,dx = \int (x+2)^{\frac{1}{2}}\,dx$

$= \frac{2}{3}(x+2)^{\frac{3}{2}}\,dx$

(6) $\int 3x^2\sqrt{x^3+6}\,dx$

$= \frac{2}{3}(x^3+6)^{\frac{3}{2}} + C$

(7) $\int (x^3+2x)^3(3x^2+2)\,dx$

$= \frac{1}{4}(x^3+2x)^4 + C$

(8) $\int (2x^2+4x-1)^4(x+1)\,dx$

$= \frac{1}{4}\int (2x^2+4x-1)^4(4x+4)\,dx$

$= \frac{1}{4\times 5}(2x^2+4x-1)^5 + C$

$= \frac{1}{20}(2x^2+4x-1)^5 + C.$

$\int \frac{-3}{\sqrt{2-3x}}\,dx = \int (-3)(2-3x)^{-\frac{1}{2}}\,dx$

$= 2(2-3x)^{\frac{1}{2}} + C$

$\int 15x(x^5 - 4)\,dx.$

$= \int 15x^6 - 60x\,dx$

$= \frac{15}{7}x^7 - \frac{60}{2}x^2 + C$

$\int x^3(x^4+1)^6\,dx$

$\int 4x^3(x^4+1)^6\,dx$

$= (\frac{1}{4})(\frac{1}{7})(x^4+1)^7 + C.$

$\int (x+2)^7\,dx$

$= \frac{1}{8}(x+2)^8 + C.$

) $\int (6x+4)\sqrt{3x+2}\,dx$

$\int 2(3x+2)(3x+2)^{\frac{1}{2}}\,dx.$

$= 2\int (3x+2)^{\frac{3}{2}}dx = \frac{2}{3}\times\frac{2}{5}(3x+2)^{\frac{5}{2}} + C.$

) $\int (1-2x)(x^2-x-3)^3\,dx$

$= -\int (2x-1)(x^2-x-3)^3\,dx$

$= -\frac{1}{4}(x^2-x-3)^4 + C.$

) $\int \frac{(x+1)}{x^3+x^2}\,dx$

$\int \frac{(x+1)}{x^2(x+1)}\,dx = \int x^{-2}\,dx = -x^{-1} + C$

$= \frac{-1}{x} + C.$

) $\int \frac{8x^2 - 10x - 3}{2x-3}\,dx$

$= \int \frac{(2x-3)(4x+1)}{(2x-3)}\,dx = \int (4x+1)\,dx$

$= 2x^2 + x + C.$

(6) $\int \frac{3x}{\sqrt{x^3-x^2}}\,dx$

$= \int \frac{3x}{x\sqrt{x-1}}\,dx = \int 3(x-1)^{-\frac{1}{2}}\,dx$

$= 3x2\,(x-1)^{\frac{1}{2}} + C$

(7) $\int (6x+4)\sqrt{3x+2}\,dx$

مثل (13) محلولة

$= \int \ldots\ldots$

) $\int 6(x-2)^2 (x^3-6x^2+12x+1)^5 dx$

$= \int 6(x^2-4x+4)(x^3-6x^2+12x+1)^5\,dx$

$= \frac{2}{6}(x^3-6x^2+12x+1)^6 + C.$

) $\int \frac{dx}{(x^2+6x+9)}$

$= \int (x+3)^{-2}\,dx = \frac{(x+3)^{-1}}{-1} + C$

$= \frac{-1}{x+3} + C.$

) $\int (x^2-14x+49)^{5/2}\,dx$

$= \int ((x-7)^2)^{5/2}\,dx = \int (x-7)^5\,dx$

$= \frac{1}{6}(x-7)^6 + C.$

$\int \frac{(x-1)}{(2x-3)^3}\,dx$

$= \frac{1}{2}\int \frac{2x-2-1+1}{(2x-3)^3}\,dx = \int \frac{1}{2}(2x-3)^{-2} + \frac{1}{2}(2x-3)^{-3}\,dx$

$= \frac{1}{4}(-1)(2x-3)^{-1} - \frac{1}{8}(2x-3)^{-2} + C$

) $\int \frac{5}{x^2}\left(1+\frac{1}{x}\right)^3 dx.$

$-5\int \frac{-1}{x^2}\left(1+x^{-1}\right)^3 dx$

$= \frac{-5}{4}\left(1+\frac{1}{x}\right)^4 + C$

(31) توضيح للمسألة

$\frac{x-1}{x-3^3} = \frac{1}{2}\cdot\frac{2x-2-1+1}{(2x-3)^3}$

$= \left(\frac{1}{2}\right)\frac{(2x-3)+1}{(2x-3)^3}$

$= \frac{1}{2}\left[\frac{(2x-3)}{(2x-3)^3} + \frac{1}{(2x-3)^3}\right]$

(17) $\int \frac{1}{\sqrt{x}}\left(1+\sqrt{x}\right)^4 dx$

$= 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^4 dx$

$= \frac{2}{5}\left(1+\sqrt{x}\right)^5 + C.$

(18) $\int \frac{1}{x^2}\left(3+\frac{1}{x}\right)^3 dx$

$= -\int (-x^{-2})\left(3+x^{-1}\right)^3 dx$

$= -\frac{1}{4}\left(3+\frac{1}{x}\right)^4 + C.$

(19) $\int \frac{x^4+1}{x^2}\,dx$

$= \int x^{4-2} + x^{-2}\,dx = \int x^2 + x^{-2}\,dx$

$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + C = \frac{1}{3}x^3 - \frac{1}{x} + C$

(20) $\int \sqrt{x^4-5x^2}\,dx = \int \sqrt{x^2(x^2-5)}\,dx$

$= \int x\,(x^2-5)^{\frac{1}{2}}\,dx = \frac{1}{2}\times\frac{2}{3}(x^2-5)^{\frac{3}{2}} + C$

$= \frac{1}{3}(x^2-5)\sqrt{x^2-5} + C.$

(21) $\int \frac{5}{\sqrt[3]{x^2+4x+4}}\,dx$

$= \int 5\left[(x+2)^2\right]^{-1/3}\,dx = \int 5\,(x+2)^{-\frac{2}{3}}\,dx$

$= 5(3)(x+2)^{\frac{1}{3}} + C.$

(22) $\int x^5\left(1-\frac{1}{x}\right)^5 dx$

$= \int \left(x\left(1-x^{-1}\right)\right)^5 dx = \int (x-1)^5\,dx$

$= \frac{1}{6}(x-1)^6 + C.$

(23) $\int \left(\frac{(x^3-8)}{(x-2)} + 5\right) dx$

$= \int \left((x^2+2x+4)+5\right) dx$

$= \frac{1}{3}x^3 + x^2 + 9x + C.$

(24) $\int (x^2+2x+1)^5\,dx$

$= \int \left[(x+1)^2\right]^5 dx = \int (x+1)^{10}\,dx$

$= \frac{1}{11}(x+1)^{11} + C.$

(25) $\int 5x\sqrt{x^2-3}\,dx$

$= \frac{5}{2}\cdot\frac{2}{3}(x^2-3)^{\frac{3}{2}} + C = \frac{5}{3}(x^2-3)^{\frac{3}{2}} + C.$

أوجدي التكاملات الآتية:

1) $\int \cos(x+2)\,dx$

$= \sin(x+2) + C$

2) $\int 3\sin 4x\,dx$

$= \frac{-3}{4}\int -4\sin 4x\,dx$

$= \frac{-3}{4}\cos 4x + C.$

3) $\int \sin(1-6x)\,dx$

$= \frac{-1}{6}\int -6\sin(1-6x)\,dx$

$= \frac{-1}{6}\cos(1-6x) + C$

4) $\int \cos 3x \sin^3 3x\,dx$

$= \int (3\cos 3x)(\sin 3x)^3\,dx$

$= \frac{(\sin 3x)^4}{3\times 4} + C = \frac{(\sin 3x)^4}{12} + C$

5) $\int \sin 2x\,(1-\cos 2x)^{-2}\,dx$

$= \frac{1}{2}\int (2\sin 2x)(1-\cos 2x)^{-2}\,dx$

$= \frac{-1}{2}(1-\cos 2x)^{-1} + C.$

6) $\int \sin x\,(1+\cos x)^2\,dx$

$= \frac{-1}{3}(1+\cos x)^3 + C$

7) $\int \frac{\sin 4x}{\cos 2x}\,dx$

$= \int \frac{2\sin 2x\cos 2x}{\cos 2x}\,dx$

$= \int 2\sin 2x\,dx = -\cos 2x + C.$

8) $\int \frac{\sin x}{\cos^4 x}\,dx$

$= \int \tan x \cdot \frac{1}{\cos^3 x}\,dx$

$= \int \tan x \sec x \sec^2 x\,dx$

$= \frac{1}{3}\sec^3 x + C$

9) $\int \frac{\cos x}{(1-\sin x)^2}\,dx = \int \cos x\,(1-\sin x)^{-2}\,dx$

$\int \sec^5 3x \tan 3x\,dx$

$= \frac{1}{3}\int 3\sec^4 3x\,(\sec 3x \tan 3x)\,dx$

$= \frac{1}{3}\cdot\frac{1}{5}\sec^5 3x + C.$

$\int \sec^2(3x-4)\,dx$

$= \frac{1}{3}\tan(3x-4) + C.$

$\int \sin^3 x \cos x\,dx$

$= \frac{\sin^4 x}{4} + C.$

$\int \frac{dx}{\sin^2 5x} = \int \csc^2 5x\,dx$

$= \frac{-1}{5}\int -5\csc^2 5x\,dx$

$= \frac{-1}{5}\cot 5x + C.$

$\int \cos^2 x\,dx$

$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C.$

$\int \frac{dx}{\tan 4x \sin 4x}$

$= \int \cot 4x \csc 4x\,dx$

$= \frac{-1}{4}\csc 4x + C.$

$\int x(\tan x^2)(\sec x^2)\,dx$

$= \frac{1}{2}\int 2x\,(\tan x^2 \sec x^2)\,dx$

$= \frac{1}{2}\sec x^2 + C.$

$\int x \cot x^2 \csc x^2\,dx$

$= \frac{1}{2}\int 2x \cot x^2 \csc x^2\,dx$

$= \frac{-1}{2}\csc x^2 + C$

$\int \sec x \cot x \csc x\,dx$

$= \int \frac{1}{\cos x}\cdot\frac{1}{\sin x}\cdot\frac{1}{\sin x}\,dx = \int \csc^2 x \cot x\,dx$

$= -\frac{1}{2}\csc^2 x + C$

) $\int \tan 4x \cot 4x \, dx$

$= \int \tan 4x \frac{1}{\tan 4x} dx = \int 1 \, dx$

$= x + C$.

) $\int \sec^5 x \tan x \, dx$

$= \int \sec^4 x \sec x \tan x \, dx$

$= \frac{\sec^5 x}{5} + C$.

) $\int \cos^3 x \, dx$

$= \int \cos x (1 - \sin^2 x) dx$

$= \int \cos x - \cos x \sin^2 x \, dx$

$= \sin x - \frac{1}{3} \sin^3 x + C$

) $\int \frac{\sec^2 x}{\tan^2 x} dx$

$= \int \frac{1}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x} dx$

$= \int \csc^2 x \, dx = -\cot x + C$.

) $\int \frac{\sec^2 x}{(1 + \tan x)^4} dx$

$= \int (1 + \tan x)^{-4} \sec^2 x \, dx$

$= \frac{-1}{3} (1 + \tan x)^{-3} + C$.

) $\int \frac{\csc^2 x}{(5 - \cot x)^3} dx$

$= \int (5 - \cot x)^{-3} \csc^2 x \, dx$

$= \frac{-1}{2} (5 - \cot x)^{-2} + C$.

) $\int \frac{\csc^2 x}{\cot^7 x} dx =$

$= \int \frac{\csc^2 x}{\csc^7 x} \cdot \sec^7 x \, dx$

$= \int \frac{\sec^5 x}{\csc^5 x} \cdot \sec^2 x \, dx$

) $\int \frac{\tan x}{\cos x} dx$ $= \int \tan^5 x \sec^2 x \, dx$

$= \frac{1}{6} \tan^6 x + C$

$= \int \tan x \sec x \, dx = \sec x + C$.

) $\int \frac{3 \sin x \cos x}{\sqrt{1 + 3 \sin^2 x}} dx = \frac{2(1 + 3\sin^2 x)^{\frac{1}{2}}}{2} + C$

$= (1 + 3 \sin^2 x)^{\frac{1}{2}} + C$

(19) $\int \frac{\csc 2x}{\sin 2x} dx$

$= \frac{1}{2} \int 2 \csc^2 2x \, dx$

$= \frac{-1}{2} \cot 2x + C$.

(20) $\int \sec^2 (3x - 4) dx$

$= \frac{1}{3} \tan (3x - 4) + C$

* (21) $\int \frac{dx}{\cos x \cot x}$.

$= \int \sec x \tan x \, dx$

$= \sec x + C$

(22) $\int \frac{\sin x \sec x}{\cos x} dx$

$= \int \frac{\sin x}{\cos x} \cdot \sec x \, dx$

$= \int \tan x \sec x \, dx = \sec x + C$

(23) $\int \frac{\cot x (1 + \cot^2 x)}{\csc x} dx$

$= \int \cot x \frac{\csc^2 x}{\csc x} dx$

$= -\int -\cot x \csc x \, dx = -\csc x + C$

(24) $\int \frac{\tan x}{\cos x} dx$

$= \int \tan x \sec x \, dx = \sec x + C$

(25) $\int (\tan^2 x + 1) dx$

$= \int \sec^2 x \, dx = \tan x + C$

(26) $\int (\cos x + \sin x)^2 dx$

$= \int (\cos^2 x + 2 \sin x \cos x + \sin^2 x) dx$

$= \int 1 \, dx + \int \sin 2x \, dx$

$= x - \frac{1}{2} \cos 2x + C$

(27) $\int \frac{6}{\csc x} dx$

$= \int 6 \sin x \, dx = -6 \cos x + C$.

$$\int \frac{\sin 2x \cos x}{\sin x}\,dx$$

$$= \int \frac{2\sin x \cos^2 x}{\sin x}\,dx = \int 2\cos^2 x\,dx$$

$$= \int \frac{1}{2}\,dx + \int \frac{1}{2}\cos 2x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\int \frac{dx}{\sin^2 x - \cos x + \cos^2 x}$$

$$\int \frac{dx}{(1-\cos x)} = \int \frac{(1+\cos x)}{(1-\cos^2 x)}\,dx$$

$$= \int \frac{1+\cos x}{\sin^2 x}\,dx = \int \csc^2 x + \cot x \csc x\,dx$$

$$\int \tan x(\sec x + \tan x)\,dx = -\cot x - \csc x + C$$

$$\int \tan x \sec x\,dx + \int \tan^2 x\,dx$$

$$\sec x + \int (-1 + \sec^2 x)\,dx$$

$$= \sec x - x + \tan x + C.$$

$$\int (\cos x - \cos^3 x)\,dx$$

$$= \int \cos x(1 - \cos^2 x)\,dx$$

$$= \int \cos x \sin^2 x\,dx = \frac{\sin^3 x}{3} + C.$$

$$\int \sec^4 x\,dx = \int \sec^2 x(\tan^2 x + 1)\,dx$$

$$= \int \sec^2 x \tan^2 x\,dx + \int \sec^2 x\,dx$$

$$= \frac{\tan^3 x}{3} + \tan x + C.$$

$$\int (\sec x + \tan x)(1 - \sin x)\,dx$$

$$= \int \left(\frac{1+\sin x}{\cos x}\right)(1 - \sin x)\,dx$$

$$= \int \frac{1 - \sin^2 x}{\cos x}\,dx = \int \cos x\,dx = \sin x + C$$

$$\int 4\csc^4 x \cot x\,dx.$$

$$= \int 4\csc^3 x \csc x \cot x\,dx$$

$$= -\csc^4 x + C.$$

$$\int (3x - \sec 2x)(3x + \sec 2x)\,dx.$$

$$= \int (9x^2 - \sec^2 2x)\,dx$$

$$= 3x^3 - \frac{\tan 2x}{2} + C$$

$$\int (\tan x + \cot x)^2\,dx.$$

(43) ...

$$= \int \ldots\ldots$$

(37) $$\int \frac{\sin 2x}{\sqrt{2 + \sin^2 x}}\,dx$$

$$= \int 2\sin x \cos x (2 + \sin^2 x)^{-\frac{1}{2}}\,dx = 2(2 + \sin^2 x)^{\frac{1}{2}} + C$$

(38)* $$\int \sqrt{\sin^2 x + \cos^2 x}\,dx$$

$$= \int (1)^{\frac{1}{2}}\,dx = x + C$$

(39) $$\int \sqrt{\tan^2 x + \tan^4 x}\,dx$$

$$= \int \tan x \sqrt{1 + \tan^2 x}\,dx$$

$$= \int \tan x \sec x\,dx = \sec x + C$$

(40) $$\int (\tan x \cot x)^8\,dx$$

$$= \int (1)^8\,dx = x + C$$

(41) $$\int \sqrt{\sec^4 x \tan^2 x}\,dx$$

$$= \int \sec^2 x \tan x\,dx = \frac{\tan^2 x}{2} + C$$

(42) $$\int (\tan^4 x - 1)\,dx$$

$$= \int (\tan^2 x - 1)(\tan^2 x + 1)\,dx$$

$$= \int \tan^2 x \sec^2 x - \sec^2 x\,dx = \frac{\tan^3 x}{3} - \tan x + C$$

(43) $$\int (\tan x + \cot x)^2\,dx.$$

$$= \int (\tan^2 x + 2 + \cot^2 x)\,dx = \int \sec^2 x + \csc^2 x\,dx$$

$$= \tan x - \cot x + C.$$

(44) $$\int \frac{\cos^2 x - \sin^2 x}{4\sin^2 x \cos^2 x}\,dx.$$

$$= \int \frac{\cos 2x}{(\sin 2x)^2}\,dx = \int \frac{\cos 2x}{\sin 2x} \cdot \frac{dx}{\sin 2x}$$

$$= \int \cot 2x \cdot \csc 2x\,dx = -\frac{\csc 2x}{2} + C$$

(45)* $$\int \sin 2x \sec^3 x\,dx.$$

$$= \int 2\sin x \cos x \ldots\,dx - \int \tan x \ldots\,dx$$